Green Investment under Policy Uncertainty and Bayesian Learning

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Abstract

Many countries have introduced support schemes to accelerate investments in renewable energy (RE). Experience shows that, over time, retraction or revision of support schemes become more likely. Investors in RE are greatly affected by the risk of such subsidy changes. This paper examines how investment behavior is affected by updating a subjective belief on the timing of a subsidy revision, incorporating Bayesian learning into a real options modeling approach. We analyze a scenario where a retroactive downward adjustment of fixed feed-in tariffs (FIT) is expected through a regime switching model. We find that investors are less likely to invest when the arrival rate of a policy change increases. Further, investors prefer a lower FIT with a long expected lifespan. We also consider an extension where, after retraction, electricity is sold in a free market. We find that if policy uncertainty is high, an increase in the FIT will be less effective at accelerating investment. However, if policy risk is low, FIT schemes can significantly accelerate investment, even in highly volatile markets.

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1. Introduction

In this paper, we study investment in a renewable energy (RE) project from a real options with learning perspective. Since policy uncertainty[1] – in the form of adverse revisions of support schemes – has a large impact on RE investments, it is important that investors assess and update this risk over time. In standard real option models, learning is an implicit consequence of postponing investment. We allow for a more realistic modeling of the investment environment, where information is received, processed and incorporated explicitly in the decision making process.

The member states of the European Union have agreed to reduce the emission of greenhouse gases substantially by 2050. Specific targets, like EU2020 and EU2030, have been set in order to reach this long-term goal. One of the objectives of The European Strategic Energy Technology Plan (SET-Plan) is to accelerate investments in renewable energy technologies. As a consequence of the deregulation of the electricity markets in Europe, it is private investors with an objective of maximizing profit who choose whether to invest in an RE project or not (Abadie and Chamorro (2014)). At the same time, the costs of electricity generation from most renewable energy sources are significantly above average European market prices of electricity (Klessmann et al. (2013)). Therefore, several European countries have changed their policies and introduced various support schemes to ensure competitiveness of renewable energy production and encourage investment.

Support schemes can be characterized as either quantity-driven[2] or price-driven[3]. The price-driven feed-in schemes are the most commonly used support mechanism. In 2015 nearly 80 countries had employed feed-in tariffs (FIT) as support policy (REN21 (2015)). FITs are considered to be the most effective scheme for accelerating development of renewable energy sources (Couture and Gagnon (2010); del Rio and Mir-Artigues (2012); Ritzenhofen

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[1]In line with other research in this field (e.g. Boomsma and Linnerud (2015); Ritzenhofen and Spinler (2016); Yang et al. (2008)), the terms "uncertainty" and "risk" will be used interchangeably in this paper.

[2]Quantity-driven schemes include electricity certificates, where producers of renewable energy are given a number of certificates based on the quantity of electricity supplied to the market.

[3]Price-driven schemes include feed-in schemes, which can be implemented either as a price premium paid on top of the electricity price, or as a fixed tariff paid to producers instead of the electricity price. The fixed tariff is independent of the electricity price.

and Spinler (2016)).

Under a FIT scheme, producers are often guaranteed a market independent and fixed price for every unit of electricity generated, over the lifetime of a project (Couture and Gagnon (2010)). However, the problem for investors is that unexpected and retroactive revisions of vital subsidy payments have occurred in several countries in recent years (REN21 (2015)). In Bulgaria, Germany, Greece, Italy, and Switzerland, the FIT rate was reduced during 2014, and in Ukraine, the tax exemption for companies that sell renewable electricity has been removed (REN21 (2015)). In Spain, Belgium, the Czech Republic, Bulgaria and Greece, the size of subsidy payments was retroactively adjusted, thereby reducing the profitability of already operating plants (Boomsma and Linnerud (2015)). According to an estimate, the revision in Spain caused a 40% cut in expected income for a large amount of RE projects (The Institute of Energy for South East Europe (2014)). These cuts made the investors unable to meet their debt payments. As a consequence, several lawsuits against the Spanish government were filed. In one lawsuit it was concluded that plaintiff investors could not legitimately expect the FIT scheme to remain unchanged throughout the life of their RE plants, and that the investors could have easily foreseen the prospect of a revision.

The possibility of an unexpected subsidy revision has introduced a new source of uncertainty for investors, since the profitability of RE investments is largely or entirely dependent on consistent government policy (Helm et al. (2003)). White et al. (2013) state that policy uncertainty is a significant challenge for actors in the renewable energy sector. This is in line with Europe's largest producer of renewable energy, Statkraft, which states in its annual report of 2014 that uncertainty related to framework conditions, such as taxes, fees and political regulations are highly accentuated in investment decisions (Statkraft (2014)). Canada's Rural Partnership stated the importance of policy support being consistent, long term, and predictable to avoid boom and bust cycles (White et al. (2013)). Investors' subjective belief regarding a change in support policy is therefore of great importance for investment decisions in the renewable energy sector.

We develop a model in which a risk-neutral profit maximizing investor, who expects a future adverse retroactive transition between two regimes of fixed FIT, has the option to invest in an RE project. The transition can be thought of as a downward adjustment of FIT received by RE producers. Furthermore, we extend the model and analyze a scenario where investors expect a retroactive transition from a regime of fixed FIT to a free-market regime,

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where the electricity produced must be sold on the spot or futures market at a price that varies over time. Similar to Boomsma et al. (2012), Adkins and Paxson (2016), Boomsma and Linnerud (2015) and Ritzenhofen and Spinler (2016), we consider a single subsidy revision. Our model distinguishes itself from the formerly mentioned in that the transition rate between the subsidy regimes is unknown. Through Bayesian learning, the investor updates her subjective belief about the value of the transition rate, based on the arrival of exogenous signals.

The specification of active learning varies among researchers. For this work, we (similar to Martzoukos (2003)) define the processing of new information and explicit updating of belief in the decision making as active learning. In our context, observations and research of markets and framework conditions give rise to active learning.

Applying a real options approach allows us to incorporate some important characteristics of RE investments. First, investment costs are often considered project specific and therefore sunk. Second, the project value is uncertain, and depends on factors such as fluctuating electricity prices and changing subsidy schemes. Third, the investor can choose to postpone the project if the current framework conditions do not justify immediate investment. The investor has an option to invest in the project, i.e. the right, but not the obligation to invest.

In standard real options models, the value of the underlying project often varies according to a stochastic variable, e.g. electricity price. In our main model, the value of the option to invest varies only with a stochastic belief process, describing the investor's expectation about the lifetime of the currently high FIT scheme. The optimal investment strategy is characterized by a threshold on the probabilistic belief of the high FIT scheme having a long lifespan. The explicit opportunity to learn about the lifetime of the subsidy scheme, motivates the investor to postpone investment. In an extension we also account for a stochastic electricity price. To analyze the results of our model we present a case study based on a wind power project in Europe. Sensitivity in the option value and the investment threshold is then examined for selected parameters.

Real options theory has been applied by several authors to problems regarding uncertain market conditions and policy change in the energy sector. With this work we contribute to two strands of literature. First, we extend the traditional real options model by including exogenous arrival of information in the decision making. Second, we allow for a more realistic analysis of

RE investments under policy uncertainty, by including an evolving subjective belief about the timing of a policy revision.

Fuss et al. (2008) analyze the effects of market and policy uncertainty on investment in a coal-fired electricity generation facility. Policy uncertainty is modeled by an uncertain drift rate in the price process of CO_2. They find that investors will postpone their decision until the true value of the CO_2 price drift revealed. Their results indicate that uncertainty related to government policies affects investment decisions more than uncertainty in market prices.

Boomsma et al. (2012) study investment timing and capacity choice for renewable energy projects under different support schemes accounting for stochastic capacity cost, electricity price and subsidy payment. Analyzing the possibility of a shift from one support scheme to another, they find that the project value under the current scheme depends on the value under the alternative scheme and the transition probability. Compared to the case without policy risk, the risk exposure of energy investors increases. Adkins and Paxson (2016) compare the effectiveness of different subsidy schemes for investment decisions in a renewable energy facility under uncertain electricity price and quantity sold. They shown that the option value is always greater in the presence of a government subsidy than in its absence. Sudden introduction or retraction of subsidies is modelled by a Poisson jump process with constant intensity factor. Adkins and Paxson (2016) conclude that a subsidy having unexpected withdrawal motivates earlier investment, compared to the case without subsidies.

Closest to our paper is the work of Ritzenhofen and Spinler (2016) and Boomsma and Linnerud (2015). Ritzenhofen and Spinler (2016) consider a regime switching model, in which regulators are considering a shift from a FIT scheme to a free market regime. Their results suggest that policy uncertainty has little impact on investment projects when current FIT regimes are sufficiently attractive. In contrast, when FIT levels are near electricity market prices, regulatory uncertainty reduces the investment rate. Boomsma and Linnerud (2015) examine how investors in energy projects respond to possible termination or revision of current support schemes. As in our paper policy uncertainty is modeled as a Markov process with a given jump intensity. Boomsma and Linnerud (2015) show that the risk of subsidy retraction will slow down the investment rate if it is retroactively applied, but otherwise increase the rate. The authors also conclude that policy uncertainty may add substantial risk to investments in the energy sector.

Neither of the aforementioned papers however consider learning. Within a framework using a time homogeneous Markov process or a Poisson jump process to model policy changes, the implicit assumption is made that investors have no information regarding the dynamics governing the changing policy scheme. Our paper contributes to the existing literature by explicitly incorporating Bayesian learning in the investment decision.

Contributions to the real options literature considering active learning are still rather limited. Among one of the first contributions is Pawlina and Kort (2005), who value an irreversible investment opportunity of a firm where the investment costs are subject can increase resulting from a policy change. Harrison and Sunar (2015) examine investment planning in a continuous-time Bayesian framework. A firm is considering investment in a project with unknown value. However, the uncertainty about project value can be reduced by several means of learning. Information gathering in any learning mode follows a Brownian motion with exogenously given drift and incurs a given cost. The optimal learning policy is dependent on the drift and corresponding cost of a learning mode, versus the signal quality. Jensen (1982) studies adoption behavior of a firm facing the option to invest in a new innovation when the probability of the innovation being profitable is unknown. In each time period the decision maker receives a signal indicating the profitability of the project, and the probabilistic belief is updated in a Bayesian manner. Thijssen et al. (2004) examine a firm with the option to invest in a project of unknown profitability. The decision maker's belief about the profitability of the project is updated based on the arrival of signals that follows a Poisson process.

Our starting point is the discrete arrival of signals, as in Thijssen et al. (2004). We then use a random walk approximation to derive a Brownian motion-driven stochastic differential equation (SDE) describing the investor's belief process where the arrival of signals is continuous. Shiryaev (1967) and Peskir and Shiryaev (2006) obtain an SDE which is similar to ours, when studying the problem of minimizing the cost of error when sequentially testing a hypothesis on the unknown drift rate of a one-dimensional Brownian motion. The SDE of Shiryaev (1967) is also the starting point for Ryan and Lippman (2003) and Kwon and Lippman (2011) who analyze decision making under Bayesian learning.

Learning related to energy has, to the best of our knowledge, only been examined in relation to global warming. Examples include Kolstad (1996) who examines optimal climate-related policy when one can learn about uncer-

tainty about CO_2-related damages, and Kelly and Kolstad (1999) who study the relationship between greenhouse gas levels and global mean temperature in a Bayesian framework. Ours is the first contribution that considers Bayesian learning about policy uncertainty in green investments.

2. The Model

We consider a continuous time model, where time is indexed by $t \geq 0$. A risk neutral and profit maximizing investor has the option to invest in an RE project. At some random point in time, regulators are expected to revise the current subsidy scheme. The current fixed FIT scheme offers a subsidy payment of K_0, while the subsequent scheme will offer a subsidy payment of K_1. A change is adverse to investors so that $K_0 > K_1$.[4]

For simplicity, we assume that the state of the world, θ, can only take two values: good ($\theta = 1$) or bad ($\theta = 0$). In the good (bad) state the duration of the current subsidy scheme is expected to be long (short). The arrival rate of subsidy reversal is denoted by λ_s, $s = G, B$, where it is assumed that $\lambda_B > \lambda_G > 0$.

The true state of the world is not known to the investor *ex-ante*. At time t, the probabilistic belief of being in state G, given all the information that the investor has received up to time t is denoted by X_t. The prior belief in the good state is given by $\mathbb{P}(\theta = 1) = X_0$.

Similarly to Harrison and Sunar (2015), we assume that the frequency of information arrivals (signals) about the true state of the world is sufficiently high to be modelled by a Brownian motion. Following a Bayesian approach, the signals are then used to continuously update the investor's belief about the world.

All our modeling assumptions presented and motivated in the following subsections are in line with the standard real options literature, as well as with the literature on sequential hypothesis testing in continuous time.

