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# FRA and EKF Based State of Charge Estimation of Zinc-nickel Single Flow Batteries

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**Abstract.** The reliable state of charge (SOC) estimation is indispensable for flow batteries to maintain the safe and reliable operation. The widely adopted Extended Kalman filter (EKF) algorithm is a model-based method, however, the uncertainties in battery model will cause large errors in SOC estimation. An accurate battery model is the essence to capture the behaviors of batteries. In this paper, a novel framework for the SOC estimation of Zinc-nickel flow batteries is proposed based on the fast recursive algorithm (FRA) and extended Kalman filter (EKF). The FRA is firstly used to determine the model structure and identify the model parameters. Due to merits of FRA, a compact and accurate model of flow battery is built. Then, the SOC is estimated using the EKF based on the proposed linear-in-the-parameter model. Experimental studies and resultant simulations manifest the modelling accuracy of the proposed SOC estimation framework.

**Keywords:** flow battery, state of charge(SOC), fast recursive algorithm (FRA), extended Kalman filter(EKF)

## 1 Introduction

Batteries have become increasingly popular in smart grid and electric vehicles (EV) applications for energy storage. The redox flow battery (RFB) is characterized by the long life cycles and high charging/discharging efficiency, and has undergone rapid development in recent years [1]. Cheng and Zhang et al. [2] have proposed a novel RFB system, namely the single flow zinc-nickel battery system. Zinc-nickel single flow batteries (ZNBs), known as electrochemical energy storage sources, have attracted a lot of attention due to their advantages of high energy density, safety, and low cost [3]. The advantages of ZNBs include moderate cost, modularity, transportability and flexible operation [4]. Similar to solid batteries and other type of flow batteries, the battery management system (BMS) is vital for zinc-nickel flow batteries to ensure the optimal, reliable and

efficient operation, and to provide accurate battery internal state information for the energy management modules is an important issue to address in the BMS [5]. In particular, the battery state of charge (SOC) estimation is an essential part of a BMS, it provides fundamental knowledge about the real-time remaining capacity and energy of the battery.

During the last decade, a large number of real-time SOC estimation methods have been developed. The methods can be classified into four groups, including the looking-up table based methods, ampere-hour integral method, model-based estimation methods, and data-driven estimation methods [6]. Among these groups of methods, the looking-up table based approach is more suitable for laboratory environment. While the reliability of the ampere-hour method is affected by measurement errors and available capacity. The data-driven methods are very sensitive to their identified parameters extracted from the training data, though they may achieve high prediction accuracy. Contrary to model-free methods, the model-based estimation methods require accurate battery models. Electrochemical model (EM), equivalent circuit model (ECM) and black-box model are among the most commonly used models. These models often require to incorporate state estimation algorithms and adaptive filters to estimate and infer the internal states of batteries, such as open circuit voltage (OCV), the state of charge (SOC), and the state of health (SOH), etc. Among these methods, the extended Kalman filter (EKF) and other Kalman filter variants are widely adopted for SOC estimation.

In this work, the fast recursive algorithm (FRA) [7] is applied for battery model identification. The relationship between battery terminal signals and SOC is first interpreted by a linear-in-the-parameters model, then EKF based SOC estimation is presented.

This paper is organized as follows. A brief review of the FRA is presented in Section 2. Then the battery state equations deduced by FRA is introduced in Section 3, and the EKF based battery SOC estimation method is also described in details. Section 4 validates the efficacy of the SOC estimation and the proposed batteries models. Finally, Section 5 concludes the paper.

## 2 Preliminaries

The accuracy of battery SOC estimation heavily depends on the accuracy of the battery model. This section first gives a brief introduction to the state of charge (SOC) of batteries, followed by the fast recursive algorithm used for the model identification.

