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Horizontal Mergers and Product Quality

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Abstract. We study the effects of a horizontal merger when firms compete on price and quality. In a Salop framework with three symmetric firms, several striking results appear. First, the merging firms reduce quality but possibly also price, whereas the outside firm increases both price and quality. As a result, the average price in the market increases, but also the average quality. Second, the outside firm benefits more than the merging firms from the merger, and the merger can be unprofitable for the merger partners, i.e., the 'merger paradox' may appear. Third, the merger always reduces total consumer utility (though some consumers may benefit), but total welfare can increase due to endogenous quality cost savings. In a generalised framework with n firms, we identify two key factors for the merger effects: (i) the magnitude of marginal variable quality costs, which determines the nature of strategic interaction, and (ii) the cross-quality and cross-price demand effects, which determines the intensity of price relative to quality competition. These findings have implications for antitrust policy in industries where quality is a key strategic variable for the firms.

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JEL classifiction: L13, L15, L41

1 Introduction

In most markets firms compete not only on price (or quantity), but also along several non-price dimensions, such as quality, variety, marketing, R&D, etc. In such markets horizontal mergers facilitate not just coordination on price (or quantity), but indeed also on the relevant non-price variables, implying that the merger effects will depend on the strategic relationship between price and non-price variables. Despite this obvious fact, within the extensive literature on mergers surprisingly few papers have explicitly analysed the effects of horizontal mergers when firms compete along both price *and* non-price variables.[1]

In this paper we offer a contribution towards filling this gap in the literature by focusing on *quality* as the key non-price variable. Product (or service) quality affects consumer choice and demand, and is therefore an important strategic variable for firms in most industries. For example, airline companies decide on service quality and frequency of flights to attract customers; pharmaceutical companies invest in R&D to improve drug quality; health-care providers ac-

[1]See, for instance, the extensive survey by Whinston (2007). Although there is a large empirical literature on horizontal mergers that accounts for product characteristics (see, e.g., Björnerstedt and Verboven, 2016, and references therein) these are treated as exogenous, and not as an endogenous strategic variable for the firms, which is a key feature of our model.

quire medical technology to offer better health services; car dealers expend promotional effort to increase sales. In such industries, horizontal mergers will affect not just prices but indeed also product (or service) quality, which jointly determines merger profitability and welfare effects.

The quality dimension has received more attention in recent antitrust practice. For example, a feared reduction of service quality was one of the elements determining the European Commission's decision to reject the proposed takeover of Aer Lingus by Ryanair in 2007.[2] Nevertheless, the effects of mergers on quality remain an under-researched issue, which poses a considerable challenge to competition policy practitioners, as pointed out by the OECD Competition Committee:

> "...the role of quality effects in merger controls, and in particular, trading off between quality and price effects, remains to be one of the most vexatious – and still unresolved – issues." (OECD, 2013, p. 1)

When analysing the (anti-)competitive effects of a horizontal merger, we ask the following set of questions: Does the merger result in higher prices and poorer quality, or is there a scope for lower prices or higher quality? Is the merger profitable, and in case for whom? Can a merger without any direct cost synergies be welfare improving? As our analysis will reveal, the answers to these questions crucially rely on two factors: (i) the strategic relationship between firms' price and quality decisions; and (ii) the intensity of competition (or demand-responsiveness) on quality relative to price.

In the first part of the paper, we apply a Salop (1979) spatial competition framework where demand is explicitly derived from individual preferences and depends on price, quality and distance, which implies that products are horizontally and (potentially) vertically differentiated. We consider a pre-merger market structure with three identical firms symmetrically located on the Salop circle. We assume that two of the three firms merge, which implies that the merger effects also include the strategic response by the non-merging (outside) firm.[3] The merger facilitates coordination of price and quality decisions by the merging firms. Thus, the post-merger market structure is a duopoly with two asymmetric firms, i.e., the two merged firms and the outside firm. There are no direct cost synergies of the merger, but the firms' costs may of course be affected by the merger through endogenous changes in price and quality.

From the Salop framework, we derive three striking results. First, the merging firms reduce quality but possibly also price, whereas the outside firm increases both price and quality. This result is surprising at first glance. If firms compete only on price (with fixed quality), a merger would lead to higher prices both for the merging and outside firms due to prices being strategic complements. Moreover, if firms compete only on quality (with fixed price), the merger would lead to lower quality at both the merging and non-merging firms due to qualities being strategic complements.[4]

[2] Commission decision of 27 June 2007 in Case No COMP/M.4439 – Ryanair / Aer Lingus I.

[3] A merger that involves all firms in the market is not considered for two reasons. First, mergers that result in a monopoly are usually prohibited by antitrust authorities, and thus rarely observed in practice. Second, such mergers ignore the strategic response by the non-merging firms, and the merger effects are usually straightforward (and equivalent to comparing monopoly to competition).

[4] As we will show later, quality decisions are strategic complements, for given prices, if firms have variable quality costs. If quality costs are only fixed costs, then quality decisions are strategically independent (when

However, when firms compete on both price and quality, the nature of strategic interaction changes, and the merger effects are qualitatively different. Whereas the merging firms always reduce quality, the non-merging firm responds by *increasing* quality.[5] The reason is that the outside firm also responds to the merger by increasing prices, and a higher profit margin makes it profitable to increase quality, thus making qualities *net strategic substitutes* among firms.[6] In fact, the quality response by the outside firm is sufficiently strong to ensure an increase in *average* quality (weighted by demand) in the market as a result of the merger.

Moreover, the non-merging firm always responds to the merger by increasing prices, whereas the merging firms increase prices only if demand responsiveness to quality is sufficiently low. The possibility that prices of merging firms may decrease is at first glance surprising and results from the fact that price and quality are *within-firm* strategic complements. A merger allows the merging parties to internalise a negative competition externality by reducing quality and increasing price. However, lower quality reduces demand, which implies that profits are maximised at a lower price. This latter effect is the dominant one, implying that the overall effect of the merger is a price reduction by the merging firms, if the demand responsiveness to quality is sufficiently strong. Nevertheless, the average price in the market always increases as a result of the merger.

Second, the non-merging firm always benefits more from the merger than the merging firms. Thus, the well-known 'merger paradox' is present also in our framework with quality competition. Furthermore, a merger is privately profitable unless demand responsiveness to quality is sufficiently high. If demand is very quality-elastic, a merger triggers a quality increase by the non-merging firm that is sufficiently strong to make the merger unprofitable. This is an interesting result considering that prices are strategic complements, which tends to make mergers profitable under *Bertrand* competition (with differentiated products).[7]

Third, because of the non-uniform effects of a merger on quality and price, the welfare effects are generally ambiguous. We show that consumer utility is reduced on average, although some consumers may actually be better off: if demand is sufficiently quality-elastic, the utility gain of the quality increase outweighs the utility loss of the price increase for consumers who buy from the non-merging firm. Perhaps surprisingly, we also find that a merger might in fact improve social welfare if demand is sufficiently responsive to quality. The reason is that a merger indirectly leads to savings of (endogenous) fixed quality costs.

In the second part of the paper, we generalise the analysis by considering an imperfect competition framework with n firms and fairly general demand and cost functions. While the merger effects are, as expected, more ambiguous, we are able to pin down two key factors that determine the effects on price and quality: (i) the magnitude of marginal variable quality costs, which determines the nature of strategic interaction along the quality and price dimen-

prices are fixed). For more details, see, for instance, Brekke et al. (forthcoming) who analyse the effects hospital mergers with regulated prices.

[5]This asymmetric quality effect of a merger is partly mirrored in the results from the simulated effects of a merger in the Minneapolis newspaper market in Fan (2013), where she finds a drop in content quality for the merged newspapers but a quality increase for two out of three competitors.

[6]The concept of net strategic substitutability/complementarity is explained in Section 3.

[7]See the seminal work by Denecker and Davidson (1985). Assuming Cournot competition, Salant et al. (1983) reported the striking result that mergers are usually not profitable for the merging firms unless sufficiently many firms take part in the merger. See also Perry and Porter (1985) and Farrell and Shapiro (1990).

sions, and (ii) the relative magnitude of cross-quality and cross-price effects on demand, which determines the relative intensity of competition along the quality and price dimensions. More precisely, we define the intensity of competition to be stronger along the quality dimension than along the price dimension if cross-quality effects (on demand) are larger (in absolute value) than cross-price effects, and *vice versa*. Using this conceptual definition, we show that if firms compete sufficiently strongly on quality relative to price, the merged firm will increase both price and quality, and, *vice versa*, if competition is sufficiently much stronger on prices than on qualities, the merged firm will reduce both price and quality. However, for intermediate cases, the merged firm's quality and price responses are generally ambiguous.

The response from non-merging firms depends on the nature of strategic interaction. If variable quality costs are sufficiently small, qualities are net strategic substitutes and prices are net strategic complements. In this case, we show that the non-merging firms' quality and price responses always go in the same direction. If the merged firm's incentives to reduce quality are sufficiently strong, the non-merging firms will respond by *increasing* both quality and price, which is the case in the Salop framework. On the other hand, if the merged firm has sufficiently strong incentives to increase prices, the non-merging firms will respond by *reducing* both quality and price. The former case arises if competition is sufficiently strong along the quality dimension, whereas the latter case requires that competition is sufficiently strong along the price dimension.

The rest of the paper is organised as follows. In Section 2 we relate our paper to the existing literature. In Section 3 and 4 we set up a three firm Salop model and derive the effects of a horizontal merger on the market outcomes and welfare. The merger effects on price and quality are generalised in Section 4, where we present a general differentiated-products framework with n firms. In Section 5 we summarise our findings and provide some concluding remarks. All proofs of the propositions in Sections 3 and 4 are provided in Appendix A and B, respectively.

2 Related literature

The economic literature on horizontal mergers is large, starting with the seminal contributions by Salant et al. (1983), Perry and Porter (1985), and Farrell and Shapiro (1990) for horizontal mergers in a Cournot oligopoly. A key result from these studies is the 'merger paradox', i.e., that horizontal mergers tend to be unprofitable for the merging firms but profitable for the non-merging firms.[8] The reason is that the non-merging firms respond by increasing production and thus capturing a larger share of the market, making the quantity reduction by the merging firms unprofitable, though the market price increases. Our study is more related to the seminal paper by Deneckere and Davidson (1985) who find in a Bertrand oligopoly with differentiated products that horizontal mergers tend to be profitable for all firms in the industry; a result that is due to the fact that prices are strategic complements. We show in our paper that the 'merger paradox' may re-appear in a model where firms sell differentiated products and compete in prices and quality. The main reason for this is that the non-merging firms respond to the merger by increasing quality and capturing a larger share of the market,

[8]Salant et al. (1983) show that if sufficiently many of the firms in the industry take part in the merger, then the merger become profitable. Farrell and Shapiro (1990) consider also efficiency gains, which of course increases the scope for profitable mergers, and also welfare improving mergers.

i.e., qualities are (net) strategic substitutes.[9]

Although the literature on horizontal mergers is generally large, surprisingly few papers explicitly study merger effects when competition is multidimensional, including price and non-price variables. A recent and rare exception is Pinto and Sibley (2016) who analyse unilateral merger effects using a differentiated Bertrand model with endogenous quality. Based on numerical simulations with five firms, they find that the merged firm's quality might increase or decrease as a result of a merger. Our paper differs in that we are able to derive analytical results on price and quality effects of a horizontal merger using a Salop framework, which also enables us to conduct a full welfare analysis of the merger effects. Our paper also differs in that we propose a generalised model with n firms, non-linear demand and non-parameterised cost structure, which allows us to identify key mechanisms that determine the nature of competition and thus merger effects in markets where firms compete on price and quality.

There exists, though, a rich literature on competition and quality, dating back at least to Swan (1970), who compared the incentives of a monopolist and a competitive firm with respect to a particular quality dimension, namely product durability. Much of the subsequent literature consists of papers that apply a vertical differentiation framework, often with firms that offer a range of products with different qualities.[10] Models of price-quality competition in a horizontal differentiation framework are fewer and include, i.e., Economides (1993), Ma and Burgess (1993), Gravelle (1999) and Brekke et al. (2010).[11] However, there is no explicit merger analysis in these papers.

In fact, theoretical studies that explicitly analyse the effects of a horizontal merger on the price and quality offered by merging and non-merging firms are almost non-existent. A notable exception is Gabszewicz et al. (2015), who analyse a game of endogenous coalition formation in a three-firm version of Mussa and Rosen's (1978) model of vertical differentiation. Because of the intrinsic asymmetry of the chosen modelling framework, the focus of this paper is mainly on identifying equilibrium coalition structures and it is therefore quite different from the present paper. Willig (2011) includes product quality in an analysis of unilateral competitive effects of horizontal mergers ('upward pricing pressure'), but there is no equilibrium analysis with strategic interaction between merging and non-merging firms. Tenn et al. (2010), which is mainly an empirical study, construct a theoretical model where firms compete in price and promotion, and show that the effects of the merger on prices are very different in the presence of promotional competition. However, there is no strategic interaction between merging and non-merging firms, since the merger is equivalent to monopolising the industry, i.e., the merger changes the market structure from duopoly to monopoly.[12]

[9] This is always the case in the linear Salop model, but also the case in the more general model for sufficiently small marginal variable quality costs.

[10] Some early key contributions to this strand of the literature include Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1982, 1983) for the case of single-product firms, and Mussa and Rosen (1979), Gal-Or (1983) and Champsaur and Rochet (1989) for the case of multi-product firms.

[11] There are also a few empirical papers studying the effect of more competition on quality; for example Mazzeo (2003), who finds a positive relationship between competition and quality in the US airline industry; and Matsa (2011) who studies the effect of competition on supermarkets' incentive to provide quality, and finds that competition from Wal-Mart decreases inventory shortfalls by up to 24 percent.

