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# A platoon based cooperative eco-driving model for mixed automated and human-driven vehicles at a signalised intersection

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## Abstract

The advancements in communication and sensing technologies can be exploited to assist the drivers in making better decisions. In this paper, we consider the design of a real-time cooperative eco-driving strategy for a group of vehicles with mixed automated vehicles (AVs) and human-driven vehicles (HVs). The lead vehicles in the platoon can receive the signal phase and timing information via vehicle-to-infrastructure (V2I) communication and the traffic state of preceding vehicle and current platoon via vehicle-to-vehicle (V2V) communication. We propose a receding horizon model predictive control (MPC) method to minimise the fuel consumption for platoons and drive the platoons to pass the intersection on a green phase. The method is then extended to dynamic platoon splitting and merging rules for cooperation among AVs and HVs in response to the high variation in urban traffic flow. Extensive simulation tests are also conducted to demonstrate the performance of the model in various conditions in the mixed traffic flow and different penetration rates of AVs. Our model shows that the cooperation between AVs and HVs can further smooth out the trajectory of the latter and reduce the fuel consumption of the entire traffic system, especially for the low penetration of AVs. It is noteworthy that the proposed model does not compromise the traffic efficiency and the driving comfort while achieving the eco-driving strategy.

*Keywords:* Cooperative driving, Platoon based operations, Eco-driving, Automated vehicles, Heterogeneous flow, Car following model

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## 1. Introduction

Transportation is one of the main sources of energy consumption and greenhouse gas emission. In the EU, transportation is responsible for 33% of energy consumption and 23% of total emissions (European Commission, 2016). Road transport represents most of it, 72.8% in total greenhouse gas emissions and 73.4% in transport energy demand. A lot of work has been done to mitigate these effects from different aspects, for example, optimised engine design,

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7 better road surface condition and more training for drivers. Due to the continually increasing  
8 number of vehicles, however, the total vehicle fuel consumption is still rising. The concept of  
9 “eco-driving” has drawn increasing attention from both researchers and government (Carsten  
10 et al., 2016). The core of eco-driving technologies is to provide drivers with a variety of advice  
11 and feedback to minimise the fuel consumption and emissions while driving.

12 Unlike continuous traffic flow on freeways, traffic flows on urban roads are regularly in-  
13 terrupted by traffic signals and conflicting traffic flows at intersections. As such the vehicles  
14 travel with strong variations in their velocity and consume more fuel. Eco-driving strategies  
15 can be designed to reduce the idling time on the red light and subsequent strong acceler-  
16 ation by advising the drivers to approach intersections using a moderate acceleration and  
17 deceleration. The development of sensing and communication technologies make Vehicle-to-  
18 Infrastructure (V2I) and Vehicle-to-Vehicle (V2V) communication possible in the near future.  
19 These technologies offer potential applications for eco-driving patterns at intersections as the  
20 connected vehicles can receive the Signal Phase and Timing (SPaT) information from the  
21 intersection controller by V2I and also receive the position and velocity information from  
22 surrounding vehicles by V2V communication. Better speed advice can be generated using  
23 this information, and thus vehicles may adjust their speed in advance, in order to avoid  
24 stopping at the stop line and subsequent strong acceleration, and consequently reduce the  
25 fuel consumption.

26 Both field experiments (Schall and Mohnen, 2017) and simulator experiments (Van der  
27 Voort et al., 2001; Staubach et al., 2014) show that eco-driving reduces the fuel consumption  
28 between 5% and 18%, and drivers exhibit a high acceptance towards an eco-driving support  
29 system. It has no negative effects on safety, but many eco-driving methods lead to low travel  
30 speed and may have a negative impact on the following vehicles (Wu et al., 2015; Staubach  
31 et al., 2014). Moreover, they may even increase the travel time of the host vehicles and  
32 following vehicles.

33 This paper proposes a real-time cooperative eco-driving strategy for a platoon including  
34 mixed automated vehicles (AVs) and human-driven vehicles (HVs) approaching a signalised  
35 intersection. It adopts a model predictive control (MPC) method to control the trajectories  
36 of AVs. Here the AVs are considered the leaders of the platoon with the aim of minimising the  
37 total fuel consumption of the whole platoon without sacrificing the travel time of the leaders.  
38 It also reduces the travel time for the following vehicles to a certain extent. The rest of the  
39 paper is organised as follows: the literature review of the eco-driving modelling is presented  
40 in section 2. Then, the proposed model structure, optimisation method, and platoon control  
41 scheme are described in section 3. In section 4, the properties of the proposed model are  
42 extensively studied and the performance of the proposed method for different penetration  
43 rates of AVs is also examined. A final section 5 summarises the paper’s findings.

## 44 2. Literature Review

45 One of the applications of speed advisory systems is Intelligent Speed Adaptation (ISA)  
46 which is widely used in several EU countries (Almqvist et al., 1991; Liu and Tate, 2004). ISA  
47 devices are primarily aimed at safer driving by advising drivers a desired speed and speed  
48 limits on specific road sections (Ngoduy et al., 2009). Experiments showed that ISA strategies  
49 can potentially mitigate congestion and reduce fuel consumption and pollutant emissions due

50 to smoother speed variations (Oei and Polak, 2002; Panis et al., 2006). In conventional ISA  
51 systems, vehicles are still driven by humans, and traffic information is usually obtained from  
52 loop detectors.

53 There are two main methods proposed in the literature which utilise the traffic signal  
54 information to reduce idle time and fuel consumption. The first approach suggests a con-  
55 stant speed or constant acceleration for an individual driver to reduce the idle time or fuel  
56 consumption. This is commonly named Green Light Optimised Speed Advisory (GLOSA)  
57 system. It is usually implemented as an optimisation model by assuming a simple speed  
58 pattern in front of the intersection. Rakha and Kamalanathsharma (2011) considered a fuel  
59 consumption model in the objective function and showed that simplified objective functions  
60 such as minimising the deceleration or idling time may not get the optimal result in terms  
61 of fuel consumption. This work is further extended to control the variable speed limit for  
62 each vehicle to minimise the fuel consumption (Kamalanathsharma et al., 2015) and inte-  
63 grate queue estimation (Yang et al., 2017). Mandava et al. (2009) developed an arterial  
64 velocity planning algorithm which provided speed advice to the drivers regarding the most  
65 fuel optimal path computed using upcoming signal information. The objective function was  
66 aimed at minimising the deceleration and acceleration rates, and 12-14% energy/emission  
67 savings were achieved. Tielert et al. (2010) conducted a large-scale simulation to identify the  
68 impact of gear choice and distance to the intersection. They found that sub-optimal gear  
69 choice can reduce the positive performance of the speed adaptation. Another finding was that  
70 the benefit of providing information to the vehicles located further than 600m is negligible.  
71 Treiber and Kesting (2014) implemented three strategies of speed adaptation: early break,  
72 early start and avoiding queue in the Improved Intelligent-Driver Model. The travel time  
73 decreases linearly with the penetration of equipped vehicles. They also found that increasing  
74 the maximum speed from 50km/h to 70km/h doubles the performance index. Katwijk and  
75 Gabriel (2015) considered the impact of different trajectories on the fuel consumption. The  
76 vehicle was advised to use a smaller deceleration, even combined with a period of constant  
77 speed, instead of a hard deceleration in front of the red light. Stebbins et al. (2017) developed  
78 a method to suggest an acceleration to the leading vehicle only in a platoon to reduce delays.  
79 It was assumed that every vehicle that is the first to pass the intersection on a green light  
80 can be selected as a leading vehicle. Instead of controlling the speed directly, Ubierno and  
81 Jin (2016) proposed a green driving strategy to control the individual advisory speed limit of  
82 connected vehicles while following their leaders at signalised intersections; it can be applied  
83 to any level of market penetration. Although no fuel consumption model was explicitly used  
84 in this modelling method, it still saved 15% in travel delays and 8% in fuel consumption and  
85 emission.

86 The second approach uses an optimal control or an MPC method to provide dynamic  
87 or real-time speed advice to an individual vehicle considering the local and predictive traffic  
88 states. This approach is thus more suitable for AVs because of the real-time detecting and  
89 speed adjustment. Asadi and Vahidi (2011) calculated the optimal speed that reduces idling  
90 at red lights using the given future state of traffic lights and developed an optimisation-based  
91 MPC model to consider multiple objectives. Kamal et al. (2013) predicted the dynamics  
92 of the preceding vehicle based on the information from inter-vehicle communication and  
93 considered the signal status of the upcoming intersections to compute the optimal vehicle  
94 control input for fuel economy by an MPC method. He et al. (2015) developed a multi-stage

95 optimal control model considering the spatial and temporal constraints by the queue in front  
96 of the intersection. They also considered the constraints to reduce the negative impact on the  
97 following vehicles, but it was only active at the terminal time step at each stage. [Wan et al.](#)  
98 [\(2016\)](#) used optimal control theory to solve the minimum fuel control problem and found  
99 that the minimal fuel driving strategy is a bang-singular-bang control, which means either  
100 maximum acceleration or engine shut down is used. By employing a sub-optimal method,  
101 the speed advisory equipped vehicle can also benefit the following conventional vehicles. [De](#)  
102 [Nunzio et al. \(2016\)](#) used a combination of a pruning algorithm and shortest path method to  
103 find the minimum energy consumption path in multi-intersections. The non-convex optimal  
104 control problem was then reduced to a convex problem which can be solved efficiently.