### 2.1 Battery state of charge (SOC)

SOC is an important internal state that describes the ratio of the remaining capacity to the present capacity of a battery. The relationship between the SOC at time instant  $k$  and SOC at time instant  $k + 1$  can be calculated below [8].

$$SOC_{k+1} = SOC_k + \frac{\eta \Delta t}{Q} I_k \quad (1)$$

where  $Q$  is the nominal capacity,  $\eta$  is the coulombic efficiency,  $\Delta t$  is the sampling interval, and  $I_k$  is the current at time instant  $t$ . When the battery is charging, the current value is assumed positive, vice versa. Since SOC calculation based on Eq.(1) is subject to a number of uncertainties associated to the initial value estimation of SOC, and noises and inaccuracies introduced into the terminal current measurements. Therefore, both a good model to capture the relationship of SOC with a set of readily available measurements and the EKF for SOC estimation subject to measurement noises and other inaccuracies are employed.

## 2.2 Fast recursive algorithm for model identification

The fast recursive algorithm (FRA) is an efficient method for nonlinear dynamic system identification and modeling developed by Kang and his co-workers [7]. FRA is able to select and determine both the model structure and the model parameters simultaneously. In this work, the FRA is used to correlate the nonlinear relationship between the battery terminal voltage and SOC, which is described by a linear-in-parameter equation.

A normal nonlinear discrete-time dynamic system can be represented in a matrix form as follows:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{\Theta} + \mathbf{\Xi} \quad (2)$$

Where  $\mathbf{y} = [y(1), \dots, y(m)]^T$  are the system outputs,  $\mathbf{\Phi} = [\varphi_1, \dots, \varphi_n]$  is the regression matrix and each  $\varphi_i = [\varphi_i(1), \dots, \varphi_i(m)]^T$ , ( $i = 1, \dots, n$ ) contains all candidate model terms,  $\mathbf{\Theta} = [\theta_1, \dots, \theta_n]^T$  and  $\theta_i$  ( $i = 1, \dots, n$ ) is the unknown parameters to be identified, and  $\mathbf{\Xi} = [\xi_1, \dots, \xi_m]^T$  is the model residual matrix. Two recursive matrixes  $M_k$  and  $R_k$ , are predefined in FRA to fulfill the forward model selection procedure as:

$$\mathbf{M}_k = \mathbf{\Phi}_k^T \mathbf{\Phi}_k \quad (3)$$

$$\mathbf{R}_k = \mathbf{I} - \mathbf{\Phi}_k \mathbf{M}_k^{-1} \mathbf{\Phi}_k^T \quad (4)$$

where  $\mathbf{\Phi}_k$  contains the first  $k$  columns of the full regression matrix  $\mathbf{\Phi}$ , additionally,  $k = 1, \dots, n$ , and  $\mathbf{R}_0 = \mathbf{I}$ . When  $\{\varphi_i, i = 1, 2, \dots, n\}$  in  $\mathbf{\Phi}$  are mutually linearly independent, the recursive matrix  $\mathbf{R}_k$  will has the following distinguished properties [9]:

$$\mathbf{R}_{k+1} = \mathbf{R}_k - \frac{\mathbf{R}_k \varphi_{k+1} \varphi_{k+1}^T \mathbf{R}_k^T}{\varphi_{k+1}^T \mathbf{R}_k \varphi_{k+1}}, \quad k = 0, 1, \dots, (n-1) \quad (5)$$

$$\mathbf{R}_k^T = \mathbf{R}_k, \quad \mathbf{R}_k \mathbf{R}_k = \mathbf{R}_k, \quad k = 0, 1, \dots, n \quad (6)$$

$$\mathbf{R}_k \mathbf{R}_j = \mathbf{R}_j \mathbf{R}_k = \mathbf{R}_k, \quad k \geq j; \quad k, j = 0, 1, \dots, n \quad (7)$$

$$\mathbf{R}_k \varphi_i = 0, \quad \forall i \in \{1, \dots, k\} \quad (8)$$

Assuming  $E_k$  denotes the cost function. When the first  $k$  columns in  $\mathbf{\Phi}$  are selected, and  $E_k$  can be expressed as

$$E_k = \mathbf{y}^T \mathbf{R}_k \mathbf{y} \quad (9)$$

To simplify the formulas and decrease the computational complexity, three quantities are consequently defined as

