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Alternative Method for Predictive Functional Control to Handle an Integrating Process

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Abstract—This work proposes an improved method for Predictive Functional Control (PFC) to handle an integrating process. Instead of assuming a constant future input, the dynamic is shaped with a first-order Laguerre polynomial so that it converges to the expected steady state value. This modification provides simpler coding and tuning compared to the conventional method in the literature. Simulation results show that the proposed controller improves the consistency of the open-loop prediction of an integrating process and thus improves closed-loop performance and constraint handling properties. The practicality of this algorithm is also validated on laboratory hardware.

Index Terms—Predictive Control, PFC, Laguerre Function, Constraints, Integrating Process, Transparent Control, Servo System.

I. INTRODUCTION

Predictive Functional Control (PFC) is a simple version of Model Predictive Control (MPC) developed in the early 1970s [1]. This algorithm only requires simple coding and low-level computation while retaining similar benefits to MPC in handling constraints and/or delays [2]. Despite its appealing characteristics, PFC receives relatively little interest in the literature [3] as it does not easily have rigorous properties such as stability assurances [4], [5] or robust feasibility [6]. However, the key selling point of PFC is the simplicity in tuning and implementation; it is a competitor to PID rather than the conventional predictive controller.

The simplistic PFC concept has several limitations, especially when dealing with an integrating process [2]. Due to their marginally stable dynamics, the constant future input assumption of PFC gives a divergent open-loop prediction [2], [7], [8]. Consequently, it may lead to poor closed-loop performance, prediction inconsistency, and also a failure in constraint implementation. Nevertheless, for low-level control applications where PID is unable to handle a constraint, PFC is still considered as an attractive option. Thus, the aim of this paper is to overcome some weaknesses while maintaining the formulation simplicity and cost effectiveness of PFC.

To deal with open-loop unstable plant, PFC practitioners often employ a cascade like structure known as transparent

control [2]. The inner loop consists of a proportional controller to stabilise the system predictions, and PFC provides the target trajectory via an outer loop. In practice, this structure often works better than PID within a constrained environment. However, the use of constant future input assumption can still lead to ill-posed decision making which impacts on both closed-loop performance [9] and constraint handling [10]. The interaction between inner and outer loops also makes the tuning and constraint implementation less transparent and, within the literature, there is no clear or systematic explanation of the approach.

Recent work has shown Laguerre PFC (LPFC) can improve closed-loop performance, prediction consistency and constraint handling for a stable system [9], [10]. This paper explores its capability to handle an integrating process. LPFC shapes the future input trajectory to converge to steady state with 1st order dynamics; this framework can stabilise the open-loop prediction of an integrating system without requiring a cascade structure, thus retaining a standard, and simpler, PFC formulation. Moreover, the Laguerre pole can be used to fine tune the closed loop performance [9], [11] and facilitate more reliable constraint management. The required modification is straightforward and thus in line with the simplicity requirement.

Section II gives some background on the nominal PFC and LPFC frameworks. Section III introduces the transparent control and LPFC law to handle an integrating process. Section IV provides a numerical example and analysis for both approaches. Section V validates the results with laboratory equipment simulations and section VI provides conclusions.

II. PFC FORMULATION FOR NOMINAL SYSTEM

This section only gives a brief review of conventional and Laguerre PFC formulations. For more detailed explanations of PFC theory and concepts can be found in these references, e.g. [2], [3], [7], [9]. To focus on the key conceptual contribution, the offset free correction is omitted from the formulation although it is applied in all the examples. Without loss of generality, all the control structures will use a general transfer function model for prediction. It is noted that the sensitivity of PFC to uncertainties constitutes future work.