105 To the best of our knowledge, most current work focuses on developing fuel economic  
106 control strategies for a single vehicle without considering the impact on the other vehicles.  
107 [HomChaudhuri et al. \(2017\)](#) considered neighbourhood information exchange and designed a  
108 decentralised control model emulating the selfish behaviour of human drivers, but their model  
109 still considers one vehicle and does not describe the interactions between platoons. [Zhou et al.](#)  
110 [\(2017\)](#); [Ma et al. \(2017\)](#) proposed a parsimonious shooting heuristic algorithm to optimise the  
111 trajectories of a stream of vehicles and considered multiple objective functions such as fuel  
112 consumption and travel time, but all vehicles are required to be AVs in their method. [Jiang](#)  
113 [et al. \(2017\)](#) proposed an eco-driving model in partially connected and automated vehicles  
114 environment; however, they did not consider the cooperation of AVs and HVs, even though  
115 the behaviour of the AV still affects the following vehicles. This indicates that there are no  
116 platoon-based dynamics in their approach. Our model will fill in this gap by showing that  
117 the cooperation between AVs and HVs can further smooth the trajectory of the latter and  
118 consequently reduce the fuel consumption of the whole platoon. The proposed method will  
119 optimise the fuel consumption for platoons and drive the platoons to pass the intersection  
120 on a green phase. The proposed model is flexible that allows multiple AVs and HVs in the  
121 platoon. Both the impact of cooperation among AVs and cooperation among AVs and HVs  
122 will be studied in detail. Most existing work uses the rolling horizon MPC method, and  
123 the optimised vehicles sometimes travel with low speed to achieve a better fuel economy.  
124 In this paper, a distinctive receding horizon MPC method is proposed to ensure that eco-  
125 driving strategies do not have an adverse impact on the traffic efficiency. On the contrary, the  
126 proposed model can increase the speed while passing the intersection and thus increase the  
127 traffic efficiency. In addition, the driving comfort is considered by using jerk as the control  
128 variable.

## 129 **Notation**

130 The notation in Table 1 is used throughout this paper.

## 131 **3. Problem formulation**

132 This paper focuses on the design of an eco-driving strategy for a group of vehicles with  
133 mixed AVs and HVs. The movements of HVs are modelled by a car-following model while the  
134 dynamics of AVs are described by an MPC method. For the sake of simplicity, in this paper,  
135 an optimal velocity model (OVM) is applied to describe the behaviour of HVs ([Bando et al.](#),

Table 1: Notation of major variables used in this paper

Symbol	Description
$t$	Time instant
$x_i^a(t), v_i^a(t), a_i^a(t)$	The position, speed, acceleration of an AV $i$ at time $t$ where the superscript $a$ denotes AV
$x_j^h(t), v_j^h(t), a_j^h(t)$	The position, speed, acceleration of an HV $j$ at time $t$ where the superscript $h$ denotes HV
$\hat{x}_{tf}, \hat{v}_{tf}, \hat{a}_{tf}$	The desired position, speed, acceleration at terminal time, respectively where subscript $tf$ denotes terminal time
$u(t)$	The jerk of an AV which is the control variable at time $t$
$J$	Total cost in the MPC objective function
$L$	Running cost in the MPC objective function
$\theta$	Terminal cost in the MPC objective function
$F_i^a(t), F_j^h(t)$	Instantaneous fuel consumption rate for AV $i$ , and HV $j$ , respectively, at time $t$
$t_i^f$	Terminal time for the vehicle $i$ and also the time to pass the stop line
$T_k^g, T_k^r$	The start time of green light, red light respectively, in cycle $k$

136 1995). Nevertheless, the proposed modelling methodology holds for any other car-following  
 137 model. Our method allows several closely running vehicles to form a platoon and pass the  
 138 intersection on the green light without stopping. A basic schematic representation is shown  
 139 in Figure 1.

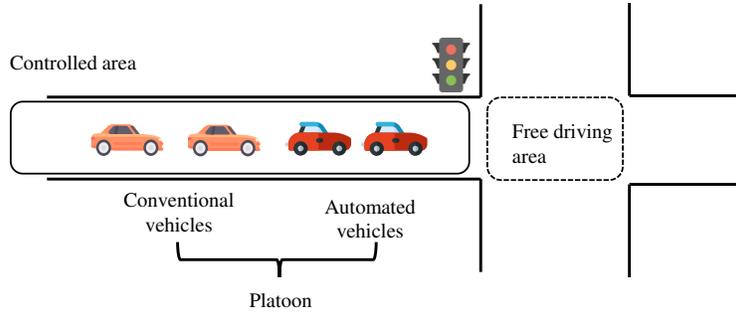


Figure 1: Schematic of eco-driving problem at a signalised intersection

### 140 3.1. Assumptions

141 To facilitate our model development, some necessary assumptions are made as follows.

- 142 1. In order to set up the cooperative behaviour between the AVs and the following HVs,  
 143 AVs have to know the positions and speeds of some following vehicles and the direct  
 144 preceding vehicle in real time. We will assume that this information is available through  
 145 either connected vehicle technology or a roadside unit (RSU) (Jia and Ngoduy, 2016a).  
 146 This assumption will be relaxed in section 4.3 where the AVs obtain this information  
 147 about the direct following vehicle via its own detectors.
- 148 2. All AVs can receive the signal timing information from the downstream intersection via  
 149 V2I.

- 150 3. No communication delay or detection error is considered in the paper; for cooperative  
 151 driving behaviour in a platoon with realistic communication, we refer to [Jia and Ngoduy](#)  
 152 [\(2016b\)](#). This assumption will be relaxed in our future work.
- 153 4. AVs in different platoons can share the information about the vehicles’ arrival time via  
 154 either V2V or RSU; hence they can predict a better arrival time.
- 155 5. This work only focuses on the longitudinal movement on the urban road.

156 It is worth noticing that AVs will interact with the downstream intersection and decide  
 157 their dynamics to get the whole platoon through the intersection during the green time  
 158 period.

159 *3.2. Optimal velocity model*

The OVM is formulated based on the presumption that a vehicle is driven to reach an optimal velocity, which depends on the headway with respect to the preceding vehicle in a continuous time step. The acceleration of vehicle in the OVM is calculated by

$$a_j^h(t) = \kappa [V_{op}(\Delta x_j(t)) - v_j^h(t)] \quad (1)$$

where  $\Delta x_j(t) = x_{j-1}(t) - x_j^h(t)$  is the distance headway between vehicles  $j$  and its preceding vehicle  $j - 1$ .  $V_{op}(\Delta x_j(t))$  defines the optimal velocity, which is a function of the distance headway.  $\kappa$  is the sensitivity. The sensitivity is the inverse of the delay time that is required to reach the optimal velocity. In this paper, the following velocity function proposed by [Helbing and Tilch \(1998\)](#) is chosen:

$$V_{op}(\Delta x_j) = V_1 + V_2 \tanh [C_1(\Delta x_j - l_c) - C_2] \quad (2)$$

160 where  $V_1, V_2, C_1, C_2$  are the parameters and  $l_c$  denotes the vehicle length. The parameters  
 161 calibrated by the empirical follow-the-leader data for city traffic in [Helbing and Tilch \(1998\)](#)  
 162 are used in this paper:  $\kappa = 0.85 \text{ s}^{-1}$ ,  $V_1 = 6.75 \text{ m/s}$ ,  $V_2 = 7.91 \text{ m/s}$ ,  $C_1 = 0.13 \text{ m}^{-1}$ ,  $C_2 = 1.57$   
 163 and  $l_c = 5 \text{ m}$ . Because the OVM may generate unrealistic high acceleration ([Helbing and](#)  
 164 [Tilch, 1998](#)), the acceleration limits shown in [Table 2](#) are applied.

165 *3.3. Model predictive control*

166 Each AV is able to receive real-time information from the preceding vehicle and following  
 167 vehicles via V2V, such as position and velocity. In the MPC method, a common assumption  
 168 is that the preceding vehicle is travelling at a constant velocity. So the time for the AV to  
 169 arrive at the intersection on green time can also be estimated. Then a receding horizon MPC  
 170 method will be used. For the safety and comfort purposes, a further assumption is made  
 171 that the AV travels across the intersection with a constant velocity, which implies that the  
 172 acceleration of the AV at the stop line should be 0. Accordingly, in our model, the control  
 173 variable is the derivative of the acceleration of the AV, which is also called “jerk” and denoted  
 174 as  $u(t)$ .