2.1. Derivation of the belief process

We start by modeling the log-likelihood ratio process of the information arrivals. We follow Dixit (1993), starting with a random walk approximation and then taking the continuous-time limit. Take a time interval $[0, T]$ and

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[4]A full glossary of parameters can be found in Appendix A.

split it up into discrete steps of length Δt. In each time step, a positive or a negative signal is received. The probability of receiving a correct signal about the true state of the world θ is $p > \frac{1}{2}$. The log-likelihood of the signals received up to time t is denoted by Y_t. Conditional on the true state of the world θ, per time step we have

$$Y_{t+\Delta t}|\theta = \begin{cases} Y_t + (2\theta - 1)\log\left(\dfrac{p}{1-p}\right) & \text{w.p. } p \\ Y_t - (2\theta - 1)\log\left(\dfrac{p}{1-p}\right) & \text{w.p. } 1-p. \end{cases}$$

The time interval $[0, T]$ is divided into n time steps of equal length Δt, so that $n = \frac{T}{\Delta t}$. We define time step i to be $[(i-1)\Delta t, i\Delta t]$.
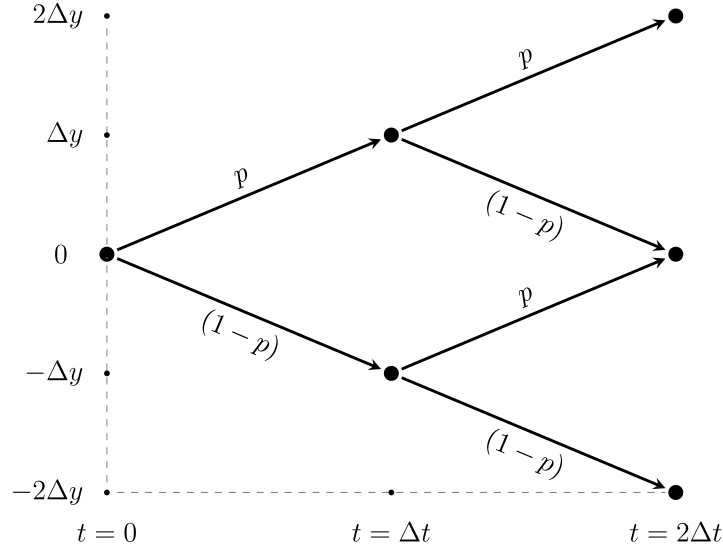

Figure 1: Binomial tree showing the possible paths of Y over two time steps conditional on $\theta = 1$.

Let $(Z_i)_{i=1}^n$ be a sequence of independent Bernoulli random variables such that $P(Z_i = 1) = p$ and $P(Z_i = -1) = 1 - p$. Then for some initial value, $Y_0 = y_0$, the process Y, conditional on $\theta = 1$, can be expressed as a sum of Bernoulli random variables:

$$Y_T = y_0 + \Delta y \sum_{i=1}^n Z_i.$$

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Since the increments of the process are independent, the expectation and variance over the time horizon are given by (see Appendix B)

$$\mathbb{E}\left[Y_T - Y_0\right] = \frac{T}{\Delta t}\Delta y\left(2p - 1\right), \tag{1}$$

$$Var\left(Y_T - Y_0\right) = \frac{T}{\Delta t}\left(\Delta y\right)^2 4p(1 - p). \tag{2}$$

Keeping the time horizon fixed at T and taking the limit $n \to \infty$, the process $(Y_i)_{i=1}^n$ converges to $(Y_t)_{t\in[0,T]}$, with $\mathbb{E}[Y_T] = \mu_X T$ and $Var(Y_T) = \sigma_X^2 T$ for some μ_X and σ_X. To accomplish this, we choose the parameters Δy and p such that the expectation and variance stay finite while taking the limit, i.e.

$$\lim_{\Delta t \to 0} \frac{T}{\Delta t}\Delta y(2p - 1) = \mu_X T, \quad \text{and} \quad \lim_{\Delta t \to 0} \frac{T}{\Delta t}(\Delta y)^2 4p(1 - p) = \sigma_X^2 T.$$

For this to hold, we must have $\frac{(\Delta y)^2}{\Delta t} = \sigma_X^2$, which yields

$$\Delta y = \sigma_X \sqrt{\Delta t}, \tag{3}$$

and

$$p = \frac{e^{\sigma_X \sqrt{\Delta t}}}{1 + e^{\sigma_X \sqrt{\Delta t}}}. \tag{4}$$

The process in fact converges point-wise to an arithmetic Brownian motion with the desired properties, given that the step size and probability are consistent with (3) and (4) (see Appendix C for derivation details). That is,

$$dY|\theta = (2\theta - 1)\mu_X dt + \sigma_X dW,$$

where dW is the increment of a Wiener process. Denoting the filtration generated by Y by $\mathcal{F}^Y$ and the *posterior process* by $X_t := \mathbb{P}(\theta = 1|\mathcal{F}_t^Y)$, and applying Ito's Lemma (see Appendix D) we obtain

$$dX|\theta = (2\theta - 1)\sigma_X^2 X^{2-\theta}(1 - X)^{\theta+1}dt + \sigma_X X(1 - X)dW.$$

Note that at time t, given the investor's belief, she expects the change in her belief to be given by

$$dX_t = X_t dX_t|\{\theta = 1\} + (1 - X_t)dX_t|\{\theta = 0\} = \sigma_X X(1 - X)dW. \tag{5}$$

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So, the investor beliefs the posterior process to be a martingale, i.e. she can't predict the information she is going to receive in advance. We interpret dX as the *rate of learning*. It is evident that the rate of change in the posterior belief is governed by the value of $\sigma_X X(1-X)$. Firstly, the rate of learning increases as the signal strength, σ_X, increases, because each individual signal carries more information. Secondly, the term $X(1-X)$ reaches its maximum at $X = \frac{1}{2}$, which means that the rate of learning is highest when the investor has an equal belief of being in either state. Lastly, the rate of learning decreases as X moves toward its upper or lower bound. If $X = 0$ or $X = 1$, then $dX = 0$ and the process is in an absorbing state.

2.2. Policy uncertainty

Policy uncertainty involves the possibility of a change or termination of the current support scheme. These events occur at discrete points in time. Policy uncertainty is modeled as a Markov process, $(\delta_t)_{t \geq 0}$, with two regimes $\{0, 1\}$, such that

$$\delta_t = \begin{cases} 1, & \text{if a policy change has occurred in the time interval } [0, t), \\ 0, & \text{otherwise,} \end{cases}$$

with $\delta_0 = 0$.

Subsidies are normally intended to accelerate investments, as a step to meet production goals from renewable sources. As technology becomes more mature and cost-effective, and production goals are met, the need for high subsidy payments decreases. Reduction of subsidy payments are therefore permanent, and will not be followed by an increase to previous levels. We only consider one revision, as in Boomsma and Linnerud (2015) and Ritzenhofen and Spinler (2016). If the relevant costs continue to decrease in the long-run, several revisions might however be expected.

The transition rates of the Markov process are denoted by λ_{ij}, where

$$\lambda_{ij} = \lambda 1_{i=0, j=1},$$

with $\lambda \in \{\lambda_G, \lambda_B\}$.

2.3. Model formulation

When an investor has obtained a license to develop and operate a power plant, she owns the exclusive right to install the project within a given time

frame. For analytical tractability we assume that once granted, this exclusive right will be available forever.[5]

Similar to Boomsma and Linnerud (2015) and Ritzenhofen and Spinler (2016), we assume that the lifetime of the project is finite and denoted by T, construction is instantaneous and the generating capacity is exogenous. The expected production is constant, and there are no operational flexibility or other options embedded in the facility.

Renewable electricity generation is largely dependent on weather conditions, making production highly variable both in the short and medium term. However, according to Boomsma et al. (2012), production is less variable over longer time scales, e.g., yearly. Less variation in production in the long-term justifies the assumption of constant expected production.

In contrast to conventional power plants, most of the costs of owning and operating RE plants are known with great certainty prior to investment (European Wind Energy Association (2009)). For wind, solar and hydropower, the operation and maintenance (O&M) costs are relatively low since the energy input is freely available. Capital costs such as interest and depreciation can be predicted with a high accuracy at the time of investment, and are known for sure once the plant is built and financed. Therefore, the risk is low with regards to cost assessments in RE plants. In the current research in this field, O&M costs are often assumed constant and included in the investment cost, as in Boomsma and Linnerud (2015), or neglected, as in Fleten et al. (2007). We assume constant operating costs and can therefore include them in the irreversible and fixed investment cost denoted by I.

Electricity markets may be considered incomplete due to the lack of suitable hedging instruments for volume risk and risk of revision/retraction of the current support scheme (Boomsma et al. (2012); Boomsma and Linnerud (2015)). As a consequence, risk-neutral valuation may not be possible. We therefore assume an exogenously given real discount rate, denoted by r. The investor is assumed to be a price-taker in the relevant markets. Furthermore, we consider subsidies in the form of fixed FIT payments. However, we believe the model can easily be extended to include static FIT degression.

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[5]A project where the investment decision must be made within a finite amount of time usually demand a numerical solution, as seen in e.g. Ritzenhofen and Spinler (2016).

2.3.1. Transition between two regimes of FIT

We consider two regimes, which are characterized by the subsidy scheme in place,

- Regime 0: A change in FIT payment has not yet occurred, the project value after investment is denoted by $V_0(X)$, the option to invest is denoted by $F_0(X)$ and the instantaneous revenue is denoted K_0,

- Regime 1: A change in FIT payment has occurred, the project value after investment is denoted by V_1, the option to invest is denoted by F_1 and the instantaneous revenue is denoted K_1, where $K_1 < K_0$.

If we are in the regime where the subsidy has been withdrawn ($\delta = 1$), then the value of investment is given by

$$V_1 = \int_0^T K_1 e^{-rs} ds = \frac{K_1}{r} \left(\frac{1 - e^{-rT}}{r} \right). \tag{6}$$

For simplicity we assume that $V_1 < I$, so that investment is not optimal with the lower subsidy.

With retroactive revision of the subsidy scheme and starting in regime 0, for a given estimate λ, the project value once invested, calculated as revenue per MWh of electricity produced, is given by (see Appendix E)

$$\begin{aligned}
V_\lambda = & \frac{K_0}{r} \left[1 - \frac{\lambda}{r + \lambda} + \left(\frac{\lambda}{r + \lambda} - 1 \right) e^{-(r+\lambda)T} \right] \\
& + \frac{K_1}{r} \left[\left(1 - \frac{\lambda}{r + \lambda} \right) e^{-(r+\lambda)T} + \frac{\lambda}{r + \lambda} - e^{-rT} \right].
\end{aligned} \tag{7}$$

Hence, the expected value of an installed project (when $\delta = 0$) is equal to

$$V_0(X) = X V_{\lambda_G} + (1 - X) V_{\lambda_B}. \tag{8}$$

We assume that $V_0(0) - I < 0$, otherwise there would be no value of waiting and the investor would invest as long as the net present value (NPV) is positive. This assumption is further motivated by the retroactive changes of the subsidy regime in 2014 in Spain, which led to a significant decrease in profitability for RE producers and a drastic slowdown in investments (CSP-World (2014); REN21 (2015)).

At every point in time the investor has to decide whether to invest, paying the investment cost I and start accumulating profits in accordance with $V_0(X)$, or to delay investment and continue learning.

We want to find the threshold of the subjective belief, X^*, beyond which it is optimal to invest. The free-boundary X^* separates the continuation region from the stopping region. In the continuation region, $(0, X^*)$, postponing investment and learning is more valuable than immediate investment. Therefore, the option value is higher than the expected payoff from immediate investment and the optimal decision is to postpone. In the stopping region, $X \geq X^*$, the expected gain from immediate investment is greater than or equal to the option value, and the optimal decision is to invest.

In the continuation region (with $\delta_t = 0$), the value of the option to invest must satisfy the Bellman equation:

$$F_0(X) = \max \left\{ V_0(X) - I, \lim_{dt\downarrow 0} \frac{e^{-rdt}}{dt} \left(\mathbb{E}[(1 - \lambda dt)F_0(X + dX)] \right. \right.$$
$$\left. \left. + \mathbb{E}[\lambda dt F_1] \right) \right\}. \tag{9}$$

Here F_1 denotes the value of investing if a regime change occurs, which, over a time interval of length dt, happens with probability λdt, i.e.

$$F_1 = \max \{V_1 - I, \, 0\} = 0.$$

The probability of a change in subsidy payment during a short time interval dt is $\mathbb{E}[\lambda \, dt]$, and the probability that a change will not occur is $\mathbb{E}[1 - \lambda \, dt]$. In regime 1, the revenue is a fixed tariff of K_1 for the remaining lifetime of the facility. The fixed tariff makes the option to postpone investment worthless, since there is no uncertainty. In addition, the net present value is assumed to be negative, therefore the value of the option to invest in regime 1 is zero.

Applying Ito's lemma and rearranging terms, we obtain the following second order linear ODE (see Appendix F), which holds when continuation is optimal

$$\frac{1}{2}\sigma_X^2 X^2(1 - X)^2 \frac{\partial^2 F_0}{\partial X^2} - \left(X\lambda_G + (1 - X)\lambda_B + r \right) F_0 = 0. \tag{10}$$

Equation (10) does not have a closed form solution. The differential equation is singular at $X = 0$ and $X = 1$, thus no solution exists for these

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values of X. We are, however, only interested in a solution on the interval $X \in (0, 1)$, since $X = 0$ and $X = 1$ are absorbing and not reachable from any other state.

