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Entry deterrence by timing rather than overinvestment in a strategic real options framework

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Abstract

This paper examines a dynamic incumbent-entrant framework with stochastic evolution of the (inverse) demand, in which both the optimal timing of the investments and the capacity choices are explicitly considered. We find that the incumbent invests earlier than the entrant and that entry deterrence is achieved through timing rather than through overinvestment. This is because the incumbent invests earlier and in a smaller amount compared to a scenario without potential entry. If, on the other hand, the size of the investment is exogenously given, the investment order changes and the entrant invests before the incumbent does.

Keywords: Game Theory, Incumbent/Entrant, Capacity choice, Investment under Uncertainty, Real-Option Games

JEL classification: C73, D92, L13

1 Introduction

Starting with the seminal paper by Spence (1977) the choice of production capacity as an instrument for entry deterrence has been extensively studied in the literature. In a standard two-stage set-up, where the incumbent chooses its capacity before the potential competitor decides about entry, entry deterrence is achieved by the incumbent through overinvestment and leads to absence of the competitor from the market. After installing a sufficiently large capacity by the incumbent, the potential entrant finds the market not profitable enough to undertake an investment. In a dynamic setting, where the demand evolves over time (with a positive trend), however, it cannot be expected that potential entrants are perpetually deterred from the market. Hence, the question arises how the investment behavior of the incumbent is affected by the threat of entry in such a setting.

This paper considers a dynamic model where both an incumbent and an entrant have the option to acquire once some (additional) production capacity. Both firms are free to choose the size of their installment, which is assumed to be irreversible and is fully used in the market competition. A situation, where some firms become active on an established market after incumbents have already invested, could arise, e.g., when these firms are new entrepreneurs, when firms did not have sufficient funds during the earlier stages, or when the market was not open to competition in the first place.

As a first result, we find that under general conditions the incumbent is most eager to undertake the investment first. In this way the incumbent accomplishes that it delays the investment of the entrant and it extends its monopoly period. The entrant reacts by waiting with investment until demand has become sufficiently large.

A second important result is that entry deterrence is not achieved via overinvestment, but via timing. The threat of entry makes the incumbent invest sooner in order to precede investment of the entrant. Since the incumbent's investment increases the quantity on the market, the output price is reduced, which in turn reduces the profitability of entering this market, and thus delays entry. Furthermore, whereas explained in, e.g., Tirole (1988, p. 315), the monopolist sets a smaller capacity than a (potential) duopolist facing a threat of entry, we find the opposite result. Since the incumbent invests early, i.e. in a market with a still relatively small demand, it pursues a small capacity expansion. In the absence of an entry threat the monopolist would wait for a market with a higher demand and invest in a larger capacity. In other words, when deterring entry, timing is of greater importance than overinvesting.

A crucial aspect of these results is that the size of the investment is flexible. Considering a variant of our model in which investment sizes are fixed, the incumbent no longer has the possibility to undertake a small investment in a small market in order to preempt the entrant. Interestingly, we find that in such a setting the investment order is reversed; the entrant undertakes an investment first. The reason is that in this situation, where the investment size and thus investment costs are equal, the entrant, which does not suffer from cannibalization, has a larger incentive to invest. Being able to choose the investment size is thus of key importance for making preemption optimal for the incumbent.

Focusing again on the framework where investment size can be chosen freely, another key result worth mentioning arises from taking into account uncertainty. For an incumbent with a small initial capacity it is found that in scenarios where the market uncertainty is small the entrant becomes market leader in the long run, i.e. produces higher output after both firms have invested, whereas for large market uncertainty the incumbent is the largest firm on the market in the long run. Uncertainty creates a value of waiting with investment (Dixit and Pindyck (1994)). This dilutes the preemption effect, so that when demand is more uncertain, the investment of the incumbent occurs later, thus when demand is higher. This implies that under large uncertainty the optimal size of the incumbent's investment is larger. For larger initial capacities of the incumbent it stays market leader irrespective of the level of uncertainty.

Next we review the relevant literature with the aim to highlight our contributions. The literature review is mainly structured around the three dimensions relevant for our paper, namely (i) *flexibility in capacity choice*, (ii) *flexibility in timing the investment*, and (iii) *uncertainty modeled via diffusion. Flexibility in capacity choice* is already present in the early contributions by Spence (1977) and Dixit (1980). Based on these papers, a rich literature has explored the rationale behind entry deterrence in two-stage games under a variety of assumptions about the mode of post-entry competition between firms. What we add to this literature is the ability of firms to determine their optimal investment time, whereas, in addition, they have to deal with a stochastically evolving demand.

In addition to flexibility in capacity choice, *flexibility in timing the investment* is present in early dynamic models of entry deterrence, like Spence (1979) or Fudenberg *et al.* (1983). These contributions focus on the dynamics of (irreversible) capacity build-up in static market environments, if investment is bounded from above. A key insight in this literature is that, in addition to equilibria which essentially correspond to a Stackelberg equilibrium with the incumbent as leader, there exist Markov Perfect equilibria in which the incumbent can strategically deter the follower from investing, thereby weakening competition. This is due to the initial asymmetry and the dynamic build-up of capacity. Robles (2011) develops a two-period game where demand is deterministic and increasing between the two periods. He characterizes conditions under which incumbents build capacities, which are partly idle in the first period, in order to deter other firms from the market. We extend this literature by considering uncertainty in future demand.

An early stochastic model is Perrakis and Waskett (1983), in which it is shown that key insights about optimality of deterrence respectively accommodation might change qualitatively if it is assumed that demand is uncertain for the firms until the time of investment. In more recent contributions to this stream of literature Maskin (1999) and Swinney *et al.* (2011) highlight that high demand uncertainty makes entry deterrence less attractive and fosters the use of accommodation strategies by incumbents. Our main contribution relative to these papers is not only that we address the role of investment timing for potential entry deterrence, but also that we consider a stochastically evolving market environment.

An important paper in this area that captures *flexibility in timing the investment* and *uncertainty modeled via a diffusion is* Boyer *et al.* (2004). They study entry deterrence in a dynamic setting with price competition and a stochastically evolving willingness to pay of consumers. They assume that firms can invest repeatedly, where the size of each investment is fixed, and point out that in such a setting an important effect of investment is the delay of the competitor's investment. It is shown that different types of equilibria might arise in such a setting. In spite of the usual logic associated with preemption under price competition, in some of these equilibria firms acquire positive rents. Concerning the timing of investment, Boyer *et al.* (2004) show that in their setting (under certain conditions) the incentives for preemption are smaller for the incumbent than for the challenger with lower capacity. A similar setting with Cournot competition is studied in Boyer *et al.* (2012). It is shown that competition induces too early first investment relative to the social optimum and that the smaller firm invests first. The market environment considered in Boyer *et al.* (2004, 2012) is closely related to our setup. However, the assumption of fixed investment units crucially distinguishes these studies from our approach, where both timing and investment size are chosen by the firms. We find that the endogeneity of investment size is crucial and leads to qualitatively different insights compared to settings with fixed investment size. Also, due to the consideration of Cournot competition, investment in Boyer *et al.* (2012) has considerably less commitment power compared to the setup we consider. A main focus of Boyer *et al.* (2004, 2012), as well as of recent studies by Besanko *et al.* (2004) and Besanko *et al.* (2010) dealing with (partly) reversible capacity investments in oligopolistic markets with stochastically evolving demand, is the long run industry structure that emerges. Considering only one investment option for each firm, our paper does not address this issue, but rather focuses on entry deterrence in the early phase of an industry with evolving demand.

As discussed above, in this paper we show that in an incumbent-entrant framework uncertainty may play an important role, in particular when the incumbent's initial capacity is relatively small. The effect of uncertainty on market leadership in a setting with endogenous choice of timing and size of investment has also been analyzed by Huisman and Kort (2015) in a setting with two symmetric firms both entering the market.

The main insight of our analysis that the incumbent invests prior to the entrant can be seen to follow the logic to "eat your own lunch before someone else does" (Deutschman (1994)). This logic has been, among others, explored in Nault and Vandenbosch (1996) in the framework of a model, where firms endogenously choose the time to launch a new product generation. Nault and Vandenbosch (1996) develop a deterministic timing game similar to Fudenberg and Tirole (1985), but where in the latter paper firms are symmetric, Nault and Vandenbosch consider an incumbent-entrant framework. Apart from the fact that their paper does not explicitly deal with capacity investment, the key difference to our approach is that the type of expansion as such is fixed and the size of the expansion cannot be chosen by the firms.

Within a strategic real options framework, investment decisions involving both capacity choice and timing have first been considered by Huisman and Kort (2015). They study this problem for two symmetric entrants on a new market. This paper differs from their analysis by considering an incumbent-entrant framework, in which one of the players has an initial capacity. The latter model characteristic enabled us to establish the new result that entry deterrence takes place by timing rather than overinvestment. Also, it allows us to characterize under which circumstances in such a dynamic market setting the incumbent ends up as the larger firm in the long run and to explore which role a flexible choice of investment size has for optimal entry deterrence behavior.

This paper is organized in the following way. Section 2 explains the model and discusses its assumptions. Section 3.1 looks at the case of exogenous firms roles, i.e. an individual firm knows beforehand whether it will be the first or second investor. Then the other firm can choose to invest at the same time or later. This is followed by Section 3.2 studying the game when endogenizing investment roles, i.e. both firms are allowed to become the first investor. Section 3.3 focuses on the size of the incumbent's investment relative to that of the entrant. An analysis on extreme parameter values is performed in Section 3.4. Section 4 shows that the incumbent does not overinvest to deter entry and analyzes a variant of the model where investment size is exogenous. The paper is concluded in Section 5. The appendices provide all proofs as well as the details of the strategy profiles underlying the considered Markov Perfect Equilibria as well as numerical robustness checks.

2 The Model

Consider an industry setting with two firms. One firm is actively producing and the other firm is a potential entrant. The first firm is the incumbent and is denoted as firm I. The potential entrant is denoted as firm E. Both firms have a one-off investment opportunity. For firm I this means an expansion of its current capacity and for the entrant an investment means starting up production and entering this market. Both firms are assumed to be rational, risk neutral and value maximizing. The inverse demand function on this market is multiplicative and equals

$$p(t) = x(t)(1 - \eta Q(t)),$$

where $p(t)$ is the output price, $x(t)$ a process that models exogenous shocks to the system, $Q(t)$ equals the total quantity sold at the market at time $t \geq 0$, and $\eta > 0$ is a fixed price sensitivity parameter.

The exogenous shock process $(x(t))_{t\geq 0}$ follows a geometric Brownian motion, i.e.

$$\mathrm{d}x(t) = \alpha x(t)\mathrm{d}t + \sigma x(t)\mathrm{d}z(t).$$

The right hand side contains two terms. The first term represents the trend of the process with trend parameter α. Although from an economic perspective the consideration of a positive α seems most relevant in our framework, formally no assumption about the sign of α is required to carry out our analysis. The second term brings in the exogenous shocks through $z(t)$, a Wiener process which has a normal distribution with expectation 0 and volatility $\sqrt{t}$. Here $\sigma > 0$ is the volatility parameter. Throughout the paper we will refer to the initial value of the process $x(t)$ as X, i.e., $X = x(0)$.

We denote by $\mathcal{F} = (\mathcal{F}_t)_{t\geq 0}$ the filtration associated with the process $x(\cdot)$ capturing the available information at time t.

Discounting takes place under a fixed positive rate $r > \alpha$. The investment cost is linearly related to the investment size, where the marginal cost parameter equals δ. The inverse demand function is chosen to be in line with e.g. Pindyck (1988), He and Pindyck (1992), Aguerrevere (2003), Wu (2007), and Huisman and Kort (2015). In this model firms are committed to produce the amount their capacity allows. This assumption is widely used in the literature on capacity constrained oligopolies (e.g. Deneckere *et al.* (1997), Chod and Rudi (2005), Anand and Girotra (2007), Goyal and Netessine (2007), and Huisman and Kort (2015)). For example, Goyal and Netessine (2007) argue that firms may find it difficult to produce below capacity due to fixed costs associated with, for example, labor, commitments to suppliers, and production ramp-up. Since firms' outputs are dictated by their capacities, we are considering, as standard in real options games, closed-loop equilibria, so that each firm is able to observe the opponent's capacity investments (see, e.g., Huisman and Kort (2015)).

To formulate the firms' objective function, we denote by $Q_I(t)$, $Q_E(t)$ the capacities of the incumbent and the entrant at time t with $Q_I(0) = q_{1I} > 0$, $Q_E(0) = q_{1E} = 0$, where q_{1I} denotes the initial capacity of the incumbent. Capacities can be changed by lumpy investments and are given by

$$Q_f(t) \quad = \quad Q_f(0) + \mathbf{1}_{[\tau_f,\infty)}(t) q_{2f}$$

for $f \in \{I, E\}$, where τ_f and q_{2f} are measurable with respect to $\mathcal{F}_t$ and determined based on both players' strategy profiles, as explained below.

Using this notation the firms' objective functions can be written as

$$J_f(x(0), Q_I(t), Q_E(t)) = \mathbb{E} \int_{t=0}^{\infty} e^{-rt} \left[Q_f(t) x(t)(1 - \eta(Q_f(t) + Q_g(t))) \mathrm{d}t - e^{-r\tau_f} \delta q_{2f} \right],$$

where $f, g \in \{I, E\},\ g \neq f$ and the expectation is taken with respect to the Wiener process $x(t)$.

The capacity dynamics $Q_I(t), Q_E(t)$ are determined by the firms' strategies each of which comprises two decisions: timing and capacity size of the investment. In our analysis we consider Markov Perfect Equilibria (MPE) of the game and hence restrict attention to strategies under which the firms' actions at each point in time t are functions of the state $(x(t), Q_I(t), Q_E(t))$. Following e.g. Riedel and Steg (2017) we interpret the game as one with three modes and formulate mode-dependent strategies. In mode m_0 no firm has invested yet, whereas in mode $m_f,\ f \in \{I, E\}$ firm $g \neq f$ has already invested, but firm f still has to invest. Each firm's strategy is then given by a triple

$$(\tilde{q}_{2f}(x(t), Q_I(t), Q_E(t), m(t)),\ \tilde{\tau}_f(Q_I(t), Q_E(t), m(t)),\ \kappa_f(x(t), Q_I(t), Q_E(t)))$$

with $f \in \{I, E\}$ and $m(t) \in \{m_0, m_f\}$. Whereas $\tilde{q}_{2f}$ is a non-negative real number the second component $\tilde{\tau}_f$ is a stopping time measurable with respect to $\mathcal{F}_t$ and κ_f is binary, i.e., $\kappa_f \in \{0, 1\}$ on the entire state space[1]. The third component is only relevant in mode m_0 and therefore does not depend on m. The actual

[1]Strictly speaking stopping times might not only condition on the current, but also on past values of $x(t)$ and might therefore not be Markovian. However, as it turns out in equilibrium all stopping times condition only on the current value of $x(t)$ and hence indeed are Markovian strategies.

investment time τ_f of firm $f \in \{I, E\}$ is then determined as follows

$$\tau_f = \begin{cases} \tilde{\tau}_f(q_{1I}, 0, m_0) & \text{if} \quad \begin{array}{l} \tilde{\tau}_f(q_{1I}, 0, m_0) < \tilde{\tau}_g(q_{1I}, 0, m_0) \\ \text{or} \\ \tilde{\tau}_f(q_{1I}, 0, m_0) \;=\; \tilde{\tau}_g(q_{1I}, 0, m_0) \quad \text{and} \\ \kappa_f(x(\tilde{\tau}_f), q_{1I}, 0) > \kappa_g(x(\tilde{\tau}_g), q_{1I}, 0), \end{array} \\ \tilde{\tau}_f(Q_I(\tilde{\tau}_g), Q_E(\tilde{\tau}_g), m_f) & \text{else.} \end{cases}$$

with $g \neq f$. If $\tilde{\tau}_f(q_{1I}, 0, m_0) = \tilde{\tau}_g(q_{1I}, 0, m_0)$ and $\kappa_f(x(\tilde{\tau}_f), q_{1I}, 0) = \kappa_g(x(\tilde{\tau}_g), q_{1I}, 0)$, then with probability 0.5 the incumbent invests first, i.e. $\tau_I = \tilde{\tau}_I(q_{1I}, 0, m_0), \tau_E = \tilde{\tau}_E(Q_I(\tilde{\tau}_I), Q_E(\tilde{\tau}_I), m_E)$ and with probability 0.5 the entrant invests first, i.e. $\tau_E = \tilde{\tau}_E(q_{1I}, 0, m_0), \tau_I = \tilde{\tau}_I(Q_I(\tilde{\tau}_E), Q_E(\tilde{\tau}_E), m_I)$. The investment quantity follows directly as

$$q_{2f} = \begin{cases} \tilde{q}_{2f}(x(\tau_f), Q_I(0), Q_E(\tau_f), m(\tau_f)) & \text{if } f = I, \\ \tilde{q}_{2f}(x(\tau_f), Q_I(\tau_f), Q_E(0), m(\tau_f)) & \text{if } f = E, \end{cases}$$

where $m(t)$ denotes the mode of the game at time t.

