Consumer confusion, obfuscation, and price regulation*

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Abstract

This paper studies firms' obfuscation choices in a duopoly setting where two firms differ in their marginal costs of production. We show that the high-cost firm chooses maximum obfuscation while the low-cost firm chooses minimal (maximal) obfuscation if the cost advantage is large (small). We argue that in this setting there is a new role for price regulation as it leads to more transparent pricing. Moreover, a price cap benefits social welfare as it shifts production to the more efficient low-cost firm.

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1 Introduction

For markets and competition to work, well-informed consumers are essential.[1] The degree of information and transparency in a market and, hence, the environment under which consumers make their purchase decisions can be heavily influenced by firm strategy. For instance, by deliberately increasing the complexity of price structure or price presentation, firms may make consumers' decision processes harder. Such strategies by firms with the intent of confusing consumers have commonly been termed obfuscation (Ellison and Ellison, 2009).

The aim of this paper is to understand firms' incentives to obfuscate in a setting where firms differ in their (marginal) costs of production and the effects of price regulation. The question is whether low efficiency firms can use obfuscation tactics to hide their high costs. To this end, we develop a duopoly model where two heterogeneous firms compete to supply a homogeneous product. There are two types of consumers: sophisticated and naive consumers. Sophisticated consumers can perfectly evaluate the firms' offers and pick the better one. Naive consumers are not able to compare the two offers, and thus randomly choose one of the two offers.

By obfuscation firms can increase the number of naive consumers in the market. In practice, this could be the use of different price formats or terms and language which makes it harder for some consumers to fully understand pricing and, hence, impedes comparisons between different offers. For example, although in many countries supermarkets are required to display unit prices for groceries, in reality "[unit prices] were often not being displayed on multi-buys or promotions, and different units were used for varieties of the same product, making them extremely difficult to compare".[2] The retail financial industry is also known for using obfuscation tactics to disorient consumers.[3] Alternatively, an increase in obfuscation

[1] Imperfect consumer decision making has been documented in many markets, such as retail financial markets (Campbell, 2006), telecommunication markets (Miravete, 2013) and electricity markets (Wilson and Waddams, 2010), among others.

[2] See the Guardian report by Smithers (2011) who cites the research done by the consumer campaigning charity *Which?*. Piccione and Spiegler (2012) and Chioveanu and Zhou (2013) provide theoretical treatments of this approach.

[3] See, e.g., Carlin and Manso (2011) who study the interaction between obfuscation and investor sophistication in mutual fund markets.

might also correspond to the number of price elements (as in mobile phone contracts) which may also make it harder for consumers to evaluate different offers and pick the best deal.[4]

We show that the high-cost firm always benefits from more naive consumers. Hence, this firm chooses to obfuscate as much as possible. In contrast, the low-cost firm has ambiguous preferences towards the share of naive consumers. On the one hand, more transparency hurts the firms as pricing becomes more competitive. On the other hand, due to its lower production costs, this firm has an advantage in competing for sophisticated consumers. This second effect is stronger the larger the cost advantage is. As a result, the more efficient firm chooses to obfuscate as much as possible if the cost advantage is small, but chooses not to obfuscate at all if its cost advantage is sufficiently large.

On the policy front, this paper argues that imposing a price cap can be an effective measure of consumer protection in this context. We show that introducing a price cap is beneficial for consumers for two reasons. First, there is an immediate positive impact as pricing by firms becomes more competitive. However, there is a second and more important positive effect which works via the obfuscation decisions. Due to the introduction of the price cap the low-cost firm may choose not to obfuscate anymore. The reason is that serving the naive consumers has become a less attractive option and competing for the sophisticated consumers a relatively more attractive option. As a consequence, the low-cost firm which has an advantage in attracting sophisticated consumers may prefer to stop obfuscating. Thus in the current setup, a price cap has the new role of leading to more transparent pricing.

Importantly, beneficial effects of price regulation also extend to social welfare. We show that with more competitive pricing from both firms, the probability that the high-cost firm's price is lower decreases. This shifts the demand of sophisticated consumers from the high-cost firm to the low-cost firm and hence, improves social welfare. Moreover, when a price cap reduces obfuscation, more consumers become sophisticated and are able to buy from the lower priced firm. This further increases social welfare be-

[4] Spiegler (2006) considers a setup where consumers are confused by a multitude of price dimensions. In addition, firms may also shroud certain price elements. See, e.g., Gabaix and Laibson (2006).

cause overall it is the low-cost firm that is more likely to have the lower price in equilibrium. This discussion of production efficiency in the presence of strategic obfuscation is a novel part of the current paper.

When policies that directly combat obfuscation are difficult to implement or easy to be circumvented, a price cap might be a feasible alternative. Indeed, the introduction of price caps in markets with incomplete consumer understanding, such as banking or telecommunications markets, has been discussed extensively among policy makers. For instance, in retail financial markets, caps on credit card interest rates, unauthorised overdraft interest rates or on surcharges charged for foreign ATM withdrawals are heavily discussed. With the purpose of helping inattentive consumers, in telecommunications markets, the EU has introduced caps on mobile data roaming charges and SMS services. This paper suggests that such policies might have been effective to help consumers make better decisions.

We point out that the effectiveness of a price cap in the current setup is in contrast to that of a model with rational and strategic consumers only and relies on the presence of behavioural consumers. For instance, based on Burdett and Judd's (1983) search model, Fershtman and Fishman (1994) and Armstrong et al. (2009) point out that under a price cap prices will likely be less dispersed and hence fewer consumers will make the effort to become better informed. Consequently, firms will face less competition and will raise their average prices. Therefore, in such models a price cap may backfire and hurt consumers. We also note that, in standard models without obfuscation, price cap regulation is neither universally optimal, especially when demand is uncertain. See, for example, Dobbs (2004) and Earle et al. (2007). Discouraged market entry and asymmetric information between regulated firms and the regulator can also be additional sources of inefficiency.

Besides price regulation, we discuss two alternative policy measures in the current setting. One consumer protection policy could simply be limiting the scope of obfuscation. While such policies are effective in reducing firm obfuscation and increase the share of sophisticated consumers, in practice they often can easily be circumvented. The other policy measure we discuss is taxation in the form of an excise. Although such a policy generally raises the price consumers pay, it can be effective in reducing the more efficient

firm's incentive to obfuscate and even in improving production efficiency. When regulators are willing to trade consumer surplus for efficiency, taxation can be a useful tool in combating obfuscation. In light of this analysis, the Air Passenger Duty might have inadvertently reduced more efficient airlines' and/or travel agencies' incentives to present air travel prices in more complicated ways.

The rest of the paper is organised as follows. We first briefly discuss the related literature in Section 2. We then introduce our model in Section 3. After the equilibrium analysis in Section 4, Section 5 studies the effect of a price cap on pricing and on consumer welfare. An analysis of social welfare is presented in Section 6. In Section 7 we discuss the two extensions. Section 8 concludes.

