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Evidence accumulation account of human operators' decisions in intermittent control during inverted pendulum balancing

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Abstract—Human operators often employ intermittent, discontinuous control strategies in a variety of tasks. A typical intermittent controller monitors control error and generates corrective action when the deviation of the controlled system from the desired state becomes too large to ignore. Most contemporary models of human intermittent control employ simple, threshold-based trigger mechanism to model the process of control activation. However, recent experimental studies demonstrate that the control activation patterns produced by human operators do not support threshold-based models, and provide evidence for more complex activation mechanisms. In this paper, we investigate whether intermittent control activation in humans can be modeled as a decision-making process. We utilize an established drift-diffusion model, which treats decision making as an evidence accumulation process, and study it in simple numerical simulations. We demonstrate that this model robustly replicates the control activation patterns (distributions of control error at movement onset) produced by human operators in previously conducted experiments on virtual inverted pendulum balancing. Our results provide support to the hypothesis that intermittent control activation in human operators can be treated as an evidence accumulation process.

Index Terms—Human operator, Motor control, Decision making, Intermittent control

I. INTRODUCTION

In a variety of situations, ranging from postural balance to car driving to aircraft landing, human operators engage in motor behavior aimed at maintaining a system under control near a target state or trajectory. There is increasing evidence that in such tasks human operators use complex, non-linear control strategies, which have been subject to intensive modeling efforts (see [1]–[3] for review). A common thread running through much of the recent empirical and theoretical research is that human control is intermittent: an operator observes the state of the controlled system continuously, but applies corrective effort only intermittently. Opposing the traditional notion of continuous optimal control, the studies on intermittent control emphasize discrete, ballistic corrective movements separated by periods of inactivity (Fig. 1). Intermittent control is observed in a variety of laboratory and real-life tasks, including visuomotor tracking [4], inverted pendulum balancing [5], [6], postural balance [7], [8], and car driving [9], [10]. Such intermittent control strategies can be more efficient

than continuous feedback in the presence of sensorimotor delays and neural noise [11]. However, despite much interest from both applied and basic research communities, the theory of human intermittent control is still in its initial stages of development. The combination of key mechanisms in play during intermittent control is generally agreed upon, and includes delays, noise, prediction, open-loop control adjustments, and event-driven control activation [1], [2], [12]. However, the details of these mechanisms remain obscure and adequate computational models are often missing. The overarching aim of this paper is to shed light on one of these key mechanisms: control activation.

The majority of human intermittent control models rely on the assumption that corrective movements are triggered when the control error exceeds a fixed threshold (e.g. [1], [13]). This is normally attributed to *sensory deadzones*, i.e., the lack of operator's awareness of the small control errors. Empirical observations, however, indicate that human operators often ignore deviations which significantly exceed the perception threshold [9], [14]. Indeed, due to the difficulty of handling small deviations and metabolic costs of high-frequency control, human operators can choose to ignore acceptable deviations as long as this does not threaten the overall goal of the control process [6], [15]. In this case, transitions from passive to active control phase can no longer be reduced to threshold-driven triggering, and therefore require more advanced models.

The problem of control activation modeling has been treated previously using two related but distinct approaches. One class of models, *noise-driven activation*, provided phenomenological description of the activation process using, e.g., the notion of double-well attractor borrowed from physics [14], [16]. These simple models closely reproduce human operators' behavior observed experimentally in virtual inverted pendulum balancing. These models were validated specifically against the key characteristic of control activation, the distribution of *action points*, i.e., the magnitudes of control error triggering the corrective movements. However, noise-driven activation models only emphasize the intrinsic stochasticity of the process, but do not provide any insights into psychophysiological mechanisms of control activation, which suggests the need for

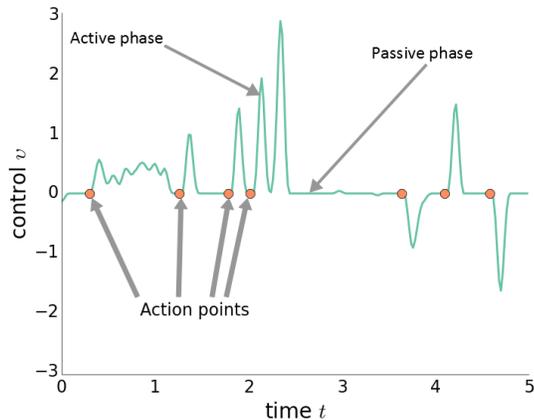


Fig. 1: Time pattern of human intermittent control observed in a virtual stick balancing experiment [14]. The *action points* are the values of control error (in this case, inverted pendulum tilt angle) triggering the onset of corrective movement (marked by orange circles). Figure file is available under CC-BY [19].

more cognitively plausible models.