$$\begin{cases} \varphi_i^{(k)} \triangleq \mathbf{R}_k \varphi_i, & \varphi_i^{(0)} \triangleq \mathbf{R}_0 \varphi_i = \varphi_i \\ a_{k,i} \triangleq \left( \varphi_k^{(k-1)} \right)^T \varphi_i^{(k-1)}, & a_{1,i} \triangleq \varphi_1^T \varphi_i \\ a_{k,y} \triangleq \left( \varphi_k^{(k-1)} \right)^T \mathbf{y}, & a_{1,y} \triangleq \left( \varphi_1^{(0)} \right)^T \mathbf{y} = \varphi_1^T \mathbf{y} \end{cases} \quad (10)$$

where  $i = 1, \dots, n$ , and  $k = 0, 1, \dots, n$ . According to the properties of  $\mathbf{R}_k$  and the new quantities definition, the net contribution of the selected model term  $\varphi_{k+1}$  to the cost function can be calculated as

$$\Delta E_{k+1} = \frac{\left( \mathbf{y}^T \varphi_{k+1}^{(k)} \right)^2}{\left( \left( \varphi_{k+1}^{(k)} \right)^T \varphi_{k+1}^{(k)} \right)} = \frac{\left( a_{k+1,y}^T \right)^2}{a_{k+1,k+1}}, \quad k = 0, 1, \dots, n-1 \quad (11)$$

By calculating the net contribution of each term, the model terms with maximum contributions will be selected. Then an effective formula will be given for model parameters identification procedure as follows:

$$\hat{\theta}_j = \frac{a_{j,y} - \sum_{i=j+1}^k \hat{\theta}_i a_{j,i}}{a_{j,j}}, \quad j = k, k-1, \dots, 1 \quad (12)$$

Equitation (11) and (12) constitute the FRA, which solves the least-squares problem recursively.

### 3 Battery SOC estimation

This section introduces the SOC estimation based on the EKF algorithm. The main idea is to use the model built by FRA to predict the terminal voltage in real-time which is then compared with the actual measured voltage signals. The estimated error is used to update the SOC estimation using the Extended Kalman Filter (EKF).

#### 3.1 Battery state space model

The detailed EKF algorithm for battery SOC estimation can be found in [10]. As discussed before, to have an accurate estimation of the SOC, a good battery model is essential. As described in Section 1, EM, ECM and data-driven black-box models are three popular types of battery models. The EMs are the most accurate, but they are hard to establish because they require detailed first principle knowledge and the computational complexity restricts its real-time applications. The ECMs are expressed by a combination of voltage and current source, capacitance, and resistance, where the resistance of the single flow Zinc-Nickle battery model changes with the battery SOC, this varying parameter

may introduce error in SOC estimation [11]. The black-box models describe the relationship between the voltage and SOC using nonlinear functions, and do not require detailed first principle knowledge of the battery. FRA is an effective method to build a mathematical model. In this paper, the state space model of the battery is deduced using the coulomb counting equation for the SOC and FRA, where the SOC is an model state and its relationship with readily measurable terminal voltages and terminal currents are established. The state equation is expressed as follows:

$$\begin{cases} SOC_{k+1} = SOC_k + \frac{\eta \Delta t}{Q} I_k + w_k \\ Z_{k+1} = h(SOC_{k+1}, I_{k+1}) + v_k \end{cases} \quad (13)$$

where  $Z_k$  is the terminal voltage at time instant  $k$ ,  $w_k$  and  $v_k$  are the process noise and measurement noise respectively,  $I_k$  denotes the terminal current signals. To capture the relationship of  $Z_k$  with SOC and terminal voltage, some linear or nonlinear functions associated to the  $SOC_k$  are selected from a predefined model candidate pool using the FRA method. The candidate pool consists of current, terminal voltage, SOC, their nonlinear forms, and nonlinear combinations of these two or three terms. Then, using the FRA approach, the system parameter identification and the selection of nonlinear terms associated to the model are conducted simultaneously.

### 3.2 EKF based SOC estimation

To produce an accurate estimation of the SOC based on the real-time measurements of terminal currents and voltages, the EKF algorithm is applied to the battery SOC model (Eq. (13)), and the EKF is briefly introduced below. The nonlinear battery state-space model of the EKF are shown in Equation (14),

$$\begin{cases} SOC_{k+1} = f(SOC_k, I_k) + w_k \\ Z_{k+1} = h(SOC_{k+1}, I_{k+1}) + v_k \end{cases} \quad (14)$$

where function  $f(SOC_k, I_k)$  is the nonlinear state transition function and function  $h(SOC_k, I_k)$  is the nonlinear measurement function. Vector  $w_k$  and  $v_k$  are uncorrelated zero-mean white Gaussian noise with covariance matrixes  $Q$  and  $R$  respectively.