2 Related literature

Recently, a new stream of literature has emerged that studies competition in the presence of behaviourally biased consumers.[5] In this context, an important question is whether and how firms might decide to confuse consumers (obfuscation) in order to increase market power. For instance, and closely related to the present paper, Carlin (2009) studies the incentives to obfuscate in an oligopoly with n symmetric firms. He shows that more competition in the form of a larger number of firms increases the incentives to obfuscate. Piccione and Spiegler (2009) and Gu and Wenzel (2014) study an asymmetric setup where firms differ in the ability to attract naive consumers (prominence). Gu and Wenzel (2014) also show that more prominent firms may have more incentives to obfuscate and policies that regulate obfuscation may not be effective due to the reaction from less prominent firms. The novelty of the present paper is to understand firm's obfuscation incentives in an asymmetric duopoly where firms differ in their productivity levels. We argue that price regulation is a sensible consumer protection policy. In particular, it reduces firms' incentives to obfuscate and thus helps to promote market sophistication and production efficiency.

[5] See Spiegler (2011) for a textbook treatment and Huck and Zhou (2011) for an overview on this literature.

Focusing on how firms may obfuscate the market, Piccione and Spiegler (2012) provide a duopoly framework in which firms independently choose from a set of available price frames with different combinations resulting in different levels of comparability for consumers. The authors show that regulatory interventions that aim to facilitate comparisons may lead to a less competitive market outcome. Chioveanu and Zhou (2013) provide a framework in which firms independently choose from two possible price frames but this simpler construction can accommodate more than two firms and allows for the distinction of frame complexity and frame differentiation. They show that an increase in the number of competitors may result in higher industry profits and lower consumer surplus. In contrast, modelling of obfuscation in our paper follows Carlin (2009) and we introduce asymmetric competitors in terms of their production costs. We show that regulatory interventions in the form of a price cap can be effective not only in protecting consumers but also in improving efficiency in production allocation.

Obfuscation also becomes relevant when consumers are unable to evaluate all characteristics or price elements of a product. Assuming a random sampling procedure by consumers, Spiegler (2006) shows that firms adopt a random pricing strategy with high prices in some dimensions and low prices along other dimensions. Gabaix and Laibson (2006) develop a model where consumers observe one price component (price of the base good), but are not aware of another component (the add-on). This leads to an equilibrium with low base-good and high add-on prices. Dahremoeller (2013) and Wenzel (2015) study shrouding incentives in asymmetric markets where firms differ in cost, but do not consider policy interventions. Spiegler (2014) provides a general duopoly framework that captures a variety of obfuscation strategies (shrouding product attributes, multi-dimensional pricing or framing in order to reduce product comparability).

In a framework with rational consumers only, an alternative description of obfuscation may also arise by firms manipulating search costs. Wilson (2010) and Ellison and Wolitzky (2012) analyse search models where firms can obfuscate by increasing consumers' search costs. Petrikaite (2015) offers a framework where a multiproduct monopolist uses search costs to screen consumers.

Although most of the contributions so far are theoretical, there have also

been a few experimental studies. Kalayci and Potters (2011) examine whether buyer confusion increases market prices and find results that support the effectiveness of buyer confusion. In particular, they find that firms offering high-quality are are less likely to employ obfuscation strategies than firms offering a low quality. Kalayci (2015) presents experimental evidence that a seller's complexity and price choices are positively correlated. This is in contrast to the findings in Sitzia and Zizzo (2011) where the authors are unable to detect a significant effect of product complexity on prices.[6] Gu and Wenzel (2015) study the effects of different policies (a consumer protection policy and policies that level the playing field among firms) on firms' incentives to strategically obfuscate the market.

3 The model

We consider a market where two firms compete to supply a homogeneous product. The two firms differ in their (constant) marginal costs of producing this product. Firm 1 has a marginal cost of c_1 and Firm 2 has a cost of c_2. We assume that $0 < c_1 < c_2$ so that Firm 1 is designated to be the low-cost firm.[7] A mass one of consumers wishes to buy at most one unit of the product if the price does not exceed the reservation value of r. This reservation value is larger than the production costs of both firms so that, in equilibrium, both firms are active $(r > c_2)$.

As in Varian (1980), there are two types of consumers: sophisticated and naive consumers. The share of naive consumers is denoted by μ whereas the share of sophisticated consumers is $(1 - \mu)$. Sophisticated consumers understand true offering prices and buy from the firm that offers the lower price. A tie is broken with an equal probability. Naive consumers are unable to compare prices and buy at random. Naive consumers choose either of the two firms equally likely.[8]

[6] While these studies conduct experiments in a laboratory setting, related questions have also been studied using field experiments. See, e.g., Hossain and Morgan (2006), Chetty et al. (2009), and Brown et al. (2010).

[7] The model can be extended to oligopoly with more than two firms. However, as this does not the main qualitative results of the paper, we focus here on the duopoly case.

[8] This assumption of random shopping of confused consumers is also employed in related papers, for instance, in Carlin (2009) or in the price-frame-competition models by Piccione

As in Carlin (2009) the share of naive consumers can be influenced by firms' obfuscation strategies. Let $k_i \in [\underline{k}, \overline{k}]$, $i = 1, 2$, be Firm i's obfuscation choice where a higher level of k_i is interpreted as a higher level of obfuscation. The share of naive consumers is then given by $\mu(k_1, k_2)$ where $\frac{\partial \mu(k_1,k_2)}{\partial k_i} > 0$. The share of sophisticated consumers is correspondingly reduced if firms chose to obfuscate more. To focus on strategic effects among firms, the obfuscation choice is costless.[9]

We consider the following game. In the first stage, the two firms simultaneously and independently decide on its own obfuscation level k_i. After knowing each other's obfuscation level and hence, the share of naive consumers, they compete in prices in the second stage.

We note that in our model obfuscation and pricing decisions are sequential. With sequential decisions, we assume pricing is more flexible while obfuscation tends to be more persistent. For example, in retail financial markets, pricing is much more flexible than obfuscation strategies such as inventing new terminologies. With simultaneous decisions, on the other hand, one would assume that price and obfuscation are joint decisions.[10]

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and Spiegler (2012) and Chioveanu and Zhou (2013). Note that we assume that naive consumers do not understand that, as it will later turn out, in equilibrium Firm 1 will charge a lower price than Firm 2 on average. This seems to be consistent with our notion of naiveness. Consumers who fail to recognize the best offer are also unlikely to consider that one firm might have a larger incentives for higher prices than another firm due to cost differences.

[9] Note that instead of interpreting $\underline{k}$ as no obfuscation it could also be interpreted as counter obfuscation or advertising, strictly increasing the number of sophisticated consumers. This alternative interpretation, however, has no qualitative effects on our main results.

[10] The approach with simultaneous decisions is followed, for instance, in Carlin (2009), Piccione and Spiegler (2012) and Chioveanu and Zhou (2013). Which approach is more suitable depends largely on context where with sequential decisions one would assume pricing is more flexible and obfuscation decisions more persistent. Assuming a simultaneous move game would, however, lead to a qualitatively similar structure of equilibrium obfuscation as under sequential moves. The high-cost firm has larger incentives to obfuscate and would choose to obfuscate more often than the low-cost firm because it clearly benefits to a larger degree from the presence of myopic consumers. This is because the high-cost firm charges on average higher prices than the low-cost firm and only naive consumer would buy at such higher prices.