Another set of models approaches control activation as a decision-making process [10], [12]. These models implicate stochastic *evidence accumulation* mechanisms in control activation. The evidence accumulation mechanisms are supported by much neuroscientific work on decision-making [17], [18], thereby providing a biologically grounded interpretation of the control onset emergence. Moreover, a general intermittent model involving as a part an evidence accumulation mechanism was shown to reproduce amplitudes and timing of steering adjustments exhibited by human drivers in a lane-keeping task [12]. However, this general model encapsulates many assumptions regarding other control mechanisms (e.g., prediction, movement generation, delays), which precludes one from disentangling the effects of these mechanisms on control activation. Furthermore, unlike the noise-driven activation model, the evidence accumulation model has not yet been confronted to human action points data, which limits its validity as a general control activation framework.

In this paper we aim to bridge gap between the two above approaches. We propose a decision-making model of control activation based on noisy bounded evidence accumulation and demonstrate that it reproduces in details the activation patterns observed in human operators in a virtual balancing task (Fig. 2). Our results provide support to the hypothesis that intermittent control activation in human operators can be treated as an evidence accumulation process.

II. DRIFT-DIFFUSION MODEL OF CONTROL ACTIVATION IN INVERTED PENDULUM BALANCING TASK

Background

The currently predominant view on human decision making is that it operates as an evidence accumulation process, which is usually described by different variants of drift-diffusion

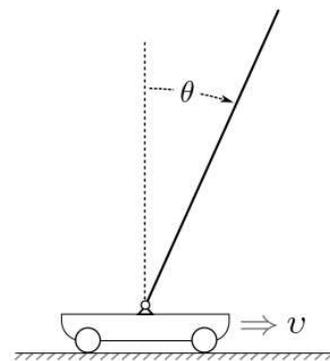


Fig. 2: Inverted pendulum on a cart. This simple mechanical system was previously used to investigate control activation patterns in virtual balancing task [14]. Figure file is available under CC-BY [20].

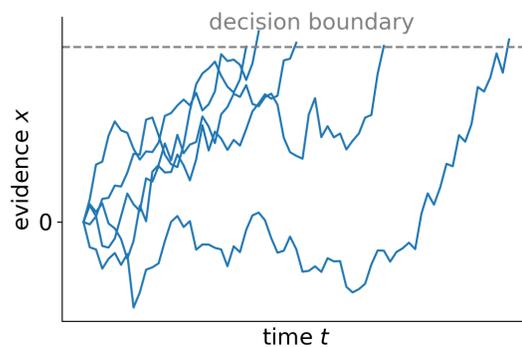


Fig. 3: Time patterns of evidence accumulation produced in five individual trials by a drift-diffusion model. The decision is made when accumulated evidence (driven by deterministic drift and random diffusion) reaches a decision boundary. Figure file is available under CC-BY [23].

model (DDM) [21], [22]. This model posits that, in a simple stimulus detection task, a decision maker continuously samples and accumulates the evidence in favor of the stimulus being present. The rate of accumulation depends on the stimulus intensity. In addition, integration of evidence over time is necessarily affected by noise in neural firing rates. When the amount of accumulated evidence exceeds a pre-defined boundary, the decision is made that the stimulus is present (Fig. 3).

It has been shown that DDM and its numerous flavors can remarkably well capture the error rates and reaction time distributions exhibited by human subjects in a variety of perceptual tasks (see [18] for a recent review). Moreover, neural recordings of brain activity in non-human primates suggest that similar accumulation-to-boundary mechanisms are implemented in lateral intraparietal cortex associated with simple perceptual decisions [17], [24], [25]. Similar accumulative processes have also been directly traced in human neuroimaging studies [26],

[27]. However, despite its simplicity and cognitive plausibility, the applications of this model to human control are still scarce, which warrants more detailed investigations.