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A. Traditional PFC

A PFC framework is based on simple human concepts and computes a required control action depending on how fast a user expects/wishes the output to reach the target. There are two main components in the PFC formulation which are the desired target trajectory and system prediction. The control law is calculated by enforcing the following equality:

$$y_{k+n|k} = R - (R - y_k)\lambda^n \quad (1)$$

where $y_{k+n|k}$ is the n -step ahead system prediction at sample time k . The right hand side of (1) represents the desired trajectory of the output from y_k to the target value R with a convergence rate λ . The two main tuning parameters are:

- The coincidence horizon n defines the point where the system prediction matches the target trajectory.
- The desired closed-loop pole $\lambda = e^{-3T/CLTR}$, with T the sampling time and $CLTR$ the closed-loop time response.

Since the n -step ahead prediction algebra of a linear transfer function mathematical model is well known in the literature (e.g. [12]), here only the key results are provided. For inputs u_k and outputs y_k , the n -step ahead linear model prediction is given as:

$$y_{k+n|k} = H_n \underline{u}_k + P_n \underline{u}_k + Q_n \underline{y}_k \quad (2)$$

where parameters H_n , P_n , Q_n depend on the model parameters and for a model of order m :

$$\underline{u}_k = \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+n-1} \end{bmatrix}; \underline{u}_k = \begin{bmatrix} u_{k-1} \\ u_{k-2} \\ \vdots \\ u_{k-m} \end{bmatrix}; \underline{y}_k = \begin{bmatrix} y_k \\ y_{k-1} \\ \vdots \\ y_{k-m} \end{bmatrix} \quad (3)$$

The control input is computed by substituting the prediction (2) directly into (1) to obtain:

$$H_n \underline{u}_k + P_n \underline{u}_k + Q_n \underline{y}_k = R - (R - y_k)\lambda^n \quad (4)$$

Adding a constant future input prediction assumption $u_{k+i|k} = u_k, i = 0, \dots, n$ and defining $h_n = \sum(H_n)$, the nominal PFC control law reduces to:

$$\underline{u}_k = \frac{R - (R - y_k)\lambda^n - (P_n \underline{u}_k + Q_n \underline{y}_k)}{h_n} \quad (5)$$

Remark 1: The tuning parameter λ should make the design process transparent, however the selection of coincidence horizon n affects the efficacy of λ due to the constant future input assumption [7]. With small horizons, the effectiveness of λ is more significant, but there may be poor prediction consistency with the target behaviour resulting in poor closed-loop behaviour. Conversely, larger horizons gives better prediction consistency but reducing the effectiveness of λ as a tuning parameter.

Remark 2: Prediction consistency is important for effective constraint handling thus, for some challenging dynamics, a constant input assumption may be ineffective [10].

B. Laguerre PFC (LPFC)

The LPFC approach utilises the expected constant steady-state input u_{ss} to eliminate the offset. The z -transform of discrete Laguerre polynomials are [11]:

$$L(z) = \sqrt{1-a^2} \frac{(z^{-1}-a)^{j-1}}{(1-az^{-1})^j}; \quad 0 < a < 1 \quad (6)$$

where j is the order of Laguerre function and a is the Laguerre pole which depends on a user selection between $0 < a < 1$. For simplicity of coding and concept, a first-order Laguerre polynomial is employed here, although high order polynomials may be used [11], [13], [14]. The function with altered scaling becomes an exponential decay as:

$$L(z) = \frac{1}{1-az^{-1}} \equiv 1 + az^{-1} + a^2 z^{-2} + \dots \quad (7)$$

Hence, the future input prediction is parametrised as:

$$u_{k+n} = u_{ss} + L_n \eta; \quad L_n = a^n; \quad L = [1, a, \dots, a^{n-1}] \quad (8)$$

where η is a degree of freedom. The parametrisation of (8) gives output predictions which converge to the steady-state exponentially with a rate a . The n step ahead output prediction is derived by substituting (8) into (2):

$$y_{k+n|k} = h_n u_{ss} + H_n L_n \eta_k + P_n \underline{u}_k + Q_n \underline{y}_k \quad (9)$$

Hence, the LPFC control law is defined by substituting prediction (9) into (1) and solving for parameter η as:

$$\eta_k = \frac{R - (R - y_k)\lambda^n - (P_n \underline{u}_k + Q_n \underline{y}_k) - h_n u_{ss}}{H_n L_n} \quad (10)$$

Due to the receding horizon principle [11] and the definition of $L(z)$ in (7), the current input u_k is:

$$u_k = u_{ss} + \eta_k \quad (11)$$

Remark 3: By shaping the future input dynamics, LPFC can improve prediction consistency, closed-loop performance and the efficacy of λ as a tuning parameter [9]. This improvement also provides a more accurate and less conservative solution when satisfying output/state limits [10].