4 Analysis

4.1 Pricing stage

We start by analysing the second stage of the game. At this stage, with a given share of naive consumers and a given pair of prices, the firms' profits are as follows. For $i = 1, 2$ and $j \neq i$,

$$\Pi_i(\mu, p_i, p_j) = \begin{cases} (p_i - c_i)\left[\frac{\mu}{2} + (1-\mu)\right] & \text{if } p_i < p_j \\ (p_i - c_i)\left[\frac{\mu}{2} + \frac{1-\mu}{2}\right] & \text{if } p_i = p_j \\ (p_i - c_i)\frac{\mu}{2} & \text{if } p_i > p_j \end{cases} . \tag{1}$$

The following proposition details the price equilibrium in mixed strategies:

Proposition 1. For any given share of naive consumers, $\mu \in (0, 1)$, there exists a unique Nash equilibrium in the pricing stage, in which Firm 1 prices according to the cumulative distribution function

$$F_1(p) = \begin{cases} 0 & \text{if } p < p_0 \\ 1 - \frac{\mu(r-p)}{2(1-\mu)(p-c_2)} & \text{if } p_0 \leq p < r \\ 1 & \text{if } p \geq r \end{cases} , \tag{2}$$

and Firm 2 prices according to the cumulative distribution function

$$F_2(p) = \begin{cases} 0 & \text{if } p < p_0 \\ 1 - \frac{\mu(r-p)}{2(1-\mu)(p-c_1)} - \frac{c_2-c_1}{p-c_1} & \text{if } p_0 \leq p < r \\ 1 & \text{if } p \geq r \end{cases} , \tag{3}$$

where $p_0 = \frac{\mu(r-c_2)}{2-\mu} + c_2$ is the lower bound of both firms' prices.

Proof: see Appendix A.1.

It is clear that there exists no pure strategy pricing equilibrium due to the presence of sophisticated consumers. The price distributions in Proposition 1 are such that given the rival's strategy a firm is indifferent regarding all prices in its support. Note that the price distribution of Firm 2 has a mass point at r. With a probability of $\frac{c_2-c_1}{r-c_1}$ this firm sets the price equal to the

reservation price.

Let P_1 and P_2 denote the two firms' equilibrium prices whose cumulative distribution functions are (2) and (3), respectively. For convenience when discussing stochastic dominance relations, we let all price distributions in this paper have the same (extended) support of $[c_2, r]$. We write $X \succeq_{st} Y$ if random variable X first order stochastically dominates Y and $X \succ_{st} Y$ if $X \succeq_{st} Y$ but $Y \not\succeq_{st} X$. Then, we have

Corollary 1. $P_2 \succ_{st} P_1$ with $F_2(p) < F_1(p)$ for all $p \in (p_0, r)$.

Proof: see Appendix A.2.

By standard results in stochastic dominance,[11] Corollary 1 implies that the high-cost firm charges on average a higher price than the low-cost firm. This confirms the intuition that more efficient firms compete more aggressively in prices.

Ex ante expected profits of the two firms are

$$\widehat{\Pi}_1 = (r - c_1)\frac{\mu}{2} + (c_2 - c_1)(1 - \mu) \text{ and} \tag{4}$$

$$\widehat{\Pi}_2 = (r - c_2)\frac{\mu}{2}. \tag{5}$$

The less efficient Firm 2 makes expected profits equal to the amount from selling only to its share of naive consumers at the reservation price, $(r-c_2)\frac{\mu}{2}$. The more efficient Firm 1's expected profits, however, also include a part which is associated with the share of sophisticated consumers, $(c_2 - c_1)(1 - \mu)$.[12] Note that this is exactly the Bertrand profit the low-cost Firm can make in a market with only a measure $(1 - \mu)$ of sophisticated consumers. This part of profit is unique to asymmetric costs as the rent from sophisticated consumers will be completely competed away if the firms had the same costs. The composition of the firms' profits has implications on their choices of obfuscation which we now analyse.

[11] For instance, Lemma 4.1 in Wolfstetter (1999).

[12] We also note that, for any given share of naive consumers, the efficient Firm 1 earns strictly higher profits than the less efficient Firm 2.

4.2 Obfuscation choices

In the first stage, Firm 2's profits are strictly increasing in the share of naive consumers:

$$\frac{\partial \widehat{\Pi}_2}{\partial k_2} = \frac{r - c_2}{2} \frac{\partial \mu}{\partial k_2} > 0. \tag{6}$$

Hence, this firm chooses obfuscation as high as possible, that is, $k_2^* = \overline{k}$. The partial derivative of Firm 1's profits with respect to the obfuscation choice k_1 is given by:

$$\frac{\partial \widehat{\Pi}_1}{\partial k_1} = \frac{r + c_1 - 2c_2}{2} \frac{\partial \mu}{\partial k_1} \gtrless 0. \tag{7}$$

This partial derivative is negative if $c_1 < 2c_2 - r$. In this case, Firm 1 benefits from a more transparent market, i.e., fewer naive consumers, and hence chooses its obfuscation to be as low as possible, that is, $k_1^* = \underline{k}$. If, however, $c_1 > 2c_2 - r$, Firm 1 prefers the market to be as least transparent as possible and, in consequence, raises its obfuscation to the maximum possible level, $k_1^* = \overline{k}$.

Define $\tilde{c}_1 := 2c_2 - r$. The following Proposition states equilibrium obfuscation.

Proposition 2.

i) The high-cost Firm 2 chooses $k_2^* = \bar{k}$.

ii) The low-cost Firm 1 chooses $k_1^* = \bar{k}$ if $c_1 > \tilde{c}_1$ and chooses $k_1^* = \underline{k}$ if $c_1 < \tilde{c}_1$. If $\tilde{c}_1 = 2c_2 - r$, Firm 1 is indifferent about the level of obfuscation.

In equilibrium, the high-cost firm always chooses maximum obfuscation. This firm's profits are strictly increasing with the number of naive consumers and so it has incentives to raise obfuscation as much as possible. By contrast, Firm 1's obfuscation strategy depends on its efficiency advantage vis-a-vis Firm 2. If the efficiency advantage is large, Firm 1 benefits from the presence of sophisticated consumers and chooses the lowest level of obfuscation. The underlying reason is that this efficiency advantage provides Firm 1 with an advantage over Firm 2 in competing for sophisticated consumers. Consequently, Firm 1 prefers to have as many sophisticated consumers as possible. If, on the other side, the efficiency advantage is small

and firms are rather symmetric, Firm 1 prefers the highest possible level of obfuscation.

The following corollary summarises equilibrium obfuscation by providing the equilibrium share of naive consumers:

Corollary 2. Define $\overline{\mu} = \mu(\overline{k}, \overline{k})$ and $\underline{\mu} = \mu(\underline{k}, \overline{k})$. The equilibrium share of naive consumers is

$$\mu^* = \begin{cases} \overline{\mu} & \text{if } c_1 > \tilde{c}_1 \\ \in (\underline{\mu}, \overline{\mu}) & \text{if } c_1 = \tilde{c}_1 \\ \underline{\mu} & \text{if } c_1 < \tilde{c}_1 \end{cases}. \tag{8}$$

At this point we would like to briefly comment on the case of identical costs, which is covered by our analysis when $c_2 \to c_1$. When $c_2 \to c_1$, both pricing distributions converge towards each other such that there is a symmetric Nash equilibrium in mixed strategies in the pricing stage (and there is no mass point on r). Also, both firms' profits converge towards $\lim_{c_2 \to c_1} \Pi_1 = \lim_{c_2 \to c_1} \Pi_2 = (r - c_1)\frac{\mu}{2}$. In this equilibrium, profits are strictly increasing in the number of naive consumers such that both firms would obfuscate at the maximum level. In this sense, the outcome resembles the case where the efficiency advantage is small, as covered in the previous results. We also note that with identical costs, the results in the following sections coincide with the case of low cost difference. The only difference arises that with identical costs there are no social welfare effects for regulation.