Model

Mathematically, DDM is described by a stochastic differential equation

$$dx = A dt + c\xi\sqrt{dt}, \quad (1)$$

where $x(t)$ is the evidence accumulated at time t , A is the drift rate parameter associated with the stimulus strength, c is the diffusion rate, or noise intensity, and ξ is white Gaussian noise. Finally, the boundary parameter b defines the critical amount of evidence at which the decision is made.

In the context of intermittent control, we assume that during the passive control phase, a human operator continuously observes the control error and accumulates evidence in favor of activating the control according to Eq. (1). This suggests that the drift rate A may be dependent on control error, making the control error analogous to the “stimulus strength” in conventional DDM paradigms [21], [26]. In our model, we simply assume

$$A = \theta,$$

where θ is control error. When the accumulated evidence reaches the decision boundary, the corrective action is launched.

We exemplify the model using a simple balancing task, in which a human operator maintains an overdamped inverted pendulum upright by adjusting the position of the moving cart connected with the pendulum via a pivot (Fig. 2). Control activation patterns in non-expert human operators were investigated previously using a virtual version of this task, where the operators controlled the cart velocity by moving a computer mouse [14], [28]. In this case, control error θ is represented simply by pendulum tilt angle. The dynamics of the task is described by the equation

$$d\theta = \theta dt + v dt, \quad (2)$$

where θ is the pendulum tilt angle, and the control variable v is the velocity of the moving cart controlled directly by human operator (here the linear coefficients specifying the temporal scale of the system dynamics are all assumed to be equal to 1 without any loss of generality). In the passive control phase ($v = 0$), θ increases exponentially until the pendulum falls (for any non-zero initial disturbance). Therefore, in the model (1), we assume

$$A = A(t) = \theta(t) = \theta_0 e^t, \quad (3)$$

where t is the time passed since the start of each passive phase, and θ_0 is the value of control error at the start of that passive phase.

The previously reported analysis of human performance in the above task revealed that for every subject the values of action points (pendulum tilt angles at which control was activated, see Fig. 1) followed exponential distribution with a peak at the angle close to zero [14]. This implies that the

operators often ignored much larger control errors than suggested by a threshold-based activation model. In what follows we investigate whether the DDM-like model can reproduce this exponential distribution of action points.

III. MODEL SIMULATION

In the simplest case $A \equiv \text{const}$, the model (1) admits analytical solution [22]. However, for time-dependent A derivation of the closed-form solution is non-trivial and deserves individual consideration. For this reason, here we study the properties of the model only via numerical simulation.

As mentioned above, our main focus here is the control activation mechanism; modeling corrective adjustments in the active control phase is outside the scope of this paper. Therefore, for the present purposes it is sufficient to perform open-loop simulations of the model (1) in a sequence of independent control activation trials, rather than embedding the model in a closed-loop simulation of the whole balancing process. In the beginning of each trial, we reset the control error θ to a baseline value, and then simulate evidence accumulation over time according to Eqs. (1–3) until the decision boundary is hit. For each tested combination of model parameters, we simulate 1000 trials, and then calculate the distribution of action point values obtained for each trial.

The model has two free parameters, diffusion rate c and decision boundary b . To estimate the range of possible behaviors of the model (in terms of action point distributions), we first performed exhaustive search over the discrete logarithmic grid based on the range of plausible parameter values $b, c \in [10^{-1}, 10^1]$. This allowed us to narrow down the plausible parameter values to $b, c \in [10^{-1}, 10^0]$. Here we report the results for the parameter combinations in this range, which yield the distributions most similar to the one obtained from the experimental data.

Note that, after controlling for the scale of pendulum fluctuations which varied with subjects’ balancing skill, the action point distribution was universal for all participants [14]. For this reason, in the model we aimed to reproduce the *z-scored* action points, i.e., measured not in radian but in standard deviations of the pendulum angle θ .