We find an analytical solution to (10) in the form of a power series (see Appendix G):

$$F_0(X) = A_1 X^c \sum_{n=0}^{\infty} a_n(c) X^n,$$

where

$$c = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(\lambda_B + r)}{\sigma_X^2}},$$

and where we recursively define

$$a_0(c) = 1,$$
$$a_1(c) = \frac{\sigma_X^2 c(c-1) - \lambda_B + \lambda_G}{\frac{1}{2}\sigma_X^2 c(c+1) - \lambda_B - r},$$

and, for $n \geq 2$

$$a_n(c) = \frac{2\left[\sigma_X^2(n+c-1)(n+c-2) - \lambda_B + \lambda_G\right] a_{n-1}(c) - \left[\sigma_X^2(n+c-2)(n+c-3)\right] a_{n-2}(c)}{\sigma_X^2(n+c)(n+c-1) - 2(\lambda_B + r)}.$$

Following Dixit and Pindyck (1994), at the free boundary X^*, the option to invest must satisfy the value-matching and smooth-pasting conditions

$$F_0(X^*) = V_0(X^*) - I, \tag{11}$$

and

$$\left.\frac{\partial F_0}{\partial X}\right|_{X=X^*} = \left.\frac{\partial V_0}{\partial X}\right|_{X=X^*}, \tag{12}$$

respectively. We solve the free-boundary problem following Pinto et al. (2009), who use a mixed analytical/numerical solution process based on the method of Frobenius.

3. Numerical results

In this section we obtain numerical results for the investment threshold and the option value for a case study based on a wind power project. We then examine sensitivity of the investment threshold and option value to changes in selected parameters. Our choice for an on-shore wind project case study is motivated by the fact that investments in on-shore wind projects are currently the most relevant in terms of size of all renewable energy projects in Europe. In 2017, for example, wind power installed more than any other form or power generation in Europe accounting for 55% of total power capacity installations, of which 80% were on-shore wind installations (Wind Europe (2017)). While the wind power project case is chosen to illustrate our results, our findings hold more generally.

3.1. Case study

Our case study focuses on an investment in a single onshore wind turbine. Wind is globally the most important source of renewable energy for electricity generation, and onshore wind represents the largest fraction (REN21 (2015)). Although we focus on a single wind turbine, the results extend to for instance an investment in a wind park containing several turbines or an investment in solar power. The parameter values used in our calculations below are summarized in Table 1.

The parameters are based on a typical 2 MW wind turbine installed in Europe (European Wind Energy Association (2009)). The investment cost and the project life of the wind power turbine are set to $I = 3\,320\,000$ EUR and $T = 20$ years, respectively. The investment cost include upfront costs and operations and maintenance (O&M) costs, and is calculated using a risk-adjusted nominal discount rate of 7.5%. The O&M costs are set equal to 15 EUR per MWh of generated electricity (McKenna et al. (2014)). The capacity factor of an electricity generating facility is the amount of electricity generated during a year divided by the amount of electricity generated with the facility running at maximum power output in all 8 760 hours of a year. For wind turbines the typical capacity factors are in the range 20 - 35%. We set the capacity factor to $F_{Cap} = 30\%$, which is in line with Boccard (2009). The exact capacity factor of a plant can be estimated to a high degree of accuracy by analytical tools and simulations, and will depend on, e.g., wind conditions and the specific technology used.

Our model is solved using dynamic programming, which entails setting an exogenous risk-adjusted discount rate in the analysis. This rate is calculated as the sum of the risk-free rate and a risk premium reflecting the risk embedded in the project. Following Boomsma and Linnerud (2015), we set the risk adjusted real discount rate equal to 5%. The real discount rate corresponds to a nominal rate of 7.5% and an inflation rate of 2.5%. Since I is constant over time, we implicitly assume that the investment cost will grow at the rate of inflation.

The FIT is $K_0 = 65$ EUR/MWh and $K_1 = 30$ EUR/MWh, for regimes 0 and 1, respectively. The FIT under regime 0 is in line with the rates in Spain and Germany as reported by the European Wind Energy Association (2009). The FIT under regime 1 corresponds to the average day-ahead price of electricity for the period April 2012 to April 2016, based on weekly data from the Nordic electricity exchange Nord Pool[6].

The transition rates are set to $\lambda_G = 0.05$ and $\lambda_B = 0.2$, implying an expected regime change in 20 years and 5 years, respectively. Hence, in the *Good* state, the investor expects to receive subsidy payments throughout the project lifetime.

The signal strength of the belief process is set to $\sigma_X = 0.3$.

3.2. Results

Based on the values in the presented case study (see Table 1), the investment threshold and option value are calculated numerically.[7]

We obtain an investment threshold of $X^* = 0.799$. Hence, the investor must have a strong belief in the subsidies of regime 0 being long-lived before she is willing to invest. We show in Figure 2 how the value of the option and the NPV varies with X. The investment threshold, X^*, lies at the tangency point of the option value and the NPV. In a now-or-never scenario, the investor will invest if X is greater than or equal to 0.693. For lower values of X, the project will be rejected even though it might turn out to be profitable at a later point in time.

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[6]http://www.nordpoolspot.com - Nord Pool is Europe's leading market for physical and financial power contracts. The day-ahead market consists of about 360 buyers and sellers of power, and is the main arena for trading. The electricity price is determined by supply and demand.

[7]All numerical results are obtained using MATLAB R2015a. $F_0(X)$ is expanded to $n = 1000$ terms, so that the error is of order $<< 10^{-10}$.

Table 1: Parameter values in base case.

Parameter	Value	Unit	Description
I	3 320 000	EUR	Investment cost
K_0	65	EUR/MWh	FIT in regime 0
K_1	30	EUR/MWh	FIT in regime 1
T	20	Years	Lifetime of RE project
λ_G	0.05	-	Rate of revision, Good state
λ_B	0.2	-	Rate of revision, Bad state
σ_X	0.3	-	Signal strength, belief process
r	0.05	-	Real discount rate
Capacity	2	MW	Capacity of power plant
F_{Cap}	0.3	-	Capacity factor

The difference between the NPV of investing at the optimal threshold and investing suboptimally, is called the value of waiting (Dixit and Pindyck (1994)). We show that the NPV rule can be very misleading and that the value of waiting can be substantial up to the optimal threshold (see Figure 2) .

3.3. Sensitivity analysis

In this section we examine the sensitivity of the option value and the investment threshold to selected parameters, and discuss the implications for investors and policy makers.

3.3.1. Sensitivity to the signal strength (σ_X)

One important difference between our model and standard real option models, is that the option dynamics are governed by the evolution of the belief process and not by a process related to the value of the project. A change in σ_X does not affect the value of the project, but does affect the rate of learning. One can interpret σ_X as the amount of information received per signal. With a higher information arrival, the rate of learning increases, leading to a higher option value as illustrated in Figure 3a.

An increase in the signal strength results in a more volatile belief process, and the belief of being in the *Good* state can therefore change more quickly. For high σ_X it is more likely that X reaches high values even when the true state of the world is *Bad*. The higher rate of learning, and the possibility of
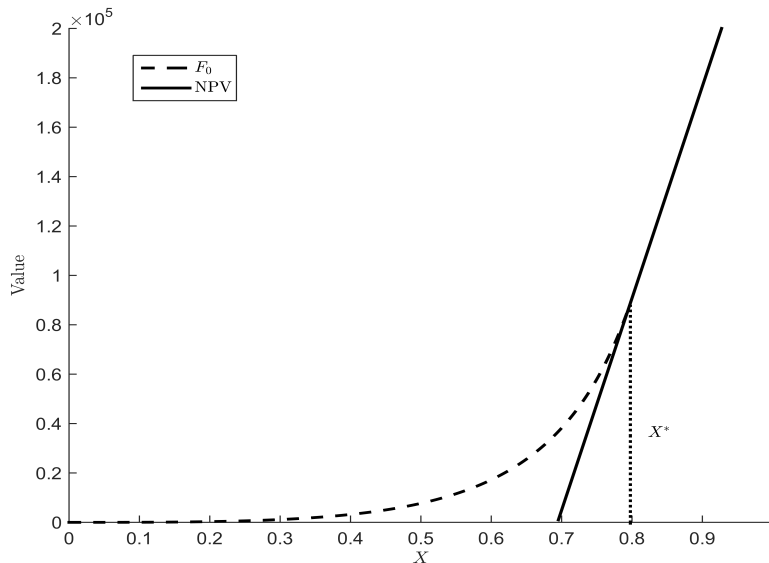
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Figure 2: Option value and NPV vs belief of being in the Good state.

a quickly changing belief, leads to an increase in the investment threshold as shown in Figure 3b. When σ_X goes to zero, no information arrives and there is no value of learning. Since the investor's initial belief about the state of the world will not change, she faces a now-or-never scenario with investment according to the NPV rule.

Generally, in real option models, a higher investment threshold is associated with a lower investment rate; see Dixit and Pindyck (1994). In our model, the effects of a higher or lower investment threshold are not as straightforward. The timing of the investment decision depends on two effects: the rate of learning and the level of the investment threshold. An increase in the investment threshold may be counteracted by an increase in the rate of learning. We might therefore observe a higher investment rate at a higher investment threshold.

The optimal policy of the investor is characterized by a single threshold. The expected time to investment is infinite due to a positive probability that the belief process will never reach this threshold (Kwon and Lippman (2011)). To illustrate how the investment rate is affected by a change in the signal strength, we have run Monte Carlo simulations of the probability process. Since the expected time to investment is infinite, the results are

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relative, however suitable for our analysis. By discretizising X as given by Equation (5), we have generated 10 000 sample paths of the belief process in the base case (Table 1), with the initial belief, X_0, set to 0.4.

We find that the relative time to investment is decreasing in σ_X. The increasing investment threshold is therefore offset by a higher rate of learning, and the result is a higher investment rate. In practice, high information arrival can correspond to a transparent government, which clearly communicates the current and intended framework conditions to RE investors. In the next sections, we will use that for constant σ_X, a lower investment threshold corresponds to a higher investment rate.
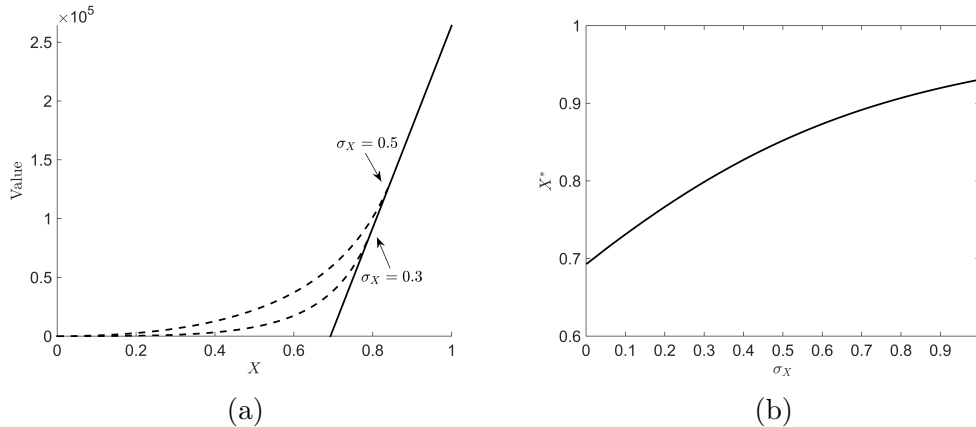

Figure 3: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the signal strength.

3.3.2. Sensitivity to the investment cost (I)

Numerical results indicate that the investment threshold increases in the investment cost (see Figure 4b). This is an intuitive and standard result in real options analysis. As the expected gain from investment decreases, the investor must be more certain of the high FIT scheme being long-lasting before investing.

Similarly, the option value naturally decreases in the investment cost. However the optimal payoff is non-monotonic in the investment cost as seen in Figure 4a. For a standard option to invest the relationship is monotonically increasing (Dixit and Pindyck (1994)). When the value of the underlying project is derived from an unbounded stochastic variable, such as price, the

optimal payoff can always increase to offset an increase in the investment cost. In our model, the stochastic variable is a probability measure and bounded between 0 and 1. Since the project value is static in both states of the world, the expected value can not exceed the value in the *Good* state. The combination of the bounded stochastic variable and the static value of the project leads to the non-monotonic relationship.

One can see from Figures 4a and 4b that the option to learn is valuable for only a limited range of investment costs. When the investment cost approaches the project value given the *Bad* state, the potential loss from investment decreases. When the potential loss is zero the investment threshold is $X^* = 0$, which means that the investor would invest immediately. The NPV in both the *Good* and the *Bad* state would be non-negative, and there would be no downside of investing. By postponing investment the investor will miss out on the higher revenues under regime 0. When the investment cost approaches the project value in the *Good* state, the potential upside from investing goes to zero and naturally the investment trigger goes to $X^* = 1$. Investment would never happen, since NPV in both states is less than or equal to zero.

From the perspective of an investor, uncertainty over payoff can be compensated by a reduction in investment cost. A lower total investment cost can be achieved through lower upfront costs and/or lower O&M costs. The investor will therefore invest at a lower subjective belief if technology progress and/or additional subsidies reduce the investment cost. From the perspective of policy makers, the investment rate can be influenced through subsidizing the investment cost by introducing for example tax credits. In the United States, RE plants are subsidized through investment tax credits (ITC)[8] and production tax credits (PTC)[9] (US Department of Energy (2015a), US Department of Energy (2015b)). Our results indicate that reducing the total investment cost of investors by issuing ITC, lowers the investment threshold and increase the investment rate in RE plants.