4.3 Scope for obfuscation

From a public policy point of view, direct measures of consumer protection against obfuscation can be envisaged. In this section we discuss whether a regulation that limits the scope for obfuscation is effective or not in the current setting. Such a policy intervention can be modelled as a cap on the maximum level of obfuscation $\overline{k}$.

Suppose that such a policy decreases the upper bound of obfuscation to a level of $\overline{k}' < \overline{k}$. Note that the conditions of whether a firm prefers more or less naive consumers are independent of the scope for obfuscation (see (6)

and (7)). Therefore, in the case where Firm 1 does not obfuscate the share of naive consumers is reduced to $\underline{\mu}'$ where $\underline{\mu}' = \mu(\underline{k}, \overline{k}')$. In the case where Firm 1 obfuscates the impact of the policy is even larger as the behaviour of Firm 1 is also affected.

Due to the decrease in the share of naive consumers, both firms' prices and the first order statistic of them decrease in the usual stochastic order. It follows that consumer surplus will unambiguously increase. Moreover, since t as defined in Lemma 1 increases in μ, following the proof of Lemma 2 in Appendix A.5, it is easily verified that when the share of naive consumers decreases, the probability of the inefficient Firm 2 having the lower price decreases. Thus, in this setting with two firms that differ in their costs, limiting the scope for obfuscation improves social welfare.

Policies that directly limit the scope for obfuscation, however, might not be effective in practice as firms may find alternative ways to confuse consumers. For example, although regulatory bodies can require airlines, hotels and travel agencies to quote prices that include all necessary taxes, fees, charges and surcharges, contingency charges and the prices of optional services often surprise many consumers. Therefore, in the remainder of the paper we focus on alternative policies and analyse whether they can be helpful in combating obfuscation.

5 Introducing a price cap

Having analysed the base model, we now investigate whether introducing a price cap is a useful policy for promoting market sophistication and raising consumer and social welfare. We consider the case where the price cap, i.e., the maximum price that can be charged κ, is neither irrelevantly high nor too strict to effectively shut down the less efficient firm: $c_2 < \kappa < r$.

When a price cap $\kappa \in (c_2, r)$ is imposed, from the firms' point of view, this is strategically equivalent to a reduction in consumers' reservation price from r to κ. Therefore, all of our previous results can be readily applied for the current purpose.

By replacing r by κ in (4) and (5), we have the expected profits of the two firms. As in Proposition 2, the high-cost Firm 2 will still choose maximum

obfuscation. The low-cost Firm 1, however, will now prefer maximum obfuscation only when $c_1 > 2c_2 - \kappa$ where $2c_2 - \kappa > \tilde{c}_1$ as $\kappa < r$. If, however, $\tilde{c}_1 < c_1 < 2c_2 - \kappa$, Firm 1's incentive to obfuscate is strictly reduced. In this case, Firm 1 would have chosen maximum obfuscation without the price cap but will now choose minimum obfuscation. If $c_1 < \tilde{c}_1$, then Firm 1 still chooses minimum obfuscation. To summarise, if the cost difference is small ($c_1 > \tilde{c}_1$) then a sufficiently strong price cap ($\kappa < 2c_2 - c_1$) strictly decreases firm obfuscation. Otherwise, firm obfuscation is unaffected. The following Proposition gives the result in terms of market sophistication.

Corollary 3. Suppose a price cap $\kappa \in (c_2, r)$ is introduced.

- If $\kappa < 2c_2 - c_1 < r$, then the equilibrium share of naive consumers decreases from $\overline{\mu}$ to $\underline{\mu}$.
- If $2c_2 - c_1 < \kappa < r$ [$\kappa < r < 2c_2 - c_1$], then the equilibrium share of naive consumers remains unchanged at $\overline{\mu}$ [$\underline{\mu}$].

The intuition for Firm 1's reduced incentive in obfuscation is the following. After the price limit is introduced, the benefit from serving naive consumers has been reduced (only a price of up to κ can be charged) and, hence, competing for sophisticated consumer has become a more attractive option. This effect benefits Firm 1 as its lower cost enables this firm to compete more successfully for sophisticated, price sensitive consumers. If this effect is sufficiently strong, i.e., the price cap sufficiently low, Firm 1 may choose not to obfuscate as to increase the share of sophisticated consumers.

5.1 Consumer welfare

Since all consumers buy one unit of product, consumer surplus is inversely related to the price they pay. The effects of a price cap on consumer welfare can hence be studied by ordering price distributions. As the equilibrium share of naive consumers can change after the price limit, we make the dependence of price distributions on μ explicit. Following (2) and (3), the cumulative distribution functions of Firm i's equilibrium price $P_i^{\kappa,\mu}$, $i = 1, 2$,

under the price cap κ and the share of naive consumers μ are respectively,

$$F_1^{\kappa,\mu}(p) = \begin{cases} 0 & \text{if } p < p_0^{\kappa,\mu} \\ 1 - \frac{\mu(\kappa - p)}{2(1-\mu)(p-c_2)} & \text{if } p_0^{\kappa,\mu} \leq p < \kappa \\ 1 & \text{if } p \geq \kappa \end{cases} \tag{9}$$

$$\text{and } F_2^{\kappa,\mu}(p) = \begin{cases} 0 & \text{if } p < p_0^{\kappa,\mu} \\ 1 - \frac{\mu(\kappa - p)}{2(1-\mu)(p-c_1)} - \frac{c_2 - c_1}{p - c_1} & \text{if } p_0^{\kappa,\mu} \leq p < \kappa \\ 1 & \text{if } p \geq \kappa \end{cases}, \tag{10}$$

where $p_0^{\kappa,\mu} = \frac{\mu(\kappa - c_2)}{2-\mu} + c_2$ is the lower bound of both firms' prices.

After imposing a price cap, the upper bound of the prices naturally decreases. Because $p_0^{\kappa,\mu}$ increases in κ and μ, irrespective whether or not firm obfuscation is affected, the lower bound of the prices also decreases. Indeed, we show that both firms' equilibrium prices decrease in terms of the usual stochastic order. Moreover, the price that sophisticated consumers pay — the first order statistic of the two firms' prices — also decreases in the usual stochastic order. Let $P_{(1)}^{\kappa,\mu}$ denote the first order statistic of $P_1^{\kappa,\mu}$ and $P_2^{\kappa,\mu}$. For clarity, we write $P_i^{r,\mu}$ and $P_{(1)}^{r,\mu}$ when no price cap is imposed. The next Proposition formalises the above result.