[12] In a related paper, Brekke et al. (forthcoming) study hospital mergers, allowing for quality competition and strategic interaction between merging and non-merging firms. However, prices are regulated and hospitals are assumed to be semi-altruistic firms. Two other papers on hospital mergers by Calem et al. (2003) and

Despite the obvious importance of the topic, the empirical literature on the effects of horizontal mergers on quality is also relatively scarce.[13] However, there are a few exceptions. Fan (2013) develops a structural model of newspaper markets and show that ignoring adjustments to product characteristics as a result of a merger substantially affects the simulated merger effects. Focusing on a merger in the Minneapolis newspaper market that was blocked by the US Department of Justice, she finds that the merger would have led to higher prices, but a reduction in content quality, local news ratio and content variety. Similar conclusions are reached by Tenn et al. (2010) based on merger simulation in the ice cream industry, where they report that a blocked merger would have led to higher prices and lower promotional effort.[14] Israel et al. (2013) conduct a merger simulation in the US airline industry, and find that prices (fares) would increase but also quality, resulting in a reduction in quality-adjusted prices due to the merger. There are also some empirical studies on (actual) mergers in the hospital industry, which tend to find large price effects and weak (though mostly negative) effects on quality (see Gaynor and Town, 2012).[15] Thus, there is great ambiguity in the empirical findings on horizontal mergers, which reflects the findings of our theoretical analysis.

Our paper is also somewhat related to the literature on horizontal mergers and product choice, which acknowledges that a merger might lead the merging (and possibly non-merging) firms to reposition their products or to change their product line. Theoretical contributions in this strand of the literature include Lommerud and Sørgard (1997), Posada and Straume (2004), Norman et al. (2005) and Gandhi et al. (2008), whereas key empirical contributions include Berry and Waldfogel (2001) and Sweeting (2010). Although in our formal analysis we do not allow, by assumption, for the possibility that a merger might affect product choices, we will return to a further discussion of this possibility in the final section of the paper.

3 A Salop model

In this section, we set up a simple Salop model where firms compete on price and quality. Consider a market for a particular good where three firms, denoted by $i = 1, 2, 3$, are equidistantly located on a circle with circumference equal to 1.[16] A total mass of 1 consumers are uniformly distributed on the same circle. The spatial dimension reflects either horizontal product differentiation or geographical distance. Each consumer demands one unit of the good from the most preferred provider. The net utility of a consumer located at z and buying the good from

Brekke (2004) allow for both price and quality competition, but consider only mergers that lead to monopoly.

[13] The empirical literature on horizontal mergers in general, though, is extensive; see, for instance, the review by Whinston (2007).

[14] They also show that ignoring promotional effort implies that one underestimates the price effects of the merger.

[15] See also Ho and Hamilton (2000), Capps (2005) and Romano and Balan (2011).

[16] The assumption of three instead of n firms is made in order to make the analysis tractable. In a market with n firms there would be ex post differences among the non-merging firms, where the incentives for a non-merging firm with respect to quality and price setting in the post-merger game depend on its relative positioning in space vis-à-vis the merged firms. However, as competition is localised, the strongest responses to a merger will always come from the merging firms' closest neighbours. Therefore, the assumption of three firms is without too much loss of generality.

Firm i, located at x_i, is given by

$$u_{z,x_i} = v + bq_i - p_i - t(z - x_i)^2, \tag{1}$$

where q_i and p_i are the quality offered and the price charged, respectively, by Firm i; $b > 0$ is the marginal utility of quality; and $t > 0$ is a transportation cost parameter.

The demand facing each firm is a function of its own price and quality, and the prices and qualities of its two competitors. When each consumer makes a utility-maximising choice, the demand for the good offered by Firm i is given by

$$D_i = \frac{1}{3} + \frac{3}{2t}\left[b(2q_i - q_{i+1} - q_{i-1}) - (2p_i - p_{i+1} - p_{i-1})\right]. \tag{2}$$

Notice that a high (low) value of b relative to t implies a high (low) *demand responsiveness to quality.*

All firms are assumed to have *ex ante* identical costs. Firm i's cost function is given by

$$C_i = cq_iD_i + \frac{k}{2}q_i^2, \tag{3}$$

where $c \in (0, b)$ and $k > 0$. This cost function allows for both variable and fixed costs of quality, and implies that quality and output are cost substitutes.[17] In order to ensure equilibrium existence with interior solutions in both the pre- and post-merger game, we impose the parameter restrictions $c < b$ and $t > 9(b-c)^2/5k$. Each firm is assumed to maximise profits, given by

$$\pi_i = (p_i - cq_i)D_i - \frac{k}{2}q_i^2. \tag{4}$$

3.1 The pre-merger game

We look for the Nash equilibrium of a game where price and quality choices are made simultaneously.[18] Firm i chooses q_i and p_i to maximise (4). The first-order conditions for optimal quality and price, respectively, are given by[19]

$$\frac{\partial \pi_i}{\partial q_i} = \frac{3(b+c)p_i - (6bc + kt)q_i}{t} + \frac{3c\Sigma_{j\neq i}(bq_j - p_j)}{2t} - \frac{c}{3} = 0, \tag{5}$$

[17] For example, higher quality implies higher variable production costs if more expensive inputs are required to produce a higher-quality product.

[18] The results are qualitatively similar if we instead assume that quality and price decisions are made sequentially. The derivation of these results are available upon request.

[19] The second-order conditions are

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -\left(\frac{6bc + kt}{t}\right) < 0, \ \frac{\partial^2 \pi_i}{\partial p_i^2} = -\frac{6}{t} < 0$$

and

$$\left(\frac{\partial^2 \pi_i}{\partial p_i^2}\right)\left(\frac{\partial^2 \pi_i}{\partial q_i^2}\right) - \left(\frac{\partial^2 \pi_i}{\partial q_i \partial p_i}\right)^2 = 3\left(\frac{2kt - 3(b-c)^2}{t^2}\right) > 0,$$

which are satisfied under our assumption of $t > 9(b-c)^2/5k$.

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{3} + \frac{3(b+c)q_i - 6p_i}{t} + \frac{3\Sigma_{j \neq i}(p_j - bq_j)}{2t} = 0. \tag{6}$$

In order to understand the merger effects, it is instructive to analyse the strategic interaction between the quality and price decisions, both *within* each firm and *between* firms. From (5) the best-quality-response function of Firm i is given by

$$q_i(p_i, p_j, q_j) = \frac{3(b+c)p_i}{(6bc+kt)} + \frac{3c\Sigma_{j \neq i}(bq_j - p_j)}{2(6bc+kt)} - \frac{ct}{3(6bc+kt)}, \tag{7}$$

whereas the best-price-response function is given by

$$p_i(q_i, q_j, p_j) = \frac{(b+c)q_i}{2} + \frac{\Sigma_{j \neq i}(p_j - bq_j)}{4} + \frac{t}{18}. \tag{8}$$

We see that both qualities and prices are *gross strategic complements* between competing firms; i.e., $\partial q_i / \partial q_j > 0$ for given prices and $\partial p_i / \partial p_j > 0$ for given qualities.

The (gross) strategic complementarity between qualities is explained as follows. If a firm increases its quality, the competing firms lose demand, which in turn reduces their marginal cost of quality provision. These firms will therefore respond by increasing their quality. Notice that this strategic complementarity is caused by the presence of variable quality costs. In contrast, if there are only fixed costs of quality provision (i.e., if $c = 0$), the firms' optimal quality choices are strategically independent.

The strategic complementarity of the firms' pricing decisions is standard. All else equal, a unilateral price increase by one firm leads to higher demand for the competing firms. Their marginal revenues (measured as a function of output) are consequently reduced and they will optimally respond by increasing their prices as well.

However, the strategic relationships described above are *partial* in the sense that one of the choice variables (price or quality) is taken to be fixed. How do quality and price decisions interact strategically? From (7) and (8) we see that *quality and price are strategic complements within firms*; i.e., $\partial q_i / \partial p_i > 0$ and $\partial p_i / \partial q_i > 0$. A higher price has two effects on the incentives for quality provision. It increases the firm's profit margin and also reduces the marginal cost of quality provision through lower demand. Both effects contribute to a higher optimal level of quality. And *vice versa*, a higher quality level leads to higher demand and also increases marginal production costs, and both effects contribute to a higher optimal price.

On the other hand, *quality and price are strategic substitutes between firms*; i.e., $\partial q_i / \partial p_j < 0$ and $\partial p_i / \partial q_j < 0$. All else equal, a unilateral price increase by one firm leads to higher demand for competing firms. As a result, their marginal costs of quality provision increase and they will optimally respond by reducing their qualities. Similarly, if a firm increases its quality, competing firms will have lower demand and their profits are therefore maximised, all else equal, at a lower price.

We can internalise the strategic relationship between quality and price within each firm by simultaneously solving (7)-(8) with respect to q_i and p_i, yielding

$$p_i(p_j, q_j) = \frac{(kt + 3c(b-c))(2t + 9\Sigma_{j \neq i}(p_j - bq_j))}{18\left(2kt - 3(b-c)^2\right)} \tag{9}$$

and

$$q_i(p_j, q_j) = \frac{(b-c)\left(2t + 9\Sigma_{j\neq i}(p_j - bq_j)\right)}{6\left(2kt - 3(b-c)^2\right)}. \tag{10}$$

Whereas the strategic complementarity between prices remains, we see that the strategic relationship between qualities changes when we take the optimal price adjustments into account. In other words, *qualities are net strategic substitutes*; i.e., $\partial q_i / \partial q_j < 0$ when p_i is optimally adjusted.[20] As explained above, the direct (gross) effect of higher quality by a firm is that rival firms will increase their qualities and lower their prices. However, as quality and price are strategic complements within firms, a lower price implies that the quality should be optimally adjusted downwards. This indirect effect outweighs the direct effect, making qualities net strategic substitutes.[21]

The various strategic relationships between the firms' price and quality decisions can be summarised as follows:

Lemma 1 *In a Salop model where firms compete on price and quality,*
(i) prices are gross strategic complements ($\partial p_i(q_i)/\partial p_j > 0$);
(ii) qualities are gross strategic complements ($\partial q_i(p_i)/\partial q_j > 0$);
(iii) prices are net strategic complements ($\partial p_i/\partial p_j > 0$);
(iv) qualities are net strategic substitutes ($\partial q_i/\partial q_j < 0$);
(v) quality and price are strategic complements within firms ($\partial p_i/\partial q_i > 0$ and $\partial q_i/\partial p_i > 0$);
(vi) quality and price are strategic substitutes across firms ($\partial p_i(q_i)/\partial q_j < 0$ and $\partial q_i(p_i)/\partial p_j < 0$).

Simultaneously solving the three pairs of best-response functions given by (9)-(10), the symmetric Nash equilibrium in the *pre-merger* game is characterised by the following qualities and prices:[22]

$$q_i^* = \frac{b-c}{3k}, \tag{11}$$

$$p_i^* = \frac{t}{9} + \frac{c(b-c)}{3k}. \tag{12}$$

3.2 The post-merger game

Consider a merger between two of the firms. We assume that the merger does not change the type and number of products in the market. In other words, a merger only implies a coordination of price and quality decisions by the merger participants.[23] In the *post-merger* game, the outside firm chooses quality and price, denoted q_o and p_o, to maximise its profits, whereas the merger participants choose quality and price for each of the merged firms' products

[20] Notice that this relationship holds also without cost substitutability ($c = 0$).

[21] In a similar type of spatial competition model, Barros and Martinez-Giralt (2002) also find that qualities are strategic substitutes under price-quality competition. In a three-firm vertical differentiation model with quality-then-price competition, Scarpa (1998) finds that quality competition is characterised by strategic complementarity between some firms and strategic substitutability between others.

[22] Expressions for equilibrium profits in the pre- and post-merger games are presented in Appendix A.

[23] We return to a further discussion of this assumption in the concluding section of the paper.

(or at each of the merged firms' plants), denoted q_m and p_m, to maximise the sum of the two firms' profits. In the asymmetric Nash equilibrium, qualities and prices are given by

$$q_m^* = \frac{(b-c)\left(5kt - 9(b-c)^2\right)}{9k\left(2kt - 3(b-c)^2\right)}, \tag{13}$$

$$p_m^* = \frac{\left(5kt - 9(b-c)^2\right)(2kt + 3c(b-c))}{27k\left(2kt - 3(b-c)^2\right)}, \tag{14}$$

for each of the merged firms, and

$$q_o^* = \frac{(b-c)\left(8kt - 9(b-c)^2\right)}{9k\left(2kt - 3(b-c)^2\right)}, \tag{15}$$

$$p_o^* = \frac{(kt + 3c(b-c))\left(8kt - 9(b-c)^2\right)}{27k\left(2kt - 3(b-c)^2\right)}, \tag{16}$$

for the outside firm. Before analysing the effects of a merger on qualities and prices, let us first check under which conditions a merger is profitable.

Proposition 1 *In a Salop model where firms compete on price and quality, a horizontal merger is profitable for the participants if*

$$t > \frac{9(b-c)^2}{4k},$$

and unprofitable otherwise.

This result is perhaps surprising. It is easily shown that a merger is always profitable when quality is the only competition variable or when price is the only competition variable (which is a standard result from the merger literature, the seminal paper being Deneckere and Davidson, 1985). However, a merger might not be profitable when competition occurs simultaneously along both dimensions. The intuition for this result will be discussed below, after deriving the equilibrium price and quality responses to the merger. For the remainder of the analysis in this section, we will assume that the condition in Proposition 1 is satisfied; i.e., we restrict attention to profitable mergers only. This condition, along with the condition $b > c$, also ensures existence and uniqueness of the Nash equilibria in the pre- and post-merger games.

Proposition 2 *In a Salop model where firms compete on price and quality, a horizontal merger leads to*

(i) lower quality offered by the merged firms, higher quality offered by the outside firm, and higher average quality in the market;

(ii) higher (lower) prices charged by the merged firms if the demand responsiveness to quality is sufficiently low (high), a higher price charged by the outside firm, and a higher average price in the market;

(iii) smaller market shares for the merged firms.