175 *3.3.1. State variables*

In order to minimise the fuel consumption for all vehicles in the platoon, the state variables of those vehicles are included in the system state. For a general platoon including  $m$  AVs and  $n$  HVs, its state is designed as

$$\mathbf{X}(t) = \underbrace{[x_i^a(t), v_i^a(t), a_i^a(t), \dots, x_m^a(t), v_m^a(t), a_m^a(t)]}_{\text{AVs}} \underbrace{[x_j^h(t), v_j^h(t), \dots, x_n^h(t), v_n^h(t)]}_{\text{HVs}}^T \quad (3)$$

$i = 1, \dots, m; j = 1, \dots, n$

The corresponding system dynamic function is

$$\dot{\mathbf{X}}(t) = \underbrace{[v_i^a(t), a_i^a(t), u_i^a(t), \dots, v_m^a(t), a_m^a(t), u_m^a(t)]}_{\text{AVs}} \underbrace{[v_j^h(t), a_j^h(t), \dots, v_n^h(t), a_n^h(t)]}_{\text{HVs}}^T \quad (4)$$

176 where the acceleration of the HV  $a_j^h(t)$  is calculated by equation 1.

177 *3.3.2. Objective function*

The total cost function for the platoon is defined as:

$$\min_u J = \theta(\mathbf{X}(\mathbf{t}^f)) + \int_{t_i^0}^{t_i^f} L(\mathbf{X}(t)) dt \quad (5)$$

The control goal is to drive AVs from the current position to the stop line with the desired velocity and acceleration. Therefore, the terminal cost is designed as the squared difference between the terminal state for the AVs and the desired terminal state:

$$\theta(\mathbf{X}(\mathbf{t}^f)) = \sum_i^m p_1(x_i^a(t_i^f) - \hat{x}_{tf})^2 + p_2(v_i^a(t_i^f) - \hat{v}_{tf})^2 + p_3(a_i^a(t_i^f) - \hat{a}_{tf})^2 \quad (6)$$

178 where  $p_1$ ,  $p_2$ ,  $p_3$  are the weights for the corresponding terms. In the paper, the desired  
 179 terminal position  $\hat{x}_{tf}$  is the downstream stop line, and the desired terminal speed  $\hat{v}_{tf}$  is the  
 180 maximum allowed velocity which is 14.66 m/s. Note that the maximum speed of the AVs is  
 181 the same as that of HVs using the described parameters. The desired terminal acceleration  
 182  $\hat{a}_{tf}$  is 0 m/s<sup>2</sup> because of the constant velocity assumption described above.

The running cost is the driving cost at every time step. In this paper, it means the total fuel consumption for all vehicles in the platoon and it is formulated as:

$$L(\mathbf{X}(t)) = \sum_i^m F_i^a(t) + \sum_j^n F_j^h(t) \quad (7)$$

An instantaneous fuel consumption model developed by Akcelik (1989) is adopted in this work. It uses the instantaneous acceleration and velocity to estimate the fuel consumption rate:

$$F = \alpha + \beta_1 P_T + (\beta_2 m a^2 v)_{a>0} \quad (8)$$

183 where  $P_T$  is the total power (kW) required to drive the vehicle, which is the sum of coast-  
 184 down drag power, inertia power and extra engine power.  $P_T$  is non-negative. The third term

185 means extra engine drag power during acceleration, which only exists when the acceleration  
 186 is larger than zero.

$$P_T = \max\{0, d_1v + d_2v^2 + d_3v^3 + mav\} \quad (9)$$

187 The parameters  $\alpha$ ,  $\beta_1$ ,  $\beta_2$ ,  $d_1$ ,  $d_2$ ,  $d_3$ ,  $m$  in equations 8 and 9 are taken from Akcelik  
 188 (1989) and are  $\alpha = 0.666 \text{ mL/s}$ ,  $\beta_1 = 0.072 \text{ mL/kJ}$ ,  $\beta_2 = 0.0344 \text{ mL/(kJ} \cdot \text{m/s}^2)$ ,  $d_1 =$   
 189  $0.269 \text{ kN}$ ,  $d_2 = 0.0171 \text{ kN/(m/s)}$ ,  $d_3 = 0.000672 \text{ kN/(m/s)}^2$ ,  $m = 1680 \text{ kg}$ .

The terminal time  $t_i^f$  is set to be the earliest time that allows the AV  $i$  to pass the intersection on the green phase. It is calculated by

$$t_i^f = \max(t_i^{f'}, t_i^g) \quad (10)$$

where  $t_i^{f'}$  denotes the earliest possible arrival time, and is calculated by

$$t_i^{f'} = \max(t_i^{\min}, t_{i-1}^f + h) \quad (11)$$

where  $t_i^{\min}$  denotes the minimum travel time by using the highest jerk,  $t_{i-1}^f$  denotes the travel time of the preceding vehicle  $i - 1$ . If the vehicle  $i - 1$  is an AV, its estimated travel time information can be available via V2V. If it is an HV that belongs to the preceding platoon, the AV (or AVs) in the preceding platoon must have the travel time information and transfer to vehicle  $i$ . If not, it can be estimated by using loop detectors (Treiber and Kesting, 2014; Guler et al., 2014; He et al., 2015) or connected vehicles (Yang et al., 2017; Zheng and Liu, 2017).  $h$  denotes the safety time headway of an AV with its preceding vehicle. Please note that it is the same as the saturation time headway of HVs using the described parameters. This is specially designed to show that the reduction of travel time is not resulting from the smaller time headway of AVs, but from the proposed eco-driving method.  $t_i^g$  denotes the start of the green light which is closest to  $t_i^{f'}$ . It is calculated by

$$t_i^g = \begin{cases} T_k^g & t_i^{f'} \in [T_k^g, T_k^r) \\ T_{k+1}^g & t_i^{f'} \in [T_k^r, T_{k+1}^g) \end{cases} \quad (12)$$

190 where  $T_k^g(T_k^r)$  denotes the start time of green (red) light in the signal cycle  $k$ .

191 Please note that when there are multiple AVs in a platoon, they have different  $t_i^0$  and  $t_i^f$   
 192 and the proposed optimal control problem is a multi-stage optimal control problem which  
 193 can be solved by GPOPS. We only discuss isolated intersection in this paper, but the pro-  
 194 posed model can be extended to multi-intersections without much trouble by taking each  
 195 intersection as a stage (He et al., 2015).

### 196 3.3.3. Constraints

$$\text{Speed constraints: } v_{\min} \leq v_i^a(t) \leq v_{\max} \quad (13a)$$

$$\text{Acceleration constraints: } a_{\min} \leq a_i^a(t) \leq a_{\max} \quad (13b)$$

$$\text{Jerk constraints: } u_{\min} \leq u_i^a(t) \leq u_{\max} \quad (13c)$$

$$\text{Safety constraints: } a_i^a(t) \leq a_i^{OVM}(t) \quad (13d)$$

197 where  $v_{min}$ ,  $v_{max}$ ,  $a_{min}$ ,  $a_{max}$ ,  $u_{min}$ ,  $u_{max}$  denote the lower and upper bounds of the velocity,  
 198 acceleration and jerk, respectively. The same speed and acceleration limits in Table 2 are  
 199 used for both MPC and OVM.  $a_i^{OVM}(t)$  is calculated by equation 1 using the speed and the  
 200 gap of AV. This implies that the car-following model (i.e. OVM) is used as the upper bound  
 201 of the acceleration for an AV. It prevents the MPC algorithm from acting too aggressively  
 202 to achieve the final goal. So basically, the upper bound of the acceleration reads:  $a_i^a(t) \leq$   
 203  $min(a_{max}, a_i^{OVM}(t))$ . It also provides the possibility of handing over to human driving more  
 204 smoothly if required.

### 205 3.4. Interactions between AVs and HVs

206 In order to provide an eco-driving strategy for the benefits of both AVs and HVs in the  
 207 platoon, several kinds of cooperation are considered in the model. The overall interactions  
 208 are shown in Figure 2. Note that in the platoon, HVs are modelled by the OVM and AVs  
 209 are controlled by the MPC method.

210 In Figure 2, there are basically two types of cooperative behaviour for AVs: (1) interact-  
 211 ing with preceding vehicles between platoons; (2) interacting with the AVs or HVs within  
 212 the platoon. If the preceding vehicle belongs to the preceding platoon, then the leading  
 213 (automated) vehicle of the preceding platoon knows the passing time of its members and  
 214 can transfer the information to the AVs in the considered platoon. Otherwise, the AVs have  
 215 to predict the arrival time of the preceding vehicles based on the data acquired by their  
 216 built-in detectors or other sources of communication such as RSU or even connected vehicle  
 217 technologies. For the vehicles in a platoon, the cooperation is designed for the purposes of  
 218 safety and fuel efficiency. The AVs in the platoon consider the dynamics of all vehicles in the  
 219 platoon and attempt to find a strategy that minimises the fuel consumption for all vehicles  
 220 in the platoon.