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[8]Investment tax credits allow eligible RE producers to subtract a percentage of the investment cost from the amount of tax owed to the government, indirectly reducing the investment cost.

[9]Production tax credits reduces the tax owed to the government for eligible RE producers, based on the amount of electricity produced.

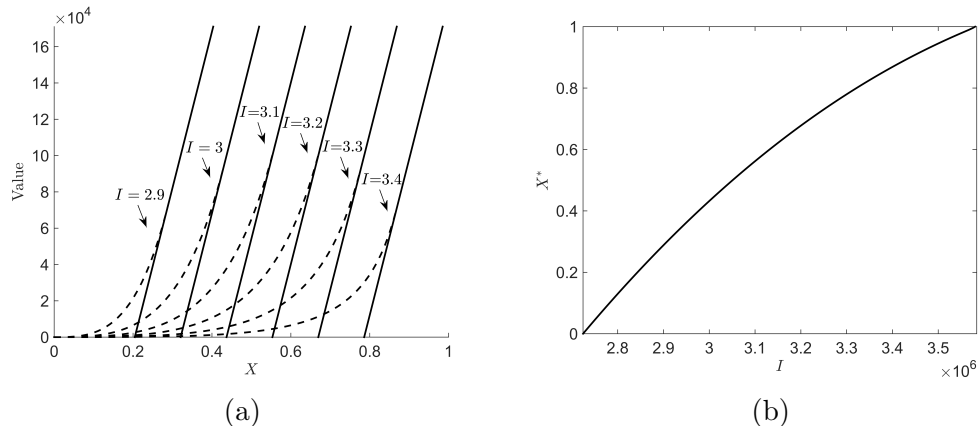Figure 4: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the investment cost (in millions in (a)).

3.3.3. Sensitivity to the transition rates (λ_G and λ_B)

We start by examining the sensitivity in λ_G. There are two effects that cause the option value to decrease in λ_G (see Figure 5a). First, a higher transition rate means that a revision of the high FIT scheme is expected to arrive sooner. This effect makes it less attractive to delay the investment. Second, as λ_G increases, the expected value of the project goes to zero since the expected lifespan of regime 0 will go to zero.

If $\lambda_G = 0$, a revision of the subsidy scheme will never occur and the project would receive the high FIT throughout its lifetime, given that the world is in the *Good* state. The difference between the NPV in the *Good* and the *Bad* state is largest, and the option value is at its maximum, all else equal. In addition, postponement has no negative effect and will eventually reveal which state the world is in. Therefore, the value of learning is at its highest.

The investment threshold is affected by two opposing effects when λ_G increases. First, the shorter expected time to a revision makes it less attractive to postpone investment. As a consequence, the investment threshold decreases. Second, the expected value of the project decreases, which causes the investment threshold to increase. The first effect is always dominated by the second effect, as illustrated by the monotonic relationship in Figure 5b.

As seen in Figures 6a and 7, the sensitivity in λ_B and λ_G is similar. However, a change in λ_B does not affect the value of the project in the *Good*

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state, and the value of waiting is non-monotonic in λ_B. As λ_B increases, the expected lifespan of the high FIT scheme decreases. Therefore the option to postpone investment has less value. In effect, for large enough λ_B, it becomes costlier to wait instead of investing (see Figure 6b).

The investment threshold increases in λ_B, by the same reasoning as for λ_G. Therefore, we conclude that a higher arrival rate of the policy change (both for the *Good* and the *Bad* state) leads to a higher investment threshold, and therefore according to the analysis presented in Section 3.3.1., to a lower investment rate.

From equation (6) and (8), we also see that the value in the *Bad* state approaches the value of the project under regime 1 for large λ_B. Since the potential downside has a lower bound and the NPV in the *Good* state is positive for all λ_B, the investment threshold is less sensitive for larger λ_B.
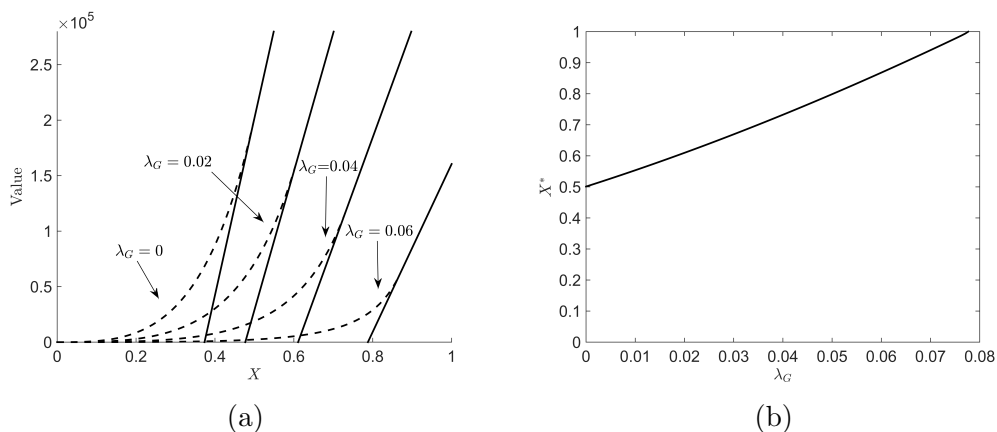

Figure 5: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the arrival rate of a policy change, given the Good state.

3.3.4. Sensitivity to the FIT (K_0 and K_1)

We start by looking at the FIT level in regime 0. As shown in Figure 8a, the option value increases in K_0. This result is intuitive, since a higher K_0 leads to a higher expected value of the project. However, the optimal payoff is non-monotonic in K_0, by the same reasoning as for I. The investment threshold decreases in K_0 (see Figure 8b). As K_0 decreases, the NPV of the project in the *Good* state goes to zero, and the investor needs to be more certain of regime 0 being long-lasting before investment.
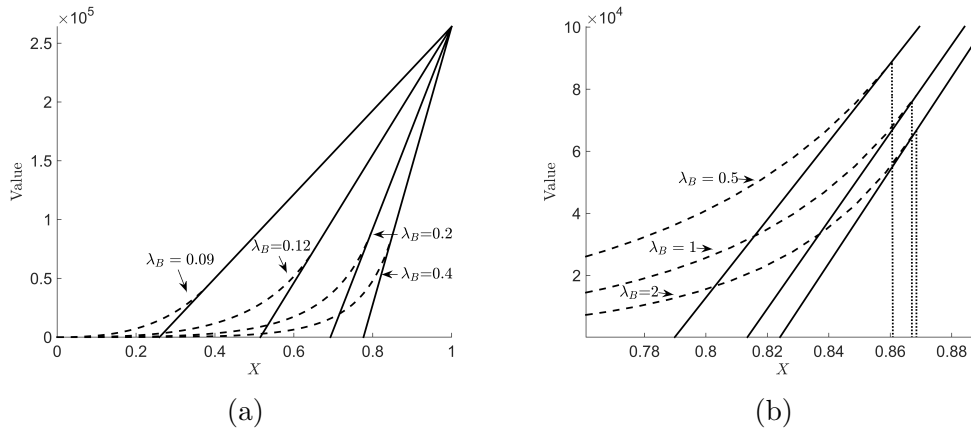
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Figure 6: The figures (a) and (b) show the sensitivity of the option value to the arrival rate of a policy change, given the Bad state. Plot (b) is zoomed in to illustrate the non-monotonic optimal NPV.

The sensitivity in investment threshold and option value and the non-monotonic optimal payoff is similar for K_1, as shown in Figure 9a and 9b. As K_1 increases, the expected value of the project increases, and naturally the option to invest becomes more valuable. Since the expected project value in regime 1 increases in K_1, the investment decision is less dependent on the lifespan of the high FIT scheme. As a result, the investment threshold decreases in K_1.

3.3.5. Relationship between FIT and transition rate

We illustrate the FIT payment needed for a constant investment threshold for different transition rates in Figure 10, 11 and 12a. As previously stated, the investment trigger increases in the arrival rate and decreases in the FIT level. Thus, in order to keep the investment trigger and investment rate constant, an increase in λ must be offset by an increase in K, and vice versa. The marginal required subsidy level decreases in λ_B (see Figures 10b and 11b). This result follows from the fact that the investment trigger becomes less sensitive to changes in λ_B as λ_B increases. Similarly, a diminishing increase in K_1 for increasing λ_G is illustrated in Figure 11a.

These results indicate that a lower subsidy payment, which is expected to be sustainable in the long term, gives the same investment rate as a higher payment which is believed to be less sustainable.

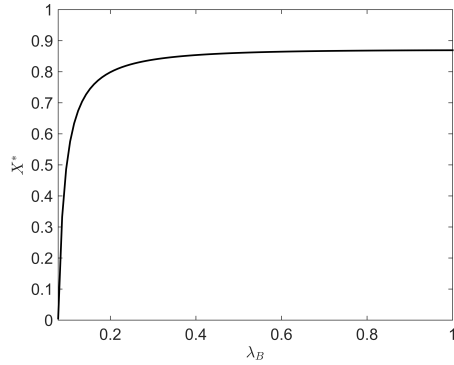In Figure 12b, we plot the expected NPV at the time of investment,

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Figure 7: Sensitivity of the investment threshold to the arrival rate of a policy change, given the Bad state.
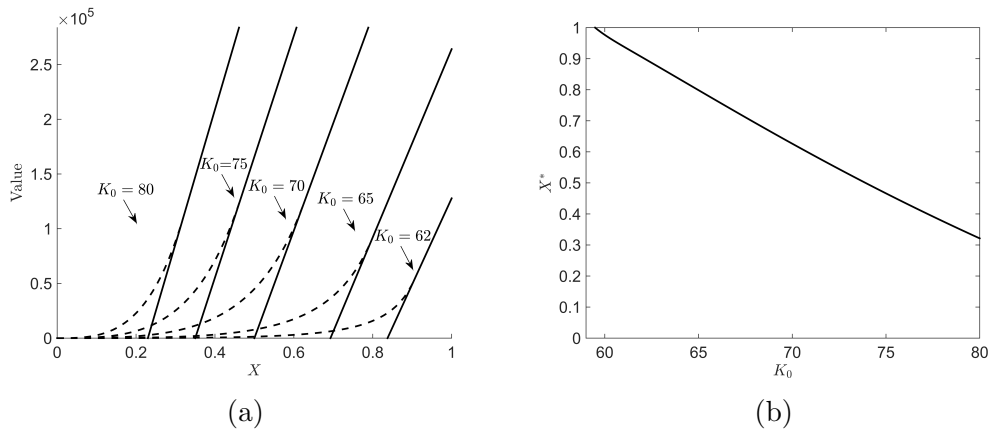

Figure 8: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the fixed feed-in tariff in regime 0.

$V_0(X^*) - I$, for the different combinations of K_0, λ_G and λ_B found in Figure 12a.[10] Interestingly, even though the expected NPV at investment varies greatly for the different combinations of subsidy payment and transition rates, the investment rate is the same. We find that the expected NPV is higher for a combination of lower K_0, λ_G and λ_B. This implies that an in-

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[10]For a given combination of λ_G and λ_B, we find the necessary K_0 for keeping the investment threshold constant. Based on the investment threshold, $X^* = 0.799$, we calculate $V_0(X^*) - I$ for this mix of K_0, λ_G and λ_B. The other parameters are given by Table 1.

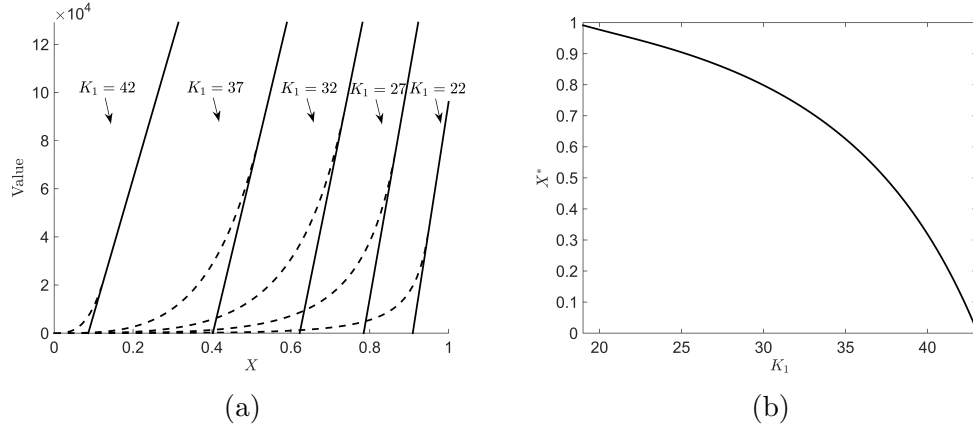(a)   (b)

Figure 9: The figures show the sensitivity of (a) the option value and (b) the investment threshold to the fixed feed-in tariff in regime 1.