Proposition 3. Suppose a price cap $\kappa \in (c_2, r)$ is introduced. Then, both firms' prices and the first order statistic of these two prices decrease in the usual stochastic order. Specifically,

- if $\kappa < 2c_2 - c_1 < r$, we have $P_1^{r,\overline{\mu}} \succ_{st} P_1^{\kappa,\underline{\mu}}$, $P_2^{r,\overline{\mu}} \succ_{st} P_2^{\kappa,\underline{\mu}}$, and $P_{(1)}^{r,\overline{\mu}} \succ_{st} P_{(1)}^{\kappa,\underline{\mu}}$;
- if $2c_2 - c_1 < \kappa < r$ [$\kappa < r < 2c_2 - c_1$], we have $P_1^{r,\mu} \succ_{st} P_1^{\kappa,\mu}$, $P_2^{r,\mu} \succ_{st} P_2^{\kappa,\mu}$ and $P_{(1)}^{r,\mu} \succ_{st} P_{(1)}^{\kappa,\mu}$ where $\mu = \overline{\mu}$ [$\underline{\mu}$].

Proof: see Appendix A.3.

A price cap affects price distributions through two channels. First, there is the direct effect on permissible prices which makes pricing more competitive. The reason is that engaging in more aggressive price competition has

become more attractive as profits from naive consumers decrease. That is, the price distributions of both firms are shifted downwards. This is the case when the price cap does not affect firm obfuscation. When it does, however, a reduction in the equilibrium share of naive consumers represents another, indirect, and more important channel through which a price cap forces prices down. This is because with other things being equal price competition is fiercer when there is a larger share of sophisticated consumers. This leads to a further downward shift of the price distributions.

Proposition 3 implies that the expected prices faced by each group of consumers strictly decrease after the price cap. Moreover, when firm obfuscation reduces, more consumers become sophisticated and are able to buy at the lower price. A price cap thus clearly improves consumer welfare.

5.2 Firm profits

After imposing a price cap, the less efficient Firm 2 will make less expected profits (see (5)). When the price cap does not induce a reduction in equilibrium μ, Firm 1's expected profits (4) also clearly decrease. When $\kappa < 2c_2 - c_1 < r$, however, we should also take into account of the indirect effect through μ. Nevertheless, a price cap reduces Firm 1's expected profits in this case too. Specifically,

$$\begin{aligned}\widehat{\Pi}_1^{\kappa} - \widehat{\Pi}_1^{r} =& (\kappa - c_1)\frac{1}{2}\underline{\mu} + (c_2 - c_1)(1 - \underline{\mu}) - (r - c_1)\frac{1}{2}\overline{\mu} - (c_2 - c_1)(1 - \overline{\mu}) \\ =& \underbrace{(\kappa - 2c_2 + c_1)}_{<0}\frac{1}{2}\underline{\mu} - \underbrace{(r - 2c_2 + c_1)}_{>0}\frac{1}{2}\overline{\mu} < 0.\end{aligned}$$

In summary, both firms' expected profits are reduced by the price cap. However, the high-cost Firm 2 suffers weakly more in terms of absolute profits and strictly more in terms of percentage losses.

6 Social welfare

We have seen that a price cap unequivocally improves consumer welfare. On the other hand, it also unequivocally reduces firm profits. In this section, we study how a price cap affects social welfare. The total benefit from

production and consumption in the current model is r. The total cost, however, depends on how the production is divided between the two firms. While each firm serves half of the naive consumers, the firm that has the lower realized price sells to all sophisticated consumers.

Let the probability of the inefficient Firm 2 having the lower price in equilibrium be denoted by

$$\alpha := \int_{p_0^{\kappa,\mu}}^{\kappa} F_2^{\kappa,\mu}(p) dF_1^{\kappa,\mu}(p). \tag{11}$$

Since Firm 2's price first order stochastically dominates that of Firm 1, intuitively, the probability that $p_2 < p_1$ shall be less than $\frac{1}{2}$. The following Lemma gives a closed form solution for α and confirms this intuition.

Lemma 1.

i) Define $t := \frac{\mu(\kappa - c_1)}{2(1-\mu)(c_2 - c_1)}$. Then,

$$\alpha = \left(\frac{\kappa - c_2}{\kappa - c_1}\right) t \left[(1+t)\ln\left(1 + \frac{1}{t}\right) - 1\right]. \tag{12}$$

ii) $\alpha < \frac{1}{2}$.

Proof: see Appendix A.4.

With the definition of α, the ex ante expected social welfare W can be written as

$$\begin{aligned} W &= r - \underbrace{\left(\frac{\mu}{2}c_1 + \frac{\mu}{2}c_2\right)}_{\text{supplying naive consumers}} - \underbrace{(1-\mu)[(1-\alpha)c_1 + \alpha c_2]}_{\text{supplying sophisticated consumers}} \\ &= r - c_1 - (c_2 - c_1)\left[\frac{1}{2}\mu + \alpha(1-\mu)\right]. \end{aligned} \tag{13}$$

From (13), it is clear that social inefficiency arises because half of the naive consumers and on average a measure α of sophisticated consumers buy from the high-cost firm.

To understand the impact of a price cap on social welfare, we need to evaluate how α changes with κ. Since α is a function of both κ and μ, we cate-

gorise two cases depending on whether or not the share of naive consumers changes in equilibrium. Suppose μ remains unchanged as κ is imposed. Since both firms' prices decrease in the usual stochastic order, it is not clear if α will decrease or increase. The same applies when in addition a reduction in μ is triggered by κ. It turns out that the introduction of a price cap will decrease α in both cases. Indeed, in Appendix A.5 we prove as a lemma that α strictly increases in κ for all $\kappa \in (c_2, r)$.

Given that the probability of Firm 2 having a lower price is decreased after the introduction of a price cap, social welfare (13) clearly increases if μ remains unchanged. If μ is also reduced as a result of κ, social welfare increases even further as more consumers can buy from the lower priced firm which in equilibrium is more likely to be Firm 1. To see this, note that (13) can be rearranged as

$$W = r - c_1 - (c_2 - c_1)\left[\left(\frac{1}{2} - \alpha\right)\mu + \alpha\right].$$

By Lemma 1, $\alpha < \frac{1}{2}$. Hence W increases if μ is reduced. To highlight,

Proposition 4. Imposing a price cap unequivocally increases social welfare.

Summarising, in this model with endogenous obfuscation, introducing a price cap has two beneficial effects on consumer surplus and social welfare. First, consumers benefit from more competitive pricing (for a given level of consumer transparency) and the probability of the high-cost firm inefficiently serving sophisticated consumers decreases. The intuition of this effect can also be seen by examining the mass point in the pricing stage. The size of this mass point is decreasing in the regulated price which implies that informed consumer are more likely to purchase from the low-cost firm after a price cap. Second, consumers benefit as firms obfuscate less and the market may become more sophisticated. This second effect also benefits the society as more consumers may be served by the more efficient low-cost firm.[13]

[13]We point out that with identical marginal cost ($c_1 = c_2$) social welfare is a constant and does not depend on the price cap.

7 Extensions

7.1 Taxation

Taxation is another widely used public policy instrument. In this section we discuss the effects of excise taxes on firms' obfuscation incentives. An excise tax is typically a per unit tax that is defined as a fixed amount for each unit of a good or service sold, such as the Air Passenger Duty.