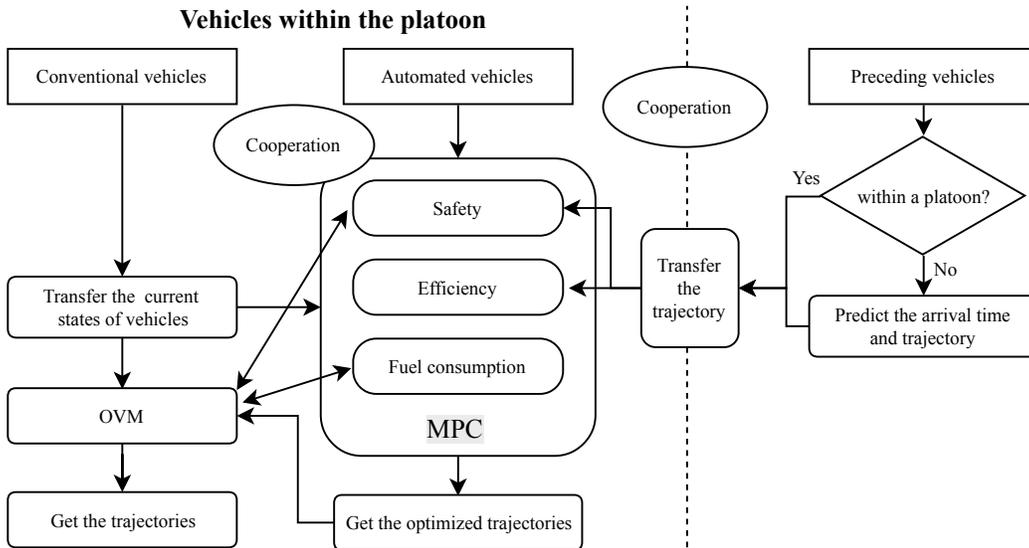


Figure 2: Interactions between AVs and HVs

221 *3.5. The control framework for platoons*

222 The proposed method is applied to a platoon instead of a single vehicle, so how to define  
 223 the platoon and how to manage the platoon dynamically are key challenges in this paper.  
 224 The platoon is usually defined as a group of vehicles that are adjacent to each other and have  
 225 similar traffic state (see [Ngoduy \(2013\)](#); [Jia and Ngoduy \(2016a,b\)](#) and references therein).  
 226 On an urban road, some vehicles can pass through the intersection on a green light and travel  
 227 with the speed that depends on the traffic conditions. Other vehicles have to stop at the  
 228 stop line when the traffic signal turns red. So it is natural to define the platoon as the group  
 229 of vehicles that can pass on the same green phase.

230 There are two criteria for a platoon:

- 231 1. All the vehicles in a platoon must pass the intersection on the same green phase.
- 232 2. The leading vehicle in a platoon must be an AV, and all AVs can only be located in  
 233 front of the HVs in each platoon.

234 Criterion 2 is essential for the proposed eco-driving method. This is because only when  
 235 the AV is in front of the HV, it can affect the following vehicles' movements by controlling its  
 236 own jerk. The platoon in this paper is different from the controversial one. It is heterogeneous  
 237 that may include AVs and HVs. The purpose of a platoon is to allow cooperation among AVs  
 238 and HVs to reduce the total fuel consumption which pass the intersection on the same green  
 239 phase. The platoon dynamics including splitting and merging are to determine which vehicle  
 240 should be considered in the cooperation loop. The setting of a platoon is not to ensure all  
 241 the vehicles in the platoon can pass the intersection on the same green phase. In fact, the  
 242 vehicles can pass the intersection on the same green phase is the necessary condition to form  
 243 a platoon, rather than the result. Different platoon settings in mixed traffic flow will be  
 244 discussed in detail in section 4.2.

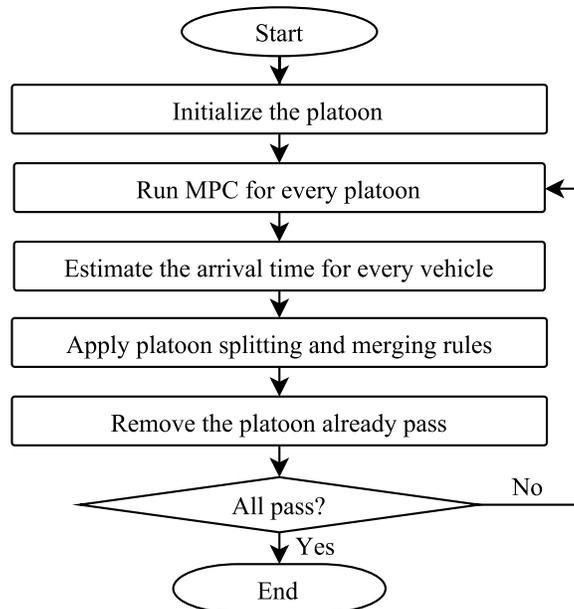


Figure 3: The overall control framework

245 The control framework for platoon dynamics is shown in Figure 3 and the main processes  
246 are described as follows:

- 247 1. Split all the vehicles on the road into several groups according to the maximum allowed  
248 number of vehicles in a platoon, and the leading AV (or AVs) in a platoon becomes the  
249 host vehicle.
- 250 2. Run the MPC algorithm for every platoon, the optimised control variables are only  
251 applied to the host vehicles for the next time step, while the behaviour of all other  
252 vehicles is governed by the OVM.
- 253 3. Apply the platoon split and merge rules every  $T_1$  time steps which is  $z$  times of the  
254 control update time step  $T$  (i.e.  $T_1 = zT$ ).

255 The platoon splitting and merging rules mainly consider the planned vehicle arrival time,  
256 signal timing information, and the defined minimum and maximum number of vehicles in a  
257 platoon. The rules are described in the following.

- 258 1. Splitting rule (see Figure 4a): After the MPC optimisation is executed, some of the  
259 vehicles in the platoon may not pass the intersection on the green time. Then the  
260 splitting rule applies. If the first vehicle that cannot pass on the same green light is  
261 an AV, then it is split from the original platoon and becomes the leading vehicle for  
262 the new platoon. Otherwise, all those that cannot pass on the same green light are  
263 discarded by the current platoon.
- 264 2. Merging rule (see Figure 4b): Merging rule is more complicated than the splitting rule  
265 as it may operate in two directions: merge with the preceding vehicles or the following  
266 vehicles. In both cases, it needs to check whether the two key criteria are still satisfied  
267 after merging. The exceptional case in figure 4b means an AV follows an HV. Please  
268 note that merging with the preceding vehicles has higher priority than merging with the  
269 following vehicles as the operations of the preceding vehicle can affect all the following  
270 vehicles and may get better performance.

271 The splitting rule is always applied before the merging rule. The discarded vehicles by  
272 the splitting rule will try to find a chance to form another platoon by the merging rule where  
273 every AV can be seen as a separate platoon with size 1. This does not mean that every HV  
274 must belong to a platoon. If an HV does not belong to any platoon, it may have to stop in  
275 front of the stop line.

### 276 3.6. Gauss pseudospectral method

277 A Matlab software package GPOPS (Rao et al., 2010) is used to solve the proposed  
278 optimal control problem. It mainly uses a numerical method, namely Gauss pseudospectral  
279 method, and is widely used in trajectory planning problems for vehicles (Wu et al., 2015;  
280 He et al., 2015) and trains (Ye and Liu, 2016). The method belongs to a direct approach  
281 (Stryk and Bulirsch, 1992) whose main idea is transforming the optimal control problem  
282 into a nonlinear programming (NLP) problem, which can then be solved by a variety of  
283 well-known solvers such as SNOPT (Gill et al., 2005) used in GPOPS. The performance of  
284 GPOPS strongly depends on the parameter settings (Ye and Liu, 2016). Usually, the user  
285 needs to try several combinations of parameter settings to find the best suitable ones. The  
286 key parameters used in GPOPS and the model are listed in Table 2.