III. PFC FORMULATION FOR INTEGRATING SYSTEM

An integrating process has marginally stable dynamics as one of the poles resides at the origin. This pole gives an extra challenge to the traditional PFC framework because the open-loop prediction does not converge when using the constant future input assumption. This section presents two alternative frameworks. The first subsection briefly reviews the proposed technique in the literature, so-called transparent control by [2], and the following subsection presents the proposed modification of LPFC for integrating processes. Currently, the concept is only introduced to a single integrator problem, while future work will consider a further generalisation for a plant with multiple integrators or marginally stable poles.

A. Transparent Control

Transparent control utilises two level of cascade structure (see Fig. 1). The inner loop employs a proportional gain with negative feedback to stabilise the open loop prediction, while nominal PFC controls the outer loop and eliminates any offset due to disturbance and enhances the overall dynamic performance [2].

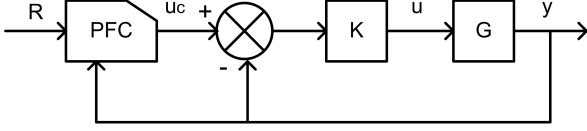


Fig. 1: Transparent PFC structure.

The inner loop with gain K will be used as a prediction model as in (12) instead of the plant model G to compute the manipulated input u_c .

$$y(s) = \frac{GK}{1 + GK} u_c(s) \quad (12)$$

The actual input u that will be send to the plant is:

$$u_k = K(u_{c,k} - y_k) \quad (13)$$

With this technique, the controlled system is able to maintain regulation during set-point changes by introducing a temporary over-compensated set-point [2]. At the same time, the outer loop will minimise the tracking error using a standard PFC formulation as discussed in section II-A.

Remark 4: Transparent PFC (TPFC) only accepts proportional gain rather than the combination with integral and/or derivative to keep the constraint implementation purely algebraic [2]. To implement input or rate constraints, a back calculation procedure is needed to transfer the information from the inner loop to the outer loop and avoiding saturation as:

$$y_k + \frac{u_{min}}{K} \leq u_{c,k} \leq y_k + \frac{u_{max}}{K} \quad (14)$$

$$y_k + \frac{\Delta u_{min} + u_{k-1}}{K} \leq u_{c,k} \leq y_k + \frac{\Delta u_{max} + u_{k-1}}{K} \quad (15)$$

Since the constraint is implemented at the current time only, there is no check that the implied predictions satisfy constraints in the future and thus recursive infeasibility may result.

Remark 5: For output or state constraints, the traditional practise utilises a multiple PFC regulators that run in parallel [2], [10]. The first regulator computes the preferred control action while the second regulator produces an input to satisfy the limit. A supervisor will choose the correct input for the plant. However, advances in computation technology mean this tedious *ad hoc* approach can be replaced with a more systematic, but simple, approach such as in [15].

B. Laguerre PFC for Integrating Process

Due to the pole on the origin, the steady state input for an integrating process is zero for a constant set point. These dynamics are still compatible with a LPFC law (section II-B) with the only required modification being to define $u_{ss} = 0$.

Theorem 1: The future input dynamics of

$$u(z) = \frac{u_{ss}}{1 - z^{-1}} + \frac{\eta_k}{1 - az^{-1}} \quad (16)$$

will give input predictions that settle exponentially at zero with a speed linked to Laguerre pole a . For an integrating process, the value η_k effects the implied steady-state outputs.