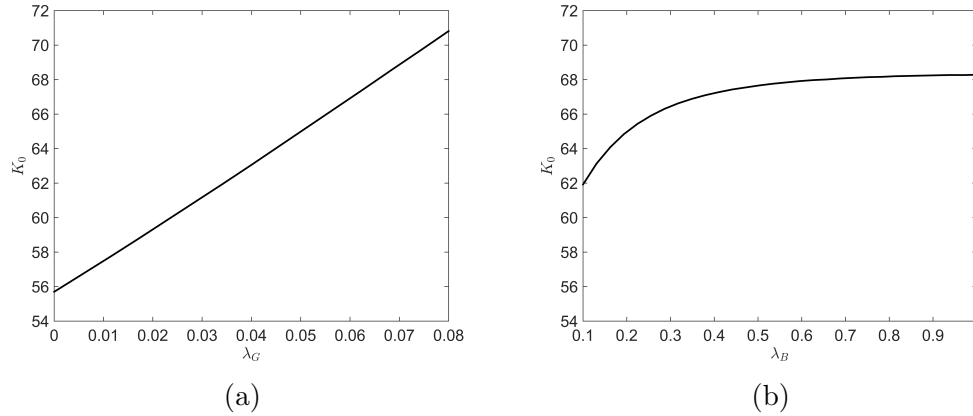(a)   (b)

Figure 10: The figures show the relationship between the fixed feed-in tariff in regime 0 and the arrival rate of a policy change for a constant investment rate, given (a) the Good state and (b) the Bad state.

vestor who chooses to invest will prefer a lower subsidy payment for a longer expected lifespan.

Examining the relationship between K_1 and the corresponding expected NPV at the time of investment showed that the same conclusions can be drawn with respect to the subsidy after the policy revision as for the case of K_0.
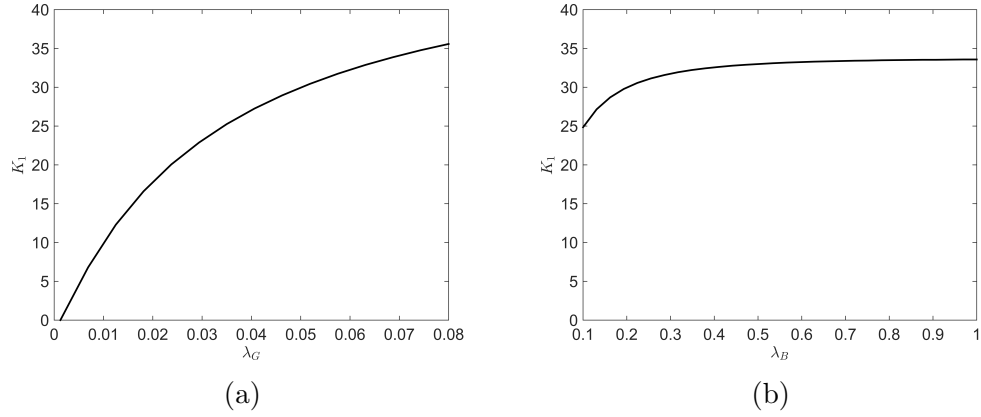
Figure 11: The figures show the relationship between the fixed feed-in tariff in regime 1 and the arrival rate of a policy change for a constant investment rate, given (a) the Good state and (b) the Bad state.
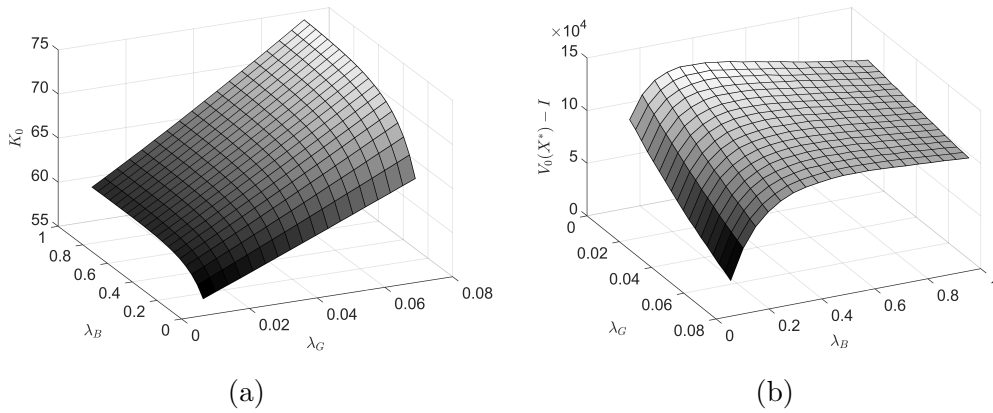

Figure 12: The figures show, for a constant investment rate, (a) the relationship between K_0, λ_G and λ_B and (b) the expected NPV at investment for different values of λ_G and λ_B (and implicitly K_0 as given in (a)).

4. Model extension

So far we have considered a policy change in the form of a retroactive downward adjustment of the FIT received by RE producers. In the following, we extend our model and examine a scenario where investors expect an adverse retroactive transition from a regime of FIT to a regime where electricity is sold in a free market. Investors are now exposed to both the policy

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uncertainty and fluctuating electricity prices.

4.1. Model formulation

We still take the perspective of a single RE investor, and consider two regimes

- Regime 0: the termination has not yet occurred, project value denoted by $V_0(X, S)$, option to invest denoted by $F_0(X, S)$ and instantaneous revenue denoted K,

- Regime 1: a termination of the subsidy scheme has occurred, project value denoted by $V_1(S)$, option to invest denoted by $F_1(S)$ and instantaneous revenue at time t denoted S_t.

We assume that the electricity price $(S_t)_{t \geq 0}$ follows a geometric Brownian motion (GBM), such that

$$dS_t = \mu_S S_t \, dt + \sigma_S S_t \, dW_{St},$$

where μ_S and σ_S are constants that represent the drift and volatility of the electricity price, respectively, and dW_{St} is the increment of a Wiener process.[11]

While Lucia and Schwartz (2002) find that two factor models[12] provide a better fit than one factor models to the data of the Nordic electricity market, Nord Pool, Schwartz and Smith (2000) claim that the short-term variations can be neglected for long-term investments. Similarly, when considering long-term commodity related investments, Pindyck (2001) states that the assumption of energy prices following a GBM will not lead to large errors. Fleten et al. (2007) argue that an investment in an RE generation unit should be treated as a long-term investment. Correspondingly, Fleten et al. (2007) assumes that long-term electricity prices follow a GBM. Other research using a GBM to model electricity prices include Boomsma and Linnerud (2015), Boomsma et al. (2012), and Ritzenhofen and Spinler (2016).

The belief process is assumed to be independent of the electricity price, so that $\mathbb{E}\left[dW_X dW_S\right] = 0$. In addition, the policy change is independent of the electricity price.

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[11]For ease of notation, we will drop the subscript t on S in the following.

[12]In two factor models of energy prices, short-term variations are often assumed to follow a mean reverting process and long-term variations are assumed to follow a GBM.

With retroactive revision of the subsidy scheme and starting in regime 0, for a given λ, the project value, calculated as revenue per MWh of electricity produced, is given by

$$V_0(S) = K\frac{1 - e^{-(r+\lambda)T}}{r + \lambda} + S\left(\frac{1 - e^{-(r-\mu_S)T}}{r - \mu_S} - \frac{1 - e^{-(r+\lambda-\mu_S)T}}{r + \lambda - \mu_S}\right).$$

This value is derived in a similar way as described in Appendix E, together with the observation that $\mathbb{E}(S_t|S_0 = S) = Se^{\mu_S t}$.

Starting in regime 0, and considering the two possible transition rates, the expected value of the project is equal to

$$V_0(X, S) = X\left[K\frac{1 - e^{-(r+\lambda_G)T}}{r + \lambda_G} + S\left(\frac{1 - e^{-(r-\mu_S)T}}{r - \mu_S} - \frac{1 - e^{-(r+\lambda_G-\mu_S)T}}{r + \lambda_G - \mu_S}\right)\right]$$
$$+ (1 - X)\left[K\frac{1 - e^{-(r+\lambda_B)T}}{r + \lambda_B} + S\left(\frac{1 - e^{-(r-\mu_S)T}}{r - \mu_S} - \frac{1 - e^{-(r+\lambda_B-\mu_S)T}}{r + \lambda_B - \mu_S}\right)\right].$$
(13)

Under regime 1, the project value is given by

$$V_1(S) = S\left[\frac{1 - e^{-(r-\mu_S)T}}{r - \mu_S}\right].$$

The value of the option to invest in the two regimes must then satisfy the following Bellman equations

$$F_0(X, S) = \max\left\{V_0(X, S) - I, \lim_{dt\downarrow 0}\frac{e^{-rdt}}{dt}\Big(\mathbb{E}[(1 - \lambda dt)F_0(X + dX, S + dS)]\right.$$
$$\left. + \mathbb{E}[\lambda dt F_1(S + dS)]\Big)\right\},$$
(14)

and

$$F_1(S) = \max\left\{V_1(S) - I, \lim_{dt\downarrow 0}\frac{e^{-rdt}}{dt}\mathbb{E}[F_1(S + dS)]\right\}.$$
(15)

Since the electricity price is stochastic, the option to invest under regime 1 has positive value, in contrast to the model in Section 2.3.1.

Applying Ito's lemma and rearranging terms, we obtain the following system of second order partial differential equations (PDEs), which must

hold when continuation is optimal:

$$\frac{1}{2}\sigma_X^2 X^2 (1-X)^2 \frac{\partial^2 F_0}{\partial X^2} + \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F_0}{\partial S^2} + \mu_S S \frac{\partial F_0}{\partial S}$$
$$- \Big(X\lambda_G + (1-X)\lambda_B\Big)\Big(F_0 - F_1\Big) - rF_0 = 0, \tag{16}$$

and

$$\frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F_1}{\partial S^2} + \mu_S S \frac{\partial F_1}{\partial S} - rF_1 = 0. \tag{17}$$

Equation (17) can be solved analytically in the usual way (see Dixit and Pindyck (1994)) to obtain

$$F_1(S) = A_1 S^{\beta_1}, \tag{18}$$

where $\beta_1 > 1$ is the positive root of the quadratic equation $\frac{1}{2}\sigma_S^2\beta(\beta-1) + \mu_S\beta - r = 0$. Since (16) has no analytical solution, we apply a finite element algorithm to solve the PDE numerically; see Appendix H for some details.

4.2. Numerical results for the case study

We now apply this model to the case study presented in Section 3.1.[13] We also examine sensitivity in the investment threshold to the volatility in electricity prices and the FIT level.

A retroactive termination of the FIT scheme will happen at a random point in time. Following the termination, the electricity produced will be sold on a free market. The FIT is set equal to $K = 65$ EUR/MWh. For the electricity price, we set $\mu_S = 0$ and $\sigma_S = 0.06$, as Boomsma and Linnerud (2015).[14] Setting the drift term equal to zero implies that the electricity price will grow according to the inflation rate. All other values are as in Table 1.

The optimal investment threshold is characterized by both the electricity price and the investor's belief in the FIT scheme being long-lived. In effect, the threshold for undertaking investment, the free-boundary, is defined by a line that separates the continuation region from the stopping region (see Figure 13). At each point in time, the investor observes the electricity price,

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[13]The variational formulation (see Appendix H) is solved by FEM using FreeFem++; see Hecht (2012). Level-set and plots have been made using MATLAB R2015a.

[14]Boomsma and Linnerud (2015) estimate σ_S by the annual standard deviation of the log returns implied by average weekly prices of three-year forward contracts traded at NASDAQ OMX for the period 1 January 2005 to 30 April 2015.
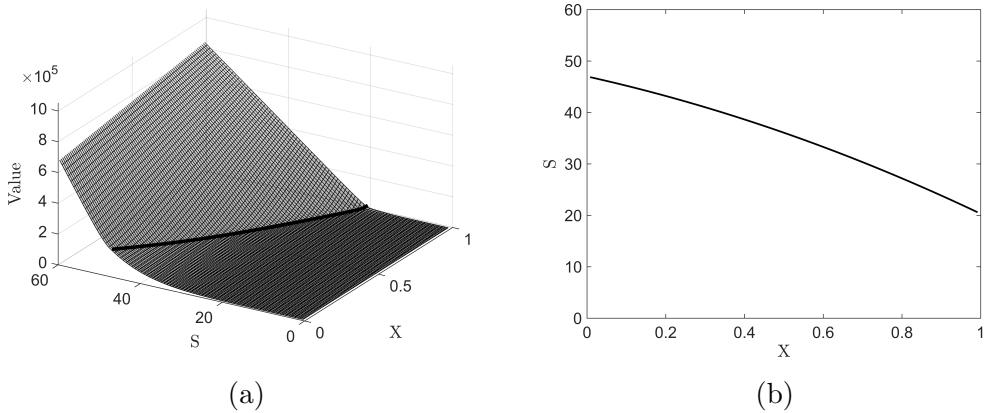
Figure 13: The figures show (a) the option value as a function of X and S (free-boundary as solid line), and (b) the free-boundary that separates the continuation region from the stopping region in two dimensions. The area below the free-boundary is called the continuation region (postponing investment is optimal) and the area above the free-boundary is called the stopping region (investment is optimal). Investment is undertaken as soon as the combination of X and observed S is above the free-boundary.

and must decide whether the combination of the expected lifespan of the FIT scheme and electricity price justifies investment. The first time this combination is at or above the free-boundary, the investor will choose to invest.