First, we analyzed the model’s behavior for the setup when in the beginning of each trial, the pendulum tilt angle θ was set to a *fixed* baseline value, $\theta_0 = 0.5$. The results for $b, c \in \{0.2, 0.4, 0.6, 0.8\}$ are represented in Fig. 4. In general, the model failed to explain the action point distribution observed in human operators. The closest fit was observed around $b = 0.8, c = 0.8$ (fourth row, fourth column in Fig. 4). Still, the best-fit model distribution differs considerably from the experimental one. Local optimization using a quasi-Newton method did not substantially improve the fit we obtained by grid search.

The failure of the model to capture human control activation pattern suggested that either 1) the DDM is not an adequate model of control activation, or 2) some factors other than evidence accumulation affect the action point distribution exhibited by human operators. To clarify this, we adopted

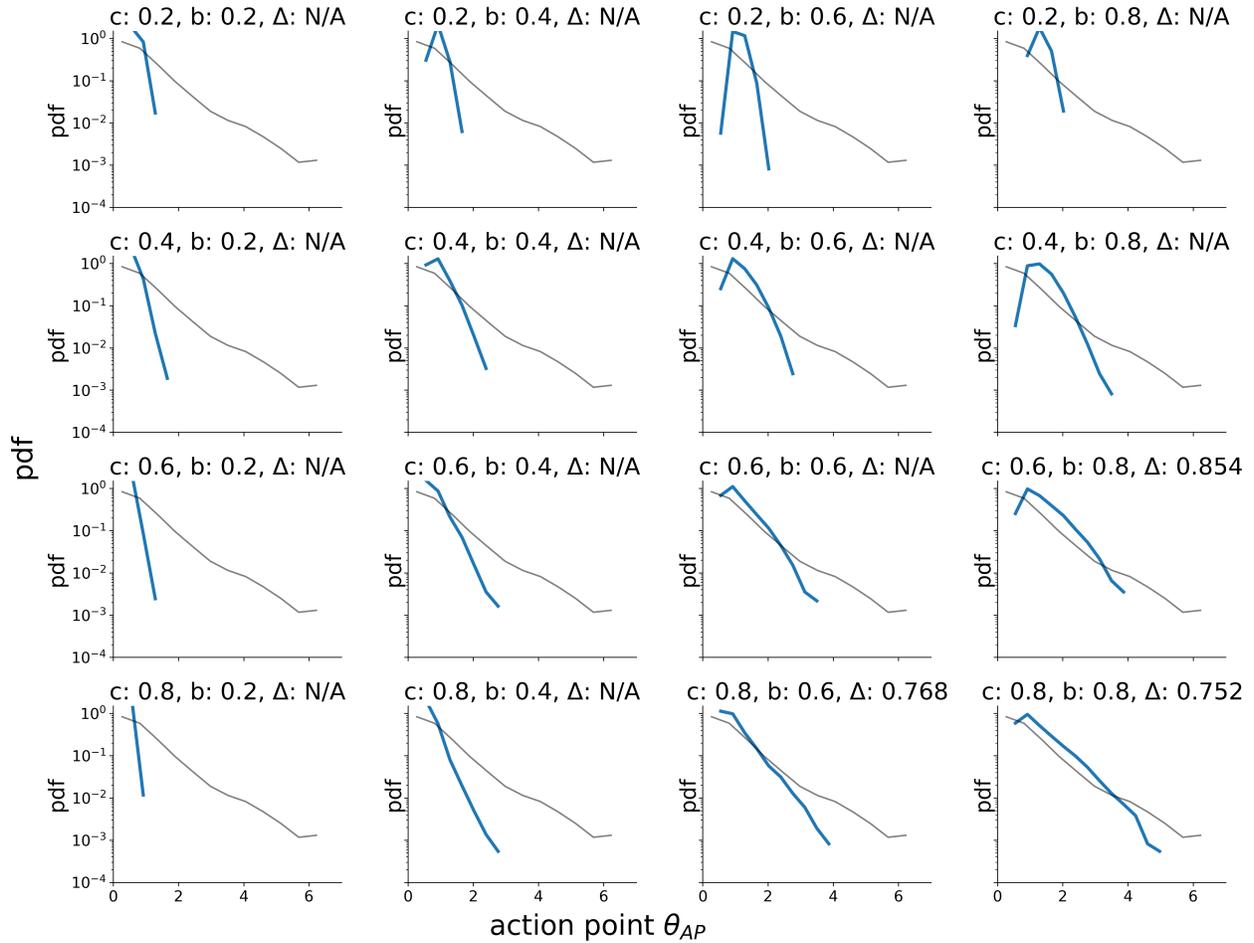


Fig. 4: Probability distribution functions (pdf) of action point values produced by the model (1) (blue lines). Open-loop numerical simulations were run, with the initial control error fixed at $\theta = 0.5$ in each trial. Black lines indicate the pdf produced by human subjects in the previously conducted experiment on virtual inverted pendulum balancing [14]. Here action points are dimensionless, i.e., measured not in radian but in standard deviations of θ , following Ref. [14]. Values of the diffusion rate c and the decision boundary parameters are indicated in each panel. An approximate measure of fitness is Δ , mean difference between the log-scaled experimental and model pdf's over the range of action point values. We did not calculate the fitness ($\Delta = \text{N/A}$) in case the range of the model action point values covered less than 50% of the experimentally obtained range. Figure file is available under CC-BY [23].

an additional assumption: the action point distribution is also affected by initial control error, i.e., the starting position of the inverted pendulum in the beginning of passive phase. In the second numerical simulation, the initial value of the pendulum tilt angle at each trial was not fixed, but instead randomly drawn from the distribution of actual starting positions observed previously in human operators¹. The results indicate that with this additional assumption, the DDM model explains the human control activation pattern observed experimentally (Fig. 5).

In the present paper we did not aim to find the best-fitting parameter values; more important was to demonstrate the overall plausibility of the model over a range of possible

parameter values. Nevertheless, it is worth noting that the model reproduced the experimental distribution best at the intermediate values of the boundary parameter b and the diffusion rate parameter c being slightly greater than b . The best fit was achieved at $c = 0.8, b = 0.6$ (fourth row, third column in Fig. 5)². The fact that the model matches the experimental data robustly for a range of possible parameter values suggests that human control activation can be described as a decision-making process based on the evidence accumulation mechanism (at least in the considered task).

¹The experimentally obtained distribution of pendulum tilt angle in the beginning of each passive phase was also exponential, but was narrower than the action point distribution

²Similarly to the case of fixed baseline control error, local quasi-Newton optimization in the vicinity of the grid nodes with the lowest Δ improved the fit only marginally