Suppose an excise tax $0 < \tau < r - c_2$ is levied to each unit of the product sold and is collected from the producers. Effectively, the two firms are now producing at the marginal costs of $c_1 + \tau$ and $c_2 + \tau$, respectively. Similar to Proposition 2, we can establish that i) the high-cost Firm 2 chooses $k_2^* = \bar{k}$ and ii) the low-cost Firm 1 chooses $k_1^* = \bar{k}$ if $c_1 > \hat{c}_1$ and chooses $k_1^* = \underline{k}$ if $c_1 < \hat{c}_1$ where $\hat{c}_1 := 2c_2 + \tau - r$.

It follows that, when $\tilde{c}_1 < c_1 < \hat{c}_1$ Firm 1 chooses full obfuscation in the absence of the excise tax while it prefers no obfuscation after its introduction. On the other hand, when $c_1 < \tilde{c}_1$ or $c_1 > \hat{c}_1$ Firm 1's obfuscation choice remains unchanged at $\underline{k}$ and $\bar{k}$, respectively.

Proposition 5. Suppose that an excise tax $\tau \in (0, r - c_2)$ is imposed.

i) If $c_1 > 2c_2 - r$ and $\tau > c_1 + r - 2c_2$, the excise tax reduces the low-cost Firm 1's obfuscation choice from $\bar{k}$ to $\underline{k}$.

ii) If $c_1 < 2c_2 - r$ or, $c_1 > 2c_2 - r$ but $\tau < c_1 + r - 2c_2$, the introduction of the excise tax has no effect on either firm's obfuscation choice.

Proposition 5 reveals that when Firm 1 originally chooses full obfuscation, a sufficiently high excise tax will reduce its incentive to obfuscate the market. The intuition is similar to that of price regulation. After the introduction of the excise tax, the profits earned by selling only to naive consumers are reduced and competition for the sophisticated consumers becomes less fierce due to the higher marginal costs of production. Therefore, when the excise tax is sufficiently high, Firm 1 finds it more profitable to compete for sophisticated consumers and consequently chooses not to obfuscate.

In contrast to a price cap, an excise tax is likely to increase the prices in the usual stochastic order. This is easily verified by inspecting the price distributions when the firms' obfuscation choices remain unchanged. When $\tilde{c}_1 < c_1 < \hat{c}_1$, however, the share of naive consumers reduces from $\overline{\mu}$ to $\underline{\mu}$. A reduction in μ tends to increase price competition and decrease the prices in the usual stochastic order. The overall effect will depend on the relative strength of these two effects. In this sense, the result on consumer welfare is less clear-cut under an excise tax than under a price cap. On the other hand, an excise tax will unambiguously improve production efficiency.

Corollary 4. The probability of the inefficient Firm 2 having the lower price in equilibrium, α, is reduced when an excise tax $0 < \tau < r - c_2$ is imposed.

Proof: see Appendix A.6.

As both α and μ can only decrease after τ is imposed, following the same analyses preceding Proposition 4, social welfare (13) will clearly increase if the collected tax revenue is not wasted. The intuition is that an excise tax affects different firms differently. In the extreme case of τ approaching $r-c_2$, Firm 2 will be forced to sell at the reservation price and as a consequence sophisticated consumers almost always buy from the more efficient firm.

However, given inevitable inefficiencies in taxation and, if consumer welfare is of significant importance, it seems price regulation is a better policy measure than taxation in the current setting.

7.2 Prominence

In this section we consider the possibility that the more efficient firm may also be more prominent among naive consumers in the sense of Gu and Wenzel (2014). To this end, let $\phi \in \left[\frac{1}{2}, 1\right)$ be the share of naive consumers that the low-cost Firm 1 attracts. The remaining share of naive consumers buy from Firm 2. Following the standard procedure, the structure of the current pricing equilibrium remains as long as $\phi < \frac{r-c_1}{2r-c_2-c_1}$. That is, Firm 2 has a mass point at r while Firm 1's price distribution stays atomless.[14]

[14]Our condition guarantees that for any value of μ the high-cost firm has a mass point at r. We note that if ϕ becomes large enough, the structure of the pricing equilibrium in Gu and Wenzel (2014) will prevail where Firm 1 instead of Firm 2 has a mass point at r.

The lower bound of the price distributions is dependent on the prominence:

$$p_0^{'} = \frac{r(1-\phi)\mu + c_2(1-\mu)}{(1-\phi)\mu + (1-\mu)}.$$

The pricing stage equilibrium profits are, respectively,

$$\Pi_1^{'} = [(r-c_1)(1-\phi)\mu + (c_2-c_1)(1-\mu)]\,\frac{\phi\mu + (1-\mu)}{(1-\phi)\mu + (1-\mu)}$$
$$\Pi_2^{'} = (r-c_2)(1-\phi)\mu$$

It follows that the high-cost Firm 2 - as in our baseline model - will choose maximum obfuscation. However, now the incentives to obfuscate for Firm 1 depends not only on the cost differences but also on the level of prominence. This can be seen from Firm 1's marginal profits of obfuscation in the obfuscation stage:

$$\frac{\partial \Pi_1^{'}}{\partial k_1} \gtrless 0 \Leftrightarrow (r + c_1 - 2c_2) - \frac{\mu(2-\mu\phi)}{2}[2(1-\phi)r + 2\phi c_1 - 2c_2] \gtrless 0. \quad (14)$$

When we consider small prominence, i.e. ϕ close to $\frac{1}{2}$, our main obfuscation result in Proposition 2 holds true because $1 > \frac{\mu(2-\mu\phi)}{2}$ and hence $(r+c_1-2c_2)$ determines the sign of $\frac{\partial \Pi_1^{'}}{\partial k_1}$. In other words, Firm 1 obfuscates only when the cost advantage is small. Consequently, our policy analysis also applies when prominence remains small.

We note that in the case of large prominence (that is, prominence plays the dominant role compared to cost differences) results are different. In essence, this case is studied in our previous work where firms only differ in prominence but have identical cost (Gu and Wenzel, 2014). In these situations, pricing and obfuscation incentives reverse. The more prominent firm has larger incentives to choose high prices and generally prefers a more obfuscated market. We found that a price cap firstly has a direct positive effect of reducing the size of the mass point in Firm 1's pricing but on the other hand it also increases Firm 2's incentive to obfuscate. For the parameter values we have checked, the direct effect dominates the other and hence even in this large prominence scenario, the overall effect of a price cap tends to be welfare improving.

8 Conclusion

This paper has studied firms' obfuscation choices when they differ in their marginal costs of production. While the high-cost firm always fully obfuscates, the obfuscation choice of the low-cost firm depends on its cost advantage. This paper has also investigated the impact of price regulation in this asymmetric setting. While, in general, price regulation may not be a good competition policy we demonstrate that, in such an environment, setting a price cap reduces the incentives of the more efficient firm to obfuscate. Thus, a price cap may be a good policy to promote market sophistication. Moreover, the more efficient low-cost firm will on average supply a larger share of the market after a price cap. This improves social welfare. While caps on the scope of obfuscation and excise taxes can also be welfare improving, price regulation seems to be a better policy measure.