If the investor expects the lifespan of the FIT scheme to be short, a higher electricity price is needed before she is willing to invest. Hence, we can conclude that either a high electricity price or a high probabilistic belief of an attractive FIT scheme being long-lived, is needed in order to motivate investment. The effect of the FIT scheme on the investment behavior is largely dependent on the perceived policy uncertainty. For high X, investors expect the lifespan of the FIT scheme to be long. Hence, a high X corresponds to a low perceived policy uncertainty.

In regime 1, where the FIT has been terminated, we find that the investor will choose to invest at an electricity price of 60 EUR/MWh (see the expression (H.11) for the threshold in Appendix H). In Figure 13b we show that FITs with a low expected lifespan will accelerate investments. However, FIT schemes are most effective when the perceived risk of a revision is low. Active learning, as modeled by an increasing X, can significantly decrease the electricity price at which it is optimal to invest.

4.3. Sensitivity analysis

4.3.1. Sensitivity to the volatility of the electricity price (σ_S)

For standard real option models, an increase in the volatility of an underlying price process will increase the value of the project; see Dixit and Pindyck (1994). Therefore, the value of the option to invest and the critical price at which it is optimal to invest increase. The critical price increases since the option value is more sensitive to changes in volatility than the project value. For a higher volatility the investment rate is expected to decrease, due to the higher investment threshold.

We conclude that the exercise boundary shifts upwards if σ_S increases; see Figure 14. For a given X, the required S at which it is optimal to invest, increases in σ_S. This effect is decreasing for larger values of X. As the investor becomes more confident in the FIT scheme being long-lived, a higher volatility in electricity prices has less effect on the investment decision.

For investors in more volatile electricity markets, the FIT scheme is less effective at accelerating RE investment when the perceived risk of a revision is high. When $X = 0$, the investor expects the revision to arrive in a relative short amount of time and the policy uncertainty is high. At this point, there is a large difference between the electricity price at which it is optimal to invest for a high and a low σ_S. Conversely, when $X = 1$, the policy uncertainty is low and a larger σ_S has little effect on the electricity price needed for investment.
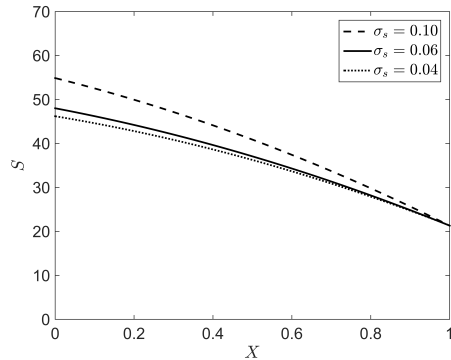

Figure 14: Sensitivity in free-boundary/investment threshold for different volatility of electricity prices.
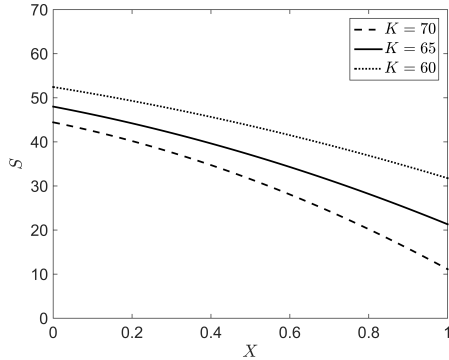
Figure 15: Sensitivity in free-boundary/investment threshold for different values of the fixed feed-in tariff.

4.3.2. Sensitivity in the FIT

We find that the exercise boundary decreases as the FIT level increases (see Figure 15). The effect is stronger when the belief in a long-lived FIT scheme increases, since the investor is increasingly eager to take advantage of the subsidies. When the perceived policy uncertainty is low (X close to 1), policy makers can have a relatively large impact on the investment rate in RE capacity by a relatively small change in the FIT even in highly volatile markets. This is due to the fact that in this case the investment decision is not sensitive to price volatility, a result shown in the previous section. The effect is significantly lower when the perceived policy uncertainty is high, which means a more generous subsidy is required to achieve the same investment rate. Therewith, we can conclude that if policy uncertainty is high, an increase in the FIT will be less effective at accelerating investment.

5. Conclusion

This paper extends standard real options models by including exogenous arrival of information in the decision making process through a Bayesian learning approach. We consider an investor with a perpetual option to invest in a renewable energy project. The profitability of the project is highly dependent on long-lasting government subsidies. Policy uncertainty in the form of adverse changes of a subsidy scheme have a large effect on the investment decision.

A support scheme of fixed feed-in tariff (FIT) is considered, where at some

random point in time, investors expect a retroactive downward adjustment of the FIT. We extend our model and examine a situation where the subsidy scheme will be retroactively terminated and electricity must be sold on a free market where the market price is uncertain.

The arrival rate of a subsidy revision is unknown, but as time passes, the investor updates her belief of the expected lifespan of the support scheme. The aim of our paper is to examine how this learning affects investor behavior.

At every point in time, the investor must weigh the benefits from exercising the investment option, against continued observation and learning. We find that the optimal investment decision is characterized by a threshold on the subjective posterior belief of the current subsidy scheme being long-lived. In an extension of the model, the investor faces both policy uncertainty and uncertain electricity prices. The optimal investment threshold is a function of both electricity price and the subjective belief of the investor.

We find that policy uncertainty may introduce risk in the environment given by fixed FIT regimes, due to the likelihood of a revision. Our results have three important implications for the designers of FITs: i) The investment threshold increases in the arrival rate of a policy change, thereby reducing the investment rate in renewable energy plants. ii) We find that investors who choose to invest will prefer a lower FIT with a long expected lifespan, while policy makers might have different preferences depending on their objectives (e.g. highest investment rate, lowest total amount of subsidies paid out given a certain investment rate that they aim for). The challenge for policy makers is to find the right mix of subsidy payment and risk that trigger the intended amount of investment. This mix should reflect the specific characteristics of a given RE project. iii) We conclude that policy makers can have a large impact on the investment rate by a relatively small change in the FIT, when the policy uncertainty is low. The effect is significantly lower when the policy uncertainty is high, so a more generous subsidy is required to achieve the same investment rate. Active learning can greatly reduce the perceived policy uncertainty, and thereby increase the effectiveness of subsidy schemes.

We can identify at least three potential directions for further research. One possibility is to examine different type subsidy schemes, e.g. feed-in premiums or green certificates, in a similar way to Boomsma and Linnerud (2015). Adding another stochastic process will however, increase the mathematical complexity of the model, which already requires advanced numerical methods for partial differential equations.

Information arrival is likely to vary. Some events might lead to a large amount of information in a short amount of time, and there might be periods of very little or no information arrival. This effect can be captured by modeling information arrival as a Poisson process or a jump-diffusion process.

Finally, it is reasonable to assume that investors do have some discretion over the magnitude of investment. Incorporating capacity choice will allow for an analysis of how policy uncertainty affects the investment rate and installed capacity at the same time.

Appendix A. Nomenclature

θ	state of the world	S	electricity price
r	(risk-adjusted) discount rate	μ_S	electricity price trend
σ_X	volatility of learning process	σ_S	electricity price volatility
Y	likelihood ratio process	K_1	subsidy after policy change
X	posterior belief process	K_0	subsidy before policy change
λ_G	regime transition rate if $\theta = 1$	T	lifetime of RE project
λ_B	regime transition rate if $\theta = 0$	I	sunk investment costs
p	probability of correct signal		

Appendix B. Expectation and variance of $Y_T - Y_0$

The expectation follows from a standard computation:

$$\mathbb{E}\left[Y_T - Y_0\right] = \mathbb{E}\left[\Delta y \sum_{i=1}^{n} Z_i\right] = \frac{T}{\Delta t}\Delta y\, \mathbb{E}[Z]$$

$$= \frac{T}{\Delta t}\Delta y\left(p - (1-p)\right) = \frac{T}{\Delta t}\Delta y\left(2p - 1\right),$$

To find the variance, we note that since Z_i are independent random variables their correlation is 0:

$$Var\left(Y_T - Y_0\right) = Var\left(\Delta y \sum_{i=1}^{n} Z_i\right) = (\Delta y)^2 \sum_{i=1}^{n} Var\left(Z_i\right)$$

$$= (\Delta y)^2 \sum_{i=1}^{n} \mathbb{E}\left[(Z_i)^2\right] - \left(\mathbb{E}\left[Z_i\right]\right)^2 = (\Delta y)^2 \sum_{i=1}^{n}\left(1 - (2p-1)^2\right)$$

$$= \frac{T}{\Delta t}(\Delta y)^2\, 4p(1-p)$$

Appendix C. Derivation of dY

The expectation and the variance of Y over the time horizon are given by

$$\mathbb{E}[Y_T] = \frac{T}{\Delta t}\Delta y(2p-1), \tag{C.1}$$

$$Var(Y_T) = \frac{T}{\Delta t}(\Delta y)^2 4p(1-p). \tag{C.2}$$

While taking the limit as $\Delta t \to 0$ we want the variance (C.2) to stay finite and independent of Δt. Thus we must have

$$\frac{(\Delta y)^2}{\Delta t} = constant \qquad \Rightarrow \qquad (\Delta y)^2 = constant \cdot \Delta t$$

Setting the constant variance equal to σ_X^2 we get

$$\Delta y = \ln\left(\frac{p}{1-p}\right) = \sigma_X\sqrt{\Delta t} \quad \Rightarrow \quad p = \frac{e^{\sigma_X\sqrt{\Delta t}}}{1 + e^{\sigma_X\sqrt{\Delta t}}} \tag{C.3}$$

Next, we want the mean, μ, to be independent of Δt. Substituting (C.3) into (C.1), we get

$$\frac{\sigma_X\sqrt{\Delta t}}{\Delta t}\left(\frac{2e^{\sigma_X\sqrt{\Delta t}}}{1 + e^{\sigma_X\sqrt{\Delta t}}} - 1\right) = \frac{\sigma_X}{\sqrt{\Delta t}}\left(\frac{-1 + e^{\sigma_X\sqrt{\Delta t}}}{1 + e^{\sigma_X\sqrt{\Delta t}}}\right) = \mu$$

Now, taking the series expansion of e, we have

$$\frac{\sigma_X}{\sqrt{\Delta t}}\left(\frac{-1 + 1 + \sigma_X\sqrt{\Delta t} + \frac{1}{2}\sigma_X^2\Delta t + \mathcal{O}\left((\Delta t)^{\frac{3}{2}}\right)}{1 + 1 + \sigma_X\sqrt{\Delta t} + \frac{1}{2}\sigma_X^2\Delta t + \mathcal{O}\left((\Delta t)^{\frac{3}{2}}\right)}\right)$$

$$= \sigma_X^2\left(\frac{1 + \frac{1}{2}\sigma_X\sqrt{\Delta t} + \frac{1}{6}\sigma_X^2\Delta t + \mathcal{O}\left((\Delta t)^{\frac{3}{2}}\right)}{2 + \sigma_X\sqrt{\Delta t} + \frac{1}{2}\sigma_X^2\Delta t + \mathcal{O}\left((\Delta t)^{\frac{3}{2}}\right)}\right) = \mu$$

Finally, we take the limit as $\Delta t \to 0$, and obtain

$$\frac{\sigma_X^2}{2} = \mu_X$$

Then, in the limit,

$$dY = \mu_X dt + \sigma_X dW$$

Appendix D. Derivation of dX

Consider a function $F(x,t)$ that is at least twice differentiable in x. Ito's Lemma gives the differential dF as (Dixit and Pindyck, 1994)

$$dF = \frac{\partial F}{\partial x}\, dx + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}\,(dx)^2$$

Our starting point is the arithmetic Brownian motion dY, given by

$$dY = \begin{cases} \mu_X\, dt + \sigma_X\, dW & \text{if } \theta = 1 \\[2mm] -\mu_X\, dt + \sigma_X\, dW & \text{if } \theta = 0 \end{cases},$$

where $Y_t = \ln\frac{X_t}{1-X_t}$

Assuming $\theta = 1$ and applying Ito's Lemma, we obtain

$$
\begin{aligned}
dX &= \frac{\partial X}{\partial t}\, dt + \frac{\partial X}{\partial Y}\, dY + \frac{1}{2}\frac{\partial^2 X}{\partial Y^2}\,(dY)^2 \\[2mm]
&= \frac{\partial X}{\partial Y}\left[\mu_X\, dt + \sigma_X\, dW\right] + \frac{1}{2}\frac{\partial^2 X}{\partial Y^2}\left[\mu_X\, dt + \sigma_X\, dW\right]^2.
\end{aligned}
$$

Using that $X = \frac{e^Y}{e^Y+1}$, we get

$$
\begin{aligned}
dX &= \frac{e^Y}{(e^Y+1)^2}\left[\mu_X\, dt + \sigma_X\, dW\right] + \frac{1}{2}\sigma_X^2\,\frac{e^Y(1-e^Y)}{(e^Y+1)^3}\, dt \\[2mm]
&= \frac{\sigma_X^2}{2}\left[\frac{e^Y}{(e^Y+1)^2} + \frac{e^Y(1-e^Y)}{(e^Y+1)^3}\right] dt + \sigma_X\,\frac{e^Y}{(e^Y+1)^2}\, dW \\[2mm]
&= \frac{\sigma_X^2}{2}\left[\frac{2e^Y}{e^Y+1}\left(1 - \frac{e^Y}{e^Y+1}\right)^2\right] dt + \sigma_X\,\frac{e^Y}{e^Y+1}\left[1 - \frac{e^Y}{e^Y+1}\right] dW \\[2mm]
&= \sigma_X^2 X(1-X)^2\, dt + \sigma_X X(1-X)\, dW,
\end{aligned}
$$

which describes the evolution of X given the Good state.

