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# Supporting Information for "Dynamic Measurement of Low Contact Angles by Optical Microscopy"

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For the refractive mode the droplet cannot be adequately modelled as a thin lens. Instead, the simplest approach is to consider the change in vergence of a paraxial beam of light as it passes through. Vergence,  $V$ , is defined as

$$V = \frac{n}{L} \quad (\text{S1})$$

where  $L$  is the distance to the source (or focus) of the light, being defined as positive for converging light and negative for diverging light. We define  $V_0$  and  $V_1$  as the vergence of light immediately before and after entering the drop, and  $V_2$  and  $V_3$  as the vergence immediately before and after leaving the drop. We can thus use the following series of equations to find the focal point a distance  $z_i^r$  above the surface, by considering the effect on  $V$  of a curved interface of optical power  $P^r$ , and of a distance of  $2h$ :

$$V_0 = -\frac{n_m}{s_0} = \frac{n_m}{h - z_o} \quad (\text{S2})$$

$$V_1 = V_0 + P^r = V_0 + \frac{n - n_m}{R} \quad (\text{S3})$$

$$\frac{1}{V_2} = \frac{1}{V_1} - \frac{2h}{n} \quad (\text{S4})$$

$$V_3 = V_2 + P^r = V_2 + \frac{n - n_m}{R} = \frac{n_m}{s_i^r} = \frac{n_m}{z_i^r - h} \quad (\text{S5})$$

Equation 9 may then be used to calculate  $d^r$ , using Equations S2–S4, and Equations 10 and 11. This leads to the following expression:

$$d^r = 2 \tan \varphi_c \left( \frac{n_m}{g(D, \theta)} + Df(\theta) \right) \quad (\text{S6})$$

where

$$g(D, \theta) = \left[ \left( \frac{n_m}{Df(\theta) - z_o} + \frac{2(n - n_m) \sin \theta}{D} \right)^{-1} - \frac{2D}{n} f(\theta) \right]^{-1} + \frac{2(n - n_m) \sin \theta}{D}. \quad (\text{S7})$$

As can be seen in Figure 4, this approximation for the refractive mode is not in such good agreement with numerical predictions as is that for the reflective mode, except for very low-

*NA* objectives. Errors are likely introduced from spherical aberration and from the reduced thickness of the lens away from the vertex, neither of which is easily accounted for using analytical methods.