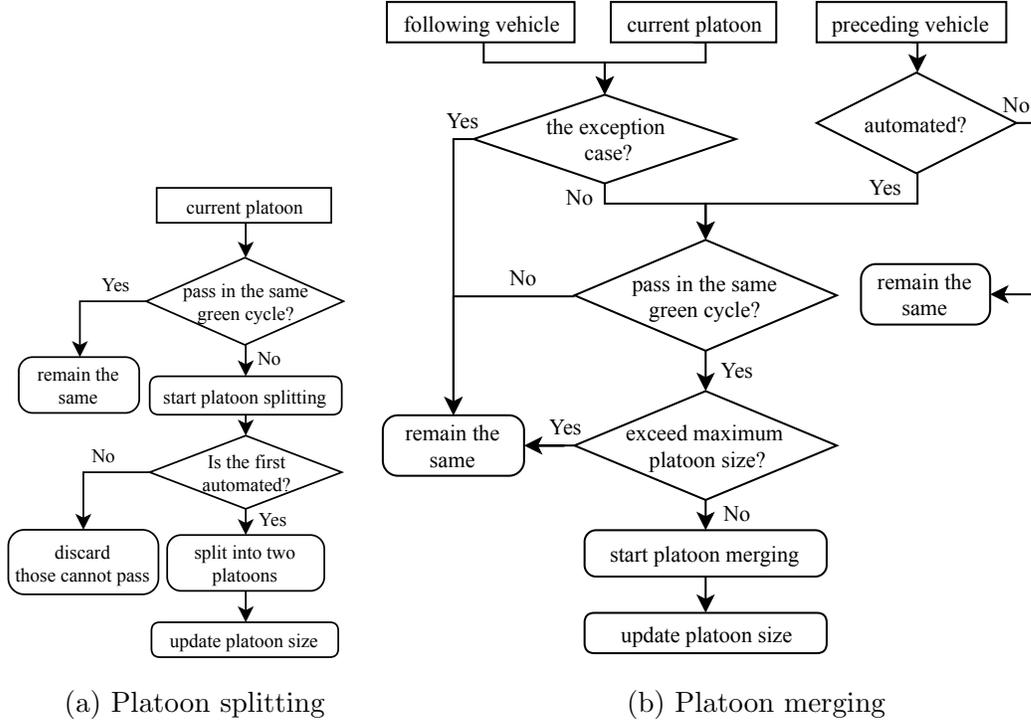


Figure 4: Platoon splitting and merging framework

## 4. Numerical studies

### 4.1. Properties of terminal cost

The terminal cost has three terms in equation 6. The first term forces the vehicle to arrive at the intersection at the terminal time. The second term maximises the speed entering the intersection. We will show that this can increase the capacity of the intersection. The third term allows the vehicle to pass the intersection with a constant speed which is the maximum speed resulting from the second term. This mainly concerns the safety when crossing the intersection. If this term were removed, the acceleration of vehicle would drop to zero suddenly due to the speed limit after the terminal time (Ntousakis et al., 2016).

In this study, three scenarios are considered to illustrate the benefits of the proposed terminal cost settings. The simulation scenario considered in this paper is a single lane road with a traffic signal light at location 250m ahead. We consider 10 vehicles driving on the road and attempting to cross the intersection. At the beginning of the simulation, all vehicles have the same velocity of 10 m/s and acceleration of 0 m/s<sup>2</sup>. The other parameters used in the MPC method are shown in Table 2.

- Scenario T1: no speed advice is given to the drivers and the accelerations of all vehicles are only calculated by the OVM. We will call this case as OVM for simplicity.
- Scenario T2: the first vehicle is an AV and only the first term in the terminal cost function 6 is considered while the running cost remains the same in function 7.
- Scenario T3: the first vehicle is an AV and the terminal cost and running cost are the same as function 6 and 7, respectively.

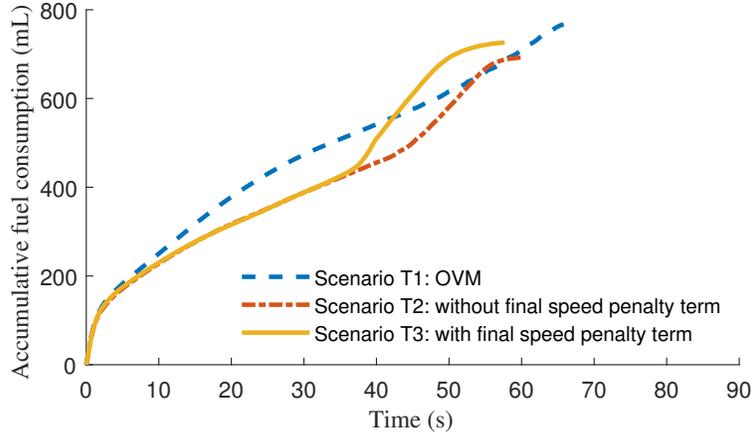
Table 2: The parameters in the proposed eco-driving method

Parameter settings in GPOPS			
Parameter	Description	Value	
setup.autoscale	Whether the optimal control problem is scaled automatically	‘on’	
setup.derivatives	Method to compute the derivatives of the objective function (gradient) and the constraints for NLP solver	‘complex’	
setup.tolerances	Optimality and feasibility tolerances for the NLP solver	[1e-3, 2e-3]	
limits.meshPoints	Locations of mesh points in the initial run	[-1,1]	
limits.nodesPerInterval	Number of allowable collocation points in a mesh interval	$2 * (t^f - t^0)$	
setup.mesh.tolerance	Mesh refinement tolerance	1e-4	
setup.mesh.iteration	Mesh refinement iterations to perform	8	
Parameter settings in model			
Parameter	Description	Value	Unit
$T_M$	Sample time for MPC method	0.5	s
$T_O$	Sample time for OVM	0.1	s
$h$	Safety time headway for an AV	2	s
$p_1$	Penalty weight for position difference	$10^5$	
$p_2$	Penalty weight for velocity difference	$10^6$	
$p_3$	Penalty weight for acceleration difference	$10^7$	
$v_{max}$	Maximum speed	14.66	m/s
$v_{min}$	Minimum speed	0	m/s
$a_{max}$	Maximum acceleration	3	m/s <sup>2</sup>
$a_{min}$	Minimum acceleration	-6	m/s <sup>2</sup>
$u_{max}$	Maximum jerk (limit for the control variable)	4	m/s <sup>3</sup>
$u_{min}$	Minimum jerk (limit for the control variable)	-4	m/s <sup>3</sup>

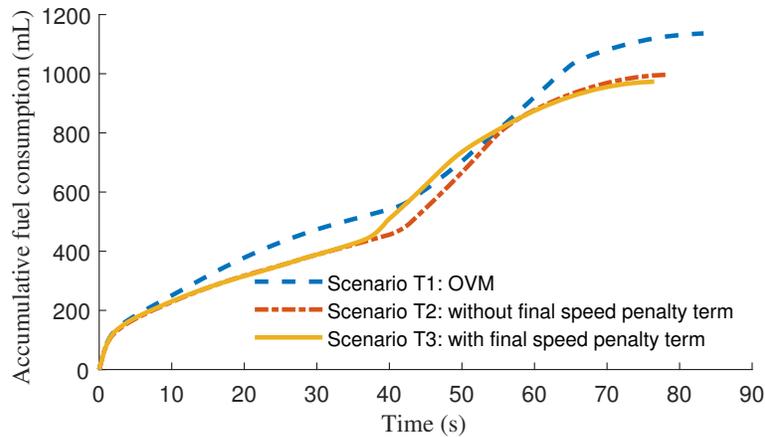
308 When all vehicles have crossed the stop line, the total fuel consumption is shown in figure  
309 5a. As expected, the fuel consumption of vehicles under MPC is much less than that in OVM.  
310 More specifically, scenario T2 reduces by 9.7% and scenario T3 reduces by 5.2% compared  
311 with scenario T1. Due to the stop in front of the intersection on red light, it also takes much  
312 more time to discharge the ten vehicles.

313 In the two scenarios of the optimal control, the model with terminal speed and acceleration  
314 penalty consumes 1.7% more fuel, as the vehicles need to accelerate more. Moreover, it also  
315 needs less green time to discharge the vehicles. The detailed data can be seen in Table 3. It  
316 takes them 20.2s and 18s in the green time window to pass in scenario T2 and scenario T3,  
317 respectively. This means that scenario T3 can let one more vehicle pass in the same signal  
318 settings. Thus, scenario T3 increases the capacity by 11.1% compared with scenario T2 and  
319 by 44.4% compared with scenario T1.

320 Figure 6 shows the detailed position and speed trajectory for every vehicle in the three  
321 scenarios. It can be seen that vehicles in both scenarios T2 and T3 can pass the intersection  
322 without stopping due to the guidance of the first vehicle. They also have a much higher final  
323 speed than vehicles in scenario T1, in which vehicles have to accelerate from a complete stop.



(a)



(b)

Figure 5: Accumulative fuel consumption (a) when all the vehicles arrive at the stop line; (b) when all the vehicles arrive at the extended distance.