Intuitively this formulation captures that if one of the two firms has an earlier stopping time in mode m_0 than its competitor it is the leader and invests first. If the two stopping times in mode m_0 coincide then only one firm actually invests, namely the one that has chosen the higher value of κ_f. In case both have chosen the same value of κ the incumbent invests. This setup allows a firm to threaten to invest at a certain stopping time $\tilde{\tau}_f(\cdot, m_0)$ conditional on the fact that the other firm does not invest at the same time. For the existence of a preemption equilibrium, like to ones we will consider in this paper, it is essential that such a conditional investment can be implemented through a Markovian strategy. Our setting allows for this and at the same time allows for a much simpler representation of the equilibrium profiles than in standard symmetric timing games[2]. Given the asymmetric setting considered in this paper and the focus on entry deterrence strategies, rather than on coordination problems in preemption games, we consider such a setup as a suitable formal framework for our analysis.

Following the standard procedure in real option games the game is solved by first determining the reaction curve of the firm investing last and then determining the optimal strategy of the firm that invests first. This means that we first consider the subgames in which the mode of the game is m_f for some $f \in \{I, E\}$. Since only one firm still can act in these subgames, they correspond to (infinite horizon) stochastic optimization

[2]Formally, the way the planned investment times $\tilde{\tau}_f$, which are the firms strategies, are transformed into the actual investment times τ_f, is part of the game-form of our considered game. Thijssen *et al.* (2012) (see also Riedel and Steg (2017)) provide an approach for a rigorous foundation of preemption-type equilibria in stochastic duopolistic timing games with symmetric firms. Their analysis is based on the original ideas of extended mixed strategies by Fudenberg and Tirole (1985). In such a setting it is not known ex-ante which firm will invest first. In the asymmetric setting considered in this paper, the situation is less complicated, since at the preemption point of the entrant, the entrant is indifferent between being leader or follower, whereas the incumbent strictly prefers to be the leader. Intuitively, it therefore is suitable to consider a game form which allows for an equilibrium where the incumbent makes the investment at the entrant's preemption point, but the entrant would invest in case no investment was made by the incumbent at that point.

problems which can be solved by standard techniques. We then use the value functions of both firms in these subgames to analyze the states of the game in mode m_0 with $Q_I(t) = q_{1I}, Q_E(t) = q_{1E} = 0$. In this analysis we will determine the investment thresholds and capacity choices of both firms in equilibrium, which will be sufficient for the economic analysis of the emerging market dynamics. A formal representation of the actual strategy profiles underlying the considered Markov Perfect Equilibrium and a proof that this profile indeed constitutes a MPE is provided in Appendix B.

Concerning the initial capacity of the incumbent, in principle, the parameter q_{1I} can take any value. As a reference case, however we will often consider the capacity value that would emerge as the long run capacity of a monopolist, which can make a single investment and is myopic in a sense that it does not take into account possible future capacity additions on this market, either by itself or by a potential entrant. We refer to such an initial capacity as the myopic investment level q_{1I}^{myop}. Following Huisman and Kort (2015) this value is given by

$$q_{1I}^{myop} = \frac{1}{\eta(\beta+1)},$$

where

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{1}$$

Due to the assumption that $r > \alpha$, we have $\beta > 1$.

3 Equilibrium Analysis

In this section we characterize the investment behavior in the Markov Perfect Equilibrium of the game described above. Employing the standard terminology in timing games (see, e.g., Fudenberg and Tirole (1985)), the first investor is called the leader and the second investor is called the follower. Before we are able to fully describe the equilibrium, in Section 3.1 we carry out two intermediate steps. We first characterize the optimal behavior of the follower firm in a situation in which the other firm, labeled as the leader, has already invested, i.e. we consider subgames in modes m_f with $f \in \{I, E\}$. Then, as a second intermediary step, we consider subgames (X, Q_I, Q_E) with $Q_f = q_{1f}, f \in \{I, E\}$, i.e., scenarios in mode m_0 prior to the first investment, and characterize a given firm's optimal investment size, if it invests instantaneously, as well as determine under which conditions it is optimal for the firm to delay its own investment, assuming that it will not be preempted by the opponent's investment. Formally, we do this by considering the game under the assumption of a fixed investment order, in which the opponent firm (the follower) cannot invest before the firm itself (the leader) has invested. The insights from these two preliminary steps carried out in Section 3.1 then allow us to characterize the equilibrium behavior on the entire state space in Section 3.2.

3.1 Leader and Follower Payoff Functions

In this section we present the set-up and a brief summary of the results describing optimal behavior of the follower, after the leader has invested, as well as optimal behavior of the leader in situations in which none of the firms has invested yet. Appendix A provides all elaborations underlying these results.

For a given sequence of investments, we denote the follower firm as firm F and similarly, the leading firm as firm L. The follower's and leader's initial capacities are denoted by q_{1F} and q_{1L}, respectively. Capacity expansion is done by installing additional quantities q_{2F} and q_{2L}[3].

Follower's Decision

The follower's optimization problem is to determine, for a given quantity $Q_L = q_{1L} + q_{2L}$ of the leader, the time and size of its own investment in order to maximize the expected payoff. Formally, the problem can be written as[4]

$$
\begin{aligned}
&V_F(X, q_{2L}) = \\
&\sup_{\tau_F \geq 0,\ q_{2F} \geq 0} \Bigg\{ \mathbb{E}\Bigg[\int\limits_{t=0}^{\tau_F} q_{1F}\ x(t)(1 - \eta(q_{1L} + q_{1F} + q_{2L}))e^{-rt}\mathrm{d}t \\
&\qquad + \int\limits_{t=\tau_F}^{\infty} (q_{1F} + q_{2F})x(t)(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}))e^{-rt}\mathrm{d}t - e^{-r\tau_F}\delta q_{2F} \;\Big|\; x(0) = X \Bigg] \Bigg\},
\end{aligned}
$$

where τ_F is the stopping time of firm $F \in \{I, E\}$ in mode m_F. The following proposition characterizes the follower's optimal investment strategy. Here, the optimal moment of investment is derived as the investment threshold $X_F^*(q_{2L})$. Investment takes place at the moment the stochastic process $x(t)$ reaches this level for the first time (see, e.g., Dixit and Pindyck (1994)). Therefore the optimal stopping time of the follower relates to $X_F^*(q_{2L})$ in the following way: $\tau_F^* = \inf\{t \geq 0 \mid x(t) \geq X_F^*(q_{2L})\}$. This means that the follower waits for $X < X_F^*(q_{2L})$, in the so-called continuation region, but invests in the stopping region, that is, for $X \geq X_F^*(q_{2L})$. It follows that the optimal strategy of the follower in these subgames can be characterized by the tuple of non-negative functions $(X_F^*(q_{2L}), q_{2F}^{opt}(X, q_{2L}))$.

[3] In principle, one could then distinguish two cases. In the first case which corresponds to mode m_E, the incumbent takes the role of the leader and the entrant takes the role of the follower, with $q_{1L} = q_{1I},\ q_{1F} = 0,\ q_{2L} = q_{2I}$ and $q_{2F} = q_E$. In the second case which corresponds to mode m_I, the entrant undertakes an investment before the incumbent expands and we have $q_{1L} = 0,\ q_{1F} = q_{1I},\ q_{2L} = q_E$ and $q_{2F} = q_{2I}$. For the analysis, both cases are analyzed simultaneously by keeping the notation open. In terms of the investment strategies introduced in Section 2, a capacity expansion choice q_{2L} respectively q_{2F} corresponds to an investment strategy $\tilde{q}_{2L}(\tilde{X}_L, q_{1L}, q_{1F}, m_0) = q_{2L}$, where $\tilde{X}_L$ is the level of $x(t)$ at which the leader invests and $\tilde{q}_{2F}(\tilde{X}_F, q_{1L} + q_{2L}, q_{1F}, m_F) = q_{2F}$, where $\tilde{X}_F$ is the level of $x(t)$ at which the follower invests.

[4] To simplify notation in the following derivations, we will not explicitly list the full state vector $(X, Q_L, Q_F, m), L, F \in \{I, E\}$, as the argument of the value function of the player, but only the parts of the state vector that conveys relevant information about the current state. In the subgames in mode m_F considered here, knowing X and the size of the leaders prior investment q_{2L} is sufficient to recover the full state vector. Hence, we only list these two values explicitly as arguments of the value function. Similar notation is used in the different types of subgames treated in the following sections.

Proposition 1 *For $X < X_F^*(q_{2L})$ the follower waits until the process $x(t)$ reaches the investment trigger $X_F^*(q_{2L})$ to install capacity $q_{2F}^*(q_{2L}) = q_{2F}^{opt}(X_F^*(q_{2L}), q_{2L})$ and for $X \geq X_F^*(q_{2L})$ the firm invests immediately and installs capacity $q_{2F}^{opt}(X, q_{2L})$. The optimal capacity level $q_{2F}^{opt}(X, q_{2L})$ and the investment trigger $X_F^*(q_{2L})$ are given by*

$$q_{2F}^{opt}(X, q_{2L}) = \frac{1}{2\eta}\left(1 - \eta(q_{1L} + 2q_{1F} + q_{2L}) - \frac{\delta(r-\alpha)}{X}\right), \tag{2}$$

$$X_F^*(q_{2L}) = \frac{\beta+1}{\beta-1}\frac{\delta(r-\alpha)}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}. \tag{3}$$

The follower's capacity in case the follower invests at the investment trigger equals

$$q_{2F}^*(q_{2L}) = q_{2F}^{opt}(X_F^*(q_{2L}), q_{2L}) = \frac{1-\eta(q_{1L}+2q_{1F}+q_{2L})}{\eta(\beta+1)}. \tag{4}$$

It then follows that the follower's value function is given by

$$V_F(X, q_{2L}) = \begin{cases} \frac{\delta}{\beta-1}\left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta} q_{2F}^*(q_{2L}) + \frac{X}{r-\alpha} q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})) & \text{if } X < X_F^*(q_{2L}), \\ \frac{X}{r-\alpha}(q_{1F}+q_{2F}^{opt}(X,q_{2L}))(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^{opt}(X,q_{2L}))) - \delta q_{2F}^{opt}(X,q_{2L}) & \text{if } X \geq X_F^*(q_{2L}). \end{cases} \tag{5}$$

Leader's Decision

From equation (3) it follows that there is a positive relation between the leader's investment quantity q_{2L} and the follower's investment threshold. This means that it will depend on the leader's choice of its capacity when the follower undertakes investment. The leader can thus delay the follower's investment by setting q_{2L} in such a way that the follower's trigger $X_F^*(q_{2L})$ exceeds the current value of $x(t)$, i.e., $X \leq X_F^*(q_{2L})$. We refer to the outcome where the leader's choice leads to this outcome as *delaying the follower*[5] and to the opposite case of $X \geq X_F^*(q_{2L})$ as *inducing immediate follower investment*[6]. For the following analysis and comparison of these two types of leader's investment strategies we denote by $V_L^{det}(X, q_{2L})$ respectively $V_L^{acc}(X, q_{2L})$ the value of the leader's objective function that can be reached under instantaneous investment by a strategy under which the follower is delayed respectively invests immediately[7].

Using this notation, for any X at which it is optimal for the leader to invest immediately, the leader's

[5]In case the incumbent is the leader, we have that the incumbent is a monopolist as long as $X < X_F^*(q_{2L})$, and as soon as x hits $X_F^*(q_{2L})$ a duopoly arises, since at that point the entrant undertakes an investment. Hence, this strategy of the incumbent corresponds to entry deterrence.

[6]In case the incumbent is the leader such behavior corresponds to an entry accommodation strategy.

[7]Since, as we will show later, in equilibrium the incumbent becomes the leader, we associate these strategies with deterrence respectively entry accommodation and we will hence use *det* respectively *acc* to signify the strategy where the leader delays respectively does not delay the follower's investment.

value function can be written as

$$V_L(X) = \sup_{q_{2L} \geq 0} J_L(X, q_{2L}) \tag{6}$$

with

$$J_L(X, q_{2L}) = \begin{cases} V_L^{det}(X, q_{2L}) & \text{if } X_F^*(q_{2L}) > X, \\ V_L^{acc}(X, q_{2L}) & \text{if } X_F^*(q_{2L}) \leq X. \end{cases} \tag{7}$$

Below we will determine for which ranges of values of X it is optimal for the leader to invest immediately, which is referred to as the stopping region. The payoff which can be obtained by the leader with a strategy under which the follower is delayed, is given by

$$\begin{aligned} V_L^{det}(X, q_{2L}) = \mathbb{E}\Bigg[& \int_{t=0}^{\tau_F^*} (q_{1L} + q_{2L}) x(t)(1 - \eta(q_{1L} + q_{1F} + q_{2L}))e^{-rt} \mathrm{d}t \\ & + \int_{t=\tau_F^*}^{\infty} (q_{1L} + q_{2L}) x(t)(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}^*(q_{2L})))e^{-rt} \mathrm{d}t \Big| x(0) = X \Bigg] - \delta q_{2L}. \end{aligned} \tag{8}$$

The first integral in this expression denotes the expected discounted revenue stream obtained by the leader before the follower has invested. Then, at the (stochastic) time $\tau_F^* \geq 0$ the follower decides to make an investment. The second integral reflects the leader's expected discounted revenue stream from that moment on. The third term is the investment outlay. Details of the calculation of the function $V_L^{det}(X)$ (as well as $V_L^{acc}(X)$) are provided in the proof of Proposition 2.

If the leader chooses a capacity such that it induces immediate investment by the follower it nevertheless acts as Stackelberg capacity leader. The highest payoff the leader can obtain by instantaneous investment without delaying the follower is given by

$$V_L^{acc}(X, q_{2L}) = \mathbb{E}\left[\int_{t=0}^{\infty} (q_{1L} + q_{2L}) x(t)(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}^{opt}(X, q_{2L})))e^{-rt} \mathrm{d}t - \delta q_{2L} \Big| x(0) = X \right]. \tag{9}$$

This expression contains two terms, the expected discounted revenue stream resulting from investment and the investment cost.

Considering the level of investment, which solves the optimization problem on the right hand side of (6), it follows from the monotonicity of $X_F^*(q_{2L})$ with respect to q_{2L} that for a fixed level of X, there exists a function $\hat{q}_{2L}(X)$ such that for $q_{2L} < \hat{q}_{2L}(X)$ we have that $X > X_F^*(q_{2L})$ and for $q_{2L} > \hat{q}_{2L}(X)$ it holds that $X < X_F^*(q_{2L})$. This means that to solve the optimization problem we need to consider two regions with respect to q_{2L}. For each state region one can find the point that yields the supremum of the payoff function. If the supremum of the payoff function associated with the delaying the follower strategy is found at q_{2L} such that $q_{2L} \leq \hat{q}_{2L}(X)$, that is, $X \geq X_F^*(q_{2L})$, then we say that the delaying the follower strategy is not *feasible*. Similarly, we say that the inducing immediate investment strategy is feasible for some X, if the maximizer of (9) is in the interval $[0, \hat{q}_{2L}(X)]$. We will show that always at least one strategy is feasible. We then find the following result.

Proposition 2 *There exist unique values $X_1 < X_2$ such that there is an interval of positive length $[X_1, X_2)$ on which both the delaying the follower investment strategy and the inducing immediate follower investment strategy are feasible for the leader. For $X < X_1$ it is optimal for the leader to delay the follower's investment and for $X \geq X_2$ optimal investment by the leader induces the follower to invest instantaneously.*

We denote the optimal capacity that solves the optimization problem as $q_L^{det}(X)$ for the strategy where the follower is delayed and, similarly, $q_L^{acc}(X)$ denotes the optimal capacity when inducing immediate follower investment. It should be noted that for low values of X the optimal investment size $q_L^{det}(X)$ might be zero even though the delaying the follower investment strategy is feasible for that value of X. However, it is evident that in such a case the leader is better off by delaying its investment rather than by (formally) carrying out his single investment option with an investment size of zero.

In Proposition 3 below, which characterizes the leader's optimal behavior, it is shown that for values of X below a certain threshold X_L^{det} respectively X_L^{acc} it is indeed optimal for the leader to delay own investment rather than making an instantaneous investment under the delaying the follower investment strategy respectively the inducing immediate follower investment strategy. Here, we assume that the follower will only be able to invest after the leader's investment, which means that the leader will not be preempted by the opponent's investment. This ensures that waiting is optimal in the continuation region. The maximum obtainable payoff for the leader which optimally delays own investment is obtained with the same method as used for the determination of follower's value function in the continuation region. Details of these calculations are provided in the proof of the proposition.

Proposition 3 *There exists a unique value X_0, $X_0 < X_2$ such that $q_L^{det}(X) > \max\{0, \hat{q}_{2L}(X)\}$ if and only if $X \in (X_0, X_2)$. Furthermore, for sufficiently small q_{1L} there exists a pair (X_L^{det}, q_L^{det*}) with $X_L^{det} \in (X_0, X_2)$ satisfying $q_L^{det*} = q_L^{det}(X_L^{det})$ and*

$$X_L^{det} = \frac{\beta}{\beta - 1} \frac{\delta(r-\alpha)}{1 - 2\eta q_{1L} - \eta q_{1F} - \eta q_L^{det*}}, \tag{10}$$

such that under the delaying follower investment strategy, for $X < X_L^{det}$ the leader waits until $x(t)$ reaches the investment threshold X_L^{det} to install q_L^{det*}, while for $X \geq X_L^{det}$ the leader invests immediately and sets capacity $q_L^{det}(X)$.