Finally, we note that we do not advocate price regulation *per se*, but rather point out that, in certain circumstances, there are positive effects of price regulation. Introducing price caps might be a sensible idea in markets where many consumers are inattentive or unable to understand the offered products and policies to tackle obfuscation can be easily avoided by firms. However, in situations with rational consumers existing research shows that price caps may have negative consequences, for instance, in search models (Fershtman and Fishman, 1994; Armstrong et al., 2009) or when the level of demand is uncertain (Dobbs, 2004). Therefore, the positive effects of price caps reported in this paper, have to be carefully weighed against other possibly negative effects, for instance, on entry or innovation.

A Appendix

A.1 Proof of Proposition 1

Proof: First, by similar arguments, Propositions 1-5 in Narasimhan (1988) also hold in the current setup. I.e., i) there exists no Nash equilibrium in pure strategies; ii) in the unique pricing equilibrium exactly one firm has one mass point at the reservation price r and it randomizes over $(p_0, r]$. The other firm randomizes over (p_0, r) with no mass points.

According to the above structure, suppose for the sake of contradiction that Firm 1 has a mass point at r. Then Firm 1 earns expected profits of $(r - c_1)\frac{\mu}{2}$ when it charges r and losing with probability 1 all sophisticated consumers. Denote the lower boundary of the support by p_0'. By charging p_0', on the other hand, Firm 1 receives all sophisticated consumers and hence makes expected profits of $(p_0' - c_1)\left[\frac{\mu}{2} + (1-\mu)\right]$. As these expected profits have to level, $p_0' = \frac{\mu(r-c_1)}{2-\mu} + c_1$. Then, Firm 2 by charging p_0' expects to earn $(p_0' - c_2)\left[\frac{\mu}{2} + (1-\mu)\right]$. However, because $(c_1 - c_2)(1-\mu) < 0$,

$$\left[\frac{\mu(r-c_1)}{2-\mu} + c_1 - c_2\right]\left[\frac{\mu}{2} + (1-\mu)\right] < (r-c_2)\frac{\mu}{2}.$$

Firm 2 hence can do strictly better by charging r and earning $(r - c_2)\frac{\mu}{2}$. This contradicts with Firm 2 randomizing over (p_0', r).

We therefore can conclude that in the unique pricing equilibrium, Firm 2 has a mass point at r and randomizes over $(p_0, r]$ while Firm 1 randomizes over (p_0, r). Firm 2's expected profits are hence $\widehat{\Pi}_2 = (r - c_2)\frac{\mu}{2}$. By Firm 2's profits at p_0, we have

$$(p_0 - c_2)\left[\frac{\mu}{2} + (1-\mu)\right] = (r-c_2)\frac{\mu}{2}$$

and hence, $p_0 = \frac{\mu(r-c_2)}{2-\mu} + c_2$.

At prices $p \in (p_0, r)$, Firm 2's expected profits are

$$(p - c_2)\left[\frac{\mu}{2} + (1 - F_1(p))(1-\mu)\right]. \tag{15}$$

Equating (15) and $\widehat{\Pi}_2$, we have $F_1(p) = 1 - \frac{\mu(r-p)}{2(1-\mu)(p-c_2)}$.

It remains only to show $F_2(p)$. Note that Firm 1's expected profits from charging p_0 are

$$\widehat{\Pi}_1 = (p_0 - c_1)\left[\frac{\mu}{2} + (1-\mu)\right] = (r-c_1)\frac{\mu}{2} + (c_2 - c_1)(1-\mu), \tag{16}$$

and its expected profits from charging prices $p \in (p_0, r)$ are

$$(p - c_1)\left[\frac{\mu}{2} + (1 - F_2(p))(1-\mu)\right]. \tag{17}$$

Equating (16) and (17), it follows that $F_2(p) = 1 - \frac{\mu(r-p)}{2(1-\mu)(p-c_1)} - \frac{c_2 - c_1}{p - c_1}$. *Q.E.D.*

A.2 Proof of Corollary 1

Proof: From (2) and (3), we have, for all $p \in (p_0, r)$,

$$F_2(p) = \frac{2(1-\mu)(p-c_2) - \mu(r-p)}{2(1-\mu)(p-c_1)} = \frac{p-c_2}{p-c_1}F_1(p)$$

$$= \left(1 - \frac{c_2 - c_1}{p - c_1}\right) F_1(p) < F_1(p),$$

where the inequality follows from $p > p_0 > c_2 > c_1$. It is then straightforward to verify that $F_2(p) \leq F_1(p)$ for all $p \in [c_2, r]$ and hence $P_2 \succeq_{st} P_1$. Since for $p \in (p_0, r)$, $F_2(p) < F_1(p)$, $P_1 \not\succeq_{st} P_2$. Therefore, $P_2 \succ_{st} P_1$. *Q.E.D.*

A.3 Proof of Proposition 3

Proof: We first prove these dominance relations when the share of naive consumers remains unchanged after the price cap. We then verify that the prices "decrease" even further when μ decreases from $\overline{\mu}$ to $\underline{\mu}$.

First, suppose $2c_2 - c_1 < \kappa < r$ or $\kappa < r < 2c_2 - c_1$. Then by Proposition 3, the equilibrium share of naive consumers is not affected by the price cap. By inspecting (9), it is easy to verify that $F_1^{r,\mu}(p) \leq F_1^{\kappa,\mu}(p)$ for all p in the extended support of $[c_2, r]$ and $F_1^{r,\mu}(p) < F_1^{\kappa,\mu}(p)$ for all $p \in (p_0^{\kappa,\mu}, \kappa)$. It then follows that $P_1^{r,\mu} \succ_{st} P_1^{\kappa,\mu}$.

Similarly, by inspecting (10), we have $P_2^{r,\mu} \succ_{st} P_2^{\kappa,\mu}$.

To show $P_{(1)}^{r,\mu} \succ_{st} P_{(1)}^{\kappa,\mu}$, note that the cumulative distribution functions of $P_{(1)}^{r,\mu}$ and $P_{(1)}^{\kappa,\mu}$ are, respectively,

$$F_{(1)}^{r,\mu}(p) = 1 - (1 - F_1^{r,\mu}(p))(1 - F_2^{r,\mu}(p))$$
$$F_{(1)}^{\kappa,\mu}(p) = 1 - (1 - F_1^{\kappa,\mu}(p))(1 - F_2^{\kappa,\mu}(p)).$$

Since $F_1^{r,\mu}(p) < F_1^{\kappa,\mu}(p)$ and $F_2^{r,\mu}(p) < F_2^{\kappa,\mu}(p)$ for $p \in (p_0^{\kappa,\mu}, \kappa)$, $F_{(1)}^{r,\mu}(p) < F_{(1)}^{\kappa,\mu}(p)$ for $p \in (p_0^{\kappa,\mu}, \kappa)$. Also, $F_{(1)}^{r,\mu}(p) = F_{(1)}^{\kappa,\mu}(p)$ for $p \in [c_2, p_0^{\kappa,\mu}] \cup [\kappa, r]$. It then follows that $P_{(1)}^{r,\mu} \succ_{st} P_{(1)}^{\kappa,\mu}$.