Table 3: Simulation results with different terminal costs

Scenario	Terminal position	Total fuel consumption (mL)	Used green time (s)	Total travel time (s)
scenario T1	stop line	767.1	25.8	527.7
	extended	1136.4		716.05
scenario T2	stop line	693.0 (-9.7%)	20.2 (-21.7%)	507.0 (-3.9%)
	extended	996.8 (-12.3%)		688.9 (-3.8%)
scenario T3	stop line	726.9 (-5.2%)	18.0 (-30.2%)	491.2 (-6.9%)
	extended	973.2 (-14.4%)		670.1 (-6.4%)

324 The speed of the first vehicle in scenario T2 is always decreasing while that in scenario T3  
325 decreases first and then increases to the maximum speed, which is the desired final speed.  
326 This also explains why scenario T3 uses more fuel than scenario T2. It is consistent with  
327 figure 5a. The total fuel consumption of scenario T2 and T3 are almost identical in the first  
328 35s. Because of the high terminal speed cost in the scenario T3, the vehicles consume much

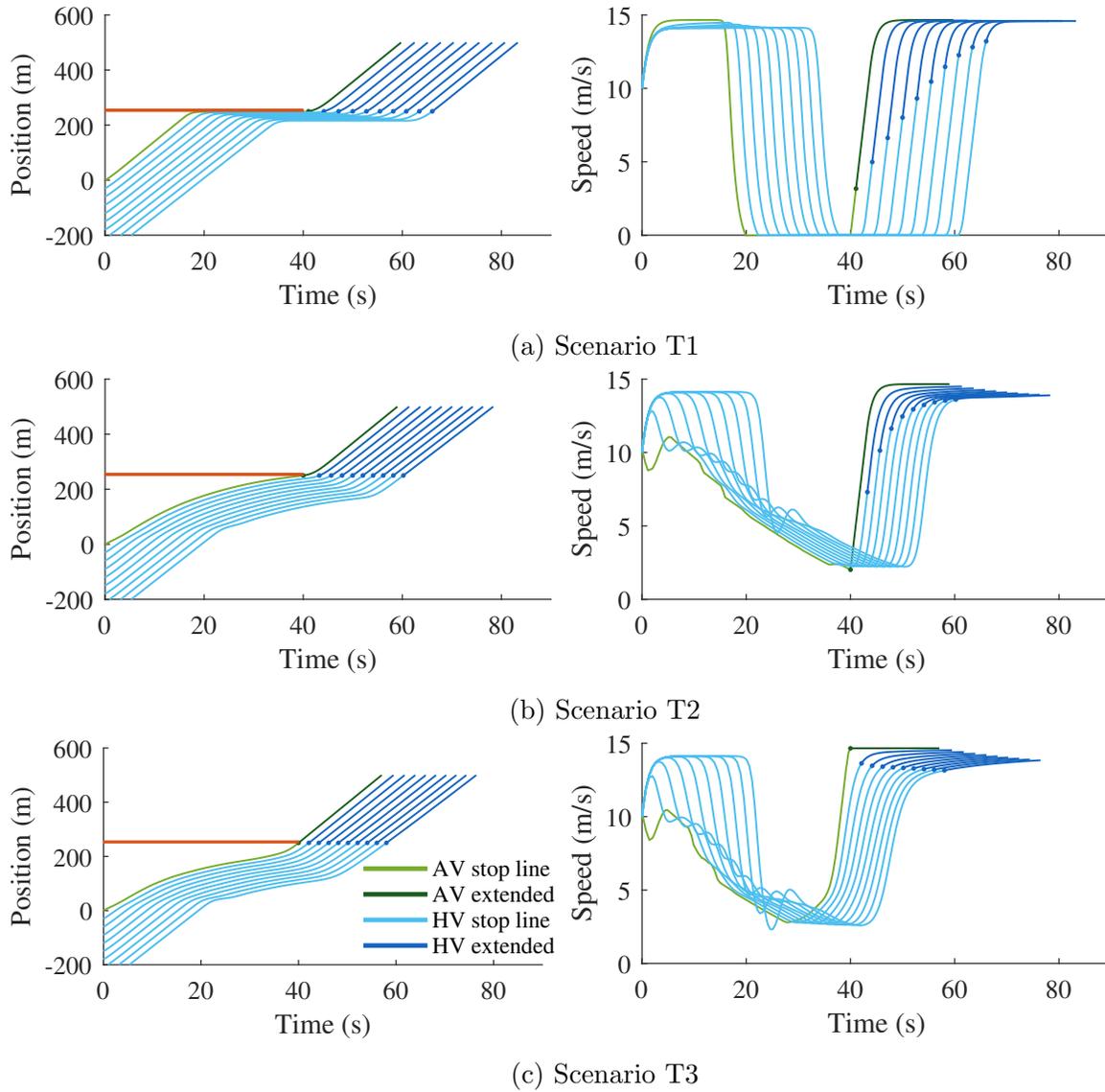


Figure 6: State trajectories of all vehicles with different terminal costs under three scenarios

329 more fuel to accelerate.

330 The terminal speed of vehicles in scenario T3 is much higher than that in scenario T1  
 331 and T2 which is the main reason that it consumes more fuel than scenario T2. This also  
 332 indicates that the vehicles in the Scenario T3 will consume much less fuel in the future. To  
 333 better understand the impact of different terminal costs, we let the vehicles keep running for  
 334 another 250m and achieve similar terminal speed. The vehicles in scenario T1 accelerate to  
 335 maximum speed quickly, but only the first vehicle in scenario T2 and T3 can achieve the  
 336 maximum speed, the following vehicles have slightly slower speed. The scenario T3 consumes  
 337 the least fuel and has the least total travel time as shown in Fig. 5b which mainly benefit  
 338 from the high terminal speed at the stop line. Thus, we conclude that the proposed terminal  
 339 cost function is a good choice for eco-driving in terms of the local benefit and future benefit.

340 *4.2. Properties of the running cost*

341 A major feature in the proposed model is that the leading AVs consider the benefits of  
 342 both themselves and the following vehicles, but the impact of this type of cooperation is  
 343 still not clear. Three typical cases in the mixed traffic flow are considered in the following  
 344 simulation studies. Only 4 vehicles will be considered in the simulations, and the platoon  
 345 setting in each case is shown in Figure 7. To facilitate the following discussion, two major  
 346 time points are defined. Let  $t_1$  denote the time when the first vehicle arrives at the stop line  
 347 and  $t_2$  denote the time when the 4th vehicle arrives at the stop line. In this section,  $t_1$  is the  
 348 start time of green light and also the time when the first AV passes the stop line, which is  
 349 40 s. Two measurements are considered here: (i) The accumulated fuel consumption during  
 350 0s and  $t_1$ ; (ii) The accumulated fuel consumption during 0s and  $t_2$  on the studied link. Let  
 351  $M_1$  and  $M_2$  denote these two measurements, respectively.

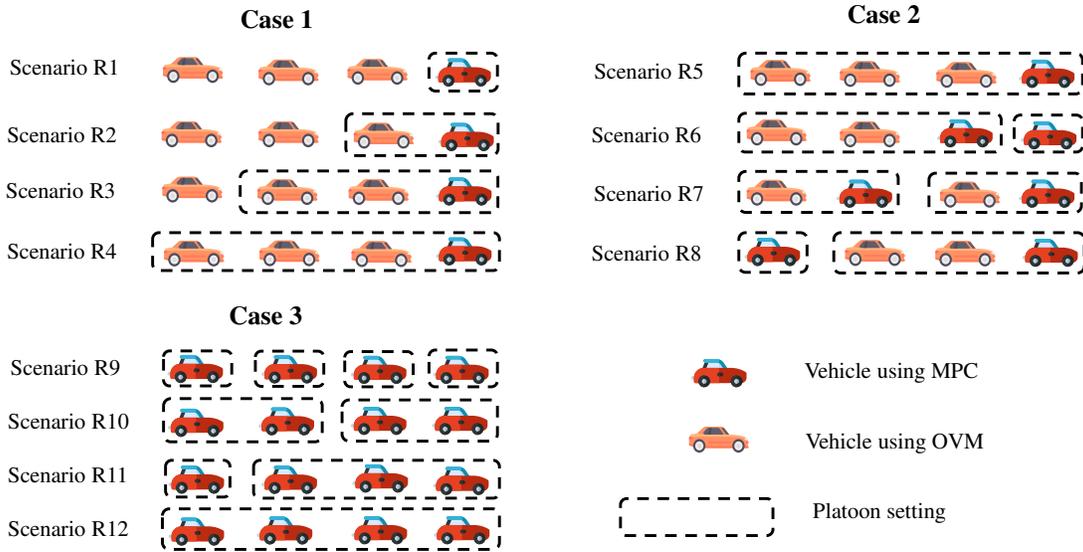


Figure 7: Platoon settings for running cost simulations

352 *4.2.1. Case 1: an AV is followed by HVs*

353 When an AV is followed by several HVs, the question is whether the AV should consider  
 354 the movements of the following vehicles, and what benefits this cooperation can bring. To  
 355 this end, four scenarios are considered in this case. In the simulations, the first vehicle is an  
 356 AV, and the following three vehicles are HVs.