The capacity $q_L^{det}(X)$ is the solution with respect to q_{2L} of

$$\frac{X}{r-\alpha}[1 - \eta(2q_{1L} + q_{1F} + 2q_{2L})] - \frac{\delta}{\beta - 1}\left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta}\left[1 - \frac{\eta\beta(q_{1L} + q_{2L})}{1 - \eta(q_{1L} + 2q_{1F} + q_{2L})}\right] - \delta = 0.$$

Finally, for sufficiently small q_{1L} there exists a pair (X_L^{acc}, q_L^{acc*}) satisfying $q_L^{acc*} = q_L^{acc}(X_L^{acc})$ and

$$X_L^{acc} = \frac{\delta(r-\alpha)\beta}{\beta - 1} \frac{q_L^{acc*} - q_{1L}}{(q_L^{acc*} - q_{1L})(1 - \eta q_{1L}) - \eta q_L^{acc*}(q_L^{acc*} + q_{1L})}, \tag{11}$$

such that under the inducing immediate follower investment strategy, for $X < X_L^{acc}$ the leader waits until $x(t)$ reaches the investment threshold X_L^{acc} to install q_L^{acc*}, while for $X \geq X_L^{acc}$ the leader invests immediately and sets capacity $q_L^{acc}(X)$, which is given by

$$q_L^{acc}(X) = \frac{1}{2\eta}\left[1 - 2\eta q_{1L} - \frac{\delta(r-\alpha)}{X}\right].$$

Although the proof of Proposition 3 assumes that q_{1L} is small, numerical analysis indicates that the range of values of q_{1L} for which the threshold X_L^{det} exists and the leader therefore eventually invests, is typically of substantial size. In case the initial capacity of the leader is large, it is optimal for the leader to abstain from any further investment, since this also blocks any further investment of the follower[8] and allows the leader to sell the quantity corresponding to its current output level at a larger price.

From Proposition 2 it follows that there exists $\hat{X} \in (X_1, X_2)$ with the property that it is the largest value $\hat{X}$ such that the delaying follower investment strategy is always optimal for $X < \hat{X}$. Extensive numerical exploration shows that $\hat{X}$ is indeed a threshold in the sense that for all $X \geq \hat{X}$ inducing immediate investment by the follower is optimal for the leader.

Furthermore, we find that $\hat{X} > \max\{X_L^{det}, X_L^{acc}\}$, which implies that the leader waits in the region $0 \leq X < X_L^{det}$ and invests $q_L^{det}(X)$ in the region $X_L^{det} \leq X < \hat{X}$, thereby delaying investment by the follower. For $X \geq \hat{X}$ it is optimal for the leader to immediately invest $q_L^{acc}(X)$, which triggers an immediate investment of the follower. The optimal payoff of the leader in mode m_0 conditional on that the other firm cannot preempt it with own investment, is therefore given by

$$V_L(X) = \begin{cases} F_L^{det}(X) & \text{if } X \in (0, X_L^{det}), \\ V_L^{det}(X) & \text{if } X \in [X_L^{det}, \hat{X}), \\ V_L^{acc}(X) & \text{if } X \in [\hat{X}, \infty). \end{cases} \tag{12}$$

Here $F_L^{det}(X)$ denotes the value of waiting for the leader in the continuation region, where it delays investment till the threshold X_L^{det} is reached[9].

Figure 1 illustrates these findings.[10] The investment of the leader, conditional on that no firm has invested before (i.e. for states with $Q_L = q_{1L}, Q_F = q_{1F}$) and that the other firm cannot preempt, can then be characterized as

$$\tilde{q}_L(X) = \begin{cases} 0 & \text{if } X \in (0, X_L^{det}), \\ q_L^{det}(X) & \text{if } X \in [X_L^{det}, \hat{X}), \\ q_L^{acc}(X) & \text{if } X \in [\hat{X}, \infty). \end{cases} \tag{13}$$

Assuming $x(0)$ to be sufficiently small, our analysis implies that for exogenous firm roles the leader waits until $x(t)$ reaches X_L^{det} and then invests q_L^{det*}. The follower waits until $x(t)$ reaches $X_F^*(q_L^{det*})$, at which point in time the follower invests.

[8]Note that we are here considering the scenario where the follower is only allowed to invest after the leader has chosen to do so.

[9]The explicit expression for $F_L^{det}(X)$ is given in the proof of Proposition 3.

[10]All examples in this paper use the following parametrization: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1,\ \delta = 1000,\ q_{1I} = \frac{1}{\eta(\beta+1)}$.

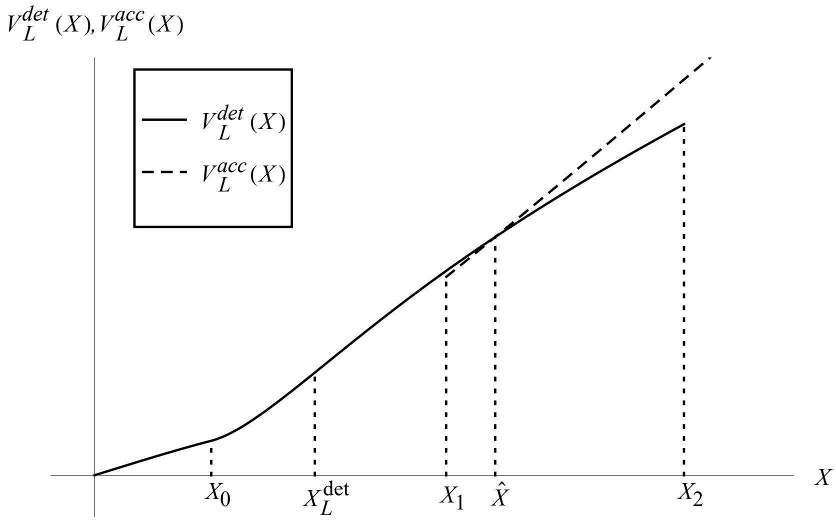


Figure 1: The leader's payoff functions while delaying the follower (solid) and while inducing immediate follower investment (dashed).

Let us, for future notation, denote $\hat{X}$ in case the entrant takes the leader role by $\hat{X}_E$ and, in case the incumbent is the leader, by $\hat{X}_I$.

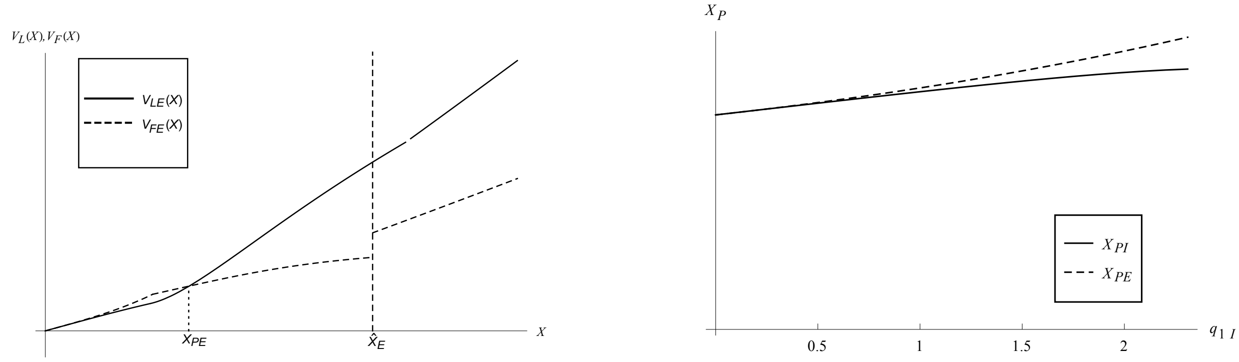
3.2 Equilibrium Investment Order

Based on the results of the previous section we can now examine the equilibrium behavior in our considered game. Hence, we consider the general scenario, where the investment order is not fixed ex-ante and both firms are allowed to invest first. Since the focus of our analysis is on the main economic effects arising in a setting with endogenous choice of timing and size of investments, rather than on the technical details arising in timing games, we abstain here from giving the full profile of Markovian strategies corresponding to the equilibrium outcome discussed below. The full description of the underlying equilibrium strategy profile is provided in Appendix B.

To characterize the firms' optimal behavior in mode m_0 we need to consider the payoff functions of a firm if it acts as leader and as follower. Figure 2a shows the two payoff functions for the entrant, denoted by $V_{LE}(X)$ and $V_{FE}(X)$, depending on the current value X of the state variable assuming that the entrant immediately invests when being leader. This means that

$$
\begin{aligned}
V_{LE}(X) &= \sup_{q_{2L}\geq 0} \begin{cases} V_{LE}^{det}(X, q_{2L}) & \text{if } X_F^*(q_{2L}) > X, \\ V_{LE}^{acc}(X, q_{2L}) & \text{if } X_F^*(q_{2L}) \leq X. \end{cases} \\
&= \begin{cases} \sup_{q_{2L}\geq 0} V_{LE}^{det}(X, q_{2L}) & \text{if } X < \hat{X}_E, \\ \sup_{q_{2L}\geq 0} V_{LE}^{acc}(X, q_{2L}) & \text{if } X \geq \hat{X}_E. \end{cases}
\end{aligned}
$$

for all X where $V_{LE}^{det}(X, q_{2L})$ and $V_{LE}^{acc}(X, q_{2L})$ are the leader payoff function as given by (8) and (9) under



(a) The entrant's payoff functions that correspond to becoming leader (solid) and follower (dashed) for the entrant. The follower is delayed for $X < \hat{X}_E$ and invests immediately for $X \geq \hat{X}_E$.

(b) Preemption points X_{PI} (solid) and X_{PE} (dashed) with free capacity choice for different values of q_{1I}.

Figure 2: Preemption points. Default parameters: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1$, and $\delta = 1000$.

state $Q_L = 0, Q_F = q_{1I}$. Analogously for $V_{FE}(X)$.

The solid curve corresponds to the outcome if the entrant takes the leader role, where the payoff of immediate investment is depicted. If the firm takes the position of the follower, one arrives at the dashed curve, corresponding to (5). For the incumbent both curves are qualitatively the same so that a comparable figure is obtained.

For small values of X investment is not profitable. Then no firm wants to invest first, which is why the follower curve lies above the leader curve. For larger values, though, each firm wants to be the first investor. Since the curves are qualitatively similar for the incumbent and the entrant firm, both firms prefer to become the leader when X is large enough, that is, when $V_{Lf}(X) > V_{Ff}(X)$ for $f \in \{I, E\}$. To prevent that the competitor undertakes an investment first, thereby making the firm end up with the follower value instead of the higher leader value, each firm prefers to invest strictly before the competitor's investment moment, i.e., it is best to preempt the other firm. Hence, assuming that the initial value of the process $x(t)$ is in the region where the follower curve exceeds the leader curve, the first moment for a firm to invest, that is, when investment as a leader becomes worth-while, is at the lowest value of X for which the leader curve no longer yields a smaller value than the follower curve. This point is called the preemption point X_P. To formally define the preemption points, let us slightly change notation. Let us denote the leader's payoff function when delaying the follower as $V_L^{det}(X, q_{2L}, q_{1L}, q_{1F})$. Similarly let $V_F(X, q_{1L} + q_{2L}, q_{1F})$ and $q_L^{det}(X, q_{1L}, q_{1F})$ denote the follower payoff function and the leader's optimal capacity while delaying the follower, respectively. Then, the preemption points of the incumbent and entrant are defined in the following way,

$$X_{PI} = \min\{X > 0 \mid V_L^{det}(X, q_L^{det}(X, q_{1I}, 0), q_{1I}, 0) = V_F(X, q_L^{det}(X, 0, q_{1I}), q_{1I})\},$$
$$X_{PE} = \min\{X > 0 \mid V_L^{det}(X, q_L^{det}(X, 0, q_{1I}), 0, q_{1I}) = V_F(X, q_{1I} + q_L^{det}(X, q_{1I}, 0), 0)\}.$$

Since $q_{1I} > 0$, the two firms are asymmetric and therefore their preemption points do not coincide. Clearly, for a firm, of which its preemption point is below that of the competitor, it can never be an equilibrium strategy to choose an investment trigger above the competitor's preemption point. If the firm would choose such a large trigger the opponent's best response would imply that the firm ends up as follower, and therefore with a smaller value compared to what it can gain as leader (see Figure 2a). If, furthermore, the optimal trigger X_L^{det} under the delaying follower investment strategy of that firm is larger than the opponent's preemption point, then the firm has no incentives to invest before the opponent's preemption point is reached. In such a situation it constitutes equilibrium behavior for the firm with the lower preemption point to set its investment trigger to the opponent's preemption point and to invest an amount which delays the opponent's investment. Following its optimal strategy the opponent chooses the follower's investment trigger and invests once this trigger is reached. Such an equilibrium is referred to as a preemption equilibrium and the following proposition shows that at least for appropriate initial capacity of the incumbent no other types of subgame-perfect equilibria exist in the considered game.

Proposition 4 *Let $q_{1I} = q_{1I}^{myop}$. Then, preemptive investment constitutes a unique subgame perfect Nash equilibrium.*

In Appendix A it is shown that Proposition 4 applies also when the initial capacity of the incumbent is sufficiently close to q_{1I}^{myop}.

As discussed in more detail in Appendix B, the planned stopping times of both firms in mode m_0 in this equilibrium coincide, i.e. $\tilde{\tau}_f(q_{1I}, 0, m_0) = \inf\{t : x(t) \geq X_{PE}\}$ with $f \in \{I, E\}$. Furthermore, we have in equilibrium $\kappa_I(q_{1I}, 0, X_{PE}) = 1$, $\kappa_I(q_{1I}, 0, X_{PE}) = 0$, which implies that the incumbent actually invests at this stopping time.

In order to clarify which firm acts as leader in the preemption equilibrium, we depict in Figure 2b the preemption points of the incumbent and the entrant for values of q_{1I}. It can be clearly seen that the preemption point of the incumbent is below that of the entrant. Furthermore, it is easy to check that the leader's investment trigger under the delaying follower investment strategy, if it is finite, is generically much larger than the entrant's preemption point (see also Lemma 2 in Appendix A).[11] Together, these two observations establish that the incumbent acts as leader in the preemption equilibrium. Hence, for $x(0) < X_{PE}$ it is optimal for the incumbent to wait and to invest the amount $q_L^{det}(X)$ when the process reaches the preemption point of the entrant. The investment is chosen in a way to delay the investment of

[11] As elaborated in Section 3.4 these two inequalities do not depend on the particular parametrization of the model chosen here but stay intact over a large range of relevant parameter settings.

the entrant and therefore it is an instrument of entry deterrence. Using this strategy the incumbent can delay the entry of its opponent till the trigger $X_F^*(q_{2I})$ is reached by $x(t)$.

Example 1 *Considering, as an illustrative example, the case* $q_{1I} = q_{1I}^{myop}$*, which under our default parametrization yields* $q_{1I} = 2.37$*, we obtain* $X_{PI} = 134$ *and* $X_{PE} = 167$*. This means that for* $X < 134$ *both firms prefer to wait, for* $134 \leq X < 167$ *the incumbent prefers to be leader and the entrant prefers to wait and for* $X \geq 167$ *both want to invest. The investment trigger* X_{LI}^{det} *is not finite in this situation since the incumbent would not undertake an investment in the case of exogenous firm roles (see Lemma 2 in Appendix A). It seems the main reason the incumbent invests at* $X = X_{PE} = 167$ *is to delay the competitor's entry: until* $x(t)$ *reaches* $X_F^* = 208$ *the incumbent is the only firm on the market.*

In this example, the incumbent expands with $q_L^{det}(X_{PE}) = 0.33$ *so that it ends up with a total capacity of* $q_{1I} + q_{2I} = 2.70$*, the entrant installs a capacity equal to* $q_E = 1.73$*.*

To understand this result one must realize that any investment reduces the output price, since this price is negatively related with the total market output. Investment by the entrant thus reduces the incumbent's value. It is then better for the incumbent to cannibalize than let the entrant reduce the price. To do so, the incumbent installs a small capacity level: small in order not to make the cannibalization effect too large, but large enough to delay investment of the entrant. To conclude, the incumbent installs a small additional capacity with the aim to protect its demand, and to prolong the period where it can profit from its monopoly position. The entrant will invest later when, for larger levels of x, demand is stronger so it is profitable to set a larger quantity on the market. This leads to the result that the incumbent invests first and expands to delay a large investment by the entrant. The entrant waits until the state variable hits the follower's investment threshold.

3.3 Market leadership

When studying industry evolution and entry deterrence, a crucial issue is the question under which circumstances early incumbents in an industry are able to maintain their market leadership as the market grows. A firm is considered the market leader when its (accumulated) capacity exceeds its competitor's. This section illustrates that in our considered setting the incumbent does not necessarily maintain its market leader position after the entrant's investment.