Proof: The signal defined in (16) has the property that

$$\lim_{k \rightarrow \infty} u_k = u_{ss} \Rightarrow \lim_{k \rightarrow \infty} y_k = R \quad (17)$$

When $u_{ss} = 0$, the steady-state output has affine dependence on the integral of the future input. \square

Remark 6: For simple first order system the value of a should be equal to λ [9]. However for a higher order system, selecting $a < \lambda$ will give faster convergence to steady state and thus can improve the prediction consistency.

Algorithm 1 (LPFC): For integrating process, a similar algorithm as in (10) is used except that u_{ss} term is removed.

$$\eta_k = \frac{R - (R - y_k)\lambda^n - (P_n u_k + Q_n y_k)}{H_n L_n} \quad (18)$$

Theorem 2: Using LPFC input predictions as defined in *Algorithm 1*, output, state and input constraints can be represented by a set of linear inequalities.

Proof: Output constraints can be constructed from the output predictions in (9) within the validation horizon n_i and comparison with the limits at each sample instant, e.g.:

$$y_{min} \leq H_{n_i} L_{n_i} \eta_k + P_{n_i} u_k + Q_{n_i} y_k \leq y_{max}, \quad \forall i > 0 \quad (19)$$

The maximum/minimum input rate/value occur at the first sample, so input constraints can be formulated as:

$$u_{min} \leq \eta_k \leq u_{max} \quad (20)$$

$$\Delta u_{min} \leq \eta_k - u_{k-1} \leq u_{max} \quad (21)$$

Combining (19,20,21) it is clear that for suitable M, v_k one can represent the satisfaction of constraints by predictions using a single vector inequality of the form:

$$M \eta_k \leq v_k \quad \square \quad (22)$$

We can now define the constraint handling algorithm which is akin to methods given in [15].

Algorithm 2: [LPFC constrained] Use the unconstrained law (18) to determine the ideal value of η_k and check each constraint in (22) using a simple loop (subscripts denote position in a vector).

Set $u_{max} = \infty, u_{min} = -\infty$.

For $i=1$:end,

if $M_i \eta_k \not\leq v_i$ & $M_i > 0$ then define $u_{max} = v_i / M_i$,

if $M_i \eta_k \not\leq v_i$ & $M_i < 0$ then define $u_{min} = v_i / M_i$,

end loop.

if $\eta_k < u_{min}$, $\eta_k = u_{min}$. if $u_{max} < \eta_k$, $\eta_k = u_{max}$.

Define $u(k)$ using (16).

Note that the upper and lower limits to ensure recursive feasibility update at each cycle in the loop but as all the inequalities are only ever tightened, changes lower down cannot contradict changes higher up throughout the horizon.

Remark 7: Infeasibility can arise due to too fast or large changes in the target. However, LPFC helps in this case because the exponential structure embedded into the input prediction automatically slows down any over aggressive input responses and thus significantly increases the likelihood of feasibility being retained. In the worst case, set point changes need to be moderated as in reference governor approaches [16].

The summary benefits of this algorithm are:

- It offers a simple and systematic framework to handle an integrating process.
- It stabilises the output prediction without a cascade structure thus no back calculation process (*Remark 4*) is needed for input/rate constraints.
- The Laguerre pole a can be utilised to control the speed of convergence to improve the prediction consistency and efficacy of constrained solution.
- The implied structure of (16) in conjunction with constraints (22) means that a recursive feasibility guarantee (nominal case) is provided [15].

IV. NUMERICAL EXAMPLES AND ANALYSIS

This section presents a numerical example to demonstrate the benefit of using LPFC compared to TPFC. A first order servo system with integrator is considered as a plant where the control objective is to track the position. The discrete mathematical model with sampling time 0.02s is:

$$G = \frac{0.0095z^{-2} + 0.0073z^{-1}}{1 - 1.45z^{-1} + 0.45z^{-2}} \quad (23)$$

This simulation will focus on the tuning process and the concept of well-posed decision making that can be observed by comparing the open-loop prediction and closed-loop behaviour of the controller. In addition, the efficacy of constraint handling is also discussed in the last subsection.