- 357 • Scenario R1: The running cost of the host vehicle is its own fuel consumption.
- 358 • Scenario R2: The running cost of the host vehicle is the sum of its own and first  
 359 following vehicle's fuel consumption.
- 360 • Scenario R3: The running cost of the host vehicle is the sum of its own and first two  
 361 following vehicles' fuel consumption.  $\square$
- 362 • Scenario R4: The running cost of the host vehicle is the sum of its own and all three  
 363 following vehicles' fuel consumption.

364 The fuel consumption for each scenario is shown in Table 4, and the state trajectories are  
 365 shown in Figure 8. In Table 4, the data are organised in the form of “ $M_1/M_2$ ” in each cell.  
 366 The bold items mean they come from AVs, and the same style will be applied in the ensuing  
 367 paper. Please note that in this case, the optimisation is only performed during 0s and 40s  
 368 and in the remaining period vehicles are driven by the OVM. It can be seen that the more  
 369 HVs are considered in the platoon, the less total fuel consumption with  $M_1$  results. The  
 370 reduction is as high as 7.3% in scenario R4 where there are three following vehicles in the  
 371 platoon. At the same time, the first vehicle consumes more fuel than in the scenarios where  
 372 there are fewer vehicles in the platoon. This is due to the fact that the AV has to modify  
 373 its trajectory to change the following vehicles’ behaviour. This can also be seen in Figure  
 374 8. As the leading vehicle sacrifices some of its energy in order to “control” the following  
 375 vehicles, some kinds of rewards may need to be introduced to incentivise the energy-efficient  
 376 behaviour, for example, providing them vouchers for cinema, social events and restaurant  
 377 visits (Schall and Mohnen, 2017).

378 If we consider the movement after 40s, we can see that when the AV cooperates with  
 379 the following vehicles, the following vehicles consume more fuel after 40s until all of them  
 380 have passed the stop line, than the scenario without cooperation. This is mainly because of  
 381 the higher acceleration calculated by the OVM after 40s. With more vehicles joining the  
 382 platoon, the saving of fuel during 0s and 40s is not sufficient to offset the increase of fuel  
 383 consumption after 40s. Actually, in a multi-intersection environment, the movement after  
 384 40s will be optimised in the next intersection. This can be seen by simply assuming that  
 385 the stop line of the upstream intersection is located at 0 m and the green light starts at 0s.  
 386 The presented results apply only to one case with the specified simulation setting. More  
 387 general simulations with various travel times are needed. Furthermore, when more vehicles  
 388 are considered in the platoon, the speed oscillations of the following vehicles are suppressed  
 389 significantly. This will contribute to better driving comfort for the following vehicles. Even  
 390 though some following vehicles are not considered in the platoon, their behaviour is also  
 391 influenced by the preceding vehicle, and their fuel consumption is reduced significantly. For  
 392 example, the fuel consumption of the 4th vehicle in scenario R3 is 10.0% less than that in  
 393 scenario R1 with  $M_1$ . This was also found by Treiber and Kesting (2014) and Wan et al.  
 394 (2016).

Table 4: Fuel consumption of different scenarios in case 1

Scenario	1st vehicle	2nd vehicle	3rd vehicle	4th vehicle	Total (mL)
scenario R1	<b>48.1 / 48.1</b>	53.1 / 57.3	55.0 / 62.9	56.2 / 67.2	212.4 / 235.5
scenario R2	<b>48.8 / 48.8</b>	50.5 / 56.4	52.1 / 61.3	53.4 / 65.2	204.8 / 231.7
scenario R3	<b>50.7 / 50.7</b>	49.3 / 56.9	49.2 / 60.9	50.6 / 64.4	199.8 / 232.9
scenario R4	<b>55.9 / 55.9</b>	49.0 / 57.4	45.9 / 60.7	46.0 / 64.1	196.7 / 238.1

#### 395 4.2.2. Case 2: an AV is followed by mixed AVs and HVs

396 When an AV is followed by mixed AVs and HVs, the question is whether the subsequent  
 397 AVs need to activate the eco-driving function or just follow the preceding vehicle. In the  
 398 simulations, the first vehicle is an AV in all scenarios. The second or third or fourth vehicle  
 399 is another AV in scenario R6, R7, and R8, respectively. All other vehicles are HVs and their

400 movements are according to the OVM. The last three vehicles in scenario R5 can either be  
401 automated or not, as their eco-driving functions are not activated and hence they behave the  
402 same as HVs.

- 403 • Scenario R5: There is only one platoon. The running cost is the sum of fuel consumption  
404 of four vehicles.
- 405 • Scenario R6: There are two platoons: the first vehicle and the last three vehicles. The  
406 running cost for the first platoon is the fuel consumption of the first vehicle, while the  
407 running cost for the second platoon is the sum of the fuel consumption of the last three  
408 vehicles.
- 409 • Scenario R7: There are two platoons: the first two vehicles and the last two vehicles.  
410 The running cost for the first platoon is the sum of the fuel consumption of the first two  
411 vehicles. The running cost for the second platoon is the sum of the fuel consumption  
412 of the last two vehicles.
- 413 • Scenario R8: There are two platoons: the first three vehicles and the last vehicle. The  
414 running cost for the first platoon is the sum of the fuel consumption of the first three  
415 vehicles. The running cost for the second platoon is the sum of the fuel consumption  
416 of the last vehicle.

417 The fuel consumption for every scenario in the simulations is shown in Table 5, and the  
418 state trajectories are shown in Figure 9. Comparing scenarios R6, R7, R8 to scenario R5,  
419 we observe that the activation of the eco-driving function in the following vehicles helps to  
420 reduce the total fuel consumption with both  $M_1$  and  $M_2$ . This is mainly due to the reduction  
421 of their own fuel consumption, which is ranging from 15.4% to 24.6% with  $M_1$  and from  
422 13.1% to 18.9% with  $M_2$ . It also helps reduce the fuel consumption of the first AV due  
423 to fewer vehicles in its platoon as discussed previously. Eventually, with another AV, the  
424 reduction of fuel consumption ranges from 3.3% to 7.4% with  $M_1$  and from 7.2% to 9.4%  
425 with  $M_2$ . This is different from the result of [Stebbins et al. \(2017\)](#) where giving speed advice  
426 to the following vehicles rarely makes a difference. This difference is mainly because in their  
427 approach only the leading vehicle can achieve the target position and speed. However, in  
428 the proposed method, the following AVs can also achieve the desired state, which can reduce  
429 the fuel consumption and travel time of the whole traffic. In Figure 9, the trajectories of  
430 the following AVs by the MPC show an obvious fallback behaviour and keep a larger gap  
431 than that in the OVM. In the OVM, the vehicle attempts to accelerate as soon as possible to  
432 achieve the optimal speed. In contrast, in the MPC method, the vehicle acts more rationally  
433 by considering the information of signal timing and state of the preceding vehicles. So, it  
434 reduces even further the fuel consumption to provide speed advice to the following AVs in  
435 the mixed AVs and HVs environment.

#### 436 4.2.3. Case 3: an AV is followed by other AVs

437 When an AV is followed by other AVs, the question is whether the leading AV needs to  
438 consider the movements of the following AVs. If it does, what kind of benefits arise from  
439 this cooperation? In the simulations, all the vehicles are AVs and arrive at the stop line with  
440 maximum speed and zero acceleration with a fixed time headway 2s.

Table 5: Fuel consumption of different scenarios in case 2

Scenario	1st vehicle	2nd vehicle	3rd vehicle	4th vehicle	Total (mL)
scenario R5	<b>55.9 / 55.9</b>	49.0 / 57.4	45.9 / 60.7	46.0 / 64.1	196.7 / 238.1
scenario R6	<b>48.1 / 48.1</b>	<b>41.5 / 49.9</b>	48.9 / 59.0	51.7 / 64.0	190.2 / 221.0
scenario R7	<b>48.8 / 48.8</b>	50.5 / 56.4	<b>37.5 / 51.4</b>	45.4 / 59.0	182.1 / 215.6
scenario R8	<b>50.7 / 50.7</b>	49.3 / 56.9	49.2 / 60.9	<b>34.7 / 52.0</b>	183.9 / 220.5

- 441 • Scenario R9: Each AV optimises its trajectory separately and minimises its own fuel  
442 consumption.
- 443 • Scenario R10: The four AVs act as two platoons. The running cost for the first platoon  
444 is the total fuel consumption of the first two vehicles and the running cost for the second  
445 platoon is the total fuel consumption of the last two vehicles.
- 446 • Scenario R11: The four AVs act as two platoons. The running cost for the first platoon  
447 is the total fuel consumption of the first three vehicles, and the running cost for the  
448 second platoon is the fuel consumption of the last vehicle.
- 449 • Scenario R12: The four AVs act as one platoon. They minimise the sum of their fuel  
450 consumption.