Following the same procedure given $\theta = 0$, we get that the process X evolves according to

$$dX = \begin{cases} \sigma_X^2 X(1-X)^2\, dt + \sigma_X X(1-X)\, dW & \text{if } \theta = 1 \\[2mm] -\sigma_X^2 X^2(1-X)\, dt + \sigma_X X(1-X)\, dW & \text{if } \theta = 0 \end{cases}.$$

Appendix E. Derivation of V_0

Define the stopping time

$$\tau_\delta := \inf\{t \geq 0 | \delta_t = 1; \delta_0 = 0\},$$

and the functions $g_{01} : [0, T] \to \mathbb{R}$ and $g_0 : [0, T] \to \mathbb{R}$ by

$$g_{01}(t) := \int_0^t K_0 e^{-rs} ds + \int_t^T K_1 e^{-rs} ds$$

$$= \frac{K_0}{r} \left(1 - e^{-rt}\right) + \frac{K_1}{r} \left(e^{-rt} - e^{-rT}\right)$$

and

$$g_0(t) := \int_0^T K_0 e^{-rs} ds = \frac{K_0}{r} \left(1 - e^{-rT}\right).$$

Note that $\tau_\delta \sim Exp(\lambda)$, with distribution function $F(t) = 1 - e^{-\lambda t}$ and density function $f(t) = \lambda e^{-\lambda t}$. The result now follows from direct computation of the expectation

$$V_0 = \mathbb{E}\left[g_{01}(\tau_\delta); \tau_\delta < T\right] + \mathbb{E}\left[g_0(\tau_\delta); \tau_\delta \geq T\right]$$

$$= \int_0^T g_{01}(t) f(t) dt + g_0(T) \mathbb{P}(\tau_\delta > T).$$

Appendix F. The Bellman equation

Starting in regime 0, the value of the option to invest must satisfy the Bellman equation

$$F_0(X) = \max \left\{ V_0(X) - I, \lim_{dt \downarrow 0} \frac{e^{-rdt}}{dt} \Big(\mathbb{E}[(1 - \lambda dt) F_0(X + dX)] \right.$$

$$\left. + \mathbb{E}[\lambda dt F_1] \Big) \right\}.$$

In the continuation region, it then holds, for small dt, that

$$(1 + r\, dt) F_0 = \mathbb{E}[1 - \lambda\, dt]\, \mathbb{E}[F_0 + dF_0] + \mathbb{E}[\lambda\, dt]\, \mathbb{E}[F_1] + o(dt),$$

where we have used the fact that the Poisson jump and the learning process are independent. Note that

$$F_1 = \max\{V_1 - I, \, 0\} = 0, \quad V_1 - I < 0 \quad \text{by assumption,}$$

and that

$$\mathbb{E}[\lambda] = X\lambda_G + (1-X)\lambda_B.$$

Applying Ito's lemma and using that X is the probabilistic belief of being in the Good state, we then get

$$(1+r\,dt)F_0 = \Big(1 - X\lambda_G\,dt - (1-X)\lambda_B\,dt\Big)\,\mathbb{E}\Big[F_0 + X\,(\sigma_X^2 X(1-X)^2\,dt + \sigma_X X(1-X)dW)\,\tfrac{\partial F_0}{\partial X}$$

$$+\ (1-X)\,(\sigma_X^2 X^2(1-X)\,dt + \sigma_X X(1-X)dW)\,\tfrac{\partial F_0}{\partial X} + \tfrac{1}{2}\sigma_X^2 X^2(1-X)^2\tfrac{\partial^2 F_0}{\partial X^2}\,dt\Big] + o(dt)$$

$$= \Big(1 - X\lambda_G\,dt - (1-X)\lambda_B\,dt\Big)\Big[F_0 + \tfrac{1}{2}\sigma_X^2 X^2(1-X)^2\tfrac{\partial^2 F_0}{\partial X^2}\,dt\Big] + o(dt).$$

We rearranging terms, dividing by dt and taking the limit $dt \downarrow 0$ we obtain the following second order ODE for the continuation region:

$$\frac{1}{2}\sigma_X^2 X^2(1-X)^2\frac{\partial^2 F_0}{\partial X^2} - \Big(X\lambda_G + (1-X)\lambda_B + r\Big)F_0 = 0. \qquad (\text{F.1})$$

Since the ODE is independent of the drift term in dX, we do not have to consider the two possible states of the world. Hence, we can reduce dX to the much simpler form

$$dX = \sigma_X X(1-X)\,dW, \qquad (\text{F.2})$$

regardless of the state of the world.

Appendix G. Solving the ODE

We seek an analytical solution of the ODE

$$\frac{1}{2}\sigma_X^2 X^2(1-X)^2\frac{\partial^2 F_0}{\partial X^2} - \Big(X\lambda_G + (1-X)\lambda_B + r\Big)F_0 = 0, \qquad (\text{G.1})$$

for $X \in (0,1)$.

Assuming a solution on the form of a Frobenius series

$$F_0(X) = X^c \sum_{n=0}^{\infty} a_n(r) \, X^n. \tag{G.2}$$

We want to find the terms and coefficients of the series solution corresponding to the differential equation at hand. Differentiating (G.2) and substituting into (G.1), we get

$$\frac{1}{2}\sigma_X^2 X^2(1-X)^2 \sum_{n=0}^{\infty}(n+c)(n+c-1)\, a_n(r)\, x^{n+c-2}$$

$$- \left(X\lambda_G + (1-X)\lambda_B + r\right) \sum_{n=0}^{\infty} a_n(r)\, x^{n+c} = 0.$$

Next, we examine the coefficients of different powers of X. For the first term of the series ($n = 0$), we get

$$\frac{1}{2}\sigma_X^2 \left(1 - 2X + X^2\right)c(c-1)\, a_0\, X^c$$
$$- (\lambda_B + r)\, a_0\, X^c - (\lambda_G - \lambda_B)\, a_0\, X^{c+1} = 0. \tag{G.3}$$

Equation (G.3) has two trivial solutions: $a_0 = 0$ and $X = 0$. We are, however, interested in finding a nontrivial solution and must, therefore, examine the three equations

$$p_0(c) = \frac{1}{2}\sigma_X^2 c(c-1) - \lambda_B - r,$$
$$p_1(c) = -\sigma_X^2 c(c-1) - \lambda_G + \lambda_B,$$
$$p_2(c) = \frac{1}{2}\sigma_X^2 c(c-1),$$

corresponding to the different powers of X.

The possible values of c are determined by $p_0(c)$, as we seek the non-trivial solution ($a_0 \neq 0$). Therefore, we get two possible values of c,

$$c_1 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(\lambda_B + r)}{\sigma_X^2}}, \tag{G.4}$$

$$c_2 = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2(\lambda_B + r)}{\sigma_X^2}}. \tag{G.5}$$

The Frobenius method states that the solution corresponding to c_2 does not exist if the difference between c_1 and c_2 is an integer (Theorem 7.5.3, Trench (2013)). Hence, if $\sqrt{\frac{1}{4} + \frac{2(\lambda_B + r)}{\sigma_X^2}}$ is an integer, only the solution corresponding to c_1 will be valid.

Assuming that the difference between c_1 and c_2 is not an integer, the general solution can be expressed as

$$F_0(X) = A_1 X^{c_1} \sum_{n=0}^{\infty} a_n(c_1) X^n + A_2 X^{c_2} \sum_{n=0}^{\infty} a_n(c_2) X^n.$$

The solution is valid and converges for $X \in (0, 1)$ (Trench, 2013).

The option to invest is worthless if $X = 0$, which is an absorbing state of the belief process. Therefore, $\lim_{X \to 0} F_0(X) = 0$ should hold. For $\lambda_B > 0$ and/or $r > 0$, we have $c_2 < 0$, and X^{c_2} goes to infinity as X goes to zero. This implies that we must have $A_2 = 0$.

We continue to examine the coefficients of different powers of X, in order to find the terms of the series. For the two first terms ($n = 0$ and $n = 1$), we get

$$\frac{1}{2}\sigma_X^2 \left(1 - 2X + X^2\right) c(c-1) a_0 X^c - (\lambda_B + r)a_0 X^c$$

$$-(\lambda_G - \lambda_B) a_0 X^{c+1} + \frac{1}{2}\sigma_X^2 \left(1 - 2X + X^2\right) c(c+1) a_1 X^{c+1}$$

$$-(\lambda_B + r) a_0 X^{c+1} - (\lambda_G - \lambda_B) a_0 X^{c+2} = 0.$$

Collecting the coefficients of X^{c+1}, we get

$$\left(\frac{1}{2}\sigma_X^2 c(c+1) - \lambda_B - r\right) a_1 - \left(\sigma_X^2 c(c-1) + \lambda_G - \lambda_B\right) a_0 = 0. \quad \text{(G.6)}$$

Choosing $a_0 = 1$, gives

$$a_1(c) = \frac{\sigma_X^2 c(c-1) + \lambda_G - \lambda_B}{\frac{1}{2}\sigma_X^2 c(c+1) - \lambda_B - r} = -\frac{p_1(c)}{p_0(c+1)}. \quad \text{(G.7)}$$

For the three first terms ($n = 0$, $n = 1$ and $n = 2$), we have

$$\frac{1}{2}\sigma_X^2 \left(1 - 2X + X^2\right) c(c-1) a_0 X^c - (\lambda_B + r) a_0 X^c$$

$$-(\lambda_G - \lambda_B) a_0 X^{c+1} + \frac{1}{2}\sigma_X^2 \left(1 - 2X + X^2\right) c(c+1) a_1 X^{c+1}$$

$$-(\lambda_B + r) a_1 X^{c+1} - (\lambda_G - \lambda_B) a_1 X^{c+2} + \frac{1}{2}\sigma_X^2 \left(1 - 2X + X^2\right)(c+2-1)(c+2) a_2 X^{c+2}$$

$$-(\lambda_B + r) a_2 X^{c+2} - (\lambda_G - \lambda_B) a_2 X^{c+3} = 0.$$

Collecting the coefficients of X^{c+2}, we get

$$\frac{1}{2}\sigma_X^2 c(c-1) a_0 - \left(\sigma_X^2 c(c+1) a_1\right.$$

$$+ \lambda_g - \lambda_B) a_1 + \left(\frac{1}{2}\sigma_X^2(c+2-1)(c+2) - \lambda_B - r\right) a_2 = 0.$$

Thus,

$$a_2(c) = \frac{2\left[\sigma_X^2 c(c+1)a_1 + \lambda_g - \lambda_B\right] a_1 - \left[\sigma_X^2 c(c-1)\right] a_0}{\sigma_X^2(c+2-1)(c+2) - \lambda_B - r}$$

$$= -\frac{p_1(c+2-1)\, a_1(c) + p_2(c+2-2)\, a_0(c)}{p_0(c+2)}.$$

Examining the terms $n-2$, $n-1$ and n and collecting the coefficients of X^{c+2}, we get the general expression for the nth coefficient

$$a_n(c) = -\frac{p_1(n+c-1)\, a_{n-1}(c) + p_2(n+c-2)\, a_{n-2}(c)}{p_0(n+c)}, \qquad n \geq 2.$$

Thus, the solution of the ODE can be expressed as

$$F_0(X) = A_1 X^{c_1} \sum_{n=0}^{\infty} a_n(c_1)\, X^n,$$

where

$$c_1 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(\lambda_B + r)}{\sigma_X^2}},$$

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and

$$a_0(c) = 1,$$

$$a_1(c) = \frac{\sigma_X^2 c(c-1) - \lambda_B + \lambda_G}{\frac{1}{2}\sigma_X^2 c(c+1) - \lambda_B - r},$$

$$a_n(c) = \frac{2\left[\sigma_X^2(n+c-1)(n+c-2) - \lambda_B + \lambda_G\right]a_{n-1}(c) - \left[\sigma_X^2(n+c-2)(n+c-3)\right]a_{n-2}(c)}{\sigma_X^2(n+c)(n+c-1) - 2(\lambda_B + r)}, \qquad n \geq 2.$$

Appendix H. Solving the system of PDEs

We want to solve the following system of PDEs

$$\frac{1}{2}\sigma_X^2 X^2(1-X)^2\frac{\partial^2 F_0}{\partial X^2} + \frac{1}{2}\sigma_S^2 S^2\frac{\partial^2 F_0}{\partial S^2} + \mu_S S\frac{\partial F_0}{\partial S} - \left(X\lambda_G + (1-X)\lambda_B\right)\left(F_0 - F_1\right) - rF_0 = 0 \tag{H.1}$$

$$\frac{1}{2}\sigma_S^2 S^2\frac{\partial^2 F_1}{\partial S^2} + \mu_S S\frac{\partial F_1}{\partial S} - rF_1 = 0 \tag{H.2}$$

From the PDE in Equation (H.1), we observe that the value of the option to invest in Regime 0, F_0, depends on the option value in regime 1, F_1. Therefore, our starting point is to find an expression for F_1.