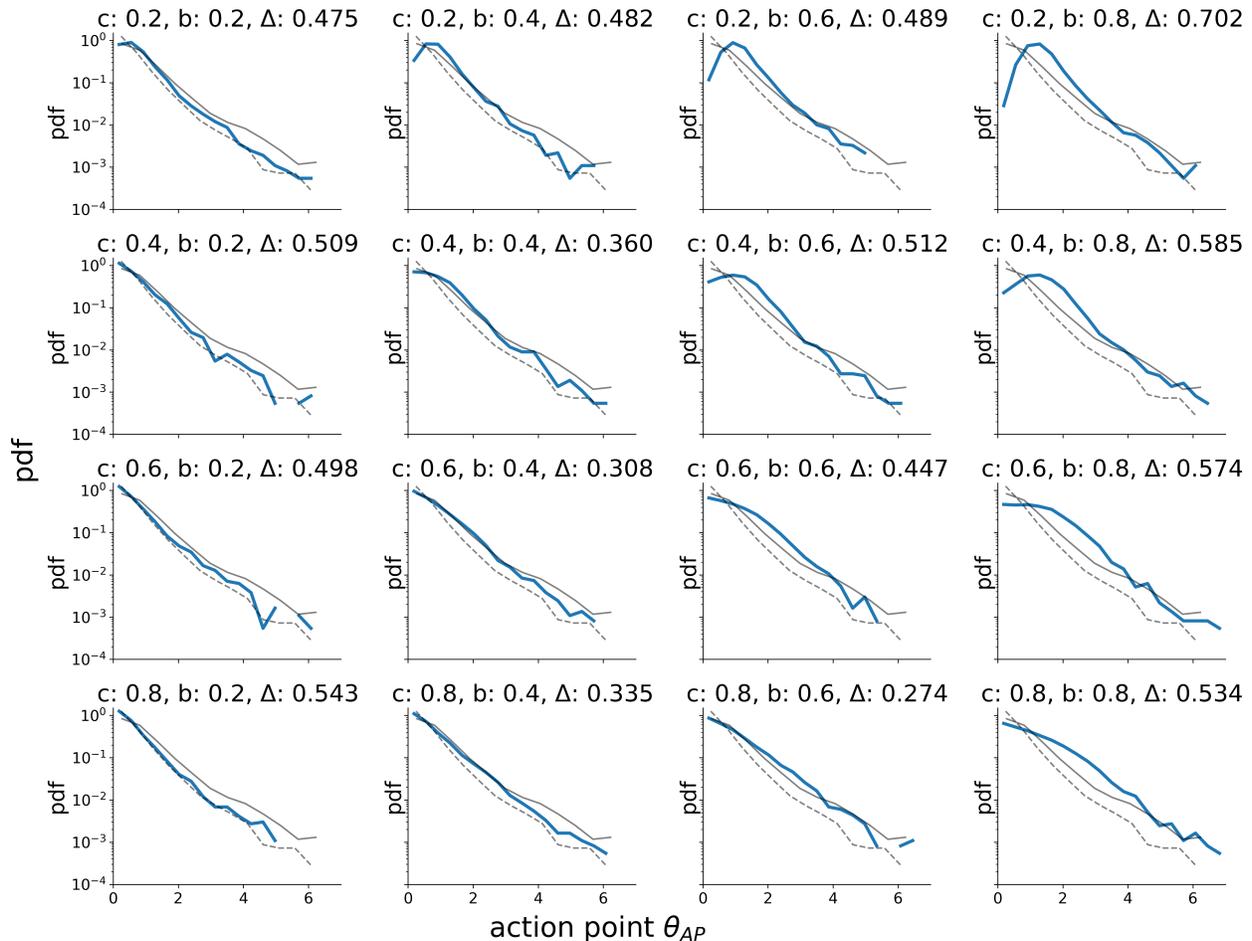


Fig. 5: Same as Fig. 4, but with the initial control error drawn from the distribution obtained from human subjects (dashed lines denote pdf of initial control error). Figure file is available under CC-BY [23].

IV. DISCUSSION

In this study we aimed to investigate whether control activation in human operators can be treated as a decision-making process. The model based on the established notion of evidence accumulation could reproduce the pattern of control activation observed in human operators, in particular, the distribution of control error triggering corrective adjustments (action points). These preliminary results may have implications for models of human performance in a wide variety of processes where human control is intermittent.

Our study is not the first one to use a drift-diffusion model in an intermittent control model. The predecessor studies included evidence accumulation as one of the building blocks in a general framework of sustained sensorimotor control, exemplified by a model of lane keeping while driving a car [10], [12]. However, these studies did not assess specifically control activation patterns, focusing instead on reproducing the characteristics of corrective adjustments (including, among others, the joint distributions of their amplitudes and inter-adjustment times). Our work addresses this issue and thereby reinforces the notion of evidence accumulation as an integral

part of future intermittent control models.

On the other hand, a class of noise-driven activation models has been previously shown to accurately reproduce in detail the control activation patterns in inverted pendulum balancing [14], [16]. This suggests that, under some conditions, double-well attractor model and drift-diffusion model might be equivalent on the computational level. This has potential implications for wider areas of decision-making research: attractor models are increasingly popular in cognitive science [29]–[31], but their connection to the evidence accumulation models remains unexplored. This is an important avenue for future studies.

In this paper, we only make on of the first steps in investigating the complex relationship between evidence accumulation and control activation. An important limitation of the present work is that the human-operated overdamped inverted pendulum is a relatively simple control system. This has allowed us to directly measure action points both in the model and human data, but may potentially limit direct applicability of our results to more sophisticated control systems, where the effects of neural delays, prediction, and motor noise are more essential. Designing a DDM-based control activation model for a more complex human-controlled process will necessarily

require meticulous search for a perceptual quantity treated as as a control error by the operators. Even in simple inverted pendulum balancing, changing the perceptual cue available to the operator can drastically change the distribution of action points [28]. This can be captured on a phenomenological level by an abstract attractor model [32]; further studies should investigate if and how the decision-making account of control activation can mechanistically explain such changes.