In order to sort out the intuition behind these results, we need to keep in mind the strategic relationships between qualities and prices – within and across firms – which we previously analysed in detail. When two firms merge they have an incentive to internalise the negative competition externality that existed between them in the pre-merger game. All else equal, they can increase their joint profits by increasing the price and reducing the quality provided. However, since price and quality are strategic complements within firms, a price increase will instigate a quality increase, whereas a quality reduction will instigate a price reduction, thus making the overall price and quality responses of the merging firms *a priori* ambiguous. On top of this, the price and quality adjustments by the merging firms will trigger a response from the outside firm, which in turn induces feedback effects on the merging firms' optimal quality and price decisions.

Proposition 2 shows that the merged firms' incentives to reduce competition along the quality dimension always dominates in the sense that a merger always leads to lower quality provision by the merging firms. The outside firm will always respond by increasing quality, which is (mainly) explained by the net strategic substitutability of qualities, as previously described. Furthermore, the increase in quality and market share for the non-merging firm implies that the (volume-weighted) *average quality* in the market goes up as a result of the merger. In other words, the 'average consumer' enjoys a higher quality in this market as a result of the merger.

On the other hand, the price response of the merging firms is ambiguous, and a merger might actually lead to lower prices if the demand responsiveness to quality is sufficiently high. If demand responds strongly to quality, competition among firms occur mainly along this dimension and a merger will therefore lead to a relatively large reduction in quality. It might then be the case that the within-firm strategic complementarity effect dominates, in the sense that the quality reduction also leads to lower prices for the merging firms. The outside firm always responds by increasing the price, though. Even if a merger leads to higher prices for the merged firms, in which case the net strategic substitutability of prices would indicate a price reduction for the outside firm, two other effects contribute to a price increase: (i) higher quality by the outside firm leads also to higher price because of within-firm strategic complementarity between price and quality, and (ii) lower quality by the merged firm leads to higher price by the outside firm because quality and price are strategic substitutes between firms. Regardless of the sign of the price effect for the merged firms, the increase in price and market share for the non-merging firm also implies that the merger leads to an increase in the (volume-weighted) *average market price.*

The intricate strategic relationships between the optimal price and quality decisions also explain the profitability result in Proposition 1. The possibility of unprofitable mergers arises from the fact that qualities are net strategic substitutes. If demand responds sufficiently strongly to quality changes, the positive quality response by the outside firm is sufficiently strong to make the merger unprofitable for the participants.[24] It is also easily confirmed that the 'merger paradox' applies here, i.e., a merger is more profitable for the firm not taking part

[24] From (10),

$$\frac{\partial q_i}{\partial q_j} = -\frac{3\,(b-c)\,b}{2\left(2kt - 3\,(b-c)^2\right)}.$$

The magnitude (in absolute value) of this effect is increasing in b.

in the merger.

Finally, notice that none of the results in Proposition 2 depend on the presence (or not) of variable quality costs. Thus, these results hold also for the special case of $c = 0$.

3.3 Welfare

In order to derive an expression for total consumer utility, notice that Firm i's demand function, given by (2), can alternatively be expressed as

$$D_i = \widehat{x}_i^{i+1}\left(p_i, p_{i+1}, q_i, q_{i+1}\right) + \widehat{x}_i^{i-1}\left(p_i, p_{i-1}, q_i, q_{i-1}\right), \tag{17}$$

where

$$\widehat{x}_i^{i+1} = \frac{1}{6} + \frac{3\left(b\left(q_i - q_{i+1}\right) - \left(p_i - p_{i+1}\right)\right)}{2t} \tag{18}$$

is the location (measured *clockwise* from Firm i) of the consumer who is indifferent between Firm i and Firm $i+1$, and

$$\widehat{x}_i^{i-1} = \frac{1}{6} + \frac{3\left(b\left(q_i - q_{i-1}\right) - \left(p_i - p_{i-1}\right)\right)}{2t} \tag{19}$$

is the location (measured *anticlockwise* from firm i) of the consumer who is indifferent between Firm i and Firm $i-1$. With a slight abuse of notation, total consumer utility is then given by[25]

$$U = \sum_{i=1}^{3}\left(\int_0^{\widehat{x}_i^{i+1}}\left(v + bq_i - p_i - ts\right)ds + \int_0^{\widehat{x}_i^{i-1}}\left(v + bq_i - p_i - ts\right)ds\right). \tag{20}$$

Social welfare also includes profits and is given by

$$W = U + \sum_{i=1}^{3}\pi_i, \tag{21}$$

which can be re-written as

$$W = \sum_{i=1}^{3}\left(\int_0^{\widehat{x}_i^{i+1}}\left(v + bq_i - ts^2\right)ds + \int_0^{\widehat{x}_i^{i-1}}\left(v + bq_i - ts^2\right)ds - cq_iD_i - \frac{k}{2}q_i^2\right). \tag{22}$$

The detailed expressions for total consumer utility and welfare in the pre- and post-merger equilibria are given in Appendix A (Section A.2). Based on these expressions, we derive the following welfare implications of a merger:

Proposition 3 *In a Salop model where firms compete on price and quality, a horizontal merger leads to*

(i) lower total consumer utility;

(ii) higher utility for more than a third of all consumers in the market if the demand responsiveness to quality is sufficiently high;

[25] Notice that, if $i = 1$, then $i - 1 = 3$, and if $i = 3$, then $i + 1 = 1$.

(iii) higher social welfare if the demand responsiveness to quality is sufficiently high; otherwise, welfare drops.

If we consider total consumer utility, or the utility of the 'average consumer' in the market, a merger has three different effects: (i) the average quality goes up, which is positive; (ii) the average price also goes up, which is negative; and (iii) total transportation costs go up (because of the asymmetry of the post-merger equilibrium), which is also negative. In our model, the second and third effects always outweigh the first effect, implying that a merger has a negative effect on total consumer utility. However, the consumers buying from the non-merging firm might still benefit from the merger. If the demand responsiveness to quality is sufficiently high, the quality response of the outside firm is sufficiently strong to make these consumers enjoy a net benefit from the merger in spite of the corresponding price increase. As these net benefits also apply (due to continuity) to some of the consumers who switch firms as a result of the merger, more than a third of the consumers in the market might potentially benefit from the merger.

Perhaps the most surprising result of our analysis is that a purely anti-competitive merger (i.e., a merger with no direct cost synergies) might improve social welfare. This will be the case if demand responds sufficiently strongly to quality changes. The intuition behind this apparently counterintuitive result is the following. Social welfare depends on a trade-off between consumer benefits of quality and two types of costs: (i) transportation costs and (ii) the cost of quality provision. Crucially, the latter type of cost is not minimised with symmetric supply of quality across all firms in the market. The reason is that there are fixed quality costs (given by the term $(k/2)q_i^2$ in the cost function), which implies that quality provision is a quasi-public good at each firm (and a pure public good if $c = 0$): quality is non-rival in consumption since consumers' benefit from quality is not affected by consumption of other consumers. This makes a merger an instrument for realising endogenous fixed-cost synergies. To see why, consider the case of symmetric quality provision as a starting point. Suppose then that quality is reduced for some firms and increased for others in such a way that the total costs of quality provision are no higher than before. Suppose also that consumers switch from low-quality to high-quality firms to such an extent that average quality provision is at least as high as before. A more asymmetric market outcome will then clearly lead to a more cost-efficient quality provision. This example serves as a general illustration of the mechanisms at play. In our specific model, where quality decreases (increases) for the merged (outside) firm and average quality goes up, it can be shown that the fixed cost per unit of average quality is identical in the pre- and post-merger equilibria. Since average quality is higher in the post-merger equilibrium, this implies that a merger shifts down the (upward sloping) average fixed cost curve, leading to more cost-efficient quality provision.[26] In other words, a merger shifts the equilibrium outcome in the direction of the outcome that minimises quality provision costs, where only one firm is active in the market.[27]

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[26]Using (11), (13), (15), and (A12) in Appendix A, we have that

$$\frac{\frac{3k}{2}\left(q_i^*\right)^2}{q_i^*} = \frac{k\left(q_m^*\right)^2 + \frac{k}{2}\left(q_o^*\right)^2}{\overline{q}} = \frac{b-c}{2}.$$

[27]Because of fixed cost duplication, the total cost of providing an average quality q^* is clearly minimised if only one firm serves the entire market and provides this quality level.

However, the cost savings from a more asymmetric quality provision must be weighed against the increase in total transportation costs that would occur in a more asymmetric outcome. If transportation costs are sufficiently low and the marginal utility of quality is sufficiently high, social welfare can be increased with a more asymmetric quality provision.[28] In our setting, the effect of a merger is precisely to make quality provision more asymmetric. Quality increases at the non-merging firm and more consumers buy from this firm in the post-merger equilibrium. Such a merger can therefore increase social welfare because of increased allocation efficiency with respect to fixed quality costs, if the demand responsiveness to quality is sufficiently high (i.e., if b is sufficiently high relative to t). We emphasise this result with the following corollary.

Corollary 1 *A merger with no direct synergy might improve social welfare due to a more cost-efficient quality provision and despite an increase in transportation costs.*

In Appendix A (section A.3) we offer a more detailed and elaborated explanation of the welfare properties of symmetric versus asymmetric market outcomes in spatial competition models. Here we show that a merger can only improve welfare when the welfare maximum is a corner solution, implying that welfare is maximised with only one active firm in the market.

4 A generalised model

In this section we apply a more general imperfect competition model with n firms and fairly general demand and cost functions. The main purpose is to provide more insight into the general mechanisms of horizontal mergers in markets with price and quality competition, and to check the robustness of the results derived within the Salop framework.

Consider a market with n single-product firms, each producing a differentiated product. Demand for good i is given by $D_i(q_1, .., q_n, p_1, .., p_n)$, where $\frac{\partial D_i}{\partial p_i} < 0$, $\frac{\partial D_i}{\partial p_j} > 0$, $\frac{\partial D_i}{\partial q_i} > 0$, $\frac{\partial D_i}{\partial q_j} < 0$, $\frac{\partial^2 D_i}{\partial q_i^2} \lesseqgtr 0$ and $\frac{\partial^2 D_i}{\partial p_i^2} \lesseqgtr 0$. We assume that the demand system is symmetric and that demand for each good is separable in all qualities and prices.[29]

The more specific cost function in (3) is generalised as follows:

$$C_i(q_i, D_i) = c(q_i) D_i + K(q_i), \tag{23}$$

where $\frac{\partial c}{\partial q_i} > 0$, $\frac{\partial^2 c}{\partial q_i^2} \lesseqgtr 0$, $\frac{\partial K}{\partial q_i} > 0$ and $\frac{\partial^2 K}{\partial q_i^2} > 0$. Thus, we retain the assumption of constant marginal production costs for a given quality level. With the above demand and cost functions, the profit of Firm i is given by

$$\pi_i(q_i, p_i) = [p_i - c(q_i)] D_i - K(q_i). \tag{24}$$

[28] Notice that the cost savings from a more asymmetric quality provision would increase the socially optimal level of quality. More asymmetric quality provision could therefore increase social welfare if consumers value the higher average quality level to a sufficient degree.

[29] The separability assumption implies that all the second-order partial cross-derivatives of D_i are zero, whereas the symmetry assumption implies that $\left|\frac{\partial D_i}{\partial q_j}\right| = \frac{\partial D_j}{\partial q_i}$ for $q_i = q_j$ and $\left|\frac{\partial D_i}{\partial p_j}\right| = \frac{\partial D_j}{\partial p_i}$ for $p_j = p_i$.

4.1 Strategic relationship between qualities and prices

The analysis in Section 3 revealed that the merger effects depend crucially on the nature of the strategic relationship between the firms' choice variables. In the generalised framework, we can characterise these strategic relationships by considering the case of $n = 2$, in which the definition of strategic substitutability/complementarity is straightforward. The symmetric Nash equilibrium is then implicitly characterised by the following pair of first-order conditions:

$$\frac{\partial \pi_i}{\partial q_i} = (p_i - c(q_i)) \frac{\partial D_i}{\partial q_i} - D_i \frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} = 0, \tag{25}$$

$$\frac{\partial \pi_i}{\partial p_i} = D_i + (p_i - c(q_i)) \frac{\partial D_i}{\partial p_i} = 0, \tag{26}$$

$i = 1, 2$. From this system of equations we can derive two different sets of best-response functions: (i) $q_i(p_i, q_j, p_j)$ and $p_i(q_i, q_j, p_j)$, which determine whether qualities (prices) are *gross* strategic substitutes or complements, and (ii) $q_i(q_j, p_j)$ and $p_i(q_j, p_j)$, which determine whether qualities (prices) are *net* strategic substitutes or complements.

The strategic relationship between qualities (prices) for given price (quality) levels are given by, respectively,

$$sign\left(\frac{\partial q_i(p_i, q_j, p_j)}{\partial q_j}\right) = sign\left(\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} = -\frac{\partial D_i}{\partial q_j}\frac{\partial c}{\partial q_i}\right) > 0 \tag{27}$$

and

$$sign\left(\frac{\partial p_i(q_i, q_j, p_j)}{\partial p_j}\right) = sign\left(\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = \frac{\partial D_i}{\partial p_j}\right) > 0. \tag{28}$$

Thus, prices are always gross strategic complements, whereas qualities are gross strategic complements as long as there is a positive relationship between quality and marginal production costs. Thus, the results on gross strategic complementarity derived in Section 3 also hold in the generalised framework. This should come as no great surprise, given the intuition behind these results, as explained in Section 3. The same is true for the within-firm strategic relationship between price and quality, which in the generalised framework is given by

$$sign\left(\frac{\partial^2 \pi_i}{\partial q_i \partial p_i} = \frac{\partial^2 \pi_i}{\partial p_i \partial q_i} = \frac{\partial D_i}{\partial q_i} - \frac{\partial D_i}{\partial p_i}\frac{\partial c_i}{\partial q_i}\right) > 0. \tag{29}$$

By internalising the above price-quality relationship, we can derive the conditions for qualities (prices) to be net strategic substitutes or complements. By differentiating (25)-(26) with respect to q_i, p_i and q_j, and applying Cramer's Rule, we have[30]

$$sign\left(\frac{\partial q_i(q_j, p_j)}{\partial q_j}\right) = sign\left(\left(\frac{\partial D_i}{\partial q_i} + \frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial p_i} + (p_i - c)\frac{\partial^2 D_i}{\partial p_i^2}\right)\right)\frac{\partial D_i}{\partial q_j}\right) \tag{30}$$

[30] See Appendix B for a more extensive derivation of (30)-(31).

and

$$sign\left(\frac{\partial p_i\left(q_j,p_j\right)}{\partial p_j}\right)=sign\left(\left(-\frac{\partial^2\pi_i}{\partial q_i^2}-\frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial q_i}-\frac{\partial D_i}{\partial p_i}\frac{\partial c}{\partial q_i}\right)\right)\frac{\partial D_i}{\partial p_j}\right). \tag{31}$$

Thus, whether qualities (prices) are net strategic substitutes or complements depends on the relative strength of two opposing forces.