Solving Equation (H.2)

In regime 1, a revision has already occurred, and the option value depends only on the stochastic electricity price. We assume that the solution of (H.2) is of the form

$$F_1(S) = A_1 S^{\beta_1} + A_2 S^{\beta_2}. \tag{H.3}$$

By substitution, we see that (H.3) satisfies Equation (H.2) if $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the characteristic equation

$$\mathcal{Q}_1(\beta) = \frac{1}{2}\sigma_S^2\beta(\beta-1) + \mu_S\beta - r. \tag{H.4}$$

Finally, $F_1(S)$ must satisfy the following boundary conditions

$$F_1(0) = 0, \tag{H.5}$$

$$F_1(S^*) = S^*\left[\frac{1 - e^{-(r-\mu_S)T}}{r - \mu_S}\right] - I, \tag{H.6}$$

$$\left.\frac{\partial F_1}{\partial S}\right|_{S=S^*} = \left[\frac{1 - e^{-(r-\mu_S)T}}{r - \mu_S}\right]. \tag{H.7}$$

Condition (H.5) arises since $S = 0$ is an absorbing state of a GBM, and the option is worthless for $S = 0$. Since $S^{\beta_2} \to \infty$ when $S \to 0$, we must have $A_2 = 0$. Condition (H.6) is a value-matching condition and condition (H.7) is a smooth-pasting condition. Solving for A_1 and S^*, we get

$$F_1(S) = A_1 S^{\beta_1}, \tag{H.8}$$

where

$$A_1 = \left(\frac{\beta_1 - 1}{I}\right)^{\beta_1 - 1} \left(\frac{e^{(\mu_S - r)T} - 1}{\beta_1(\mu_S - r)}\right)^{\beta_1}, \tag{H.9}$$

$$\beta_1 = \frac{1}{2} - \frac{\mu_S}{\sigma_S^2} + \sqrt{\left(\frac{\mu_S}{\sigma_S^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_S^2}}, \tag{H.10}$$

$$S^* = \frac{\beta_1}{\beta_1 - 1} I \frac{\mu_S - r}{e^{-(r - \mu_S)T} - 1}. \tag{H.11}$$

Rewriting Equation (H.1)

Next, we substitute (H.8) into (H.1) and let $u = F_0$, $x = X$ and $s = S$. Equation (H.1) can then be written more compactly as

$$a(x)\frac{\partial^2 u}{\partial x^2} + b(s)\frac{\partial^2 u}{\partial s^2} + c(s)\frac{\partial u}{\partial s} + d(x)\,u + e(x, s) = 0, \tag{H.12}$$

where

$$a(x) = \frac{1}{2}\sigma_X^2 x^2 (1 - x)^2, \qquad b(s) = \frac{1}{2}\sigma_S^2 s^2, \qquad c(s) = \mu_s s,$$

$$d(x) = (\lambda_B - \lambda_G)x - \lambda_B - r, \qquad e(x, s) = \left((\lambda_G - \lambda_B)x + \lambda_B\right)A_1 s^{\beta_1}.$$

Boundary conditions

On the bottom boundary, the electricity price is 0, and the option value must be 0. On the top boundary we are in the stopping region, and the option value must equal the payoff. We get

$$u(x, 0) = 0 \qquad\qquad \text{on } \Gamma_1, \tag{H.13}$$

$$u(x, s) = V_0(x, s) \qquad\qquad \text{on } \Gamma_4. \tag{H.14}$$

On the left boundary we must solve Equation (H.12) for $x = 0$ and on the right boundary for $x = 1$ (see below for derivation), which gives

$$u(0, s) = C_1 s^{\gamma_1} + A_1 s^{\beta_1} \qquad \text{on } \Gamma_6, \qquad \text{(H.15)}$$

$$u(0, s) = V_0(0, s) \qquad \text{on } \Gamma_5, \qquad \text{(H.16)}$$

$$u(1, s) = D_1 s^{\eta_1} + A_1 s^{\beta_1} \qquad \text{on } \Gamma_2, \qquad \text{(H.17)}$$

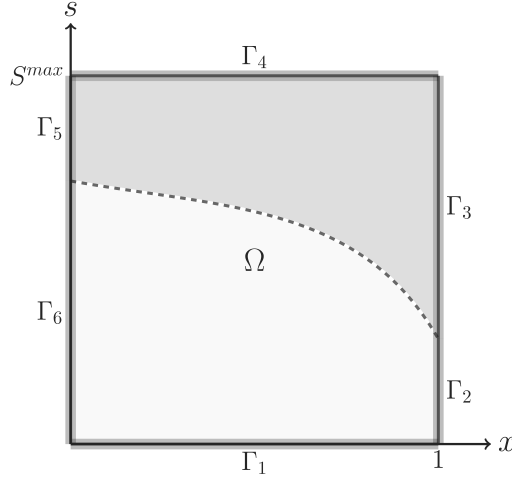$$u(1, s) = V_0(1, s) \qquad \text{on } \Gamma_3. \qquad \text{(H.18)}$$

Figure H.16: Domain $\Omega = (0, 1) \times (0, S^{max})$. Dashed line illustrates the free-boundary and separates the continuation region (light shade) and stopping region (dark shade).

Boundary conditions at $X = 0$

When $X = 0$, the transition rate is λ_B, and the differential equations that must be satisfied by F_0 and F_1, is reduced to

$$\frac{1}{2}\sigma_S^2 S^2 F_{0SS} + \mu_S S F_{0S} - \lambda_B(F_0 - F_1) - rF_0 = 0 \qquad \text{(H.19)}$$

$$\frac{1}{2}\sigma_S^2 S^2 F_{1SS} + \mu_S S F_{1S} - rF_1 = 0 \qquad \text{(H.20)}$$

The solution of (H.20) is given by equation (H.8). The solution to equation (H.19) takes the form

$$F_0(0, S) = C_1 S^{\gamma_1} + C_2 S^{\gamma_2} + A_1 S^{\beta_1}$$

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where A_1 and β_1 are specified by equation (H.9) and (H.10), respectively, and $\gamma_1 > 1$ and $\gamma_2 < 0$ are the roots of the characteristic equation

$$\mathcal{Q}_2(\gamma) = \frac{1}{2}\sigma_S^2\gamma(\gamma - 1) + \mu_S\gamma - (r + \lambda_B) \tag{H.21}$$

Finally, $F_0(0, S)$ must satisfy the following boundary conditions

$$F_0(0, 0) = 0, \tag{H.22}$$

$$F_0(0, S^*) = K\frac{1 - e^{-(r+\lambda_B)T}}{r + \lambda_B} + S^*\left[\frac{\left(1 - e^{-(r-\mu_S)T}\right)}{r - \mu_S} + \frac{\left(e^{-(r+\lambda_B-\mu_S)T} - 1\right)}{r + \lambda_B - \mu_S}\right] - I, \tag{H.23}$$

$$\left.\frac{\partial F_0}{\partial S}\right|_{S=S^*} = \frac{1}{r - \mu_S}\left(1 - e^{-(r-\mu_S)T}\right) + \frac{1}{r + \lambda_B - \mu_S}\left(e^{-(r+\lambda_B-\mu_S)T} - 1\right), \tag{H.24}$$

Since the option is worthless for $S = 0$, we must have $C_2 = 0$. We therefore have

$$F_0(0, S) = C_1 S^{\gamma_1} + A_1 S^{\beta_1} \tag{H.25}$$

where

$$\gamma_1 = \frac{1}{2} - \frac{\mu_S}{\sigma_S^2} + \sqrt{\left(\frac{\mu_S}{\sigma_S^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda_B + r)}{\sigma_S^2}},$$

and C_1 and S^* are solved for numerically. Note that S^* defines the left endpoint of the free boundary, separating Γ_5 from Γ_6.

Boundary conditions at $X = 1$

When $X = 1$ the transition rate is λ_G and the same system of PDEs as for the case with $X = 0$ must be solved, only with λ_G in stead of λ_G. We get

$$F_0(1, S) = D_1 S^{\eta_1} + A_1 S^{\beta_1} \tag{H.26}$$

where

$$\eta_1 = \frac{1}{2} - \frac{\mu_S}{\sigma_S^2} + \sqrt{\left(\frac{\mu_S}{\sigma_S^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda_G + r)}{\sigma_S^2}}.$$

and D_1 and S^{**} are solved for numerically from

$$F_0(1, S^{**}) = K\frac{1 - e^{-(r+\lambda_G)T}}{r + \lambda_G} + S^{**}\left[\frac{\left(1 - e^{-(r-\mu_S)T}\right)}{r - \mu_S} + \frac{\left(e^{-(r+\lambda_G-\mu_S)T} - 1\right)}{r + \lambda_G - \mu_S}\right] - I,$$

(H.27)

$$\left.\frac{\partial F_0}{\partial S}\right|_{S=S^{**}} = \frac{1}{r - \mu_S}\left(1 - e^{-(r-\mu_S)T}\right) + \frac{1}{r + \lambda_G - \mu_S}\left(e^{-(r+\lambda_G-\mu_S)T} - 1\right),$$

(H.28)

Note that S^{**} defines the right endpoint of the free-boundary, separating Γ_2 from Γ_3.

Variational formulation

FEM requires the PDE to be expressed in its variational form. To arrive at the variational formulation we multiply Equation (H.12) with a test function $v(x, s) \in H_0^1(\Omega) = \{v \in H^1(\Omega) : v = 0 \text{ on } \partial\Omega\}$, to get

$$a(x)\frac{\partial^2 u}{\partial x^2}v + b(s)\frac{\partial^2 u}{\partial s^2}v + c(s)\frac{\partial u}{\partial s}v + d(x)\,u\,v + e(x, s)\,v = 0. \qquad \text{(H.29)}$$

Then integrating over the domain yields

$$\int_\Omega a(x)\frac{\partial^2 u}{\partial x^2}v + \int_\Omega b(s)\frac{\partial^2 u}{\partial s^2}v + \int_\Omega \frac{\partial u}{\partial s}v + \int_\Omega d(x)\,u\,v + \int_\Omega e(x, s)\,v = 0 \quad \text{(H.30)}$$

Applying Green's Theorem to the first integral gives

$$\int_\Omega a(x)\frac{\partial^2 u}{\partial x^2}v = \int_{\partial\Omega} a(x)\frac{\partial u}{\partial x}n_x v - \int_\Omega \frac{\partial u}{\partial x}\frac{\partial}{\partial x}\left(a(x)\,v\right), \qquad \text{(H.31)}$$

$$= \int_{\partial\Omega} a(x)\frac{\partial u}{\partial x}n_x v - \int_\Omega \frac{\partial u}{\partial x}\left(\frac{\partial a(x)}{\partial x}v + a(x)\frac{\partial v}{\partial x}\right), \qquad \text{(H.32)}$$

$$= -\int_\Omega a(x)\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} - \int_\Omega \frac{\partial u}{\partial x}\frac{\partial a(x)}{\partial x}v. \qquad \text{(H.33)}$$

The last equality follows from v being defined to be zero on Dirichlet boundaries.

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Applying Green's Theorem to the second integral gives

$$\int_\Omega b(s)\frac{\partial^2 u}{\partial s^2}v = \int_{\partial\Omega} b(s)\frac{\partial u}{\partial s}n_s v - \int_\Omega \frac{\partial u}{\partial s}\frac{\partial}{\partial s}\left(b(s)\,v\right) \tag{H.34}$$

$$= \int_{\partial\Omega} b(s)\frac{\partial u}{\partial s}n_s v - \int_\Omega \frac{\partial u}{\partial s}\left(\frac{\partial b(s)}{\partial s}v + b(s)\frac{\partial v}{\partial s}\right) \tag{H.35}$$

$$= -\int_\Omega b(s)\frac{\partial u}{\partial s}\frac{\partial v}{\partial s} - \int_\Omega \frac{\partial u}{\partial s}\frac{\partial b(s)}{\partial s}v. \tag{H.36}$$

Now, substitute (H.33) and (H.36) back into (H.30) to get

$$-\int_\Omega a(x)\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} - \int_\Omega \frac{\partial u}{\partial x}\frac{\partial a(x)}{\partial x}v$$

$$-\int_\Omega b(s)\frac{\partial u}{\partial s}\frac{\partial v}{\partial s} - \int_\Omega \frac{\partial u}{\partial s}\frac{\partial b(s)}{\partial s}v + \int_\Omega c(s)\frac{\partial u}{\partial s}v + \int_\Omega d(x)\,u\,v + \int_\Omega e(x,s)\,v = 0.$$

Rearranging, gives

$$\int_\Omega \left(a(x)\frac{\partial v}{\partial x}\frac{\partial u}{\partial x} + b(s)\frac{\partial v}{\partial s}\frac{\partial u}{\partial s}\right) +$$

$$\int_\Omega \frac{\partial a(x)}{\partial x}\frac{\partial u}{\partial x}v + \int_\Omega \left(\frac{\partial b(s)}{\partial s} - c(s)\right)\frac{\partial u}{\partial s}v = \int_\Omega d(x)uv + \int_\Omega e(x,s)v. \tag{H.37}$$

The variational formulation can then be written as,

> Find u such that
> $u = g_{\Gamma_i}$ on $\partial\Omega$ for $i = 1, ..., 6$
> Equation (H.37) holds for all v, such that $v = 0$ on $\partial\Omega$

where g_{Γ_i} is a given function on the Dirichlet boundary Γ_i.

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