A. Tuning and Performance of TPFC

The first step in implementing TPFC is to tune the proportional gain before selecting the coincidence horizon n . This gain K will determine the convergence speed and the steady state error of the inner-loop. Small gain leads to slower responses, while too large a gain causes oscillatory behaviour. The root-locus (continuous time) plot shows that the choice $K = 6.45$ is around a critical value in that higher K would give oscillatory poles (see Fig. 2).

The coincidence horizon is selected by comparing the step response with a desired first order target trajectory r with $\lambda = 0.74$ (refer to Fig. 2). Since the inner loop is second order (due to the added integrator), it is necessary [7] to choose a coincidence horizon in the range $3 \leq n \leq 8$; lower values are often preferable so here $n = 3$.

Fig. 3 shows the closed-loop and predicted (at sample $k = 0$) performance of TPFC with different values of gain K . The actual closed-loop behaviour is expressed as y (output) and u (input), while the implied predictions are denoted by signals output y_p and input u_p (corresponding to first value

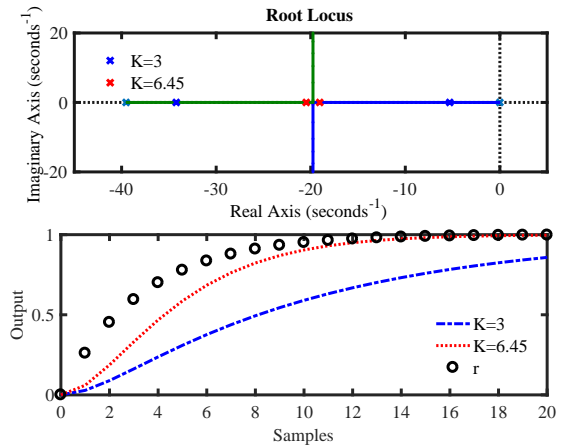


Fig. 2: Root locus and step response of $\frac{GK}{1+GK}$ with different gain K .

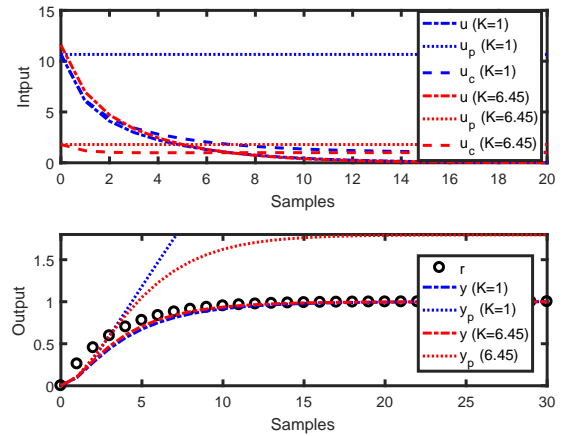


Fig. 3: Closed-loop and open-loop behaviour of G for TPFC with different K .

of input produced by PFC u_c instead of the actual input u). For both choices of K , in the unconstrained case, the closed-loop outputs y track the trajectory set point r and with almost equivalent speed. However, with $K = 1$, the implied prediction y_p has a very slow convergence in response to the constant input dynamics u_p and is inconsistent with the closed-loop behaviour y that results. With a higher gain value ($K = 6.45$), the consistency is improved but still poor. This inconsistency is likely to lead to severely flawed decision making should constraint handling be required.

B. Improvement on Prediction Consistency with LPFC.

For LPFC, due to the presence of an integrator, the coincidence horizon is selected based on the the impulse response (see Fig. 4) using the guidance of [7]; this suggests a value in the region of $n = 3$.

Ideally, for a first order system, the value of Laguerre pole a should be equal to the desired closed loop pole λ [10].