Regarding qualities, they are, on the one hand, gross strategic complements, as indicated by the second term in equation (30). On the other hand, if competing Firm j increases quality, then Firm i will have lower demand and its profits are therefore maximised, all else equal, at a lower price. Since price and quality are within-firm strategic complements, the quality level will also be adjusted downwards (first term in (30)). If latter effect dominates the former, qualities are net strategic substitutes.

Similarly, regarding prices, they are gross strategic complements, as indicated by the first term in (31). However, a price increase by competing Firm j leads to higher demand for Firm i. As a result, Firm i's marginal costs of quality provision will increase and it will optimally respond by reducing its quality. Since price and quality are within-firm strategic complements, the price level will also be adjusted downwards (second term in (31)). If the former effect dominates the latter, prices are net strategic complements.

Notice that the relative strength of the direct and indirect effects depends on the size of marginal variable quality costs ($\partial c/\partial q_i$). If these costs are sufficiently small, prices are net strategic complements and qualities are net strategic substitutes, as in the Salop model analysed in Section 3. However, in the generalised framework, these results can potentially be reversed if marginal variable quality costs are sufficiently large.[31]

The above analysis can be summarised as follows:

Lemma 2 *If marginal variable quality costs are sufficiently small, qualities are net strategic substitutes whereas prices are net strategic complements.*

4.2 Quality and price effects of a merger

Consider now a merger between two of the n firms in the industry. In a differentiated products model, given that the merger does not affect the number of goods produced, the post-merger game is an asymmetric game between one multi-product firm (the merged firm) and $n-2$ single-product firms. Thus, a merger is a discrete change of market structure that, in a general (non-parameterised) model, makes it hard to use standard comparative statics tools to assess the effects of the merger. One way to overcome this problem is to consider a 'marginal merger'. Suppose that the objective functions of the merger candidates (denoted i and j) are $\Pi_i := \pi_i + \alpha\pi_j$ and $\Pi_j := \pi_j + \alpha\pi_i$, respectively, where $\alpha \in (0,1)$.[32] The pre- and post-merger games appear then as the special cases of $\alpha = 0$ and $\alpha = 1$, respectively.

Let $N = \{1,..,n\}$ be the set of pre-merger firms/products in the industry, let $M = \{i,j\}$ be the set of merger participants, and let $O = N\backslash M$ be the set of outside (non-merging) firms.

[31] In the Salop model, this possibility is ruled out by the condition $b > c$, which is necessary to ensure equilibrium existence.

[32] This approach is somewhat similar to the concept of an 'infinitesmal merger' proposed by Farrell and Shapiro (1990), where such a merger is defined as a small change in the output of the merger participants (the insiders).

For the merging Firm i, which merges with Firm j, the first-order conditions for optimal quality and price are given by

$$\frac{\partial \Pi_i}{\partial q_i} = (p_i - c(q_i)) \frac{\partial D_i}{\partial q_i} - D_i \frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} + \alpha (p_j - c(q_j)) \frac{\partial D_j}{\partial q_i} = 0, \tag{32}$$

$$\frac{\partial \Pi_i}{\partial p_i} = D_i + (p_i - c(q_i)) \frac{\partial D_i}{\partial p_i} + \alpha (p_j - c(q_j)) \frac{\partial D_j}{\partial p_i} = 0. \tag{33}$$

Because of symmetry, $p_j = p_i$ and $q_j = q_i$ in equilibrium, which implies that (32)-(33) can be re-written as

$$(p_i - c(q_i)) \left(\frac{\partial D_i}{\partial q_i} + \alpha \frac{\partial D_j}{\partial q_i} \right) - D_i \frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} = 0, \tag{34}$$

$$D_i + (p_i - c(q_i)) \left(\frac{\partial D_i}{\partial p_i} + \alpha \frac{\partial D_j}{\partial p_i} \right) = 0. \tag{35}$$

For the non-merging Firm k, the first-order conditions are

$$\frac{\partial \pi_k}{\partial q_k} = (p_k - c(q_k)) \frac{\partial D_k}{\partial q_k} - D_k \frac{\partial c}{\partial q_k} - \frac{\partial K}{\partial q_k} = 0, \tag{36}$$

$$\frac{\partial \pi_k}{\partial p_k} = D_k + (p_k - c(q_k)) \frac{\partial D_k}{\partial p_k} = 0, \quad k \in O. \tag{37}$$

The Nash equilibrium is thus implicitly given by a system of four equations, (34)-(37), where all demand functions and their first-order derivatives are evaluated at the quality-price vector

$$(q_i, q_i, q_k, ..., q_k, p_i, p_i, p_k, ...p_k).$$

By differentiating the system (34)-(37) with respect to (q_i, p_i, q_k, p_k) and α, and applying Cramer's Rule, we can derive the equilibrium effects of the merger on the qualities and prices of all firms in the industry.

4.2.1 Quality and price responses of the merging firms

In qualitative terms, the effects of a merger on the merging firms' equilibrium choices of quality and price, respectively, are given by[33]

$$sign(\frac{\partial q_i}{\partial \alpha}) = sign \left(\begin{array}{c} \left[-\frac{\partial D_j}{\partial q_i} \frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial D_j}{\partial p_i} \left(\frac{\partial (D_i - D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i} \frac{\partial (D_i + D_j)}{\partial p_i} \right) \right] \Phi \\ +2(n-2) \frac{\partial D_k}{\partial p_i} \left(\frac{\partial D_j}{\partial q_i} + \frac{\partial c}{\partial q_i} \frac{\partial D_j}{\partial p_i} \right) \left(\frac{\partial D_i}{\partial p_k} \Omega_p - \frac{\partial D_i}{\partial q_k} \Omega_q \right) \end{array} \right) \tag{38}$$

and

$$sign(\frac{\partial p_i}{\partial \alpha}) = sign \left(\begin{array}{c} \left[-\frac{\partial D_j}{\partial p_i} \frac{\partial^2 \pi_i}{\partial q_i^2} + \frac{\partial D_j}{\partial q_i} \left(\frac{\partial (D_i + D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i} \frac{\partial (D_i - D_j)}{\partial p_i} \right) \right] \Phi \\ -2(n-2) \frac{\partial D_k}{\partial q_i} \left(\frac{\partial D_j}{\partial q_i} + \frac{\partial c}{\partial q_i} \frac{\partial D_j}{\partial p_i} \right) \left(\frac{\partial D_i}{\partial p_k} \Omega_p - \frac{\partial D_i}{\partial q_k} \Omega_q \right) \end{array} \right), \tag{39}$$

[33]See Appendix B for full details of the derivations of all expressions in this section.

where $\Phi > 0$ is a function of the equilibrium variables (see Appendix B for an explicit definition)[34], and

$$\Omega_p := \frac{\partial^2 \pi_k}{\partial q_k^2} + \frac{\partial c}{\partial q_k}\left(\frac{\partial D_k}{\partial q_k} - \frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial p_k}\right), \tag{40}$$

$$\Omega_q := \frac{\partial D_k}{\partial q_k} + \frac{\partial c}{\partial q_k}\left(\frac{\partial^2 \pi_k}{\partial p_k^2} - \frac{\partial D_k}{\partial p_k}\right). \tag{41}$$

Notice, by comparing (40)-(41) with (30)-(31), that $\Omega_p > (<)\, 0$ if prices are net strategic substitutes (complements), whereas $\Omega_q > (<)\, 0$ if qualities are net strategic substitutes (complements).

For both the quality response and the price response, we can classify the various sub-effects into two categories: (i) first-order effects through the merged firm's quality and price setting, and (ii) second-order (feedback) effects through the strategic responses of non-merging firms. The two sets of effects are grouped together in the two terms on the right-hand side of (38) and (39), respectively, where, in both equations, the first (second) term represents the first-order (second-order) effects.

The two first-order effects that we described in Section 3 are given by the two terms in the square brackets of (38) and (39), respectively. The *internalisation-of-competition effect*, which implies lower quality (in (38)) and higher prices (in (39)), is captured by the first term, whereas the *within-firm strategic complementarity effect*, which implies higher quality (in (38)) and lower prices (in (39)), is captured by the second term.

These first-order effects of a merger are complemented by feedback effects through the non-merged firms' price and quality responses. These effects are given by the second term in (38) and (39), respectively, with an *a priori* indeterminate sign. Whether the feedback effects counteract or reinforce the first-order effects depends on the relative strength of quality and price competition, and on the strategic nature of competition along these two dimensions.

The next proposition identifies two cases in which first-order and second-order effects go in the same direction and produce unambiguous results. These two cases are identified by the relative size of cross-quality effects (i.e., $|\partial D_j/\partial q_i|$) and cross-price effects (i.e., $\partial D_j/\partial p_i$) on demand.

To characterise the effect of a merger, we assume that the equilibrium outcomes (prices and qualities) are monotonic in α, so that the sign of a marginal merger has qualitatively the same sign of a merger with α going from 0 (no merger) to 1 (merger). The assumption of monotonicity is plausible since the sign of the partial derivatives of equilibrium prices and qualities with respect to α does not depend directly on the value of α but only indirectly through the effect of α on prices and qualities.

Proposition 4 *Suppose that marginal variable quality costs are sufficiently low, such that $\Omega_p < 0$ and $\Omega_q > 0$. We can then identify two different cases in which the quality and price responses of the merged firm are unambiguous:*

(i) If the cross-quality effect is sufficiently small relative to the cross-price effect, a merger leads to higher prices and higher quality by the merged firm.

(ii) If the cross-price effect is sufficiently small relative to the cross-quality, a merger leads to lower prices and lower quality by the merged firm.

[34]The positive sign of Φ follows from the assumption of a negative definite Jacobian matrix.

The first case identified in Proposition 4 defines a situation in which firms compete much harder on price than on quality. In this case, the *internalisation-of-competition effect* is much stronger for price than for quality, which in turn means that the *within-firm strategic complementarity effect* is much stronger for the quality decision than for the price decision. Thus, in sum, this gives the merged firm an incentive to increase both price and quality, and these (first-order) effects are reinforced by the second-order effects, as long as prices are net strategic complements ($\Omega_p < 0$) and qualities are net strategic substitutes ($\Omega_q > 0$).

The second case identifies the reverse scenario, where firms compete much more fiercely on quality than on price. In this case, the results are driven by the *internalisation-of-competition effect* being sufficiently much stronger for quality, and the merger leads to a reduction of both price and quality for the merged firm. Also in this case, the second-order effects reinforce the first-order effects. This case produces perhaps the most counterintuitive result, where a merger leads to a price reduction. As shown in the previous section (cf. Proposition 2), this result is captured in the Salop model for a set of parameter values where the demand responsiveness to quality is sufficiently high, which, in that framework, implies that firms compete sufficiently stronger on quality than on price.

For the remaining potential cases, where either the marginal variable quality costs are so large that the strategic nature of the game is changed, or if the degree of competition is not much stronger along one of the two dimensions (price and quality), the effects of a merger on the merged firm's price and quality responses are genuinely ambiguous. Notice also that the Salop model analysed in the previous section is only able to capture the second of the two cases described in Proposition 4, where the merged firm reduces both price and quality. This illustrates, first, that the specific parameterisation of the Salop model has clear limitations, and, second, that the two cases specified in Proposition 4 do not cover all relevant cases.

4.2.2 Quality and price responses of non-merging firms

The quality and price responses of the non-merging firms are given by, respectively,

$$sign(\frac{\partial q_k}{\partial \alpha}) = sign\left(\Omega_q\left[\frac{\partial D_k}{\partial q_i}\Psi_q + \frac{\partial D_k}{\partial p_i}\Psi_p\right]\right) \tag{42}$$

and

$$sign(\frac{\partial p_k}{\partial \alpha}) = sign\left(-\Omega_p\left[\frac{\partial D_k}{\partial q_i}\Psi_q + \frac{\partial D_k}{\partial p_i}\Psi_p\right]\right), \tag{43}$$

where

$$\Psi_p := -\frac{\partial D_j}{\partial p_i}\frac{\partial^2 \pi_i}{\partial q_i^2} + \frac{\partial D_j}{\partial q_i}\left(\frac{\partial(D_i + D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial(D_i - D_j)}{\partial p_i}\right), \tag{44}$$

$$\Psi_q := -\frac{\partial D_j}{\partial q_i}\frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial D_j}{\partial p_i}\left(\frac{\partial(D_i - D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial(D_i + D_j)}{\partial p_i}\right). \tag{45}$$

Notice here that $\Psi_p > (<)\, 0$ and $\Psi_q > (<)\, 0$ if the first-order effects of the merger on the merged firm's price and quality are positive (negative).