We find that for a small initial capacity level the entrant becomes the market leader. This means that the entrant's capacity exceeds the incumbent's initial capacity together with its expansion. However, when the incumbent starts with a sufficiently large capacity level, it keeps its position as market leader after the second firm's entry. This is illustrated in Figure 3, which shows the level q_{1I}^{ML} of the incumbent's initial capacity for which the total incumbent's capacity equals the amount set by the entrant. Market leadership thus depends on the initial capacity size. Intuitively, a larger initial capacity level has two contradictory effects on the expansions. First, a larger initial capacity, makes the expansion size decrease, for the cannibalization effect is larger for the firm already owning a larger capital stock. Second, since investment is delayed, a larger market is observed at the moment of investment, which gives an incentive to increase investment size. The former effect, however, is dominant and one observes that a larger initial capacity makes the size of the expansion decrease. The incumbent's total capacity, however, increases when the initial market's output size is larger.

In a framework with two potential entrants, i.e. no firm possesses an initial capacity, Huisman and Kort (2015) point out that market leadership is dependent on uncertainty. In particular, they show that for large demand uncertainty the first investor becomes market leader, while the second investor will invest in a larger capacity when the demand uncertainty is low. Their model considers symmetric firms, which, in our setting, would come down to $q_{1I} = 0$. Combining this with our findings implies that market leadership depends on both initial capacity and demand uncertainty.

As illustrated in Figure 3, q_{1I}^{ML} decreases when uncertainty increases. Larger uncertainty makes the incumbent delay investment, which results in a larger expansion investment, making it market leader for smaller values of q_{1I} relative to the case of smaller uncertainty. In this figure one can clearly observe for which combinations of the initial capacity level and the uncertainty level the incumbent is market leader and in which region the entrant becomes market leader.

One key observation would be that in the situation of Huisman and Kort (2015) where firms are symmetric, i.e. $q_{1I} = 0$, we would find that the entrant is market leader as long as the market uncertainty is not high. However, when considering asymmetric firms this is already no possibility anymore when $q_{1I} > 0.62$ for our basic parametrization. Comparing this value to the myopic value of the incumbent's initial capacity ($q_{1I}^{myop} = 2.37$) suggests that only for rather small values of the incumbent's initial capacity the entrant in equilibrium can become the market leader.

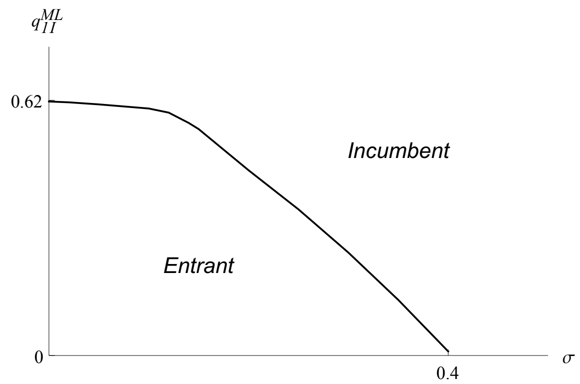


Figure 3: Market leader regions for different σ. Default parameters: $\alpha = 0.02$, $r = 0.1$, $\sigma = 0.1$, $\eta = 0.1$, and $\delta = 1000$.

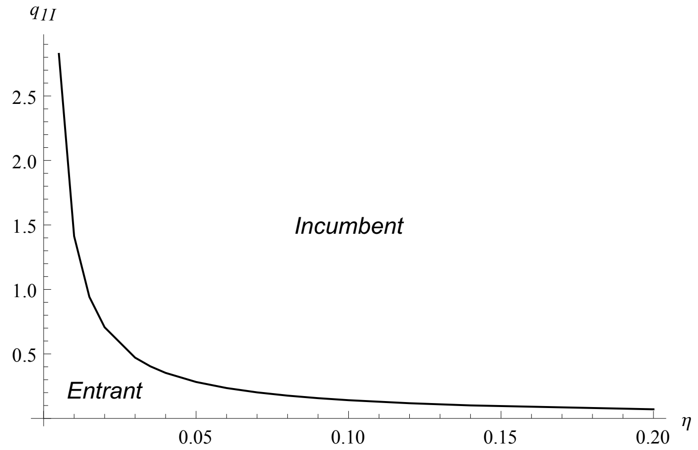


Figure 4: Regions where the incumbent invests first (above the curve) and where the entrant invests first (below the curve). Default parameters: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1$, and $\delta = 1000$.

3.4 Parameter variations

In order to inspect the effect of changes in parameter values on the investment order, the difference between the two preemption points as well as the difference between the incumbent's investment threshold and the entrant's preemption trigger is shown for a variation of all parameters in Appendix C. This makes clear that the insight that the incumbent invests first to delay the entrant's investment is very robust. There is a single exception, which occurs when the sensitivity of the market clearing price with respect to the supplied quantity (η) is very small or when there is an almost negligible size of the incumbent's initial capacity. In such a setting the entrant's preemption trigger might fall below the one of the incumbent. The trade-off between the initial capacity and the sensitivity parameter is depicted in Figure 4. This figure shows the two regions where either of the firms invest first. The curve in between depicts all values of η and q_{1L} for which both firms' preemption triggers are identical. We see that the incumbent invests first, except for a small region close to both axes where the entrant is the first investor. In fact, it holds that for $\eta \cdot q_{1L} > 0.01413$ the incumbent is leader and the entrant invests first for $\eta \cdot q_{1L} < 0.01413$. Intuition behind this result is that for the situation where η and q_{1L} are small the cannibalization effect is small. The incentives to preempt the entrant vanish the moment there is almost nothing to protect.

4 Investment Size

In this section we discuss in more detail two important aspects of the investment size in our setting. First, we analyze the question, whether the threat of entry by a competitor makes the incumbent invest more than it would in the absence of such a threat. Second, we show that the results obtained in the previous section significantly change if we assume that investment size is not a choice variable of the firm, but rather exogenously given, e.g., due to technical requirements. Finally, we briefly focus on two alternative scenarios:

one where we have two incumbents and one with product differentiation.

4.1 Overinvestment

In the literature on entry deterrence incumbents mainly deter entrants by means of overinvestment (e.g., Spence (1979) and Dixit (1980)). That is, by building large capacities on the market, it becomes unprofitable for other firms to enter this market. These entry deterrence models suggest that, apart from cases where markets are blocked (e.g. due to high entry costs), the quantity put on the market under an entry threat exceeds the amount that would be optimal for the firm in case that there is no potential entrant. This section investigates whether this notion of overinvestment also applies to the dynamic stochastic market framework presented in Section 2. Thereto, we compare the outputs in the long run, i.e. after the incumbent has made its expansionary investment, for two models: the model with (potential) competition, analyzed in Section 3, and a model where there is no threat of a potential entrant.

Overinvestment is defined as the difference between the quantity an incumbent sets on the market in the long run when there exists a threat of entry and the quantity it would set in the long run when this threat would not be present. In other words, the incumbent's expansion in the duopoly setting as presented in the previous section is compared to the incumbent's expansion in case it is a monopolist forever. To this end, the monopolist's model is presented and analyzed.

The value function of the monopolist is given by

$$V_M(X) = \sup_{\tau_M \geq 0,\ q_2 \geq 0} \Bigg\{ \mathbb{E}\Bigg[\int_{t=0}^{\tau_M} q_1\ x(t)(1-\eta q_1)e^{-rt}\mathrm{d}t + \int_{t=\tau_M}^{\infty} (q_1+q_2)x(t)(1-\eta(q_1+q_2))e^{-rt}\mathrm{d}t - e^{-r\tau_M}\delta q_2 \ \Bigg|\ x(0)=X \Bigg]\Bigg\},$$

in which q_1 is the initial capacity, q_2 corresponds to the capacity acquired by investment, and τ_M is a stopping time. The optimal expansion is denoted by $q_2^{mon}(X)$. Standard analysis, see Appendix A.2, shows that for the expansion, the threshold and capacity size equal

$$X_M^* = \frac{\beta+1}{\beta-1}\frac{\delta(r-\alpha)}{1-2\eta q_1},$$

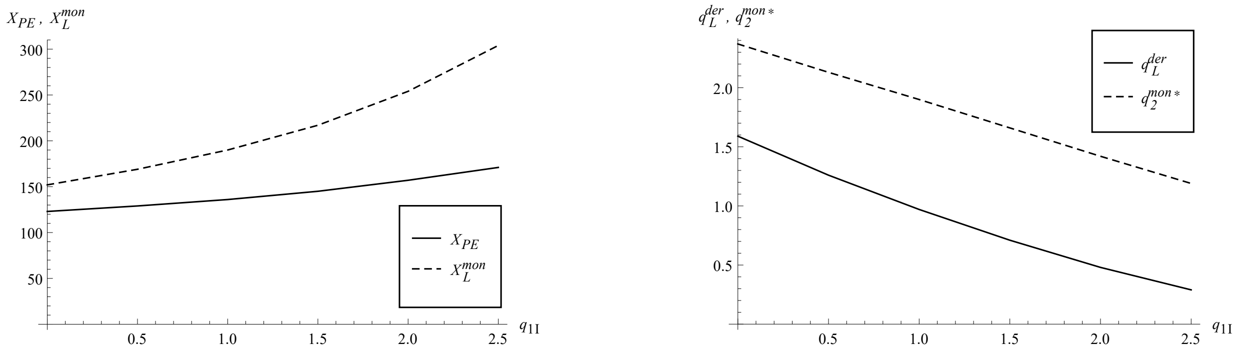
$$q_2^{mon*} = \frac{1-2\eta q_1}{\eta(\beta+1)},$$

where $q_2^{mon*} = q_2^{mon}(X_M^*)$ is the realized expansion at investment. For $X > X_M^*$ this quantity equals

$$q_2^{mon}(X) = \frac{1}{2\eta}\left(1 - 2\eta q_1 - \frac{\delta(r-\alpha)}{X}\right).$$

This means that τ_M^* is the first hitting time of $x(t)$ reaching X_M^*.

To measure overinvestment, the difference between $q_L^{det}(X_{PE})$ and q_2^{mon*} needs to be considered, which represent the realized outputs in the case entry threat is present and not, respectively. Figure 5 illustrates this difference for our standard parameter setting. In Figures (a) and (b), the optimal investment moment



(a) Investment moments with and without entry threat.

(b) Investment quantities with and without entry threat.

Figure 5: Expansions made by the incumbent with and without entry threat for different values of q_{1I}. Default parameters: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1$, and $\delta = 1000$.

and the optimal investment size are given for different values of the initial investment size. Overinvestment would occur if $q_L^{det}(X_{PE}) > q_2^{mon*}$. However, the figure illustrates the opposite. To explain this, one must realize that the investment threshold values of the monopolist are higher than the ones of the incumbent in a duopoly setting. The incumbent, by all force, prefers to keep its monopoly position as long as possible and thereto it delays investment of the entrant by preempting the entrant's preferred investment moment. This leads to an investment in a market that is still small at the moment of investment. For this reason the capacity investment of the firm is small as well. The monopolist, however, has the flexibility to wait for a price that has grown to a considerable level before investing. We conclude that, under consideration of endogenous timing as well as endogenous investment size, entry deterrence is not so much about the size but more about the timing of the investment.

4.2 Fixed Investment Size

In order to highlight the importance of the endogenous choice of investment size for our main finding that the incumbent invests prior to the entrant, in this section, we consider a scenario where the size of investment is fixed. Apart from improving our understanding of the role of endogenous investment size, the main motivation for considering a scenario with fixed investment is that for industries where expansion has to be typically carried out in fixed units, for example the establishment of an additional laboratory in the pharmaceutical industry, the assumption of a fixed investment size seems more appropriate than that of complete flexibility in the size of investment. This section shows that whether investment size is exogenous or endogenous is indeed crucial for the emerging investment order.

Consider the model presented above, but assume investment size is fixed such that $q_{2I} = q_E = K$. Now the situation is one of a stochastic timing game with asymmetric firms in the sense that one firm is the incumbent with initial capacity q_{1I}, whereas the other firm is the entrant that has thus no initial capacity.

Technically the problem is similar as in Pawlina and Kort (2006). That paper also studies a duopoly with asymmetric firms, but there asymmetry is due to investment costs being different for the firms.

The incumbent's optimal payoff functions as leader in the stopping region (under the deterrence strategy) and follower are then similar to what was found previously,

$$V_{LI}^{det}(X) = \frac{X}{r-\alpha}(q_{1I}+K)(1-\eta(q_{1I}+K)) - \frac{X_{FE}^*}{r-\alpha}\eta K(q_{1I}+K)\left(\frac{X}{X_{FE}^*}\right)^\beta - \delta K,$$

$$V_{FI}(X) = \begin{cases} \frac{\delta K}{\beta-1}\left(\frac{X}{X_{FI}^*}\right)^\beta + \frac{X}{r-\alpha}q_{1I}(1-\eta(q_{1I}+K)) & \text{if } X < X_{FE}^*, \\ \frac{X}{r-\alpha}(q_{1I}+K)(1-\eta(q_{1I}+2K)) - \delta K & \text{if } X \geq X_{FE}^*, \end{cases}$$

where $X_{FE}^* = \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{1-\eta(q_{1I}+2K)}$ and $X_{FI}^* = \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{1-2\eta(q_{1I}+K)}$ are the investment triggers of the entrant and the incumbent as follower. In a similar way one can determine the payoff functions of the entrant.

Next, one can calculate the preemption points. In Section 3.2 it was shown that under endogenous choice of the investment size the incumbent invests first, where it expands by an adequate amount such that the entrant's investment is temporarily hold off. Figure 6 shows the preemption points for the model presented in this section, i.e. where investment size is fixed. The relative position of the curves has changed compared to Figure 2b, which depicts the case with endogenous investment size: the entrant's curve now lies below the incumbent's curve, signifying that in this model the entrant precedes the incumbent in undertaking an investment. Thus, the entrant takes the leader role and the incumbent becomes follower.

If firms are free to choose the size of their installment, the incumbent has the largest incentive to invest first, for it can undertake a small investment in order to delay a large investment by the entrant. When fixing capacity for both firms at an equal level, this no longer applies: since capacity size is fixed, the incumbent cannot make a small investment to delay a large investment by the follower. Then the incentive to invest is higher for the entrant, since it does not suffer from cannibalization. As a result, the incumbent is more eager

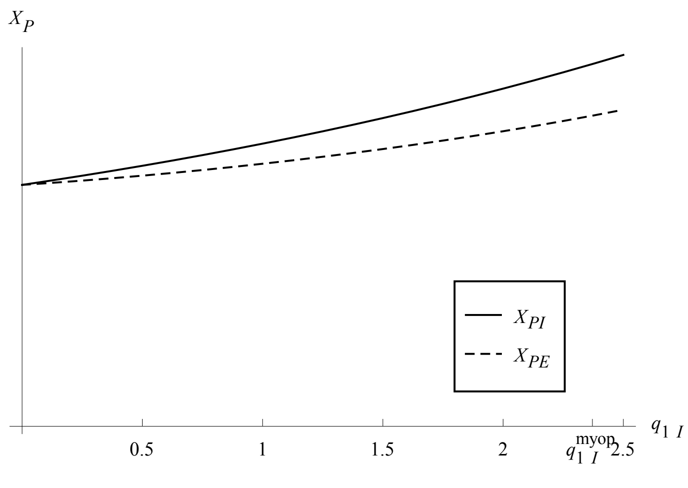


Figure 6: Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with fixed capacity for different values of q_{1I} with $K = 2.5$. Default parameters: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1$, and $\delta = 1000$.

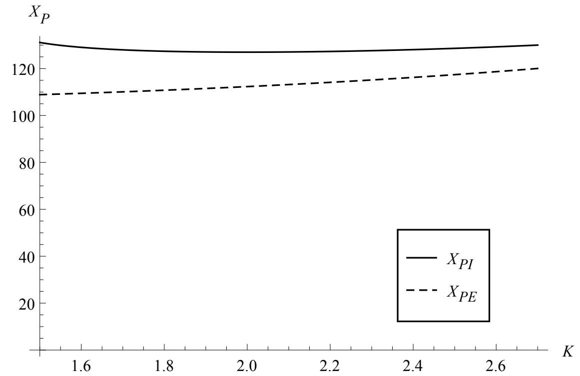


Figure 7: Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with fixed capacity for different values of K with $q_{1I} = q_{1I}^{myop}$. Default parameters: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1$, and $\delta = 1000$.

to delay its own investment and the entrant is the investment leader. Figure 7 shows that the observation that the entrant invests first when capacity size is fixed, is robust with respect to changes in the size of investment K.

5 Conclusions

The main message of this paper is that the interaction between timing and size of investment plays a crucial role in the strategy of an incumbent facing the threat of entry in a dynamic market environment. Where entry deterrence is generally understood to ward off entrants by overinvesting, we find that entry is delayed by accelerating the investment. This induces an investment, which is smaller than that of an incumbent in a comparable market without an entry threat. This implication of our analysis is well suited to explain the empirical observations reported in Leach *et al.* (2013). These authors show that, contrary to the predictions of the standard entry deterrence literature, the entry threat generated by the deregulation of the U.S. telecommunication industry did not result in an increase of capacity investments by incumbents. As the telecommunications industry in this period clearly has the characteristics of an expanding market, it fits well with the setup of our model. Therefore, our insight that in the presence of choices about both timing and size the incumbent's investment should be smaller than without an entry threat, provides a clear theoretical guidance for understanding these empirical observations. Also our result that, depending on whether investment size is flexible or fixed, the incumbent or the entrant invests first, is not only a new insight in the theoretical literature, but also gives rise to potentially testable empirical implications.