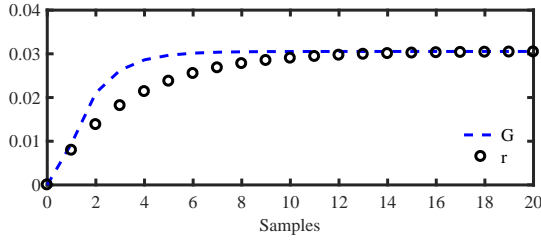


Fig. 4: Impulse response of G with target trajectory r

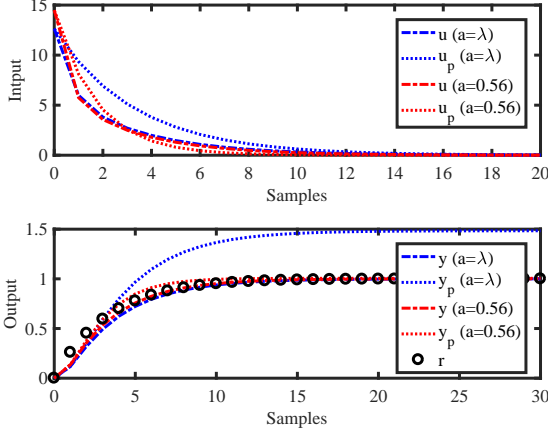


Fig. 5: Closed-loop and open-loop behaviour of G for LPFC with varying Laguerre pole a .

However, for a higher order system, this value needs to be further tuned as it has an impact on the convergence rate of the output prediction when tracking the first order target trajectory (*Theorem 1*). Fig. 5 shows that with a choice of $a = \lambda$, while the controller still tracks the set point well, there is still noticeable inconsistency between predictions y_p and the closed-loop behaviour y . However, reducing the pole to $a = 0.55$ improves the prediction consistency and the overall closed-loop performance is still good (*Remark 6*).

C. Improvement in Constrained Performance with LPFC

One of the key selling points of PFC is the computationally simple (low cost) constraint handling ability. When the system input is bounded to $u_{max} = 8$ (see Fig. 6), both of the controllers manage to track the set point and satisfy the given limit although LPFC gives a slightly better closed-loop performance due to the well posed decision. Moreover, the LPFC formulation is more straightforward to implement and does not require back calculation methods (*Remark 4*).

Fig. 7 shows the system response of both controllers when the output is limited to $y_{max} = 0.8$. The validation horizon is selected at $n_i = 10$ to cover most of the transient period and prevent constraint violation at the early stage. In this case the closed-loop response of TPFC is slower and more conservative in satisfying the limit due to the prediction inconsistency demonstrated in Fig 3. Conversely, LPFC which is based on more consistent predictions (see Fig 5) converges much faster

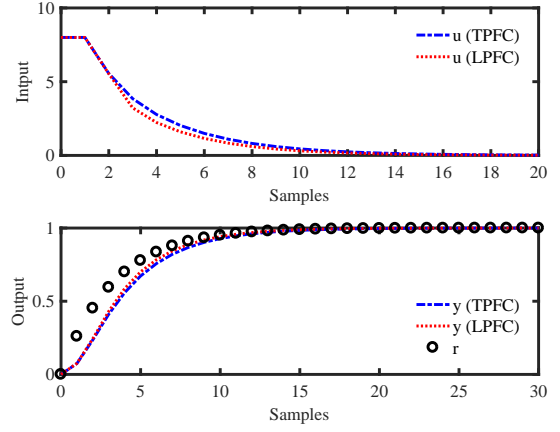


Fig. 6: Closed-loop response of G for LPFC and TPFC with bounded input ($u_{max} = 8$).

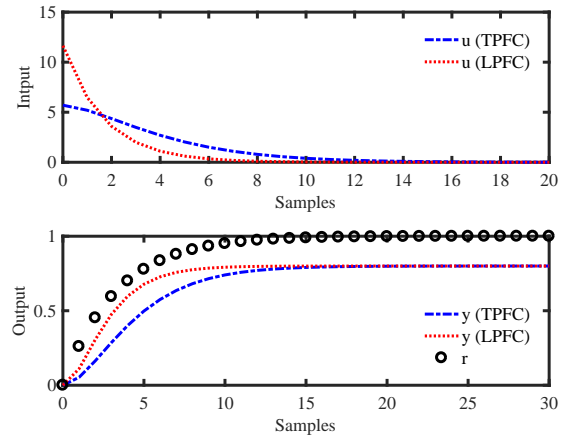


Fig. 7: Closed-loop response of G for LPFC and TPFC with bounded output ($y_{max} = 0.8$).

compared to TPFC. Clearly the constrained solution of LPFC is more accurate and less conservative.