At each time step, the function  $f(SOC_k, I_k)$  and  $h(SOC_k, I_k)$  are linearized using Taylor-series expansion, assuming that they are differentiable at all operating points, the elements of the state vector matrix are defined as:

$$\mathbf{A}_k = \left. \frac{\partial f(SOC_k, I_k)}{\partial SOC_k} \right|_{SOC_k = \hat{SOC}_k^-} \quad (15)$$

$$\mathbf{C}_k = \left. \frac{\partial h(SOC_k, I_k)}{\partial SOC_k} \right|_{SOC_k = \hat{SOC}_k^-} \quad (16)$$

To start filtering, the first step is to set initial values, the initialization of the state and error covariance at  $k = 0$  are made as follows:

$$\begin{cases} \hat{SOC}_0 = E[SOC_0] \\ \mathbf{P}_0 = E[(SOC_0 - \hat{SOC}_0)(SOC_0 - \hat{SOC}_0)^T] \end{cases} \quad (17)$$

Then using Equation (18) and (19) we can predict the state and error covariance at time instant  $k$ :

$$S\hat{O}C_k^- = f(S\hat{O}C_{k-1}, I_{k-1}) \quad (18)$$

$$\mathbf{P}_k^- = \mathbf{A}_k \mathbf{P}_{k-1} \mathbf{A}_k^T + \mathbf{Q} \quad (19)$$

According to the predictions and the measurements, the Kalman gain can be calculated, and the state and error covariance are corrected:

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{C}_k^T (\mathbf{C}_k \mathbf{P}_k^- \mathbf{C}_k^T + \mathbf{R})^{-1} \quad (20)$$

$$S\hat{O}C_k = S\hat{O}C_k^- + \mathbf{K}_k (Z_k - h(S\hat{O}C_k^-, I_k)) \quad (21)$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{C}_k) \mathbf{P}_k^- \quad (22)$$

## 4 Experiment and results

### 4.1 Data acquisition

In order to verify the proposed model determined by FRA and the EKF-based SOC estimation method, experiments were conducted. The data was obtained from a handmade ZNBs. During the tests, the rating capacity of the battery is around 3.7Ah. Additionally, 1mol ZnO and 20g/L LiOH are dissolved by the 10mol/L KOH support solution as the prepared electrolyte. The operating flow rate remains constantly at 19cm/s. The testing current signals are generated by the NEWARE CT-3008 which are fed into the ZNBs. The measurement ranges of voltage and current are 15V and 3A, respectively, and nominal measurement error bounds are within 0.1%.

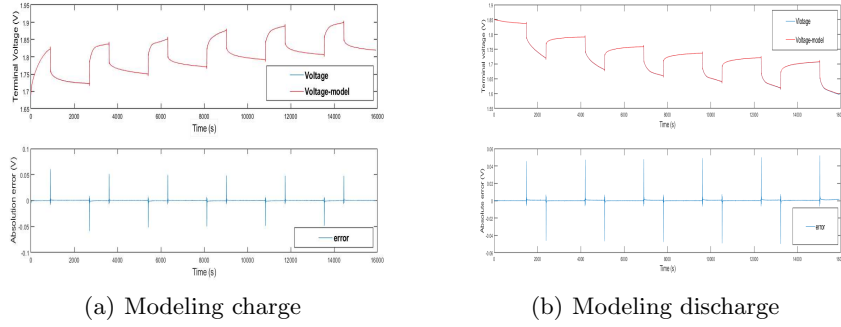
### 4.2 Experiment results

The battery charging data are used to build a model by FRA to correlate the terminal voltage with SOC and terminal current, and the candidate model terms for selection are  $V_k$ ,  $SOC_k$ ,  $I_k$  as well as their nonlinear variants. The terms which have the maximum contributions are selected as the optimal model inputs. Besides, the weights of each terms are calculated to complete the model. As shown in Eq.(23), the selected model terms are  $V_{k-1}$ ,  $\ln\sqrt{V_{k-1}}$ ,  $\sin(SOC_k)$  and  $\sin(\sqrt{SOC_k})$ .

$$V_k - 1.0053V_{k-1} + 0.0282\ln\sqrt{V_{k-1}} = 0.0031\sin(SOC_k) - 0.0041\sin(\sqrt{SOC_k}) \quad (23)$$

The results are shown in Fig. 1(a), the model output fits well to the measured terminal voltage, and the error is acceptable. Use the same model in the discharging procedure, it also fits well, as shown in Fig. 1(b).