Our model considers the situation of one incumbent and one entrant. One could wonder how our results change when the initial capacity of the entrant is not set to be equal to zero, i.e., the scenario where $q_{1E} > 0$. A model with two incumbents is very similar to the the model treated in Huberts (2017, Chapter 3). Applying this paper's model to that setting shows us that qualitatively our results remain intact. In this case it is the larger firm that preempts the smaller firm rather than the incumbent preempting the entrant.

Alternatively, one could consider a situation where instead of expanding, the firms have the option to invest in an innovative product that is differentiation from the existing product. Findings in Huberts (2017, Chapter 3) suggest that our result that the incumbent invests first carry over to scenarios where the considered investments are not made for exactly the product the incumbent already has on the market, but for a differentiated (new) product. A more extensive treatment of optimal investment patterns in scenarios with product innovations by incumbent firms in our dynamic stochastic setting in any case is an interesting avenue for future research.

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Appendix A: Proofs

A.1 Equilibrium Analysis

Proof of Proposition 1

We will first determine the payoff in the stopping and continuation region, which is followed by verification. After that we determine for which values of X the firm optimally stops.

In the stopping region the firm realizes the accumulated and discounted expected profits $V_F(X, q_{2L})$ which maximizes, with respect to q_{2F},

$$\mathbb{E}\left[\int_{t=0}^{\infty} x(t)(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}))e^{-rt}\mathrm{d}t \;\Big|\; x(0)=X\right] - \delta q_{2F} \tag{14}$$

$$\begin{aligned}
&= (q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}))\mathbb{E}\left[\int_{t=0}^{\infty} x(t)e^{-rt}\mathrm{d}t \;\Big|\; x(0)=X\right] - \delta q_{2F} \\
&= (q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}))\int_{t=0}^{\infty} x(0)e^{(\alpha-r)t}\mathrm{d}t - \delta q_{2F} \\
&= \frac{X}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})) - \delta q_{2F}.
\end{aligned} \tag{15}$$

The follower's payoff function consists of two terms. The expected discounted cash inflow stream resulting from selling goods on the market is reflected by the first term. The involved cost, when making the investment, is captured by the second term. The optimal size of the investment is found by first optimizing the payoff function, i.e. the firm chooses its capacity $q_{2F} \geq 0$ such that it maximizes its profits. Thereto we first find the solution for the first order condition,

$$0 = \frac{X}{r-\alpha}\left[1-\eta(q_{1L}+2q_{1F}+q_{2L})-2\eta q_{2F})\right] - \delta$$

$$\Leftrightarrow \tag{16}$$

$$q_{2F} = \frac{1}{2\eta}\left[1-\eta(q_{1L}+2q_{1F}+q_{2L}) - \frac{\delta(r-\alpha)}{X}\right]. \tag{17}$$

The right hand side of (17) is positive for $X > \frac{\delta(r-\alpha)}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}$ and is an increasing function of X. As we will show later, this forms the solution of the optimization problem in the stopping region. Let us therefore already denote (17) by $q_{2F}^{opt}(X, q_{2L})$. Since the control variable is measurable, continuous, and $\{\mathcal{F}_t\}$-adapted, we conclude that it is admissible.

The second order condition reassures us that this is indeed a maximum, $-2\eta\frac{X}{r-\alpha} < 0$.

In the continuation region, in which it is optimal for the firm to delay investment, the value function for the leader, following standard real options analysis (see e.g. Dixit and Pindyck (1994)), is given by the value of waiting, which follows from the following differential equation,

$$r\phi_F(X, q_{2L}) = Xq_{1F}(1-\eta(q_{1F}+q_{1L}+q_{2L})) + \alpha X\frac{\partial}{\partial X}\phi_F(X, q_{2L}) + \tfrac{1}{2}\sigma^2X^2\frac{\partial^2}{\partial X^2}\phi_F(X, q_{2L}), \tag{18}$$

with the restriction that $\phi_F(0) = 0$. The solution is given by

$$V_F(X, q_{2L}) = A_FX^{\beta} + \frac{X}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})), \tag{19}$$

where β is the positive root (see e.g. Dixit and Pindyck (1994)) following from,

$$\sigma^2\beta^2 + (2\alpha - \sigma^2)\beta = 2r, \tag{20}$$

which gives us (1). This function equals the sum of two terms reflecting the value of waiting and the value of current production.

Showing optimality requires a verification theorem. Here we follow Gozzi and Russo (2006). First, we observe that the integrand of (14) is continuous and bounded (since $r > \alpha$). Secondly, (19) is a solution of the HJB in the continuation region (18), and (15) solves the HJB in the stopping region,

$$\begin{aligned} r\phi_F(X, q_{2L}) = X(q_{1F} + q_{2F})(1 - \eta(q_{1F} + q_{1L} + q_{2L} + q_{2F})) \\ + \alpha X \frac{\partial}{\partial X}\phi_F(X, q_{2L}) + \tfrac{1}{2}\sigma^2 X^2 \frac{\partial^2}{\partial X^2}\phi_F(X, q_{2L}), \end{aligned} \tag{21}$$

with $\phi_F(0) = 0$ (see Dixit and Pindyck (1994, p. 181-182) for the restrictions on $\phi_F(X)$ as $X \to \infty$). Moreover, (5) is finite and C^1 (continuity at $X = X_F^*(q_{2L})$ is shown below), and the first terms on the right hand side of (18) and (21) are well defined, finite, and continuous. Finally, (17) is the solution of the first order condition of (15) with respect to q_{2L}. Then, optimality follows according to Theorem 2.8 (Gozzi and Russo, 2006, p. 1534).

The investment trigger and the value of the parameter $A_F(q_{2L})$ can be found by applying the value matching and smooth pasting conditions[12],

$$\begin{aligned} \frac{X}{r-\alpha}(q_{1F} + q_{2F}^{opt}(X, q_{2L}))(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}^{opt}(X, q_{2L}))) - \delta q_{2F}^{opt}(X, q_{2L}) \\ = A_F X^\beta + \frac{X}{r-\alpha} q_{1F}(1 - \eta(q_{1L} + q_{1F} + q_{2L})), \end{aligned} \tag{22}$$

$$\begin{aligned} \frac{1}{r-\alpha}(q_{1F} + q_{2F}^{opt}(X, q_{2L}))(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}^{opt}(X, q_{2L}))) \\ = A_F \beta X^{\beta-1} + \frac{1}{r-\alpha} q_{1F}(1 - \eta(q_{1L} + q_{1F} + q_{2L})). \end{aligned} \tag{23}$$

Together they make

$$\begin{aligned} \frac{X}{r-\alpha}(q_{1F} + q_{2F}^{opt}(X, q_{2L}))(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F}^{opt}(X, q_{2L}))) \left(1 - \frac{1}{\beta}\right) \\ - \frac{X}{r-\alpha} q_{1F}(1 - \eta(q_{1L} + q_{1F} + q_{2L})) \left(1 - \frac{1}{\beta}\right) = \delta q_{2F}^{opt}(X, q_{2L}), \end{aligned}$$

which leads to

$$X = \frac{\beta}{\beta - 1} \frac{\delta(r - \alpha)}{1 - \eta(q_{1L} + 2q_{1F} + q_{2L} + q_{2F}^{opt}(X, q_{2L}))}.$$

Substituting $q_{2F}^{opt}(X, q_{2L})$ and rewriting leads to the unique solution

$$X_F^*(q_{2L}) = \frac{\beta + 1}{\beta - 1} \frac{\delta(r - \alpha)}{1 - \eta(q_{1L} + 2q_{1F} + q_{2L})}.$$

[12] Following the related real options literature we use value matching and smooth pasting conditions, which implicitly rely on regularity assumptions with respect to the value function, for characterizing the investors' inter-temporally optimal behavior and abstain from providing explicit verification theorems for the optimality of the determined strategies.

Notice that since $\frac{\beta+1}{\beta-1} > 1$ we have that $X_F^*(q_{2L}) > \frac{\delta(r-\alpha)}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}$. This means that for investments undertaken in the stopping region we have that $q_{2F}^{opt}(X, q_{2L}) > 0$ and that therefore (17) is in the control space. For $X < X_F^*(q_{2L})$ the firm waits to invest

$$\begin{aligned} q_{2F}^*(q_{2L}) &= q_{2F}(X_F^*(q_{2L}), q_{2L}) \\ &= \frac{1-\eta(q_{1L}+2q_{1F}+q_{2L})}{\eta(\beta+1)}. \end{aligned}$$

Since (17) is positive for values of X around $X_F^*(q_{2L})$ we have that (22)-(23) is sufficient, despite the fact that strictly speaking $\max\{0, q_{2F}^{opt}(X, q_{2L})\}$ should have been used instead of $q_{2F}^{opt}(X, q_{2L})$.

Moreover,

$$\begin{aligned} A_F \cdot (X_F^*(q_{2L}))^\beta &= \frac{X_F^*(q_{2L})}{r-\alpha} q_{2F}^*(q_{2L})(1-\eta(q_{1L}+2q_{1F}+q_{2L}+q_{2F}^*(q_{2L}))) - \delta q_{2F}^*(q_{2L}) \\ &= \frac{\delta q_{2F}^*(q_{2L})}{\beta-1}. \end{aligned}$$

This leads to

$$V_F(X, q_{2L}) = \frac{\delta}{\beta-1}\left(\frac{X}{X_F^*(q_{2L})}\right)^\beta q_{2F}^{opt}(X_F^*(q_{2L}), q_{2L}) + \frac{X}{r-\alpha} q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L}))$$

for the continuation region. Due to the assumption that $r > \alpha$ we have $\beta > 1$. The value function $V_F(X, q_{2L})$ consists of two terms. The second term represents the current profit stream. In case the incumbent is follower, this stream is positive with $q_{1F} = q_{1I}$. When the entrant is the follower one has $q_{1F} = 0$ leading to zero current profits. The first term is the current value of the option to invest.

To show that there is a single threshold with respect to the current value of X, separating the follower's continuation from the stopping region, we need to study

$$Xq_{1F}(1-\eta(q_{1F}+q_{1L}+q_{2L})) - rV_F(X, q_{2L}) + \alpha X \frac{\partial}{\partial X} V_F(X, q_{2L}) + \tfrac{1}{2}\sigma^2 X^2 \frac{\partial^2}{\partial X^2} V_F(X, q_{2L}). \quad (24)$$

This is the difference between the value of waiting and the termination payoff (see e.g. Dixit and Pindyck (1994, p. 130)). For X below

$$\frac{\delta(r-\alpha)}{1-\eta(q_{1L}+2q_{1F}+q_{2L})} \quad (25)$$

we have that (17)< 0 so that the optimal follower capacity is equal to $q_{2F}^{opt}(X, q_{2L}) = 0$. For these values of X we find that (24) is equal to zero. For X above (25), equation (24) can be rewritten as

$$\frac{\delta^2(r-\alpha)}{4\eta X}(\sigma^2 - r - \alpha) - \frac{X}{4\eta}(1-\eta(q_{1L}+2q_{1F}+q_{2L}))^2 + r\frac{\delta}{2\eta}(1-\eta(q_{1L}+2q_{1F}+q_{2L})).$$

Since the second term is always nonnegative it follows that this function is decreasing in X if $\sigma^2 > r+\alpha$. If $\sigma^2 < r+\alpha$ the following holds. For X between (25) and

$$\frac{\delta}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}\sqrt{(r-\alpha)(r+\alpha-\sigma^2)} \quad (26)$$

it holds that (24) is increasing and for X above (26), (24) is decreasing. Finally, for X equal to (25) we have that (24) is equal to

$$\frac{\delta}{4\eta}(1-\eta(q_{1L}+2q_{1F}+q_{2L}))\sigma^2>0.$$

At this point it is strictly profitable to marginally wait, so investing cannot be optimal. From this, as the second order condition is strictly negative for all X, we can conclude that in the region where $q_{2F}^{opt}(X,q_{2L})>0$, (24) has a unique root such that (24) is decreasing in that point. It now follows from Dixit and Pindyck (1994, p. 130) that therefore the follower waits for $X<X_F^*(q_{2L})$ and invests for $X\geq X_F^*(q_{2L})$. □

The following lemma shows that the price remains positive after the follower's investment.

Lemma 1 *If* $1-\eta(q_{1L}+q_{1F}+q_{2L})>0$, *then* $1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^{opt}(X,q_{2L}))>0$

Proof of Lemma 1

Assume $1-\eta(q_{1L}+q_{1F}+q_{2L})>0$. Rewriting gives

$$1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^{opt}(X,q_{2L}))=\tfrac{1}{2}(1-\eta(q_{1L}+q_{1F}+q_{2L}))+\tfrac{1}{2}\left(\eta q_{1F}+\frac{\delta(r-\alpha)}{X}\right)$$

Since both terms are positive we conclude that the price after the follower's investment remains positive. □

Proof of Proposition 2

In order to prove this proposition, we first have to derive the expressions for $V_L^{det}(X,q_{2L})$ and $V_L^{acc}(X,q_{2L})$ in case of instantaneous investment of the leader. Considering $V_L^{det}(X,q_{2L})$ we have

$$\begin{aligned}
V_L^{det}(X,q_{2L}) &= \mathbb{E}\Bigg[\int_{t=0}^{\tau_F^*}(q_{1L}+q_{2L})x(t)(1-\eta(q_{1L}+q_{1F}+q_{2L}))e^{-rt}\mathrm{d}t \\
&\qquad +\int_{t=\tau_F^*}^{\infty}(q_{1L}+q_{2L})x(t)(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^*(q_{2L})))e^{-rt}\mathrm{d}t\Big|x(0)=X\Bigg]-\delta q_{2L} \\
&= \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}) \\
&\qquad -\eta(q_{1L}+q_{2L})q_{2F}^*(q_{2L})\frac{X_F^*(q_{2L})}{r-\alpha}\left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta}-\delta q_{2L} \\
&= \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}))-\frac{\delta}{\beta-1}(q_{1L}+q_{2L})\left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta}-\delta q_{2L}
\end{aligned} \tag{27}$$

where $\tau_F^*=\inf\{t\geq 0 \mid x(t)\geq X_F^*(q_{2L})\}$ is a stopping time of the follower. Here, as shown in Dixit and Pindyck (1994) (Chapter 9, Section 3), we use that

$$\mathbb{E}\left[\int_{t=\tau_F^*}^{\infty}x(t)e^{-rt}\mathrm{d}t \;\Big|\; x(0)=X,\ x(\tau_F^*)=X_F^*(q_{2L})\right]=\left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta}\frac{X_F^*(q_{2L})}{r-\alpha}.$$

Since $\beta>1$ we have that X^{β} is (at least) C^2 for all $X>0$ and therefore we have that (27) is continuous in $X_F^*(q_{2L})$.

Expression (27) can be interpreted as the expected revenue stream in case the follower will never invest minus the adjustment of the cash flow stream from the moment the second firm makes an investment, followed by the investment cost. The second term includes the discount factor $\mathbb{E}\left[e^{-r\tau_F}\right] = \left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta}$, where again τ_F is the time of investment of the follower (see Dixit and Pindyck (1994), Chapter 9, for derivations).

Similarly, for $V_L^{acc}(X, q_{2L})$ for the case of instantaneous investment:

$$\begin{aligned} V_L^{acc}(X, q_{2L}) &= \mathbb{E}\left[\int_{t=0}^{\infty}(q_{1L}+q_{2L})x(t)(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^{opt}(X,q_{2L})))e^{-rt}\mathrm{d}t - \delta q_{2L}\middle| x(0)=X\right] \\ &= \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^{opt}(X,q_{2L}))) - \delta q_{2L} \\ &= \frac{X}{r-\alpha}\tfrac{1}{2}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{2L})) - \tfrac{1}{2}\delta(q_{2L}-q_{1L}). \end{aligned} \tag{28}$$

In the remainder of the proof we denote by $q_L^{det}(X)$ respectively $q_L^{acc}(X)$ the maximizers of (27) respectively (28) without taking into account the constraints on q_{2L}. For any X for which the corresponding strategy is feasible this value coincides with the definition of $q_L^{det}(X)$ and $q_L^{acc}(X)$ in the main text.

The capacity $q_L^{det}(X)$ can be found by solving the first order condition,

$$\frac{X}{r-\alpha}[1-\eta(2q_{1L}+q_{1F}+2q_{2L})] - \frac{\delta}{\beta-1}\left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta}\left[1-\frac{\eta\beta(q_{1L}+q_{2L})}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}\right] - \delta = 0. \tag{29}$$

From the second order condition

$$\frac{X}{r-\alpha}2\eta\left[\underbrace{\frac{\beta}{\beta+1}}_{<1}\underbrace{\left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta-1}}_{<1}\underbrace{\frac{2-\eta(4q_{1F}+(\beta+1)(q_{1L}+q_{2L}))}{2-\eta(4q_{1F}+2(q_{1L}+q_{2L}))}}_{<1}-1\right] < 0$$

we can conclude that $V_L^{det}(X, q_{2L})$ is concave with respect to q_{2L} in the region where q_{2L} is such that $X < X_F^*(q_{2L})$.