It follows from (42)-(43) that the quality and price responses of non-merging firms depend on two different factors: (i) the size of marginal variable quality costs, which determines whether qualities and prices are net strategic substitutes or complements (i.e., the signs of Ω_q

and Ω_p), and (ii) the relative magnitude of cross-quality and cross-price effects on demand, which determines the direction of the first-order effects on the merged firm's quality and price (i.e., the signs of Ψ_q and Ψ_p).

Notice that whether the non-merging firms' quality and price responses go in the same direction or not depends only on the net strategic substitutability/complementarity of qualities and prices. If qualities are net strategic substitutes ($\Omega_q > 0$) and prices are net strategic complements ($\Omega_p < 0$), or *vice versa* ($\Omega_q < 0$ and $\Omega_p > 0$), then quality and price responses from the non-merging firms always go in the same direction. Otherwise, if qualities and prices are both net strategic substitutes or net strategic complements, quality and price responses go in opposite directions.

Once more, the direction of price and quality responses can be unambiguously pinned down only for the case where marginal variable quality costs are sufficiently low and where competition is much stronger along one of the two dimensions. The next proposition summarises the price and quality responses of the non-merging firms:

Proposition 5 *(i) If prices are net strategic complements and qualities are net strategic substitutes (i.e., $\Omega_p < 0$ and $\Omega_q > 0$), or vice versa (i.e., $\Omega_p > 0$ and $\Omega_q < 0$), then non-merging firms' quality and price responses always go in the same direction. Otherwise, the quality and price responses of non-merging firms go in opposite directions. (ii) If marginal variable quality costs are sufficiently small, such that $\Omega_p < 0$ and $\Omega_q > 0$, non-merging firms will respond to the merger by increasing both quality and price if the difference between cross-quality effects and cross-price effects is sufficiently large in either direction.*

Table 1 summarises the price and quality effects of a merger for the case of sufficiently low marginal variable quality costs, which implies that qualities are net strategic substitutes ($\Omega_q > 0$) and prices are net strategic complements ($\Omega_p < 0$). In this case there are three different regimes, depending on the relative strength of competition along the two different dimensions (price and quality).

Table 1: Price and quality responses to a merger when $\partial c/\partial q_i$ is small

Dimensions of competition	Merged firm	Non-merged firms
(1) Mainly price	$p\uparrow$ and $q\uparrow$	$p\uparrow$ and $q\uparrow$
(2) Both price and quality	??	$(p\uparrow; q\uparrow)$ or $(p\downarrow; q\downarrow)$
(3) Mainly quality	$p\downarrow$ and $q\downarrow$	$p\uparrow$ and $q\uparrow$

The direction of price and quality responses are unambiguously determined for all firms in Regimes 1 and 3, in which the degree of competition is sufficiently strong on one dimension relative to the other. The merged firm will increase both price and quality if the firms compete mostly on price (Regime 1), and reduce both price and quality if competition is mainly on quality (Regime 3). In both regimes, the non-merging firms respond by increasing both price and quality. In the remaining Regime 2, in which competition is relatively strong along both dimensions, the quality and price effects of a merger is generally indeterminate, although we know that the price and quality responses of the non-merged firms always go in the same direction.

The above analysis suggests that there is a large number of possibilities regarding the equilibrium quality and price responses by merging and non-merging firms. The Salop model analysed in Section 3 captures two of these possibilities: (i) if competition along the quality dimension is sufficiently strong, the merged firm reduces both quality and price, whereas the non-merged firm increases both quality and price; (ii) if demand is less quality-responsive, the merged firm reduces quality but increases the price, whereas the non-merged firm still increases both quality and price. The former case, which corresponds to Regime 3 in Table 1, is characterised by $\Psi_p < 0$. The latter case, which corresponds to Regime 2 in Table 1, is characterised by $\Psi_p > 0$. In both cases, $\Omega_q > 0$, $\Omega_p < 0$, $\Psi_q < 0$ and $(\partial D_k/\partial q_i)\,\Psi_q + (\partial D_k/\partial p_i)\,\Psi_p > 0$.[35] One interesting result from the Salop model is that, although the merged firm always reduces quality, the *average quality* in the market goes up. This illustrates a more general point. If qualities are net strategic substitutes, a reduction in the merged firm's quality provision does not necessarily imply that average quality provision in the market is reduced.

5 Implications for empirical studies

What are the implications of our results for empirical studies of merger effects? A key lesson is that the estimated merger effects will be biased if the estimation does not account for quality being endogenous. As our analysis has highlighted, a merger may indeed result in lower prices and lower quality. Thus, empirical studies that focus only on price effects may falsely conclude that the merger is not anti-competitive, which may also induce the competition authority to approve the merger despite substantial quality reductions.

This point is highlighted by the recent studies of Tenn et al. (2010) and Fan (2013) who both show using *structural* models that failing to account for endogenous quality (or promotion) leads to serious estimation bias in simulating the merger effects. It is indeed common in the empirical Industrial Organization literature to *simulate* the effect of a merger along the lines of the seminal paper by Nevo (2000). If the researcher has access to individual data on consumption, then demand can be modelled through a multinomial conditional logit approach where the choice of a product by each consumer is a function of price and quality attributes offered by all firms in the sample (or a mixed logit one to allow for unobserved consumer preferences under some distributional assumptions). The model allows to recover the demand responsiveness to quality and price and the cross-price and cross-quality effects of the rivals. Endogeneity of prices needs to be addressed for example through an instrumental variable approach. Our model highlights that endogeneity of quality also needs to be addressed. The same empirical literature allows to recover the marginal costs under Nash-Bertrand equilibrium assumption. If quality is not treated as exogenous, an analogous marginal cost can also be recovered for quality, along the lines suggested by Tenn et al. (2010).[36] A merger can then be simulated and the demand responsiveness to quality and price of the rivals can be computed under the assumption that the two merging firms choose quality and price as one entity. A merger will raise stronger concerns if demand responsiveness to price and/or quality drastically falls as a result of the simulated merger compared to the status quo.

[35] Recall that, in the Salop model, qualities are net strategic substitutes whereas prices are net strategic complements.

[36] Most studies with a structural approach assume that the marginal cost is constant. As suggested by our model, the marginal cost may be increasing in quality. A structural model which allows for marginal cost increasing in quality and endogenous price and quality could be developed in future research.

The above papers focus on ex-ante merger evaluations. Other studies instead focus on ex-post merger evaluation, estimating the effect of mergers on price that actually took place for example within a reduced-form difference-in-difference econometric framework. Our analysis highlights that a fuller evaluation requires the estimation of the mergers on both price *and* quality. Moreover, it is critical to control for quality in the price equation and to control for price in the quality equation. Since price and quality are endogenous, quality in the price equation would have to be instrumented for example through an exogenous shifter in the quality equation. Similarly, price in the quality equation could be instrumented with an exogenous shifter in the price equation. An alternative approach is to *not* to include quality in the price equation and quality in the price equation, to avoid endogeneity, but at least to allow for correlation in the error terms of the price and the quality equation using a Seemingly Unrelated Regression (SUR) approach.

Our analysis also highlights that the effect of mergers depend on the extent to which qualities and prices are strategic complements or substitutes in quality and prices, ie the degree to which quality or price of a firm depends on the quality and price of the rival firms. A natural approach to test for this is a spatial econometrics one, where the quality is regressed against the rivals' quality and price; and similarly, price is regressed against rivals' quality and price, as measured by the average quality or price in a given catchment area (say 40km) weighted by the distance between the firm and each rival within the catchment area. Under this type of specification, OLS is biased and inconsistent but can be estimated by maximum likelihood, which is consistent and efficient in the presence of the spatial lag terms (Anselin, 1988; Mobley, 2003 for an example).

6 Concluding remarks

In this paper we have analysed the effects of horizontal mergers when firms compete along two different dimensions: price and quality. We have shown that the effects of a merger are quite involved because of the strategic relationship between quality and price, and we report some perhaps surprising results. First, whether a merger induces the merger participants to increase or reduce prices and qualities is far from clear-cut. If firms compete sufficiently strongly on quality, a merger might lead to lower prices, and if firms compete sufficiently strongly on price, a merger might lead to higher quality. In a Salop framework with three firms, the merging firms always reduce quality, but possibly also price if demand responsiveness to quality is sufficiently high, i.e., if firms compete sufficiently fiercely on quality relative to price. Second, the non-merging firms' price and quality responses are also far from obvious. In the general framework, we show that, if variable quality costs are sufficiently small, a merger will induce non-merging firms either to increase both price and quality, or to reduce both price and quality, depending on the relative strength of price and quality competition. However, in the Salop model, the non-merging firm always increases both quality and price as a response to the merger.

The general pattern of our results suggest that welfare implications of mergers are much less clear-cut when firms compete along two different dimensions, compared with the standard case of price competition. As we have discussed above, there are two reasons for this. First, a merger will most likely result in higher quality or lower price for some firms in the industry, which makes the welfare assessment of a merger *a priori* more ambiguous. Second, the presence of fixed quality costs implies that a merger might improve welfare through endogenous fixed-cost savings, something that we explicitly confirm in the Salop framework.

Our analysis has implications for empirical studies of merger effects. The key lesson is that the estimated merger effects will be biased if the estimation does not account for quality being endogenous. Empirical studies assessing the effects of mergers either ex-ante or ex-post need to take quality into account or they risk to underestimate the price increase arising from a merger and to approve the merger despite substantial quality reductions.

Although we have conducted our analysis within a very general framework, we have nevertheless been forced to make a couple of simplifications in order to make the analysis feasible. We have assumed that demand is separable in all qualities and prices, and we have assumed constant marginal production costs for given quality levels. Although these simplifications somewhat reduce the generality of the analysis, we still believe that we have been able to capture the most important mechanisms that determine the strategic choices in markets where firms compete on both price and quality.

Besides the above mentioned simplifications, our analysis also rests on some other key assumptions which deserve some further discussion. First, we have assumed that a merger does not affect the choice of products offered by the merging firms. By relaxing this assumption, we could in principle allow the merged firm to relocate their products (in the spatial framework) or to change the number of products offered. Regarding the latter possibility, the most intuitive effect of a merger would be that the merged firm continues to produce only one of the two products originally produced by the merger partners (or, alternatively, that the merged firm closes down one of its plants, in the geographical interpretation of the model). This might be a profitable strategy if product-fixed (or plant-fixed) costs are sufficiently large, which would make the realisation of fixed-cost savings an important motivation for a merger. In the Salop framework with three firms, this would make also the post-merger game symmetric and would be completely equivalent to a reduction in the number of (single-product) firms from three to two. We know from the literature (Economides, 1993) that this would lead to higher prices and higher quality (for all firms). A main mechanism behind this result is that demand for each product would increase, which would result in higher equilibrium prices and, in turn, higher quality (because of the within-firm strategic complementarity between price and quality).

Second, we have assumed that all costs of quality provision are product specific (or plant specific). However, it might be that some types of quality investments (such as R&D, for example) are costs that are incurred at firm level and where the benefits of the investment apply to all products (or plants) of the firm. Incorporating these types of firm-specific quality investments would have at least two different effects on our results. It would make mergers more profitable and it would also likely increase the scope for mergers to affect quality positively, since a merger would then be an instrument for rationalising the costs of quality provision.

Finally, we have chosen to conduct our analysis within a horizontal differentiation framework where the pre-merger equilibrium is symmetric and firms provide the same level of quality. On one hand, this approach is advantageous since the effects of a merger do not depend on the choice of merger participants. One the other hand, our model is probably not able to capture all relevant effects of mergers in markets where high-quality and low-quality firms coexist. Future research could investigate if some of our main results extend to a vertical differentiation framework with individuals differing in the willingness to pay for quality, and firms offering products of different qualities.

Appendix A

This Appendix complements our merger analysis based on the Salop model, presented in Section 3. Here we report the pre- and post-merger equilibrium values of profits, consumer utility and welfare; the proofs of Propositions 1-3; and further details regarding endogenous fixed-cost savings caused by asymmetric market outcomes.

A.1. Equilibrium values of profits, consumer utility and welfare

In the *pre-merger* equilibrium, profits, consumer utility and welfare are given by

$$\pi_i^* = \frac{t}{27} - \frac{(b-c)^2}{18k}, \tag{A1}$$

$$U(q_i^*, p_i^*) = v - \left(\frac{13kt - 36(b-c)^2}{108k} \right), \tag{A2}$$

and

$$W(q_i^*, p_i^*) = v - \left(\frac{kt - 18(b-c)^2}{108k} \right). \tag{A3}$$

In the *post-merger* equilibrium, profits earned by the outsider and each of the merged firms, respectively, are given by

$$\pi_o^* = \frac{\left(8kt - 9(b-c)^2\right)^2}{486k\left(2kt - 3(b-c)^2\right)} \tag{A4}$$

and

$$\pi_m^* = \frac{\left(4kt - 3(b-c)^2\right)\left(5kt - 9(b-c)^2\right)^2}{486k\left(2kt - 3(b-c)^2\right)^2}, \tag{A5}$$

whereas consumer utility and welfare are given by, respectively,

$$U(q_m^*, p_m^*, q_o^*, p_o^*) = v - \left(\frac{kt\left(5589(b-c)^4 + 4kt\left(175kt - 873(b-c)^2\right)\right) - 2916(b-c)^6}{972k\left(2kt - 3(b-c)^2\right)^2} \right) \tag{A6}$$

and

$$W(q_m^*, p_m^*, q_o^*, p_o^*) = v - \left(\frac{kt\left(2025(b-c)^4 + 4kt\left(11kt - 198(b-c)^2\right)\right) - 1458(b-c)^6}{972k\left(2kt - 3(b-c)^2\right)^2} \right). \tag{A7}$$

A.2. Proofs

Proof of Proposition 1. A comparison of (A1) and (A5) yields

$$\pi_m^* - \pi_i^* = \frac{t\left(7kt - 12(b-c)^2\right)\left(4kt - 9(b-c)^2\right)}{486\left(2kt - 3(b-c)^2\right)^2}. \tag{A8}$$

Imposing the restriction $t > 9(b-c)^2/5k$, it follows that

$$\pi_m^* > (<)\, \pi_i^* \quad if \quad t > (<)\, \frac{9(b-c)^2}{4k}. \tag{A9}$$

Q.E.D.