This model is compared with the model established by orthogonal matching pursuit (OMP) algorithm, which selects model terms from an over-complete dictionary. We use 'wmpdictionary' function in Matlab to generate a dictionary for



**Fig. 1.** Results of modeling using FRA

OMP, this dictionary contains the shifted Kronecker delta subdictionary, polynomial subdictionary, cosine and sine subdictionary, and discrete cosine transform-ii orthonormal basis, etc. The 'wmpalg' function is then used to directly return the approximation of terminal voltage in the dictionary. The comparison of these two models is shown in Fig. 2 with absolute error on the horizontal and time on the vertical. The mean square error (MSE) for this model is  $6.4378e-05$ , while for the FRA-based model is  $9.2432e-06$ . It is clear that the error of the model identified by FRA is less than that of the model identified by OMP. Considering the accuracy of the FRA method and its ability to describe the relationship between terminal voltage and SOC, FRA is a good choice for battery model identification.

In Eq.(23), we use  $Z_k$  to represent the left side of this equation, the battery state-space model are:

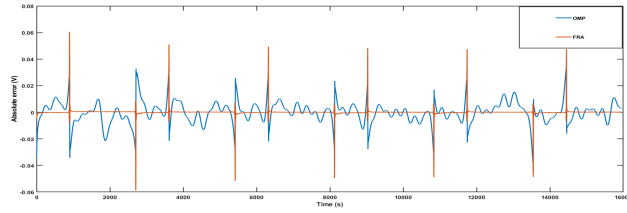
$$\begin{cases} SOC_{k+1} = SOC_k + \frac{\eta \Delta t}{Q} I_k + w_k \\ Z_{k+1} = 0.0031 \sin(SOC_{k+1}) - 0.0041 \sin(\sqrt{SOC_{k+1}}) \end{cases} \quad (24)$$

The EKF algorithm is then used for SOC estimation. Fig.3 depicts the performance of the proposed method. It can be found that the SOC value estimated by the EKF method is very close to the reference values. The absolute error between these two values is less than 2%, even when the estimation tend to steady, the error is less than 0.7%. By setting the initial value to 20% while the actual initial value is 0, the robustness of this algorithm is verified, as Fig. 3 shows, the estimated SOC converge to the actual value quickly.

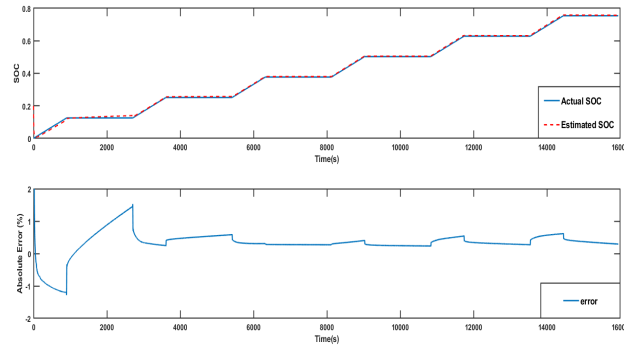
## 5 Conclusion

In this paper, an accurate terminal voltage model of flow battery has been built for EKF-based SOC estimation by applying the fast recursive algorithm (FRA), which is a computationally efficient and stable model identification method. The model performs well and is used as a measurement equation in





**Fig. 2.** Model error



**Fig. 3.** SOC estimation results using EKF

EKF-based SOC estimation. Future works will both focus on improving model accuracy and improving the estimation method. Some advanced algorithms, such as neural networks and support vector machines, will be studied for further improvement of model accuracy, while dual estimation methods and other methods will be studied to decrease the SOC estimation error.

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