Second, suppose now that $\kappa < 2c_2 - c_1 < r$. By Proposition 3, the equilibrium share of naive consumers decreases from $\overline{\mu}$ to $\underline{\mu}$. Note that $F_i^{\kappa,\mu}$, $i = 1, 2$, decreases strictly in μ for all $p \in (p_0^{\kappa,\mu}, \kappa)$. Therefore, a reduction in μ further increases $F_i^{\kappa,\mu}$ and a similar proof as the above holds. *Q.E.D.*

A.4 Proof of Lemma 1

Proof: i) We first note the following two preliminary results:

$$\int \frac{\kappa - p}{(p - c_1)(p - c_2)^2} dp = \frac{\kappa - c_1}{(c_2 - c_1)^2} \ln \frac{p - c_1}{p - c_2} - \frac{\kappa - c_2}{(c_2 - c_1)(p - c_2)} + C,$$
$$\int \frac{1}{(p - c_1)(p - c_2)^2} dp = \frac{1}{(c_2 - c_1)^2} \ln \frac{p - c_1}{p - c_2} - \frac{1}{(c_2 - c_1)(p - c_2)} + C.$$

Then by definition (11) and making use the above results, we have $\alpha =$

$$
\begin{aligned}
&\int_{p_0^{\kappa,\mu}}^{\kappa} \left[1 - \frac{\mu(\kappa - p)}{2(1-\mu)(p-c_1)} - \frac{c_2 - c_1}{p - c_1}\right] \frac{\mu(\kappa - c_2)}{2(1-\mu)(p-c_2)^2} dp \\
=&\left\{ F_1|_{p_0^{\kappa,\mu}}^{\kappa} - \frac{\mu^2(\kappa - c_2)}{4(1-\mu)^2} \int_{p_0}^{\kappa} \frac{\kappa - p}{(p-c_1)(p-c_2)^2} dp \right. \\
&\qquad \left. - \frac{\mu(\kappa - c_2)(c_2 - c_1)}{2(1-\mu)} \int_{p_0^{\kappa,\mu}}^{\kappa} \frac{1}{(p-c_1)(p-c_2)^2} dp \right\} \\
=&\left\{ F_1 + \left[\frac{\mu(\kappa - c_2)}{2(1-\mu)(c_2 - c_1)} + 1\right] \frac{\mu(\kappa - c_2)}{2(1-\mu)(p-c_2)} \right. \\
&\qquad \left. - \left[\frac{\mu(\kappa - c_1)}{2(1-\mu)(c_2 - c_1)} + 1\right] \frac{\mu(\kappa - c_2)}{2(1-\mu)(c_2 - c_1)} \ln \frac{p - c_1}{p - c_2} \right\} \Bigg|_{p_0^{\kappa,\mu}}^{\kappa} \\
=&\frac{\mu(\kappa - c_2)}{2(1-\mu)(c_2 - c_1)} \cdot \\
&\qquad \left[\left(1 + \frac{\mu(\kappa - c_1)}{2(1-\mu)(c_2 - c_1)}\right) \ln \left(1 + \frac{2(1-\mu)(c_2 - c_1)}{\mu(\kappa - c_1)}\right) - 1\right]
\end{aligned}
$$

which is (12) after substitution.

ii) From Proposition 1, we know that $F_2^{\kappa,\mu}(p) < F_1^{\kappa,\mu}(p)$ for all $p \in (p_0^{\kappa,\mu}, r)$. Then,

$$\alpha = \int_{p_0^{\kappa,\mu}}^{\kappa} F_2^{\kappa,\mu}(p) dF_1^{\kappa,\mu}(p) < \int_{p_0^{\kappa,\mu}}^{\kappa} F_1^{\kappa,\mu}(p) dF_1^{\kappa,\mu}(p) = \frac{1}{2}.$$

Q.E.D.

A.5 Proof of Lemma 2

Lemma 2. α strictly increases in κ for all $\kappa \in (c_2, r)$.

Proof: We first show that

$$h(t) := t\left[(1+t)\ln\left(1 + \frac{1}{t}\right) - 1\right] \tag{18}$$

is strictly increasing in t for all $t > 0$. Note that by definition, $t = \frac{\mu(\kappa - c_1)}{2(1-\mu)(c_2 - c_1)} \in (0, +\infty)$. With this result, we then verify that α increases in κ when μ remains unchanged and when μ increases as κ increases.

Note that $\frac{dh}{dt} = (1+2t)\ln\left(1 + \frac{1}{t}\right) - 2$. To show $(1+2t)\ln\left(1 + \frac{1}{t}\right) > 2$, we let

$$g(t) := (1+2t)\ln\left(1 + \frac{1}{t}\right).$$

Since $\frac{d^2g}{dt^2} = \frac{-2}{t^2+t} - \frac{2(t^2+t)-(1+2t)^2}{(t^2+t)^2} = \frac{1}{(t^2+t)^2} > 0$ for all $t > 0$ and

$$\lim_{t\to+\infty} \frac{dg}{dt} = \lim_{t\to+\infty} 2\ln\left(1+\frac{1}{t}\right) - \frac{1+2t}{t^2+t} = 0,$$

$\frac{dg}{dt} < 0$ for all $t > 0$. Furthermore, by l'Hôpital's rule,

$$\lim_{t\to+\infty} g(t) = \lim_{t\to+\infty} \frac{\ln\left(1+\frac{1}{t}\right)}{\frac{1}{1+2t}} = \lim_{t\to+\infty} \frac{(1+2t)^2}{2(t^2+t)} = 2.$$

Therefore, $g(t) > 2$ for all $t > 0$. This proves $\frac{dh}{dt} > 0$ for all $t > 0$.

Now we check how α changes when the price limit κ increases. Note that $t = \frac{\mu(\kappa-c_1)}{2(1-\mu)(c_2-c_1)}$ strictly increases in κ and μ.

Firstly, if μ remains unchanged, as the price limit κ increases, t increases and therefore $h(t)$ increases. Because $\alpha = \left(\frac{\kappa-c_2}{\kappa-c_1}\right) h(t)$ and $\frac{\kappa-c_2}{\kappa-c_1}$ increases in κ, α also increases.

Secondly, if the increase in the price limit κ results in a switch from $\kappa < 2c_2 - c_1 < r$ to $2c_2 - c_1 < \kappa < r$, then the equilibrium obfuscation will increase from $\underline{\mu}$ to $\overline{\mu}$ (see Proposition 3). Nevertheless, the above still holds because t also increases in μ. *Q.E.D.*

A.6 Proof of Corollary 4

Proof: Suppose that an excise tax $0 < \tau < r - c_2$ is imposed. Note first that t as defined in Lemma 1 decreases after the introduction of τ. This is because $t(\tau) = \frac{\mu(\kappa-c_1-\tau)}{2(1-\mu)(c_2-c_1)}$ is clearly decreasing in τ while μ can only either remain unchanged or decrease following the introduction of the tax.

In section A.5, we have established that $h(t)$ is increasing in t. Moreover, $\frac{\kappa-c_2-\tau}{\kappa-c_1-\tau} < \frac{\kappa-c_2}{\kappa-c_1}$. It follows that

$$\alpha(\tau) = \left(\frac{\kappa-c_2-\tau}{\kappa-c_1-\tau}\right) h(t(\tau)) < \left(\frac{\kappa-c_2}{\kappa-c_1}\right) h(t(0)) = \alpha(0),$$

and hence α decreases after the excise tax τ is imposed. *Q.E.D.*

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