Proof of Proposition 2. (i): Comparing (13) and (11) yields

$$q_m^* - q_i^* = -\frac{(b-c)\,t}{9\left(2kt - 3(b-c)^2\right)} < 0. \tag{A10}$$

Comparing (15) and (11) yields

$$q_o^* - q_i^* = \frac{2t(b-c)}{9\left(2kt - 3(b-c)^2\right)} > 0. \tag{A11}$$

The average quality in the market in the post-merger equilibrium is

$$\begin{aligned}\overline{q} &:= 2D_m\left(q_m^*, p_m^*, q_o^*, p_o^*\right) q_m^* + D_o\left(q_m^*, p_m^*, q_o^*, p_o^*\right) q_o^* \\ &= \frac{\left(81(b-c)^4 + 2kt\left(19kt - 54(b-c)^2\right)\right)(b-c)}{27k\left(2kt - 3(b-c)^2\right)^2}.\end{aligned} \tag{A12}$$

Comparing (A12) and (11) yields

$$\overline{q} - q_i^* = \frac{2kt^2(b-c)}{27\left(2kt - 3(b-c)^2\right)^2} > 0. \tag{A13}$$

(ii): Comparing (14) and (12) yields

$$p_m^* - p_i^* = t\left(\frac{4kt - 3(3b-2c)(b-c)}{27\left(2kt - 3(b-c)^2\right)}\right) > (<)\, 0 \quad if \quad t > (<)\, \frac{3(3b-2c)(b-c)}{4k}. \tag{A14}$$

Merger profitability requires $t > \frac{9(b-c)^2}{4k}$. As $\frac{3(3b-2c)(b-c)}{4k} - \frac{9(b-c)^2}{4k} = \frac{3c(b-c)}{4k} > 0$, the parameter space defined by $t < \frac{3(3b-2c)(b-c)}{4k}$ is non-empty. Regarding the non-merging firm, comparing (16) and (12) yields

$$p_o^* - p_i^* = \frac{2t\left(kt + 3c(b-c)\right)}{27\left(2kt - 3(b-c)^2\right)} > 0. \tag{A15}$$

The average price in the market in the post-merger equilibrium is

$$\begin{aligned}\overline{p} &:= 2D_m\left(q_m^*, p_m^*, q_o^*, p_o^*\right)p_m^* + D_o\left(q_m^*, p_m^*, q_o^*, p_o^*\right)p_o^* \\ &= \frac{729c\left(b-c\right)^5 + kt\left(81\left(5b-17c\right)\left(b-c\right)^3 + 2kt\left(82kt - 9\left(28b-47c\right)\left(b-c\right)\right)\right)}{243k\left(2kt-3\left(b-c\right)^2\right)^2}.\end{aligned} \tag{A16}$$

Comparing (A16) and (12) yields

$$\overline{p} - p_i^* = \frac{2t\left(81\left(b-c\right)^4 + kt\left(28kt - 9\left(b-c\right)\left(10b-11c\right)\right)\right)}{243\left(2kt-3\left(b-c\right)^2\right)^2}. \tag{A17}$$

The numerator is monotonically increasing in t for all $t > \frac{9(b-c)^2}{4k}$. Setting t at the lowest level that still guarantees profitable mergers, $t = \frac{9(b-c)^2}{4k}$, the numerator in (A17) reduces to $\frac{729b(b-c)^3(b-c)^2}{8k} > 0$. Thus, $\overline{p} > p_i^*$. (iii): Inserting the equilibrium values of qualities and prices into (2) and comparing the pre- and post-merger equilibria, yields

$$D_m\left(q_m^*, p_m^*, q_o^*, p_o^*\right) - D_i\left(q_i^*, p_i^*\right) = -\frac{tk}{9\left(2kt-3\left(b-c\right)^2\right)} < 0. \tag{A18}$$

Q.E.D.

Proof of Proposition 3. (i): Comparing (A2) and (A6) yields

$$U\left(q_m^*, p_m^*, q_o^*, p_o^*\right) - U\left(q_i^*, p_i^*\right) = -2t\left(\frac{kt\left(29kt - 99\left(b-c\right)^2\right) + 81\left(b-c\right)^4}{243\left(2kt-3\left(b-c\right)^2\right)^2}\right). \tag{A19}$$

The sign of (A19) is determined by the sign of the numerator, which is monotonically increasing in t for $t > \frac{9(b-c)^2}{4k}$. Setting $t = \frac{9(b-c)^2}{4k}$, the numerator is $\frac{81(b-c)^4}{16} > 0$. Thus, $U\left(q_m^*, p_m^*, q_o^*, p_o^*\right) < U\left(q_i^*, p_i^*\right)$ for all parameter configurations that yield profitable mergers. (ii): Consumers buying from the non-merging firm in the pre-merger equilibrium (these consumers constitute one third of the market) can potentially benefit from the merger due to higher quality (if the quality increase outweighs the utility loss of higher prices). The individual utility effect of the merger for each of these consumers is

$$\Delta u := b\left(q_o^* - q_i^*\right) - \left(p_o^* - p_i^*\right) = -\frac{2t\left(kt-3\left(b-c\right)^2\right)}{27\left(2kt-3\left(b-c\right)^2\right)} < (>) \quad if \quad t > (<)\, \frac{3\left(b-c\right)^2}{k}. \tag{A20}$$

If $\Delta u > 0$, the merger will also increase the utility of some consumers who switch from the merged firms to the outside firm as a result of the merger, and who are located sufficiently close to the consumers who were indifferent between a merged and a non-merged firm in the pre-merger equilibrium. (iii) Comparing (A3) and (A7) yields

$$W\left(q_m^*, p_m^*, q_o^*, p_o^*\right) - W\left(q_i^*, p_i^*\right) = -\frac{\left(2kt - 9(b-c)^2\right)kt^2}{243\left(2kt - 3(b-c)^2\right)^2} < (>)\, 0 \quad if \quad t > (<)\, \frac{9(b-c)^2}{2k}. \tag{A21}$$

Thus, $W\left(q_m^*, p_m^*, q_o^*, p_o^*\right) > W\left(q_i^*, p_i^*\right)$ if b is sufficiently large relative to t. *Q.E.D.*

A.3. Welfare properties of symmetric versus asymmetric market outcomes

Our result in Proposition 3 that a merger can improve welfare because it results in a more asymmetric market outcome, can be further explained and elaborated on by considering a simplified two-firm version of our model. Consider a Hotelling model with two firms located at each end of a unit line, where Firm 1 is located at 0 and Firm 2 is located at 1. All other assumptions are identical to the Salop model in Section 3. Let x denote the location of the consumer who is indifferent between the two firms. The expression for total welfare can then be written as

$$W = \int_0^x \left(v + bq_1 - ts^2\right) ds + \int_x^1 \left(v + bq_2 - t(1-s)^2\right) ds - c\left(xq_1 + (1-x)\, q_2\right) - \frac{k}{2}\left(q_1^2 + q_2^2\right). \tag{A22}$$

Maximising this function with respect to x, q_1 and q_2 yields the following first-order conditions

$$\frac{\partial W}{\partial q_1} = bx - kq_1 - cx = 0, \tag{A23}$$

$$\frac{\partial W}{\partial q_2} = b - c - kq_2 - bx + cx = 0, \tag{A24}$$

$$\frac{\partial W}{\partial x} = t + bq_1 - bq_2 - cq_1 + cq_2 - 2tx = 0. \tag{A25}$$

This system of equations has a unique solution given by

$$q_1 = q_2 = \frac{b-c}{2k} \text{ and } x = \frac{1}{2}. \tag{A26}$$

It is straightforward to show that this outcome is also the Nash equilibrium outcome of a game where the two firms choose quality and price simultaneously. Thus, simultaneous decision making yields the first-best outcome, as previously shown by Ma and Burgess (1993). This result also holds for a Salop model with $n > 2$ firms, as confirmed by Economides (1993), as long as the number of firms is exogenously given. However, this solution is the first-best outcome only for a subset of the parameter values for which the Nash equilibrium exists.

Defining the Hessian matrix of the welfare maximisation problem, its determinant is given by

$$\begin{vmatrix} \frac{\partial^2 W}{\partial q_1^2} & \frac{\partial^2 W}{\partial q_1 \partial q_2} & \frac{\partial^2 W}{\partial q_1 \partial x} \\ \frac{\partial^2 W}{\partial q_1 \partial q_2} & \frac{\partial^2 W}{\partial q_2^2} & \frac{\partial^2 W}{\partial q_2 \partial x} \\ \frac{\partial^2 W}{\partial q_1 \partial x} & \frac{\partial^2 W}{\partial q_2 \partial x} & \frac{\partial^2 W}{\partial x^2} \end{vmatrix} = -2k\left(tk - (b-c)^2\right) < (>)\, 0 \text{ if } kt > (<)\, (b-c)^2 . \tag{A27}$$

Thus, the determinant is negative (guaranteeing concavity) only if $tk > (b-c)^2$. Otherwise, the solution given by (A26) is a saddle point, and the first-best solution is a corner solution where all consumers buy from one firm and the other firm offers zero quality. In this case, if qualities are optimally chosen (from a welfare perspective) for a given value of x, *any asymmetric interior solution with positive qualities for both firms is welfare-superior to the symmetric solution* with $x = \frac{1}{2}$.

The intuition behind this result is the following. A more asymmetric outcome implies that a given average quality level can be provided at lower costs, so the optimal average quality level increases. This can be seen by solving (A23) and (A24) for q_1 and q_2, respectively, which gives the optimal quality levels for given market shares:

$$q_1(x) = \frac{(b-c)\,x}{k} \text{ and } q_2(x) = \frac{(b-c)(1-x)}{k}. \tag{A28}$$

Optimal average quality, as a function of market shares, is then given by

$$xq_1(x) + (1-x)\,q_1(x) = \frac{1}{k}(b-c)(1-2x(1-x)). \tag{A29}$$

This expression is minimised at $x = \frac{1}{2}$ and is monotonically decreasing in x over the interval $\left[0, \frac{1}{2}\right]$ and monotonically increasing in x over the interval $\left[\frac{1}{2}, 1\right]$, which illustrates that a more asymmetric outcome reduces the fixed cost of quality provision and therefore increases the optimal average quality. On the other hand, total transportation costs are minimised in a symmetric outcome with $x = \frac{1}{2}$. Thus, from a welfare perspective, there is a trade-off between minimising the costs of quality provision (which are lower in asymmetric outcomes) and minimising transportation costs (which are higher in asymmetric outcomes). An asymmetric outcome is therefore welfare optimal if the marginal utility of quality is sufficiently high relative to the cost of travelling; in the Hotelling example, if $tk < (b-c)^2$.

Appendix B

In this Appendix we present the details of the comparative statics results derived with the general model in Section 4, and the proofs of Propositions 4 and 5.

B.1. Strategic relationship between qualities and prices

Total differentiation of the first-order conditions (5)-(6) with respect to q_i, p_i and q_j yields

$$\begin{bmatrix} \frac{\partial^2 \pi_i}{\partial q_i^2} & \frac{\partial^2 \pi_i}{\partial p_i \partial q_i} \\ \frac{\partial^2 \pi_i}{\partial q_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial p_i^2} \end{bmatrix} \begin{bmatrix} dq_i \\ dp_i \end{bmatrix} + \begin{bmatrix} \frac{\partial^2 \pi_i}{\partial q_j \partial q_i} \\ \frac{\partial^2 \pi_i}{\partial q_j \partial p_i} \end{bmatrix} dq_j = 0. \tag{B1}$$

By applying Cramer's Rule, the strategic relationship between q_i and q_j, when p_i is optimally adjusted, is given by

$$\begin{aligned} \frac{\partial q_i (q_j, p_j)}{\partial q_j} &= \frac{\begin{vmatrix} -\frac{\partial^2 \pi_i}{\partial q_j \partial q_i} & \frac{\partial^2 \pi_i}{\partial p_i \partial q_i} \\ -\frac{\partial^2 \pi_i}{\partial q_j \partial p_i} & \frac{\partial^2 \pi_i}{\partial p_i^2} \end{vmatrix}}{\Delta} = \frac{\begin{vmatrix} \frac{\partial c}{\partial q_i} \frac{\partial D_i}{\partial q_j} & \frac{\partial D_i}{\partial q_i} - \frac{\partial c}{\partial q_i} \frac{\partial D_i}{\partial p_i} \\ -\frac{\partial D_i}{\partial q_j} & 2\frac{\partial D_i}{\partial p_i} + (p_i - c(q_i)) \frac{\partial^2 D_i}{\partial p_i^2} \end{vmatrix}}{\Delta} \\ &= \frac{1}{\Delta} \left(\frac{\partial D_i}{\partial q_i} + \frac{\partial c}{\partial q_i} \left(\frac{\partial D_i}{\partial p_i} + (p_i - c) \frac{\partial^2 D_i}{\partial p_i^2} \right) \right) \frac{\partial D_i}{\partial q_j}, \end{aligned} \tag{B2}$$

where

$$\Delta := \frac{\partial^2 \pi_i}{\partial q_i^2} \frac{\partial^2 \pi_i}{\partial p_i^2} - \frac{\partial^2 \pi_i}{\partial q_i \partial p_i} \frac{\partial^2 \pi_i}{\partial p_i \partial q_i} > 0. \tag{B3}$$

Similarly, differentiating (5)-(6) with respect to q_i, p_i and q_j yields

$$\begin{bmatrix} \frac{\partial^2 \pi_i}{\partial q_i^2} & \frac{\partial^2 \pi_i}{\partial p_i \partial q_i} \\ \frac{\partial^2 \pi_i}{\partial q_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial p_i^2} \end{bmatrix} \begin{bmatrix} dq_i \\ dp_i \end{bmatrix} + \begin{bmatrix} \frac{\partial^2 \pi_i}{\partial p_j \partial q_i} \\ \frac{\partial^2 \pi_i}{\partial p_j \partial p_i} \end{bmatrix} dp_j = 0. \tag{B4}$$