V. IMPLEMENTATION ON REAL HARDWARE

To validate the practicality of LPFC, the algorithm is tested on a Quanser SRV02 servo based unit powered by a Quanser VoltPAQ-X1 amplifier (see Fig. 8). This system is operated by National Instrument ELVIS II+ multifunctional data acquisition. The plant is connected to a computer via a USB connection using NI LabVIEW software. The control objective is to track the servo position $\theta(t)$ by manipulating the supplied voltage $u(t)$. The mathematical model of this system is given as (for more details, refer to [17] user manual):

$$0.0254\ddot{\theta}(t) = 1.53u(t) - \dot{\theta}(t) \quad (24)$$

where $\ddot{\theta}(t)$ and $\dot{\theta}(t)$ are both servo angular acceleration and speed, respectively. Converting the continuous model in (24) to discrete form with sampling time 0.02 s, the transfer function of angular speed to voltage input becomes G as in (23).



Fig. 8: Quanser SRV02 servo based unit.

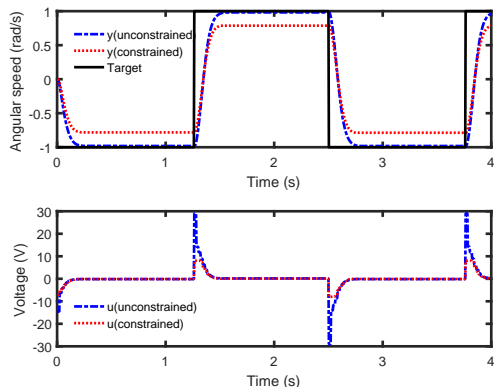


Fig. 9: Unconstrained and constrained performances of LPFC in tracking the Quanser SRV02 servo position.

The algorithm is employed with similar tuning parameters as in the previous numerical example ($\lambda = 0.74$, $n = 3$ and $a = 0.55$). Fig. 9 demonstrates the unconstrained and constrained performances of LPFC to track an alternating set point between -1 rad/s to 1 rad/s. For the unconstrained case, a similar performance to the simulation studies are obtained. The controller manages to provide a smooth tracking to the desired target while retaining the intuitive link between the target dynamic λ and the closed-loop convergences speed (CLTR = $0.2s$). In the constrained case, the implied input limits ($-8v \leq u_k \leq 8v$) and output limits ($-0.8 \leq y_k \leq 0.8$) are satisfied without any conflict by employing the systematic constraint method (*Algorithm 2*).

VI. CONCLUSIONS

This work proposes an alternative Laguerre PFC approach to control an integrating process. Since the traditional PFC formulation for integrating processes is unable to give a stable open loop prediction, the *transparent control* approach is often used. Although this cascade structure can stabilise the plant using a proportional controller, the decision making process may still be poorly posed, and notably can lead to a highly conservative solution in the presence of constraints. Conversely, by shaping input predictions using a Laguerre

polynomial, the nominal PFC method can be employed without a cascade structure. Besides, the improved prediction consistency of LPFC enables the constrained solution to become more accurate and less conservative, thus improving performance. This paper has also demonstrated the efficacy of the proposed LPFC algorithm on laboratory hardware with active constraints. Critically, the proposed algorithm is very simple to code and implement which in line with the core markets for PFC approaches.

Nevertheless, there is a potential weakness with LPFC especially when the independent model structure is used. A small offset error may occur if there is a model mismatch or the real plant is not in fact integrating. Future work aims to look more closely at this issue while providing a formal sensitivity analysis and systematic design of LPFC in handling uncertainty. Another important consideration is to analyse the alternative shaping methods which may be better tailored to deal with higher order and/or challenging dynamical.

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