Another potentially fruitful direction for future work is the extension of the behavioral study of human control with concurrently recorded neuroimaging data, since the evidence accumulation account can be leveraged to make direct predictions about brain activation as well [26], [27].

Despite their simplicity and cognitive plausibility, evidence accumulation models are rarely used in applied studies on human performance. We believe that empirical validation of these models in processes controlled by human operators will be beneficial to the wider field of human-machine interaction.

REFERENCES

- [1] P. Gawthrop, I. Loram, M. Lakie, and H. Gollee, "Intermittent control: a computational theory of human control," *Biological cybernetics*, vol. 104, no. 1-2, pp. 31–51, 2011.
- [2] J. Milton, "Intermittent Motor Control: The 'drift-and-act' Hypothesis," in *Progress in Motor Control*. Springer, 2013, pp. 169–193.
- [3] Y. Asai, S. Tateyama, and T. Nomura, "Learning an Intermittent Control Strategy for Postural Balancing Using an EMG-Based Human-Computer Interface," *PLoS One*, vol. 8, no. 5, p. e62956, 2013.
- [4] R. Bormann, J.-L. Cabrera, J. G. Milton, and C. W. Eurich, "Visuomotor tracking on a computer screen—an experimental paradigm to study the dynamics of motor control," *Neurocomputing*, vol. 58, pp. 517–523, 2004.
- [5] J. Cabrera and J. Milton, "On-off intermittency in a human balancing task," *Physical Review Letters*, vol. 89, no. 15, p. 158702, 2002.
- [6] I. Loram, H. Gollee, M. Lakie, and P. Gawthrop, "Human control of an inverted pendulum: is continuous control necessary? Is intermittent control effective? Is intermittent control physiological?" *The Journal of Physiology*, vol. 589, no. 2, pp. 307–324, 2011.
- [7] J. J. Collins and C. J. De Luca, "Open-loop and closed-loop control of posture: a random-walk analysis of center-of-pressure trajectories," *Experimental Brain Research*, vol. 95, no. 2, pp. 308–318, 1993.
- [8] I. Loram, C. Maganaris, and M. Lakie, "Human postural sway results from frequent, ballistic bias impulses by soleus and gastrocnemius," *The Journal of Physiology*, vol. 564, no. 1, pp. 295–311, 2005.
- [9] P. Wagner and I. Lubashevsky, "Empirical basis for car-following theory development," *arXiv preprint cond-mat/0311192*, 2003.
- [10] G. Markkula, "Modeling driver control behavior in both routine and near-accident driving," in *Proceedings of the Human Factors and Ergonomics Society Annual Meeting*, vol. 58, no. 1. SAGE Publications Sage CA: Los Angeles, CA, 2014, pp. 879–883.
- [11] J. Milton, R. Meyer, M. Zhvanetsky, S. Ridge, and T. Insperger, "Control at stability's edge minimizes energetic costs: expert stick balancing," *Journal of The Royal Society Interface*, vol. 13, no. 119, p. 20160212, 2016.
- [12] G. Markkula, E. Boer, R. Romano, and N. Merat, "Sustained sensorimotor control as intermittent decisions about prediction errors: Computational framework and application to ground vehicle steering," *Biological Cybernetics*, Feb 2018. [Online]. Available: <https://doi.org/10.1007/s00422-017-0743-9>
- [13] J. Milton, T. Ohira, J. Cabrera, R. Fraiser, J. Gyorffy, F. Ruiz, M. Strauss, E. Balch, P. Marin, and J. Alexander, "Balancing with vibration: a prelude for "drift and act" balance control," *PLoS One*, vol. 4, no. 10, p. e7427, 2009.
- [14] A. Zgonnikov, I. Lubashevsky, S. Kanemoto, T. Miyazawa, and T. Suzuki, "To react or not to react? Intrinsic stochasticity of human control in virtual stick balancing," *Journal of the Royal Society Interface*, vol. 11, no. 99, p. 20140636, 2014.