Applying Cramer's Rule:

$$\begin{aligned} \frac{\partial p_i (q_j, p_j)}{\partial p_j} &= \frac{\begin{vmatrix} \frac{\partial^2 \pi_i}{\partial q_i^2} & -\frac{\partial^2 \pi_i}{\partial p_j \partial q_i} \\ \frac{\partial^2 \pi_i}{\partial q_i \partial p_i} & -\frac{\partial^2 \pi_i}{\partial p_j \partial p_i} \end{vmatrix}}{\Delta} = \frac{\begin{vmatrix} \frac{\partial^2 \pi_i}{\partial q_i^2} & \frac{\partial c}{\partial q_i} \frac{\partial D_i}{\partial p_j} \\ \frac{\partial D_i}{\partial q_i} - \frac{\partial c}{\partial q_i} \frac{\partial D_i}{\partial p_i} & -\frac{\partial D_i}{\partial p_j} \end{vmatrix}}{\Delta} \\ &= \frac{1}{\Delta} \left(-\frac{\partial^2 \pi_i}{\partial q_i^2} - \frac{\partial c}{\partial q_i} \left(\frac{\partial D_i}{\partial q_i} - \frac{\partial D_i}{\partial p_i} \frac{\partial c}{\partial q_i} \right) \right) \frac{\partial D_i}{\partial p_j}. \end{aligned} \tag{B5}$$

B.2. Comparative statics of the Nash equilibrium of the general model

The Nash equilibrium is implicitly given by a system of four equations, given by (34)-(37), which are here redefined as

$$F_1 := (p_i - c(q_i))\left(\frac{\partial D_i}{\partial q_i} + \alpha\frac{\partial D_j}{\partial q_i}\right) - D_i\frac{\partial c}{\partial q_i} - \frac{\partial K}{\partial q_i} = 0, \tag{B6}$$

$$F_2 := D_i + (p_i - c(q_i))\left(\frac{\partial D_i}{\partial p_i} + \alpha\frac{\partial D_j}{\partial p_i}\right) = 0, \tag{B7}$$

$$F_3 := (p_k - c(q_k))\frac{\partial D_k}{\partial q_k} - D_k\frac{\partial c}{\partial q_k} - \frac{\partial K}{\partial q_k} = 0, \tag{B8}$$

$$F_4 := D_k + (p_k \ - c(q_k))\frac{\partial D_k}{\partial p_k} = 0. \tag{B9}$$

where the demand functions are given by

$$\begin{aligned}
&D_i\left(p_i, p_i, p_k, ..., p_k, q_i, q_i, q_k, ..., q_k\right) \\
&D_j\left(p_i, p_i, p_k, ..., p_k, q_i, q_i, q_k, ..., q_k\right) \\
&D_k\left(p_i, p_i, p_k, ..., p_k, q_i, q_i, q_k, ..., q_k\right), \quad k \in O.
\end{aligned}$$

By differentiating (B6)-(B9) with respect to q_i, p_i, q_k, p_k and α, we can write the system on matrix form as

$$\begin{bmatrix} \frac{\partial F_1}{\partial q_i} & \frac{\partial F_1}{\partial p_i} & \frac{\partial F_1}{\partial q_k} & \frac{\partial F_1}{\partial p_k} \\ \frac{\partial F_2}{\partial q_i} & \frac{\partial F_2}{\partial p_i} & \frac{\partial F_2}{\partial q_k} & \frac{\partial F_2}{\partial p_k} \\ \frac{\partial F_3}{\partial q_i} & \frac{\partial F_3}{\partial p_i} & \frac{\partial F_3}{\partial q_k} & \frac{\partial F_3}{\partial p_k} \\ \frac{\partial F_4}{\partial q_i} & \frac{\partial F_4}{\partial p_i} & \frac{\partial F_4}{\partial q_k} & \frac{\partial F_4}{\partial p_k} \end{bmatrix} \begin{bmatrix} dq_i \\ dp_i \\ dq_k \\ dp_k \end{bmatrix} + \begin{bmatrix} \frac{\partial F_1}{\partial \alpha} \\ \frac{\partial F_2}{\partial \alpha} \\ \frac{\partial F_3}{\partial \alpha} \\ \frac{\partial F_4}{\partial \alpha} \end{bmatrix} d\alpha = 0, \tag{B10}$$

where

$$
\begin{aligned}
\frac{\partial F_1}{\partial q_i} &= -\frac{\partial^2 K_i}{\partial q_i^2} - \frac{\partial^2 c}{\partial q_i^2} D_i - \frac{\partial c}{\partial q_i}\left(2\frac{\partial D_i}{\partial q_i} + (1+\alpha)\frac{\partial D_j}{\partial q_i}\right) + (p_i - c(q_i))\frac{\partial^2 D_i}{\partial q_i^2}, \\
\frac{\partial F_1}{\partial p_i} &= \left(\frac{\partial D_i}{\partial q_i} + \alpha\frac{\partial D_j}{\partial q_i}\right) - \frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial p_i} + \frac{\partial D_j}{\partial p_i}\right), \quad \frac{\partial F_1}{\partial q_k} = -\frac{\partial c}{\partial q_i}\frac{\partial D_i}{\partial q_k}(n-2), \\
\frac{\partial F_1}{\partial p_k} &= -\frac{\partial c}{\partial q_i}\frac{\partial D_i}{\partial p_k}(n-2), \quad \frac{\partial F_2}{\partial q_i} = \frac{\partial D_i}{\partial q_i} + \frac{\partial D_j}{\partial q_i} - \frac{\partial c}{\partial q_i}\left(\frac{\partial D_i}{\partial p_i} + \alpha\frac{\partial D_j}{\partial p_i}\right), \\
\frac{\partial F_2}{\partial p_i} &= 2\frac{\partial D_i}{\partial p_i} + (1+\alpha)\frac{\partial D_j}{\partial p_i} + (p_i - c(q_i))\frac{\partial^2 D_i}{\partial p_i^2}, \quad \frac{\partial F_2}{\partial q_k} = \frac{\partial D_i}{\partial q_k}(n-2), \\
\frac{\partial F_2}{\partial p_k} &= \frac{\partial D_i}{\partial p_k}(n-2), \quad \frac{\partial F_3}{\partial q_i} = -2\frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial q_i}, \quad \frac{\partial F_3}{\partial p_i} = -2\frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial p_i}, \qquad \text{(B11)} \\
\frac{\partial F_3}{\partial q_k} &= -\frac{\partial^2 K}{\partial q_k^2} - \frac{\partial^2 c}{\partial q_k^2} D_k + (p_k - c(q_k))\frac{\partial^2 D_k}{\partial q_k^2} - \frac{\partial c}{\partial q_k}\left(2\frac{\partial D_k}{\partial q_k} + (n-3)\left.\frac{\partial D_k}{\partial q_l}\right|_{q_l=q_k}\right), \\
\frac{\partial F_3}{\partial p_k} &= \frac{\partial D_k}{\partial q_k} - \frac{\partial c}{\partial q_k}\left(\frac{\partial D_k}{\partial p_k} + (n-3)\left|\frac{\partial D_k}{\partial p_l}\right|_{p_l=p_k}\right), \\
\frac{\partial F_4}{\partial q_i} &= 2\frac{\partial D_k}{\partial q_i}, \quad \frac{\partial F_4}{\partial p_i} = 2\frac{\partial D_k}{\partial p_i}, \quad \frac{\partial F_4}{\partial q_k} = \frac{\partial D_k}{\partial q_k} + (n-3)\left|\frac{\partial D_k}{\partial q_l}\right|_{q_l=q_k} - \frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial p_k}, \\
\frac{\partial F_4}{\partial p_k} &= 2\frac{\partial D_k}{\partial p_k} + (n-3)\left|\frac{\partial D_k}{\partial p_l}\right|_{p_l=p_k} + (p_k - c(q_k))\frac{\partial^2 D_k}{\partial p_k^2}, \\
\frac{\partial F_1}{\partial \alpha} &= (p_i - c(q_i))\frac{\partial D_j}{\partial q_i}, \quad \frac{\partial F_2}{\partial \alpha} = (p_i - c(q_i))\frac{\partial D_j}{\partial p_i}, \quad \frac{\partial F_3}{\partial \alpha} = \frac{\partial F_4}{\partial \alpha} = 0,
\end{aligned}
$$

where we have exploited the fact that, by symmetry, $\left|\frac{\partial D_i}{\partial q_j}\right|_{q_j=q_i} = \frac{\partial D_j}{\partial q_i}$, $\left|\frac{\partial D_i}{\partial p_j}\right|_{p_j=p_i} = \frac{\partial D_j}{\partial p_i}$, $\left|\frac{\partial D_k}{\partial q_j}\right|_{q_j=q_i} = \frac{\partial D_k}{\partial q_i}$ and $\left|\frac{\partial D_k}{\partial q_j}\right|_{q_j=q_i} = \frac{\partial D_k}{\partial q_i}$.

B.2.1. The effect of a merger on the merged firm's quality

Assuming that the Jacobian matrix is negative definite, the sign of $\partial q_i/\partial\alpha$ is given by the sign of

$$
\begin{vmatrix}
-\frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial p_i} & \frac{\partial F_1}{\partial q_k} & \frac{\partial F_1}{\partial p_k} \\
-\frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial p_i} & \frac{\partial F_2}{\partial q_k} & \frac{\partial F_2}{\partial p_k} \\
0 & \frac{\partial F_3}{\partial p_i} & \frac{\partial F_3}{\partial q_k} & \frac{\partial F_3}{\partial p_k} \\
0 & \frac{\partial F_4}{\partial p_i} & \frac{\partial F_4}{\partial q_k} & \frac{\partial F_4}{\partial p_k}
\end{vmatrix}. \qquad \text{(B12)}
$$

This determinant can be written as

$$
\begin{aligned}
&-\left(\frac{\partial F_2}{\partial p_i}\frac{\partial F_1}{\partial \alpha}-\frac{\partial F_1}{\partial p_i}\frac{\partial F_2}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_k}-\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_k}\right)\\
&-\left(\frac{\partial F_1}{\partial \alpha}\frac{\partial F_2}{\partial p_k}-\frac{\partial F_2}{\partial \alpha}\frac{\partial F_1}{\partial p_k}\right)\left(\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_k}-\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_i}\right)\\
&-\left(\frac{\partial F_1}{\partial q_k}\frac{\partial F_2}{\partial \alpha}-\frac{\partial F_2}{\partial q_k}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial p_k}-\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial p_i}\right),
\end{aligned} \tag{B13}
$$

or, when substituting from (B11),

$$
\begin{aligned}
&(p_i-c(q_i))\left[-\frac{\partial D_j}{\partial q_i}\frac{\partial^2 \pi_i}{\partial p_i^2}+\frac{\partial D_j}{\partial p_i}\left(\frac{\partial\left(D_i-D_j\right)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial (D_i+D_j)}{\partial p_i}\right)\right]\Phi\\
&+(p_i-c(q_i))\,2(n-2)\frac{\partial D_i}{\partial p_k}\frac{\partial D_k}{\partial p_i}\left(\frac{\partial D_j}{\partial q_i}+\frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial p_i}\right)\Omega_p\\
&-(p_i-c(q_i))\,2(n-2)\frac{\partial D_i}{\partial q_k}\frac{\partial D_k}{\partial p_i}\left(\frac{\partial D_j}{\partial q_i}+\frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial p_i}\right)\Omega_q,
\end{aligned} \tag{B14}
$$

where

$$
\frac{\partial^2 \pi_i}{\partial p_i^2}=2\frac{\partial D_i}{\partial p_i}+(p_i-c(q_i))\frac{\partial^2 D_i}{\partial p_i^2}<0, \tag{B15}
$$

$$
\Phi:=\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_k}-\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_k}>0, \tag{B16}
$$

$$
\Omega_p:=\frac{\partial^2 \pi_k}{\partial q_k^2}+\frac{\partial c}{\partial q_k}\left(\frac{\partial D_k}{\partial q_k}-\frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial p_k}\right), \tag{B17}
$$

$$
\Omega_q:=\frac{\partial D_k}{\partial q_k}+\frac{\partial c}{\partial q_k}\left(\frac{\partial^2 \pi_k}{\partial p_k^2}-\frac{\partial D_k}{\partial p_k}\right). \tag{B18}
$$

Notice that $\Phi>0$ by the assumption of a negative definite Jacobian matrix. Since $(p_i-c(q_i))>0$ in equilibrium, we can factor this out of (B14) and arrive at:

$$
sign\left(\frac{\partial q_i}{\partial \alpha}\right)=sign\left(\begin{array}{c}\left[-\frac{\partial D_j}{\partial q_i}\frac{\partial^2\pi}{\partial p_i^2}+\frac{\partial D_j}{\partial p_i}\left(\frac{\partial(D_i-D_j)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial(D_i+D_j)}{\partial p_i}\right)\right]\Phi\\ +2(n-2)\frac{\partial D_k}{\partial p_i}\left(\frac{\partial D_j}{\partial q_i}+\frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial p_i}\right)\left(\frac{\partial D_i}{\partial p_k}\Omega_p-\frac{\partial D_i}{\partial q_k}\Omega_q\right)\end{array}\right). \tag{B19}
$$