To find the value of $q_L^{acc}(X)$ we solve the first order condition $\frac{\partial V_L^{acc}(X,q_{2L})}{\partial q_{2L}} = 0$ which gives

$$q_L^{acc}(X) = \frac{1}{2\eta}\left[1-2\eta q_{1L} - \frac{\delta(r-\alpha)}{X}\right]. \tag{30}$$

It is easily checked that $q_L^{acc}(X) \geq 0$ if and only if $1-2\eta q_{1L} \geq \frac{\delta(r-\alpha)}{X}$, i.e., if and only if

$$X \geq \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}.$$

The second order condition again makes sure that we obtain a maximum, $-2\eta\frac{X}{r-\alpha} < 0$.

Equation (3) gives the relation between the leader's investment quantity q_{2L} and the follower's investment threshold $X_F^*(q_{2L})$. To that extent, there exists $\hat{q}_{2L}(X)$ such that for $q_{2L} > \hat{q}_{2L}(X)$ it holds that $X < X_F^*(q_{2L})$. From (3) one obtains

$$\hat{q}_{2L}(X) = \frac{1}{\eta}\left[1-\eta(q_{1L}+2q_{1F}) - \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)X}\right].$$

If the leader chooses an investment q_{2L} at a trigger X_L such that $q_{2L} > \hat{q}_{2L}(X_L)$, then the follower invests with a delay after the leader's investment and if the leader sets q_{2L} at a trigger X_L such that $q_{2L} \leq \hat{q}_{2L}(X_L)$

then the follower invests immediately. Note that since $\hat{q}_{2L}(X)$ is a strictly increasing function of X and $\hat{q}_{2L}(X) = 0$ for $X = \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)(1-\eta(q_{1L}+2q_{1F}))}$, it follows that the follower's investment is always delayed if the leader invests at $X < \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)(1-\eta(q_{1L}+2q_{1F}))}$.

The optimization problem given on the right hand side of (6) can then be solved as follows. Let $X \geq \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)(1-\eta(q_{1L}+2q_{1F}))}$. The objective function is a piecewise function of $V_L^{det}(X, q_{2L})$ for $q_{2L} > \hat{q}_{2L}(X)$ and $V_L^{acc}(X, q_{2L})$ for $q_{2L} \leq \hat{q}_{2L}(X)$. Since (27) and (28) are both concave functions of q_{2L} we can make the following observations:

- If $q_L^{det}(X) < \hat{q}_{2L}(X)$, then $q_{2L} = \hat{q}_{2L}(X)$ gives the supremum of the payoff function $V_L^{det}(X, q_{2L})$ in the region where $q_{2L} > \hat{q}_{2L}(X)$.
- If $q_L^{acc}(X) > \hat{q}_{2L}(X)$, then $q_{2L} = \hat{q}_{2L}(X)$ gives the supremum of the payoff function $V_L^{acc}(X, q_{2L})$ in the region where $q_{2L} \leq \hat{q}_{2L}(X)$.

We then have four cases. In the first case $q_L^{det}(X) < \hat{q}_{2L}(X)$ and $q_L^{acc}(X) \geq \hat{q}_{2L}(X)$, in which case the supremum is reached at $\hat{q}_{2L}(X)$. As we will show later this case does not arise in our set-up. If $q_L^{det}(X) > \hat{q}_{2L}(X)$ and $q_L^{acc}(X) \geq \hat{q}_{2L}(X)$, then the leader delays the follower and sets $q_L^{det}(X)$. The follower invests immediately after the leader if $q_L^{det}(X) < \hat{q}_{2L}(X)$ and $q_L^{acc}(X) \leq \hat{q}_{2L}(X)$, where the leader sets $q_L^{acc}(X)$. In the last case where $q_L^{det}(X) > \hat{q}_{2L}(X)$ and $q_L^{acc}(X) \leq \hat{q}_{2L}(X)$ the optimum is found by comparing the payoff functions at $q_L^{det}(X)$ and $q_L^{acc}(X)$.

To show that for $X > X_2$ delaying follower investment is not optimal we show that there exists X_2 such that $q_L^{det}(X) > \hat{q}_{2L}(X)$ if and only if $X < X_2$. The threshold X_2 is determined by the condition $q_L^{det}(X_2) = \hat{q}_{2L}(X_2)$. Total differentiating (29) with respect to X gives

$$\frac{\mathrm{d}q_L^{det}(X)}{\mathrm{d}X} = \frac{1 - \eta(2q_{1L} + q_{1F} + 2q_L^{det}) - \frac{\beta}{\beta+1}\left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta-1}(1 - 2\eta q_{1F} - (1+\beta)\eta(q_{1L} + q_L^{det}))}{X\left[2 - \frac{\beta}{\beta+1}\left(\frac{X}{X_F^*(q_{2L})}\right)^{\beta-1}\left(1 - \frac{1-\eta(2q_{1F}+\beta q_{1L}+\beta q_L^{det})}{1-\eta(2q_{1F}+q_{1L}+q_L^{det})}\right)\right]} > 0.$$

Since at X_2 we have $X_F^*(q_{2L}^{det}(X_2)) = X_2$ the first order condition (29) reduces to,

$$\begin{aligned} 0 &= \frac{\delta}{\beta-1}\frac{1 - 2\eta q_{1L} + \eta(\beta-1)q_{1F} - 2\eta q_{2L}}{1 - \eta(q_{1L} + 2q_{1F} + q_{2L})} \\ &\Leftrightarrow \\ q_{2L}^R &= \frac{1}{2\eta}(1 - 2\eta q_{1L} + (\beta-1)\eta q_{1F}). \end{aligned} \tag{31}$$

By plugging the latter expression into $X_F^*(q_{2L})$ one obtains X_2,

$$X_2 = X_F^*(q_{2L}^R) = \frac{\beta+1}{\beta-1}\frac{2\delta(r-\alpha)}{1-(\beta+3)\eta q_{1F}}.$$

Considering now the feasibility of the inducing immediate follower investment strategy, we show that there exists X_1 such that $q_L^{acc}(X) \geq 0$ and $q_L^{acc}(X) \leq \hat{q}_{2L}(X)$ if and only if $X \geq X_1$. A similar argument as above applies here that this is sufficient. Solving $q_L^{acc}(X) = \hat{q}_{2L}(X)$ leads to

$$q_L^{acc}(X) = \frac{1}{2\eta}\left[1 - 2\eta q_{1L} - \frac{\delta(r-\alpha)}{X}\right] = \frac{1}{\eta}\left[1 - 2\eta q_{1F} - \eta q_{1L} - \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)X}\right] = \hat{q}_{2L}(X)$$

$$\Leftrightarrow$$

$$1 - 4\eta q_{1F} = \frac{\delta(r-\alpha)(\beta+3)}{(\beta-1)X} \Leftrightarrow X = \frac{\beta+3}{\beta-1}\frac{\delta(r-\alpha)}{1-4\eta q_{1F}}.$$

Hence,

$$X_1 = \max\left\{\frac{\beta+3}{\beta-1}\frac{\delta(r-\alpha)}{1-4\eta q_{1F}}, \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}\right\}.$$

Assume $\frac{\beta+3}{\beta-1}\frac{\delta(r-\alpha)}{1-4\eta q_{1F}} \geq \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}$. Then, we observe that

$$\frac{X_1}{X_2} = \underbrace{\frac{\beta+3}{2\beta+2}}_{<1}\underbrace{\frac{1-(3+\beta)\eta q_{1F}}{1-4\eta q_{1F}}}_{<1} < 1.$$

Assume $\frac{\beta+3}{\beta-1}\frac{\delta(r-\alpha)}{1-4\eta q_{1F}} < \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}$. Then,

$$\frac{X_1}{X_2} = \frac{1-(\beta+3)\eta q_{1F}}{1-2\eta q_{1L}}\underbrace{\frac{\beta-1}{2\beta+2}}_{<1}$$

In case $q_{1L} = 0$ it follows straight that the first fraction is smaller than 1. For the case where $q_{1F} = 0$, rewriting $\frac{X_1}{X_2} < 1$ gives

$$q_{1L} < \frac{1}{\eta}\frac{\beta+3}{4\beta+4}.$$

Since the right hand side is larger than q_{1I}^{myop} we can conclude that this inequality holds and therefore $X_1 < X_2$.

Furthermore, for $X = X_1$ we have by definition $q_L^{acc}(X) = \hat{q}_{2L}(X)$ (ignoring the trivial case where $q_L^{acc}(X) = 0$) and, due to $X_1 < X_2$, $q_L^{det}(X_1) > \hat{q}_{2L}(X_1)$. Since $q_L^{det}(X)$ is the maximizer of $V_L^{det}(X, q_{2L})$, where this function denotes the expression in (27) for a general investment size q_{2L}, this yields

$$V_L^{det}(X_1, q_L^{det}(X_1)) > V_L^{det}(X_1, q_L^{acc}(X_1)) = V_L^{acc}(X_1, q_L^{acc}(X_1)),$$

where the last equality follows from the observation that at $q_{2L} = q_L^{acc}(X_1) = \hat{q}_{2L}(X_1)$ the payoff functions for both investment strategies coincide. Similarly, we obtain for $X = X_2$

$$V_L^{acc}(X_2, q_L^{acc}(X_2)) > V_L^{acc}(X_2, q_L^{det}(X_2)) = V_L^{det}(X_2, q_L^{det}(X_2)),$$

because it holds that $q_L^{det}(X_2) = \hat{q}_{2L}(X_2)$. Since the delaying follower investment strategy is feasible for $X \in [0, X_2]$ and the inducing immediate follower investment strategy for $X \in [X_1, \infty)$ we conclude that for $X \leq X_1$ the leader optimally delays the follower's investment and for $X \geq X_2$ the follower is induced to invest immediately.

A verification theorem shows (27), (28), (29), and (30) are indeed solutions and optimal controls for the problem in the stopping region posed in the main text. This verification theorem is equivalent to the one described in the proof op Proposition 1 where we follow Gozzi and Russo (2006). The HJB in the stopping region is given by

$$r\phi_L(X,q_{2L}) = X(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^{opt}(X,q_{2L}))) \\ +\alpha X\frac{\partial}{\partial X}\phi_L(X,q_{2L})+\tfrac{1}{2}\sigma^2X^2\frac{\partial^2}{\partial X^2}\phi_L(X,q_{2L}),$$

with $\phi_L(0)=0$ (see Dixit and Pindyck (1994, p. 181-182) for the restrictions on $\phi_L(X)$ as $X\to\infty$) for the case where the follower invests immediately after the leader and

$$r\phi_L(X,q_{2L}) = X(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}))+\alpha X\frac{\partial}{\partial X}\phi_L(X,q_{2L})+\tfrac{1}{2}\sigma^2X^2\frac{\partial^2}{\partial X^2}\phi_L(X,q_{2L}),$$

with $\phi_L(0)=0$ and

$$\phi_L(X,q_{2L}) = \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^*(q_{2L}) \text{ as } X\uparrow X_F^*(q_{2L})$$

when the follower is delayed. As the payoff functions and investment sizes are continuous in X, we have that the value function is continuous as well. Since the other verification conditions for problems of the follower and leader are very similar on a technical level, we refrain from further elaboration. The analysis on the continuation region is done in the proof of Proposition 3.

□

Technically speaking, the values of X_2 and X_1 are not strictly positive for all parameter settings. Since $X_2>0$ if $X_1>0$ let us look at the case where $X_1<0$. If the incumbent is the leader (as we will see later, this is indeed the equilibrium outcome), it follows from the arguments in the proof of Proposition 2 that $X_1>0$. When the incumbent is the follower, cases arise where $X_1<0$. This simply means that for all values of X the optimal investment size of the leader determined under the assumption that the follower invests immediately indeed induces immediate investment by the follower. A similar analysis can be done for $X_2<0$. Nevertheless, since, as we will see later, the incumbent will take the leader role, we refrain from further analyzing these cases.

Proof of Proposition 3 First, we establish the existence of the threshold X_0. In the case X is so small that $q_L^{det}(X)=0$, no investment is made at this point. We define a corresponding lower bound for X, under which the optimal investment, under the assumption that the follower is delayed, is zero, by

$$X_0=\inf\{X \mid q_L^{det}(X)>0\}.$$

The conditions determining X_0 follows from the first order condition (29) by setting $q_{2L}=0$. To show that there exists a unique point X_0, it is sufficient to do the following. Define $\psi(X)=\frac{\partial V_L^{det}(X,q_{2L})}{\partial q_{2L}}|_{q_{2L}=0}$, this function dictates the first order conditions for the value of X yielding zero capacity,

$$\psi(X) = \frac{X}{r-\alpha}[1-\eta(2q_{1L}+q_{1F})] \\ - \frac{\delta}{\beta-1}\left(\frac{\beta-1}{\beta+1}\frac{1-\eta q_{1L}-2\eta q_{1F}}{\delta(r-\alpha)}X\right)^{\beta}\left[1-\frac{\eta\beta q_{1L}}{1-\eta q_{1L}-2\eta q_{1F}}\right]-\delta.$$

Then,

$$\begin{aligned}
\psi(0) =& -\delta < 0,\\
\psi(X_F^*) =& \frac{\delta}{\beta-1}\frac{1-2\eta q_{1L}+(\beta-1)\eta q_{1F}}{1-\eta q_{1L}-2\eta q_{1F}},\\
\psi'(X) =& \frac{1-\eta(2q_{1L}+q_{1F})}{r-\alpha}\left[1-\frac{\beta}{\beta+1}\left(\frac{X}{X_F^*(q_{2L})}\Big|_{q_{2L}=0}\right)^{\beta-1}\right]\\
&+\frac{\beta}{\beta+1}\left(\frac{X}{X_F^*(q_{2L})}\Big|_{q_{2L}=0}\right)^{\beta-1}\frac{\eta q_{1F}+(\beta-1)\eta q_{1L}}{r-\alpha}.
\end{aligned}$$

From (31) it follows that $\psi(X_F^*) > 0$. Since $\psi'(X) > 0$ one can conclude that, according to the Mean Value theorem, there exists a unique $X_0 \in (0, X_F^*)$ such that $q_L^{det}(X_0) = 0$.

In order to obtain the thresholds X_L^{det} and X_L^{acc} we compare the value for the leader when investing immediately, which was calculated in the proof or Proposition 2 with the value obtained by the leader if it delays investment to a later point in time. Assuming the leader uses the delaying follower investment strategy in the stopping region, the leader's payoff function is given by (27). If X is in the continuation region, in which the leader does not invest immediately, the leader's value is obtained (following the same arguments as used in the determination of the follower's value function in its continuation regions) by setting

$$F_L^{det}(X) = A_L^{det}X^{\beta} + \frac{X}{r-\alpha}q_{1L}(1-\eta(q_{1L}+q_{1F})).$$

Applying the value matching and smooth pasting conditions shows that waiting increases the leader's payoff up to a value of X_L given in (10) and

$$A_L^{det} = (X_L^{det})^{-\beta}\frac{\delta q_L^{det}(X_L^{det})}{\beta-1} - \frac{\delta}{\beta-1}(q_{1L}+q_L^{det}(X_L^{det}))\,(X_F^*(\,q_L^{det}(X_L^{det})))^{-\beta}. \tag{32}$$

In order to show that for sufficiently small q_{1L} there exists a pair (X_L^{det}, q_L^{det*}) satisfying (10) and the first order condition for the leader, we insert (10) into (29). We treat the following two cases separately. First we look at the scenario where the incumbent is the investment leader. Then, $q_{1F} = 0$ and one obtains the equivalent condition

$$\beta\left(1-\frac{1-\eta(2q_{1L}+2q_{2L})}{1-\eta(2q_{1L}+q_{2L})}\right) = 1-\frac{1-\eta(\beta+1)(q_{1L}+q_{2L})}{1-\eta(q_{1L}+q_{2L})}\left(\frac{\beta}{\beta+1}\right)^{\beta}. \tag{33}$$

After rewriting this equation, one could similarly say that it is required that $H(q_{2L}) = 0$, where,

$$H(q_{2L}) = 1-\eta(q_{1L}+q_{2L})+\beta\left[\frac{1-\eta(2q_{1L}+2q_{2L})}{1-\eta(2q_{1L}+q_{2L})}-1\right](1-\eta(q_{1L}+q_{2L})) \\ -(1-\eta(\beta+1)(q_{1L}+q_{2L}))\left(\frac{\beta}{\beta+1}\frac{1-\eta(q_{1L}+q_{2L})}{1-\eta(2q_{1L}+q_{2L})}\right)^{\beta}.$$

Then,

$$H(0) = (1-\eta q_{1L})\left[1 - \underbrace{\left(\frac{\beta}{\beta+1}\frac{1-\eta q_{1L}}{1-2\eta q_1}\right)^\beta}_{<1 \text{ if } q_{1L}<\frac{1}{\eta(\beta+2)}} \underbrace{\frac{1-\eta(\beta+1)q_{1L}}{1-\eta q_{1L}}}_{<1}\right] > 0,$$

$$H\left(\frac{1-2\eta q_{1L}}{2\eta}\right) = -\frac{1}{2}(\beta-1)\left[1-\left(\frac{\beta}{\beta+1}\frac{1}{1-2\eta q_{1L}}\right)^\beta\right] < 0, \text{ and}$$

$$\frac{\mathrm{d}H(q_{2L})}{\mathrm{d}q_{2L}} = \underbrace{-\eta + \eta(\beta+1)\left(\frac{\beta}{\beta+1}\frac{1-\eta(q_{1L}+q_{2L})}{1-\eta(2q_{1L}+q_{2L})}\right)^\beta}_{(*)} - \frac{\beta\eta q_{2L}}{1-\eta(2q_{1L}+q_{2L})}\underbrace{\left[\frac{1-\eta(q_{1L}+q_{2L})}{1-\eta(2q_{1L}+q_{2L})}-1\right]}_{>0}$$

$$-\beta\left(\frac{\beta}{\beta+1}\frac{1-\eta(q_{1L}+q_{2L})}{1-\eta(2q_{1L}+q_{2L})}\right)^\beta \frac{\eta^2 q_{1L}(1-\eta(\beta+1)(q_{1L}+q_{2L}))}{1-\eta(2q_{1L}+q_{2L})} - \eta\beta\frac{1-\eta(q_{1L}+q_{2L})}{1-\eta(2q_{1L}+q_{2L})} < 0.$$

We observe that $H(0)$ is positive for at least $q_{1L} < \frac{1}{\eta(\beta+2)}$. The first order condition should show that $H(q_{2L})$ as a unique root. For the first two terms, labeled by $(*)$, one can show that these together are negative for small values q_{2L} and positive for high value of q_{2L} when q_{1L} is below $\frac{1}{\eta(\beta+2)}$. Moreover, $(*)$ is a strictly increasing function with respect to q_{2L}. All the other terms in the first order condition are negative. It can be concluded that, according to the Mean Value Theorem, there exists a q_{2L} on the interval $\left(0, \frac{1-2\eta q_{1L}}{2\eta}\right)$, such that $H(q_{2L}) = 0$ for sufficiently small value of q_{1L}. This value is denoted by q_L^{det*}. Earlier we showed that q_L^{det} is an increasing function. Then, since $\frac{1}{2\eta}(1-2\eta q_{1L}) = q_{2L}^R$, it follows that $X_L^{det} > X_0$ and $X_L^{det} < X_2$. Note that this also implies $X_0 < X_2$.