- [15] E. Boer, "Car following from the driver's perspective," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 2, no. 4, pp. 201–206, 1999.
- [16] A. Zgonnikov and I. Lubashevsky, "Double-well dynamics of noise-driven control activation in human intermittent control: the case of stick balancing," *Cognitive Processing*, vol. 16, no. 4, pp. 351–358, 2015.
- [17] J. I. Gold and M. N. Shadlen, "The neural basis of decision making," *Annu. Rev. Neurosci.*, vol. 30, pp. 535–574, 2007.
- [18] R. Ratcliff, P. L. Smith, S. D. Brown, and G. McKoon, "Diffusion Decision Model: Current Issues and History," *Trends in cognitive sciences*, vol. 20, no. 4, pp. 260–281, 2016.
- [19] A. Zgonnikov, "Example of cart velocity time series in virtual stick balancing task." [Online]. Available: <http://dx.doi.org/10.6084/m9.figshare.3969111>
- [20] —, "Overdamped stick balancing task." [Online]. Available: <http://dx.doi.org/10.6084/m9.figshare.3505778>
- [21] R. Ratcliff, "A theory of memory retrieval," *Psychological review*, vol. 85, no. 2, p. 59, 1978.
- [22] R. Bogacz, E. Brown, J. Moehlis, P. Holmes, and J. D. Cohen, "The physics of optimal decision making: a formal analysis of models of performance in two-alternative forced-choice tasks." *Psychological review*, vol. 113, no. 4, p. 700, 2006.
- [23] A. Zgonnikov and G. Markkula, "Figures for "Evidence accumulation account of human operators' decisions in intermittent control during inverted pendulum balancing";" Jul 2018. [Online]. Available: <http://dx.doi.org/10.6084/m9.figshare.6854681>
- [24] M. N. Shadlen and W. T. Newsome, "Neural basis of a perceptual decision in the parietal cortex (area LIP) of the rhesus monkey," *Journal of neurophysiology*, vol. 86, no. 4, pp. 1916–1936, 2001.
- [25] J. D. Roitman and M. N. Shadlen, "Response of neurons in the lateral intraparietal area during a combined visual discrimination reaction time task," *The Journal of neuroscience*, vol. 22, no. 21, pp. 9475–9489, 2002.
- [26] R. G. O'Connell, P. M. Dockree, and S. P. Kelly, "A supramodal accumulation-to-bound signal that determines perceptual decisions in humans," *Nature neuroscience*, vol. 15, no. 12, p. 1729, 2012.
- [27] B. M. Turner, C. A. Rodriguez, T. M. Norcia, S. M. McClure, and M. Steyvers, "Why more is better: Simultaneous modeling of EEG, fMRI, and behavioral data," *NeuroImage*, vol. 128, pp. 96–115, 2016.
- [28] A. Zgonnikov, S. Kanemoto, I. Lubashevsky, and T. Suzuki, "How the Type of Visual Feedback Affects Actions of Human operators: The Case of Virtual Stick Balancing," in *Systems, Man, and Cybernetics (SMC), 2015 IEEE International Conference on*. IEEE, 2015, pp. 1100–1103.
- [29] M. M. van Rooij, L. H. Favela, M. Malone, and M. J. Richardson, "Modeling the Dynamics of Risky Choice," *Ecological Psychology*, vol. 25, no. 3, pp. 293–303, 2013.
- [30] S. Scherbaum, S. Frisch, S. Leiberg, S. J. Lade, T. Goschke, and M. Dshemuchadse, "Process dynamics in delay discounting decisions: An attractor dynamics approach," *Judgment and Decision Making*, vol. 11, no. 5, pp. 472–495, 2016.
- [31] A. Zgonnikov, A. Aleni, P. Piironen, D. O'Hora, and M. di Bernardo, "Decision landscapes: visualizing mouse-tracking data," *Royal Society open science*, vol. 4, no. 11, p. 170482, 2017.
- [32] A. Zgonnikov and I. Lubashevsky, "Non-equilibrium phase transition in the model of human virtual stick balancing," in *Proceedings of the ISCIE International Symposium on Stochastic Systems Theory and its Applications*, vol. 2017. The ISCIE Symposium on Stochastic Systems Theory and Its Applications, 2017, pp. 20–24.