B.2.2. The effect of a merger on the merged firm's price

The sign of $\partial p_i/\partial\alpha$ is given by the sign of

$$
\begin{vmatrix}
\frac{\partial F_1}{\partial q_i} & -\frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial q_k} & \frac{\partial F_1}{\partial p_k}\\
\frac{\partial F_2}{\partial q_i} & -\frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial q_k} & \frac{\partial F_2}{\partial p_k}\\
\frac{\partial F_3}{\partial q_i} & 0 & \frac{\partial F_3}{\partial q_k} & \frac{\partial F_3}{\partial p_k}\\
\frac{\partial F_4}{\partial q_i} & 0 & \frac{\partial F_4}{\partial q_k} & \frac{\partial F_4}{\partial p_k}
\end{vmatrix}. \tag{B20}
$$

This determinant can be written as

$$\begin{aligned}
&-\left(\frac{\partial F_1}{\partial q_i}\frac{\partial F_2}{\partial \alpha}-\frac{\partial F_2}{\partial q_i}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_k}-\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_k}\right)\\
&-\left(\frac{\partial F_2}{\partial \alpha}\frac{\partial F_1}{\partial p_k}-\frac{\partial F_1}{\partial \alpha}\frac{\partial F_2}{\partial p_k}\right)\left(\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial q_k}-\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial q_i}\right)\\
&-\left(\frac{\partial F_2}{\partial \alpha}\frac{\partial F_1}{\partial q_k}-\frac{\partial F_2}{\partial q_k}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_i}-\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_k}\right),
\end{aligned} \tag{B21}$$

or, when substituting from (B6),

$$\begin{aligned}
&-\left(p_i-c(q_i)\right)\left[\frac{\partial D_j}{\partial p_i}\frac{\partial^2 \pi_i}{\partial q_i^2}-\frac{\partial D_j}{\partial q_i}\left(\frac{\partial\left(D_i+D_j\right)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial\left(D_i-D_j\right)}{\partial p_i}\right)\right]\Phi\\
&-\left(p_i-c(q_i)\right)2(n-2)\frac{\partial D_i}{\partial p_k}\frac{\partial D_k}{\partial q_i}\left(\frac{\partial D_j}{\partial p_i}\frac{\partial c}{\partial q_i}+\frac{\partial D_j}{\partial q_i}\right)\Omega_p\\
&+\left(p_i-c(q_i)\right)2(n-2)\frac{\partial D_i}{\partial q_k}\frac{\partial D_k}{\partial q_i}\left(\frac{\partial D_j}{\partial p_i}\frac{\partial c}{\partial q_i}+\frac{\partial D_j}{\partial q_i}\right)\Omega_q,
\end{aligned} \tag{B22}$$

where

$$\frac{\partial^2 \pi_i}{\partial q_i^2}=-\frac{\partial^2 K_i}{\partial q_i^2}-\frac{\partial^2 c}{\partial q_i^2}D_i-2\frac{\partial c}{\partial q_i}\frac{\partial D_i}{\partial q_i}+\left(p_i-c(q_i)\right)\frac{\partial^2 D_i}{\partial q_i^2}<0. \tag{B23}$$

After factoring out $(p_i - c(q_i)) > 0$ from (B22) we arrive at

$$sign(\frac{\partial p_i}{\partial \alpha})=sign\left(\begin{array}{c}\left[-\frac{\partial D_j}{\partial p_i}\frac{\partial^2 \pi_i}{\partial q_i^2}+\frac{\partial D_j}{\partial q_i}\left(\frac{\partial (D_i+D_j)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial (D_i-D_j)}{\partial p_i}\right)\right]\Phi\\ -2(n-2)\frac{\partial D_k}{\partial q_i}\left(\frac{\partial D_j}{\partial q_i}+\frac{\partial c}{\partial q_i}\frac{\partial D_j}{\partial p_i}\right)\left(\frac{\partial D_i}{\partial p_k}\Omega_p-\frac{\partial D_i}{\partial q_k}\Omega_q\right)\end{array}\right). \tag{B24}$$

B.2.3. The effect of a merger on the non-merging firms' qualities

The sign of $\partial q_k/\partial\alpha$ is given by the sign of

$$\left|\begin{array}{cccc}\frac{\partial F_1}{\partial q_i}&\frac{\partial F_1}{\partial p_i}&-\frac{\partial F_1}{\partial \alpha}&\frac{\partial F_1}{\partial p_k}\\ \frac{\partial F_2}{\partial q_i}&\frac{\partial F_2}{\partial p_i}&-\frac{\partial F_2}{\partial \alpha}&\frac{\partial F_2}{\partial p_k}\\ \frac{\partial F_3}{\partial q_i}&\frac{\partial F_3}{\partial p_i}&0&\frac{\partial F_3}{\partial p_k}\\ \frac{\partial F_4}{\partial q_i}&\frac{\partial F_4}{\partial p_i}&0&\frac{\partial F_4}{\partial p_k}\end{array}\right|. \tag{B25}$$

This determinant can be written as

$$\begin{aligned}
&-\left(\frac{\partial F_1}{\partial p_i}\frac{\partial F_2}{\partial \alpha}-\frac{\partial F_2}{\partial p_i}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_4}{\partial p_k}\frac{\partial F_3}{\partial q_i}-\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial q_i}\right)\\
&-\left(\frac{\partial F_1}{\partial q_i}\frac{\partial F_2}{\partial \alpha}-\frac{\partial F_2}{\partial q_i}\frac{\partial F_1}{\partial \alpha}\right)\left(\frac{\partial F_3}{\partial p_k}\frac{\partial F_4}{\partial p_i}-\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial p_k}\right)\\
&-\left(\frac{\partial F_1}{\partial \alpha}\frac{\partial F_2}{\partial p_k}-\frac{\partial F_2}{\partial \alpha}\frac{\partial F_1}{\partial p_k}\right)\left(\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_i}-\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_i}\right).
\end{aligned} \tag{B26}$$

Notice, however, that

$$\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_i}-\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_i}=-2\frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial q_i}2\frac{\partial D_k}{\partial p_i}+2\frac{\partial c}{\partial q_k}\frac{\partial D_k}{\partial p_i}2\frac{\partial D_k}{\partial q_i}=0,$$

which eliminates the third term in (B26). The remaining two terms can, after substituting from (B11), be written as

$$\begin{aligned}&(p_i-c(q_i))\left(-\frac{\partial D_j}{\partial q_i}\frac{\partial^2\pi_i}{\partial p_i^2}+\frac{\partial D_j}{\partial p_i}\left(\frac{\partial(D_i-D_j)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial(D_i+D_j)}{\partial p_i}\right)\right)2\frac{\partial D_k}{\partial q_i}\Omega_q\\&+(p_i-c(q_i))\left(-\frac{\partial D_j}{\partial p_i}\frac{\partial^2\pi_i}{\partial q_i^2}+\frac{\partial D_j}{\partial q_i}\left(\frac{\partial(D_i+D_j)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial(D_i-D_j)}{\partial p_i}\right)\right)2\frac{\partial D_k}{\partial p_i}\Omega_q.\end{aligned}\tag{B27}$$

Factoring out $2\,(p_i-c(q_i))>0$ from (B27), we get

$$sign\left(\frac{\partial q_k}{\partial\alpha}\right)=sign\left(\Omega_q\left(\frac{\partial D_k}{\partial q_i}\Psi_q+\frac{\partial D_k}{\partial p_i}\Psi_p\right)\right),\tag{B28}$$

where

$$\Psi_q:=-\frac{\partial D_j}{\partial q_i}\frac{\partial^2\pi_i}{\partial p_i^2}+\frac{\partial D_j}{\partial p_i}\left(\frac{\partial(D_i-D_j)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial(D_i+D_j)}{\partial p_i}\right)\tag{B29}$$

and

$$\Psi_p:=-\frac{\partial D_j}{\partial p_i}\frac{\partial^2\pi_i}{\partial q_i^2}+\frac{\partial D_j}{\partial q_i}\left(\frac{\partial(D_i+D_j)}{\partial q_i}-\frac{\partial c}{\partial q_i}\frac{\partial(D_i-D_j)}{\partial p_i}\right).\tag{B30}$$

B.2.4. The effect of a merger on the non-merging firms' prices

The sign of $\partial p_k/\partial\alpha$ is given by the sign of

$$\begin{vmatrix}\frac{\partial F_1}{\partial q_i}&\frac{\partial F_1}{\partial p_i}&\frac{\partial F_1}{\partial q_k}&-\frac{\partial F_1}{\partial\alpha}\\\frac{\partial F_2}{\partial q_i}&\frac{\partial F_2}{\partial p_i}&\frac{\partial F_2}{\partial q_k}&-\frac{\partial F_2}{\partial\alpha}\\\frac{\partial F_3}{\partial q_i}&\frac{\partial F_3}{\partial p_i}&\frac{\partial F_3}{\partial q_k}&0\\\frac{\partial F_4}{\partial q_i}&\frac{\partial F_4}{\partial p_i}&\frac{\partial F_4}{\partial q_k}&0\end{vmatrix}.\tag{B31}$$

This determinant can be written as

$$\begin{aligned}&-\left(\frac{\partial F_1}{\partial q_i}\frac{\partial F_2}{\partial\alpha}-\frac{\partial F_2}{\partial q_i}\frac{\partial F_1}{\partial\alpha}\right)\left(\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_k}-\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial p_i}\right)\\&-\left(\frac{\partial F_1}{\partial p_i}\frac{\partial F_2}{\partial\alpha}-\frac{\partial F_2}{\partial p_i}\frac{\partial F_1}{\partial\alpha}\right)\left(\frac{\partial F_3}{\partial q_k}\frac{\partial F_4}{\partial q_i}-\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial q_k}\right)\\&-\left(\frac{\partial F_1}{\partial q_k}\frac{\partial F_2}{\partial\alpha}-\frac{\partial F_2}{\partial q_k}\frac{\partial F_1}{\partial\alpha}\right)\left(\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_i}-\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_i}\right),\end{aligned}\tag{B32}$$

but we already know that

$$\frac{\partial F_3}{\partial q_i}\frac{\partial F_4}{\partial p_i}-\frac{\partial F_3}{\partial p_i}\frac{\partial F_4}{\partial q_i}=0,$$

so the third term in (B32) vanishes. After substituting from (B11), the remaining two terms can be written as

$$\begin{aligned} & -\left(p_i - c(q_i)\right)\left(-\frac{\partial^2 \pi_i}{\partial q_i^2}\frac{\partial D_j}{\partial p_i} + \frac{\partial D_j}{\partial q_i}\left(\frac{\partial\left(D_i + D_j\right)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial\left(D_i - D_j\right)}{\partial p_i}\right)\right) 2\frac{\partial D_k}{\partial p_i}\Omega_p \\ & -\left(p_i - c(q_i)\right)\left(-\frac{\partial^2 \pi_i}{\partial p_i^2}\frac{\partial D_j}{\partial q_i} + \frac{\partial D_j}{\partial p_i}\left(\frac{\partial(D_i - D_j)}{\partial q_i} - \frac{\partial c}{\partial q_i}\frac{\partial(D_i + D_j)}{\partial p_i}\right)\right) 2\frac{\partial D_k}{\partial q_i}\Omega_p. \end{aligned} \tag{B33}$$

After factoring out $2\left(p_i - c(q_i)\right) > 0$, we get

$$sign\left(\frac{\partial p_k}{\partial \alpha}\right) = sign\left(-\Omega_p\left[\frac{\partial D_k}{\partial p_i}\Psi_p + \frac{\partial D_k}{\partial q_i}\Psi_q\right]\right). \tag{B34}$$

B.3. Proofs

Proof of Proposition 4 (i) The sign of $\partial q_i/\partial\alpha$ is determined by the sign of the sum of the first-order effects (first line in (38)) and the second-order effects (second line in (38)). The sign of the first-order effects is positive if $|\partial D_j/\partial q_i|$ is sufficiently small relative to $\partial D_j/\partial p_i$. If $\partial c/\partial q_i$ is sufficiently low, the sign of the second-order effects is determined by the (inverse of the) sign of $(\partial D_i/\partial p_k)\,\Omega_p - (\partial D_i/\partial q_k)\,\Omega_q$, with $\Omega_p < 0$ and $\Omega_q > 0$. This difference is negative, implying that the second-order effects are positive, if $|\partial D_i/\partial q_k|$ is sufficiently small relative to $\partial D_i/\partial p_k$. Thus, if marginal variable quality costs are sufficiently small, and if cross-quality effects are sufficiently small relative to cross-price effects, the first-order and second-order effects go in the same direction and yields $\partial q_i/\partial\alpha > 0$. The sign of $\partial p_i/\partial\alpha$ is determined in a similar way. (ii) The proof of the second part of the Proposition follows exactly the same logic as the first part. *Q.E.D.*

Proof of Proposition 5 (i) The expressions in the square brackets of (42) and (43) are identical. Thus, $\partial q_k/\partial\alpha$ and $\partial p_k/\partial\alpha$ have equal (opposite) signs if Ω_p and Ω_q have opposite (equal) signs. (ii) If marginal variable quality costs are sufficiently small, such that $\Omega_p < 0$ and $\Omega_q > 0$, then $\partial q_k/\partial\alpha$ and $\partial p_k/\partial\alpha$ have the same sign, which is determined by the sign of $(\partial D_k/\partial q_i)\,\Psi_q + (\partial D_k/\partial p_i)\,\Psi_p$, where Ψ_q and Ψ_p are the first-order effects of the merged firm's quality and price responses, respectively. If cross-quality effects are sufficiently small relative to the cross-price effects, then $|(\partial D_k/\partial q_i)\,\Psi_q| < |(\partial D_k/\partial p_i)\,\Psi_p|$ and $\Psi_p > 0$, which implies that $\partial q_k/\partial\alpha > 0$ and $\partial p_k/\partial\alpha > 0$. On the other hand, if cross-price effects are sufficiently small relative to the cross-quality effects, then $|(\partial D_k/\partial q_i)\,\Psi_q| > |(\partial D_k/\partial p_i)\,\Psi_p|$ and $\Psi_q < 0$, which again implies that $\partial q_k/\partial\alpha > 0$ and $\partial p_k/\partial\alpha > 0$. *Q.E.D.*

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