In a similar way one can prove this for the scenario where the entrant is the leader. Here, one shows that for $q_{2L} = 0$ the function H takes a positive value, while for $q_{2L} = \frac{1}{2\eta} < q_{2L}^R$ the function becomes negative. In this scenario there is a unique root for all values of $q_{1F} > 0$.

The leader's payoff function under the inducing immediate follower investment strategy is determined in the same way as before. Upon immediate investment the leader's optimal payoff is given by (28), where the leader's optimal investment is given by (30).

With respect to the value before investment, assuming again a function of the form

$$F_L^{acc}(X) = A_L^{acc}X^\beta + \frac{X}{r-\alpha}q_{1L}(1-\eta(q_{1L}+q_{1F}))$$

one can apply the value matching and smooth pasting conditions while applying the envelope theorem. Using $q_{1L}\cdot q_{1F} = 0$, to simplify the term for $X_L^{acc}(q_{2L})$ resulting from these two conditions one ends up with (11). Moreover,

$$\begin{aligned} A_L^{acc}(X_L^{acc})^\beta &= \frac{X_L^{det}}{r-\alpha}[(q_{1L}+q_L^{acc})(1-\eta(q_{1L}+q_{1F}+q_L^{acc}+q_{2F}^*)) - q_{1L}(1-\eta(q_{1L}+q_{1F}))] - \delta q_L^{acc} \\ &= \frac{\delta\beta}{\beta-1}q_L^{acc} - \delta q_L^{acc} = \frac{\delta q_L^{acc}}{\beta-1}. \end{aligned}$$

To show existence of (q_L^{acc*}, X_L^{acc}) for sufficiently small q_{1L} we insert $q_{1L} = 0$ and (11) into the equation $q_L^{acc*} = q_L^{acc}(X_L^{acc})$. Solving for q_{2L} gives $q_L^{acc*} = \frac{1}{3\beta-1} > 0$. Therefore, by continuity, we have $X_L^{acc} > \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}$ for sufficiently small q_{1L}.

To verify optimality we need to check that the value function solves the HJB. Optimality of the control variable, which is set at $q_{2f} = 0, f \in \{I, E\}$ in the continuation region, follows from the same arguments as before. The HJB in the continuation region is given by

$$r\phi_L(X, q_{2L}) = Xq_{1L}(1 - \eta(q_{1L} + q_{1F})) + \alpha X \frac{\partial}{\partial X}\phi_L(X, q_{2L}) + \tfrac{1}{2}\sigma^2 X^2 \frac{\partial^2}{\partial X^2}\phi_L(X, q_{2L}).$$

where the first term on the right hand side is well defined, finite and continuous. This extends and completes the verification of the leader's optimization problem. □

For the proof of Proposition 4, we follow Pawlina and Kort (2006) who show that there are four types of equilibria in these types of games:

- preemption equilibria,
- sequential investment,
- simultaneous investment, and
- joint investment.

Preemptive equilibria are explained in the text. Sequential equilibria would imply that one firm is investing strictly before the preemption point of the competitor, which occurs when the investment trigger X_L^{det} of the firm with the smallest preemption point lies between the two firms' preemption points. In this type of equilibrium the entrant has no influence on the timing of the first investor. The remaining two types of equilibria involve tacit collusion among the firms. When firms decide to collude, they wait for the market to expand, that is, wait for a larger value of X, before investment is undertaken together at the same time. One can discriminate two types of collusion, distinguished by the order in which firms determine their capacity size. In the first type, one firm is Stackelberg capacity leader and decides upon the amount first where subsequently the second firm makes an immediate investment. The second investor sets its capacity after the first firm decided upon its investment scale. This type is called simultaneous investment. The second type, referred to as joint investment, is the category where there is no colluded investment order. Firms simultaneously decide upon capacities, leading to a Cournot type of equilibrium.

Proof of Proposition 4

Existence of the preemption equilibrium follows from the arguments given in the text. The following three lemmas rule out the existence of sequential investment, simultaneous investment, and joint investment equilibria. Hence, for this setting only the preemptive type of equilibrium exists.

Notice that since $X_L^{acc} < \hat{X}$ we have that $V_L^{det}(X) > V_L^{acc}(X)$ at X_L^{acc} and therefore also at X_L^{det}.

Lemma 2 *Assume that $q_{1I} = q_{1I}^{myop}$. Then for the incumbent the leader's investment threshold $X_L^{det}(q_{1I}^{myop}, 0)$ does not exist. Hence, it is optimal for the incumbent to delay investment as much as possible and to invest just before the entrant's preemption point X_{PE}.*

Proof of Lemma 2

Recall that before investment, the option value is given by

$$F_L^{det}(X) = \frac{X}{r-\alpha} q_{1L}(1 - \eta(q_{1L} + q_{1F})) + A_L^{det} X^{\beta}.$$

Then A_L^{det} reflects the net gain from investment and can be found by solving the value matching condition,

$$\lim_{X \downarrow X_L^{det}} V_L^{det}(X) = \lim_{X \uparrow X_L^{det}} V_L^{det}(X).$$

Let X_L^{det} and $X_F^*(q_{2L})$ be defined as in equations (10) and (3). Let $q_{1L} = q_{1I}^{myop} = \frac{1}{\eta(\beta+1)}$ and $q_{1F} = 0$. Then we get, from (32),

$$\begin{aligned} A_L^{det} &= (X_L^{det})^{-\beta} \frac{\delta q_{2I}^{det}}{\beta - 1} - \frac{\delta}{\beta - 1}(q_{1I}^{myop} + q_{2I}^{det})(X_F^*)^{-\beta} \\ &= \frac{\delta}{\beta - 1}\left[q_{2I}^{det}\left[\left(\frac{1}{X_L^{det}}\right)^{\beta} - \left(\frac{1}{X_F^*(q_{2L})}\right)^{\beta}\right] - q_{1I}^{myop}\left(\frac{1}{X_F^*(q_{2L})}\right)^{\beta}\right]. \end{aligned}$$

Next,

$$\frac{X_L^{det}}{X_F^*(q_{2L})} = \frac{\beta - 1 + \frac{1}{\beta+1} - \beta\eta q_{2I}^{det}}{\beta - 1 - (\beta+1)\eta q_{2I}^{det}} > 1,$$

so that $X_L^{det} > X_F^*(q_{2L})$, and therefore

$$\left(\frac{1}{X_L^{det}}\right)^{\beta} - \left(\frac{1}{X_F^*(q_{2L})}\right)^{\beta} < 0.$$

If follows that $A_L^{det} < 0$. This means that investment decreases the incumbent's payoff and the incumbent would never choose this strategy as a leader, if investment roles were exogenously determined. Hence, X_L^{det} does not exist and under endogenous investment roles it is optimal for the incumbent to delay investment as long as possible without jeopardizing the role as leader. $\square$

Lemma 3 *Simultaneous investment does not yield an equilibrium.*

Proof of Lemma 3

For the resulting payoff functions, the curves in Figure 2a should be considered. Here, the Stackelberg leader utilizes the inducing immediate follower investment strategy, denoted by *acc.* As a result, the competitor receives the follower value, being smaller than the leader value. For this reason neither of the firms would prefer to be a follower in the outcome and they would, consequently, preempt each other in taking the leader role. This forces the firms to end up in the region where the leader delays the follower's investment and the sole resulting equilibrium is the preemptive equilibrium where the follower prefers to wait rather than invest at the same time. Hence, simultaneous investment is not an equilibrium. $\square$

Lemma 4 *Joint investment does not yield an equilibrium.*

Proof of Lemma 4

Let $J(X, q_{2L}, q_{2F})$ be the firm value for joint investment, then,

$$J(X, q_{2L}, q_{2F}) = \frac{X}{r-\alpha}(q_{1L} + q_{2L})(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F})) - \delta q_{2L}.$$

Optimal capacities equal

$$\begin{aligned} q_{2L}^{join} &= \frac{1}{3\eta}\left(1 - \frac{\delta(r-\alpha)}{X}\right) - q_{1L}, \\ q_{2F}^{join} &= \frac{1}{3\eta}\left(1 - \frac{\delta(r-\alpha)}{X}\right) - q_{1F}. \end{aligned}$$

This leads to

$$\begin{aligned} V_L^{acc}(X, q_L^{acc}) &= \frac{X}{r-\alpha}\frac{1}{8\eta}\left(1 - \frac{\delta(r-\alpha)}{X}\right)^2 + \delta q_{1L}, \\ J(X, q_{2L}^{join}, q_{2F}^{join}) &= \frac{X}{r-\alpha}\frac{1}{9\eta}\left(1 - \frac{\delta(r-\alpha)}{X}\right)^2 + \delta q_{1L}. \end{aligned}$$

Hence, it holds that $V_L^{acc}(X, q_L^{acc}) > J(X, q_{2L}^{join}, q_{2F}^{join})$ for all $X > \delta(r-\alpha)$. This is sufficient to show that joint investment does not yield an equilibrium. □

Intuition behind this result is that when a firm is the leader, it can set a larger capacity which leads to a higher payoff.

This concludes the proof of the proposition.

A.2 Overinvestment

The value function of the monopolist, in the stopping region, is given by

$$\begin{aligned} V_M(X) &= \sup_{q_2 \geq 0}\left\{\mathbb{E}\left[\int_0^\infty (q_1 + q_2)X(t)(1 - \eta(q_1 + q_2))e^{-rt}\mathrm{d}t \;\middle|\; x(0) = X\right] - \delta q_2\right\} \\ &= \sup_{q_2 \geq 0}\left\{\frac{X}{r-\alpha}(q_1 + q_2)(1 - \eta(q_1 + q_2)) - \delta q_2\right\}, \end{aligned}$$

in which q_1 is the initial capacity and q_2 corresponds to the capacity acquired by investment. Maximizing the monopolist's payoff function leads to the optimal capacity expansion size,

$$q_2^{mon}(X) = \max\left\{0, \frac{1}{2\eta}\left(1 - 2\eta q_1 - \frac{\delta(r-\alpha)}{X}\right)\right\}.$$

Hence, one obtains,

$$V_M(X) = \begin{cases} \frac{X}{r-\alpha}q_1(1-\eta q_1) + \left(\frac{X}{X_M^*}\right)^\beta \frac{\delta}{\beta-1}q_2^{mon*} & \text{if } X < X_M^*, \\ \frac{(X(1-2\eta q_1) - \delta(r-\alpha))^2}{4\eta(r-\alpha)X} + \frac{Xq_1(1-\eta q_1)}{r-\alpha} & \text{if } X \geq X_M^*, \end{cases}$$

where β is defined as in (1). The optimal moment of expansion is defined as the value of x for which the option to wait no longer yields a larger value than immediate investment. The value matching and smooth pasting conditions give the threshold and capacity size

$$X_M^* = \frac{\beta+1}{\beta-1}\frac{\delta(r-\alpha)}{1-2\eta q_1},$$
$$q_2^{mon*} = \frac{1-2\eta q_1}{\eta(\beta+1)}.$$

Appendix B: Markov Perfect Equilibrium Strategies

In order to provide the profile of equilibrium strategies corresponding to the outcomes discussed in the treatment of endogenous firm roles in Section 3.2 we consider a scenario, in which, in accordance with the presented numerical evidence, we have $X_{PI} < X_{PE}$ and $X_L^{det}(q_{1I}, 0) > X_{PE}$. This means that the preemption point of the incumbent is below that of the entrant and the incumbent has no incentive to invest below the opponent's preemption point. Furthermore, we denote by $\hat{X}_I$ and $\hat{X}_E$ the thresholds separating the regions where delaying the follower investment and inducing immediate follower investment are optimal from the perspective of the incumbent and the entrant. The two inequalities above imply that $X_{PE} < \min\{\hat{X}_I, \hat{X}_E\}$. Consider now for a given initial capacity of the incumbent, q_{1I}, the following profile of Markovian strategies

$$\tilde{q}_{2I}(X, Q_I, Q_E, m) = \begin{cases} q_{2F}^{opt}(X, Q_E, 0; q_{1I}) & m = m_I \wedge X \geq X_F^*(Q_E; 0, q_{1I}), \\ q_L^{det}(X, q_{1I}, 0) & m = m_0 \wedge X_{PI} \leq X < \hat{X}_I, \\ q_L^{acc}(X, q_{1I}, 0) & m = m_0 \wedge \hat{X}_I \leq X, \\ 0 & else. \end{cases}$$

In order to be able to distinguish between the cases in which the incumbent and the entrant are the follower, we write here the optimal quantity of the follower, given in (2), as $q_{2F}^{opt}(X, q_{2L}; q_{1L}, q_{1F})$, explicitly listing the initial quantities of leader and follower as arguments. Similarly we write the follower's investment threshold, given in (3), as $X_F^*(q_{2L}; q_{1L}, q_{1F})$. Note that the first line corresponds to the mode where the incumbent is the follower, whereas the second and third line determine behavior in mode m_0 where both firms have not invested yet. By definition, the incumbent is the leader in these scenarios if it invests. Similarly, the entrant is the leader if it invests in this mode and we have

$$\tilde{q}_{2E}(X, Q_I, Q_E, m) = \begin{cases} q_{2F}^{opt}(X, Q_I - q_{1I}; q_{1I}, 0) & m = m_E \wedge X \geq X_F^*(Q_I - q_{1I}; q_{1I}, 0), \\ q_L^{det}(X, 0, q_{1I}) & m = m_0 \wedge X_{PE} \leq X < \hat{X}_E, \\ q_L^{acc}(X, 0, q_{1I}) & m = m_0 \wedge \hat{X}_E \leq X, \\ 0 & \text{else}. \end{cases}$$

The planned stopping times of the firms are given by

$$\tilde{\tau}_f(Q_I, Q_E, m) = \begin{cases} \inf\{t \geq 0 | x(t) \geq X_F^*(Q_g - q_{1g}, q_{1g}, q_{1f})\} & m = m_f, g \neq f \\ \inf\{t \geq 0 | x(t) \geq X_{PE} & m = m_0. \end{cases}$$

To complete the profile we also have

$$\kappa_f(X, Q_I, Q_E) = \begin{cases} 0 & X \leq X_{Pf} \\ 1 & X > X_{Pf} \end{cases}, \quad f \in \{I, E\}.$$

To see that this profile is indeed an equilibrium profile, first note that it follows directly from our analysis of the follower's problem that both firms act optimally in all subgames where the opponent has already invested. Considering the subgames where the opponent has not invested yet and $X > X_{PE}$, both firms face an opponent which instantaneously invests. Hence, both are the follower for sure if they do not invest, whereas with probability 0.5 they are the leader if they choose a positive investment. Since the leader's payoff function is above the follower's payoff function for both firms, it is optimal for each firm to try to invest instantaneously and to choose the investment according to the optimal leader value. Hence, both choose a positive $\tilde{q}_{2F}$ and $\kappa_f = 1$ in all these subgames. For the subgame where $X = X_{PE}$ the entrant is indifferent between being the leader and the follower. Hence, it is optimal for this firm to choose $\tilde{q}_{2E}$ according to the optimal value as a leader and at the same time $\kappa_E = 0$. Given the strategy of the incumbent this implies that the entrant does not invest at this point and receives the follower value, which is the optimal value it could gain in this subgame. On the other hand, any deviation of the incumbent from $\kappa_I = 1$ would induce a positive probability to become the follower, and therefore reduce the value of the incumbent (since the incumbents leader payoff function is above the follower payoff function at $X = X_{PE}$). Hence, the optimal strategy of the incumbent must have a positive value $\tilde{q}_{2I}$ and $\kappa_I = 1$ in this subgame. The actual value of $\tilde{q}_{2I}$ then follows directly from the analysis of the leader's problem in Section 3.1. Finally, for all subgames with $X < X_{PE}$ neither of the firms can gain by instantaneous investment and hence the given strategies are optimal. Overall, these arguments show that the strategy profile given above indeed corresponds to a Markov Perfect equilibrium of the game and yields the behavior and outcome discussed in Section 3.2.

Finally, we like to point out that for $x(0) < X_{PE}$, which is the only initial condition relevant for our economic analysis, it is not only ruled out that any subgame with $X > X_{PE}$ and no prior investment is ever reached, but also no firm can make it possible that such a subgame is reached by a unilateral deviation of its strategy. Hence, in principle the fact that the discussed behavior for such initial conditions corresponds to a Markov Perfect equilibrium could be established without determining equilibrium behavior and payoff functions for the subgames with $X > X_{PE}$ and no prior investment. In particular, this highlights that the exact values of $\hat{X}_E$ or $\hat{X}_I$ are irrelevant for our economic analysis given that they are above X_{PE}.

Appendix C: Robustness

Robustness of the preemption equilibrium

In Figures 8 and 9 we show the differences in preemption points ($X_{PE} - X_{PI}$) for variations of all model parameters in a relevant range. This is done for both $q_{1I} = q_{1I}^{myop}$ and $q_{1I} = 0.5$. Similarly, in Figure 10, the difference between the leader's investment trigger under the delaying follower investment strategy under the entrant's preemption point ($X_L^{det} - X_{PE}$) is shown for the same parameter variations and $q_{1I} = 0.5$. For $q_{1I} = q_{1I}^{myop}$ the investment trigger X_L^{det} does not exist and hence the incumbent does not have an incentive to invest before the entrant's preemption point is reached (see Lemma 2 in Appendix A). These figures confirm the claim that, apart from the hardly relevant case where η is extremely small (discussed in Section 3.4), under all parameter variations the preemption equilibrium with the incumbent as leader exists.

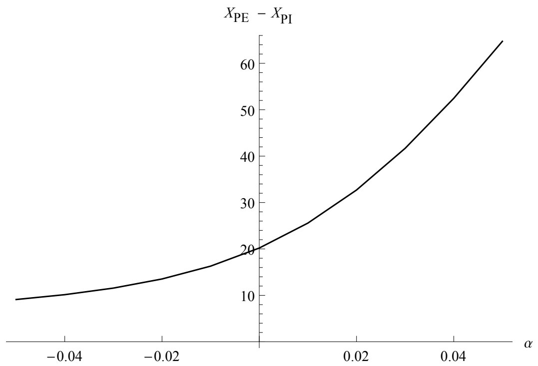
Sensitivity Analysis

The aim of this section is to briefly study the effect of the model parameters on the equilibrium. In this model there are six parameters to be taken a closer look at. First of all, the sensitivity parameter η capturing the negative relation between prices and output. The second parameter is the discount rate r. Then, the drift parameter α and the volatility parameter σ reflecting the market's uncertainty, both present in the geometric Brownian motion describing the state variable's path. Subsequently, we have the marginal investment cost δ.

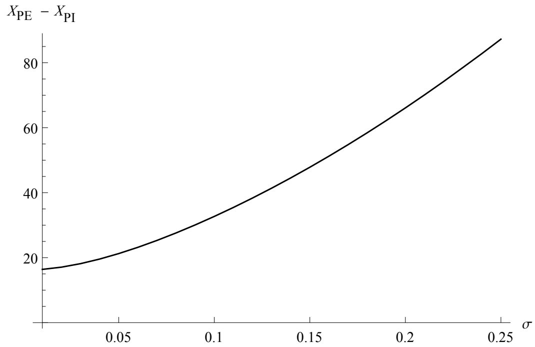
	η	r	α	δ	σ
X_{PE} ($q_{1I} = q_{1I}^{myop}$)	0	+	-/+	+	+
q_{2I}^{det} ($q_{1I} = q_{1I}^{myop}$)	-	+/-	+/-	0	+/-
X_{PE} (q_{1I} fixed)	+	+	-	+	+
q_{2I}^{det} (q_{1I} fixed)	-	-	+	0	+

Table 1: Effect of an increase in parameter values on triggers and capacities.

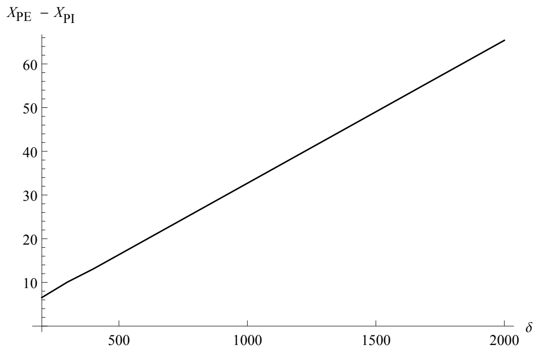
When η increases the output q_{2I} decreases exactly canceling out the increase in η, i.e. the product $\eta \cdot q_{2I}$ remains constant. Similarly $\eta \cdot q_{1I}^{myop}$ and $\eta \cdot q_E$ remain constant. In this way, when assuming $q_{1I} = q_{1I}^{myop}$, neither the investment threshold X_L^{det}, nor the preemption trigger are affected by an increase in η. However, when one assumes q_{1I} to be fixed, triggers are affected. An increase in η means an increase in ηq_{1I} and therefore a decrease in the price, which, hence, makes firms delay investment. Nevertheless, the total effect on the investment size is negative, considering the different effects. When discounting is done under a higher rate, firms value future revenues relatively less, become more concerned about current profits, and therefore delay investment. In the first place, this increases the myopic capacity size on the initial market. In the second place, since there are two effects that influence the optimal investment size for the expansion - delaying increases the capacity level, but a larger old market decreases it - it is found that the change is ambiguous. For small r the installment increases, but for relatively large r it decreases. When one fixes the initial capacity, the effect of the old market dominantly influences the capacity leading to decreasing installments. As standard in literature, the drift parameter has an opposite effect: a larger α makes firms invest earlier. The main line of reasoning is the same, when the drift parameter increases. Market demand,



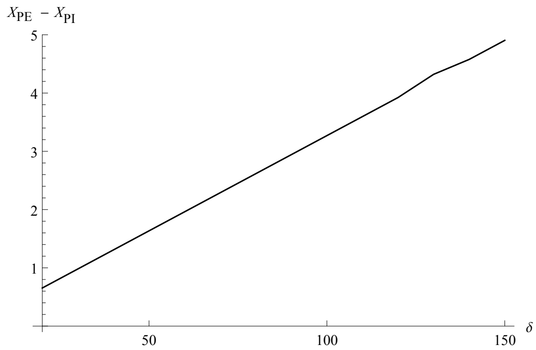
(a) Variation of α.



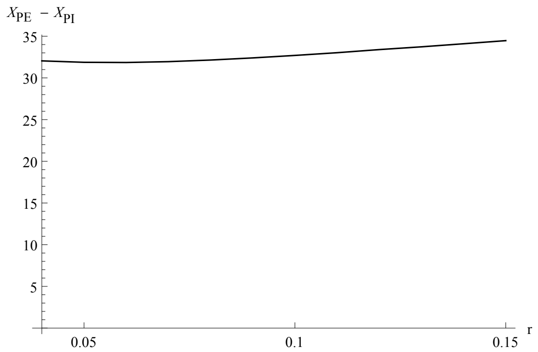
(b) Variation of σ.



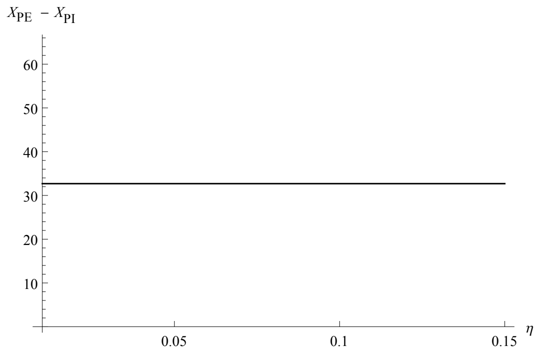
(c) Variation of δ.



(d) Small values of δ.

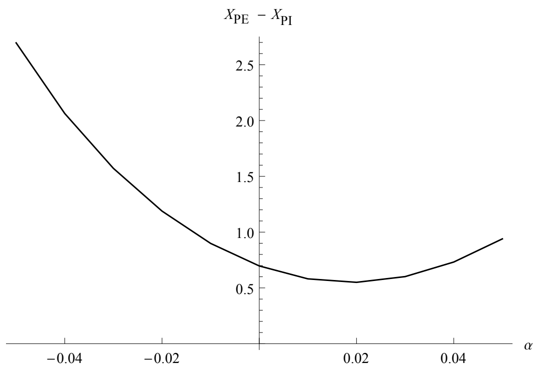


(e) Variation of r.

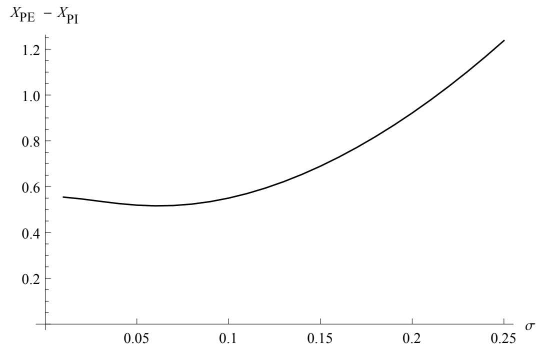


(f) Variation of η.

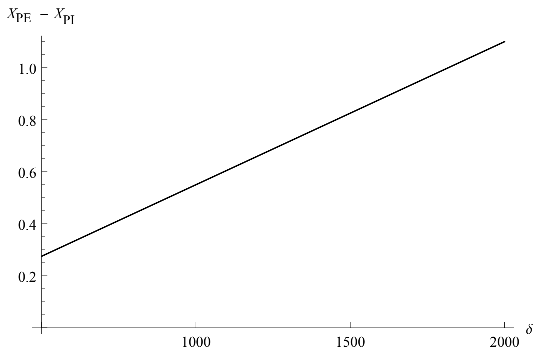
Figure 8: Difference between preemption points with $q_{1I} = q_{1I}^{myop}$. Default parameters: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1$, and $\delta = 1000$.



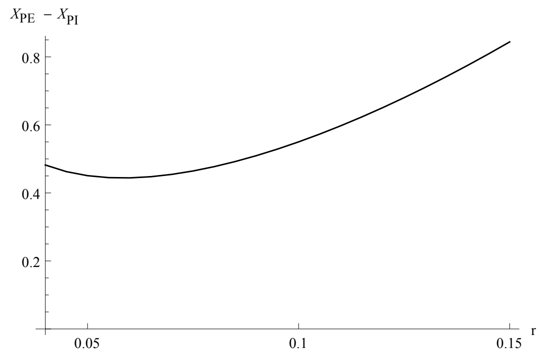
(a) Variation of α.



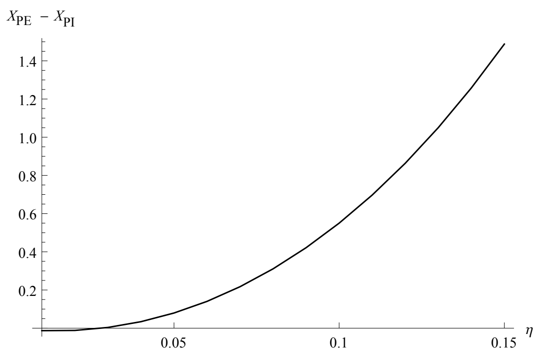
(b) Variation of σ.



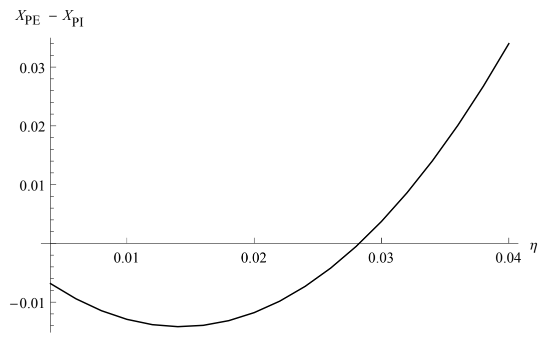
(c) Variation of δ.



(d) Variation of r.

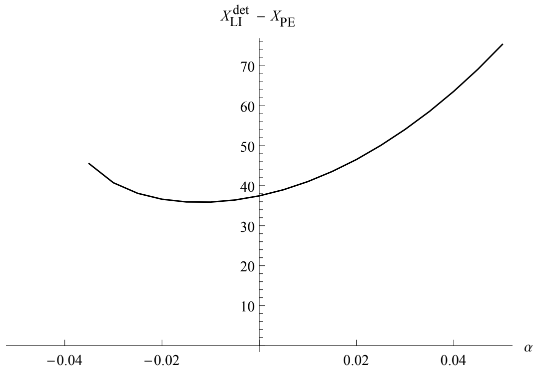


(e) Variation of η.

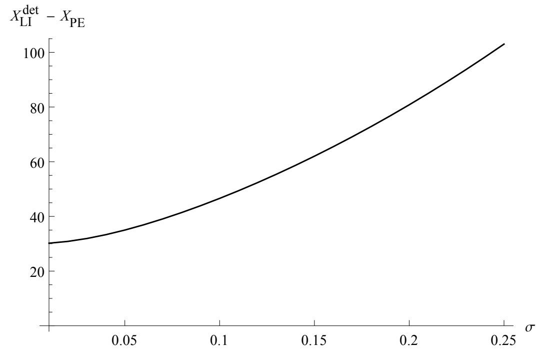


(f) Small values of η.

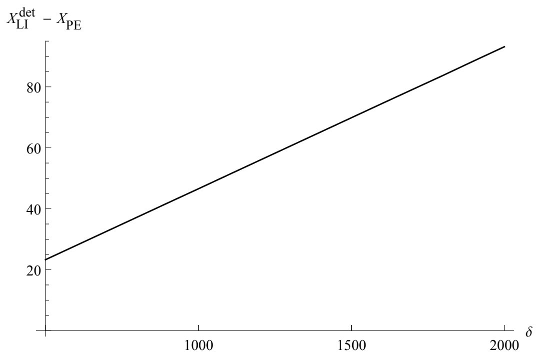
Figure 9: Difference between preemption points with $q_{1I} = 0.5$. Default parameters: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1$, and $\delta = 1000$.



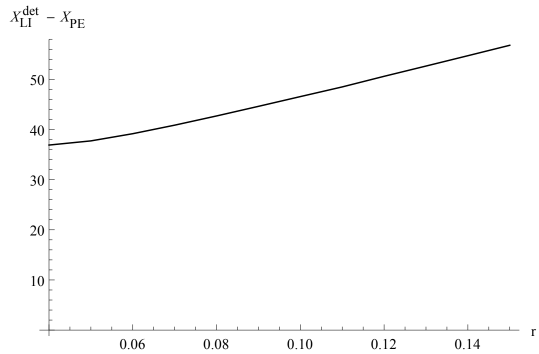
(a) Variation of α.



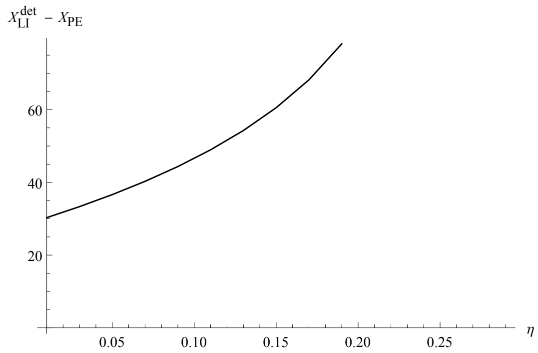
(b) Variation of σ.



(c) Variation of δ.



(d) Variation of r.



(e) Variation of η.

Figure 10: Difference between the incumbent's investment triggers and the entrant's preemption point for $q_{1I} = 0.5$. Default parameters: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1$, and $\delta = 1000$.

and therefore profits, are expected to increase more rapidly; one is then prepared to invest earlier to meet the same expectations concerning expected revenues. Nevertheless, when the initial capacity also changes under a change in parameter values,[13] a second effect comes in, similar to the analysis of η: A larger drift increases the initial capacity which leads to a delay of the investment. The effect on the optimal capacity is similar to the effect of r when the initial capacity is determined endogenously as q_{1I}^{myop}, but is, as expected, opposite to r when fixing it. The marginal investment cost has a positive effect on the investment trigger. When investing becomes more expensive, firms prefer to wait for a market where a larger output is required in order to meet the larger costs. The optimal capacities, both when fixing the initial market size and taking it myopically, are not affected. Finally, in a more uncertain market, i.e., a larger σ, future realizations become more important. Waiting gives more information. This leads to the decision to wait for a higher price, in other words, the firm is only prepared to invest for a larger value of x. This leads to an increase in the optimal capacity size. However, as in the case of r and α, the effect is ambiguous when assuming a myopic initial market size.

[13]Note that, since the initial capacity equals the myopic investment level, i.e. $q_{1I} = \frac{1}{\eta(\beta+1)}$, its level depends on the other parameter values.