451 The fuel consumption of every scenario in the simulation is shown in Table 6 and the state  
452 trajectories are shown in Figure 10. In the four scenarios, the results change very slightly  
453 and fall within 2% for every vehicle and 1% for the total in most cases. Nevertheless, the  
454 resulting trajectories may differ slightly. This outcome suggests that the cooperation among  
455 AVs does not make any obvious difference in the fuel consumption and the travel time. This  
456 conclusion is only valid for the current simulation setting and more simulation scenarios with  
457 different travel time and speed are needed, which will be shown in the following section.

Table 6: Fuel consumption of different scenarios in case 3

Scenario	1st vehicle	2nd vehicle	3rd vehicle	4th vehicle	Total (mL)
scenario R9	<b>48.1 / 48.1</b>	<b>41.8 / 49.7</b>	<b>38.7 / 50.9</b>	<b>36.8 / 52.1</b>	165.4 / 200.7
scenario R10	<b>48.6 / 48.6</b>	<b>42.5 / 49.6</b>	<b>38.9 / 51.0</b>	<b>36.6 / 52.4</b>	166.6 / 201.6
scenario R11	<b>49.0 / 49.0</b>	<b>42.7 / 50.2</b>	<b>39.1 / 51.4</b>	<b>36.2 / 52.1</b>	167.0 / 202.6
scenario R12	<b>48.6 / 48.6</b>	<b>42.1 / 49.7</b>	<b>38.8 / 51.0</b>	<b>36.5 / 52.5</b>	165.9 / 201.8

### 458 4.3. Simulations with different penetrations of AVs

459 In this part, a simulation investigation is presented to show the performance of the pro-  
460 posed method in different penetrations of AVs. Please note that the cooperation of vehicles  
461 in a platoon relies on the vehicles' state information sharing. This can be achieved in the  
462 connected vehicle environment. If the vehicles are not fully connected, we assume that the  
463 AV can still detect the state of the first direct following vehicle via its built-in detectors. So  
464 the platoon size is limited to 2 in that condition. Another scenario without cooperation is  
465 also included for comparison.

- 466 • Scenario P1: All AVs consider their own fuel consumption only. It can also be seen as  
467 setting the maximum platoon size to be 1;
- 468 • Scenario P2: All AVs consider the fuel consumption of themselves and the first directly  
469 following vehicle. This can also be seen as setting the maximum platoon size to be 2;
- 470 • Scenario P5: All AVs consider the fuel consumption of themselves and all the following  
471 vehicles within the limit of maximum platoon size. The maximum platoon size is set  
472 to be 5.

473 When determining the maximum platoon size, we should trade off the calculation burden  
474 and communication reliability in practice. Large platoon size also implies that the AVs need  
475 to sacrifice more and have a higher probability to stop in order to “control” the vehicles far  
476 away. The stopping behaviour will also be discussed later.

477 The simulation of every scenario in every penetration rate lasts for 600s and is repeated  
478 twice. In all simulations, the cycle time is 60s with green time 30s and red time 30s. Traffic  
479 demand is 850 veh/h. The type of vehicle is determined by comparing the penetration rate  
480  $p$  and a new generated random number between 0 and 1 when it enters the road. The time  
481 headway follows a truncated exponential distribution to ensure that no time-headway is less  
482 than 2s. The initial speed follows a normal distribution  $N(10, 1)$  bounded by the speed limits  
483 and ensures that no collision happens at the entrance of the road (Ubiergo and Jin, 2016).

484 The average fuel consumption and travel time produced by the simulations are shown in  
485 Table 7 and Figure 11. Overall, both fuel consumption and travel time decrease with the  
486 increasing penetration of AVs under all scenarios. In any penetration studied, the scenario  
487 with cooperation outperforms or at least equals the scenario without cooperation. In general,  
488 as more vehicles join the cooperation, more benefits are gained in terms of fuel consumption  
489 and travel time. The benefits of cooperation are most evident for lower penetration rates, and  
490 a platoon size of 5 (P5) can reduce the fuel consumption by 22% with only a 60% penetration  
491 rate, which is better than the scenario of P1 with 100% penetration. However, as more AVs  
492 are brought into the system, the additional benefit from cooperation then decreases as there  
493 is not much room for further improvement. This is in line with the previous results from case  
494 3 in section 4.2.3, where the benefit of cooperation for all four AVs was minor.

495 The travel time benefits are less significant than fuel consumption benefits and are not  
496 really present with 20% and 100% penetration of AVs. They then increase in a similar  
497 pattern to the fuel consumption and the effects of cooperation are similar. The reduction of  
498 travel time is mainly caused by the reduction of start-up lost time and queue discharge time  
499 as more vehicles pass on the green light and fewer vehicles stop on the red light thanks to  
500 the cooperation. However, when the penetration is 20%, the AVs are frequently interrupted  
501 by the preceding HVs, but when the penetration becomes 100%, there is no more room to  
502 reduce the travel time. Figure 11 shows that with the increasing penetration of AVs, the  
503 number of outliers in the fuel consumption is greatly reduced. Scenarios P2 and P5 also have  
504 much fewer outliers than scenario P1. This demonstrates that the cooperation can stabilise  
505 the traffic flow. This is also shown in Figure 12. No outliers are detected in the travel time.  
506 When the initial state is fixed, the travel time can only imply the terminal state, but all the  
507 intermediate states affect the fuel consumption. That is why the fuel consumption can show  
508 more information about the vehicles’ movements.

509 The trajectories of vehicles with 20%, 60% and 100% penetration of AVs in three scenarios  
510 are shown in Figure 12. We can see that only optimising AVs themselves is not enough to  
511 achieve a system optimum. Sometimes the selfish behaviour of an AV can even worsen the  
512 traffic. This is especially serious when the penetration is low. When the leading AV attempts  
513 to slow down to save fuel, it causes a shock-wave along the link, which triggers some following  
514 HVs to stop. Even when the penetration of the AVs becomes 100%, sometimes this kind  
515 of selfish deceleration can still occur. In the cooperation scenarios (i.e., P2 and P5), the  
516 vehicles' trajectories are largely smoothed. The negative impact of eco-driving by the AVs  
517 on the following vehicles is also reduced. This is mainly because the fuel consumption of  
518 the following vehicles is directly included in the objective function of the AVs. The leading  
519 vehicles in each platoon also help the following vehicles to reach a high speed when crossing  
520 the stop line and avoid idling on the red light.

521 We also notice that the AV may stop in the middle of the road segment even in the  
522 cooperative scenarios with low probability. There are two main reasons, (1) the planned  
523 travel time is too long and (2) the following vehicles in the same platoon are widely dispersed.  
524 But please note that the stopping behaviour of the AV in cooperation scenarios does not harm  
525 the system. It does not increase the travel time or fuel consumption for the platoon. The  
526 AV never stops close to the stop line and does not block the following vehicles from passing  
527 the stop line. If stopping behaviour is not acceptable, one may add a larger minimum speed  
528 limit constraint on the AVs (Yang et al., 2017), but this may lead to infeasible result when  
529 the planned travel time is larger than the maximum travel time by applying the minimum  
530 speed limit. Then the speed advisory system will fail and the AV has to stop around the  
531 stop line. It results in higher fuel consumption and travel time for all the following vehicles.  
532 Thus, we choose not to include the larger minimum speed limit and allow the seldom stop  
533 behaviour.

## 534 5. Conclusions

535 Providing signal information to the vehicles on signalised urban roads is demonstrated to  
536 be an effective way to reduce the idle time and the fuel consumption. However, many eco-  
537 driving strategies have a negative impact on the efficiency of the intersection, and even cause  
538 a shock-wave in the middle of the road section. In this paper, a distributed and cooperative  
539 eco-driving method has been proposed for platoons to address these issues. The proposed  
540 eco-driving method has been designed for mixed traffic flow on an urban road, which consists  
541 of HVs and various penetrations of AVs. AVs attempt to pass the intersection on the earliest  
542 possible green time with a maximum desired speed and zero acceleration. All these settings  
543 are to maximise the traffic efficiency. In the paper, the jerk has been set as the control  
544 variable in order to increase the driving comfort. In the proposed control method, the fuel  
545 consumption of AVs and some following HVs is minimised over the horizon to achieve the  
546 eco-driving benefit to more vehicles. This cooperation largely smooths out the trajectory and  
547 suppresses any shock-wave. Then a platoon formation method has been proposed to apply  
548 the distributed and cooperative eco-driving strategy to achieve a better performance for the  
549 overall traffic. Three typical cases in mixed traffic have been studied with different platoon  
550 settings. Moreover, different penetrations of AVs have been studied in the simulation to show  
551 that the proposed method can adapt to various mixed traffic conditions.

Table 7: Simulation results and differences in various penetrations of AVs

Penetration	Scenario	Fuel consumption		Travel time	
		Mean (mL)	Diff	Mean (s)	Diff
0.2	P1	55.3	-	37.4	-
	P2	52.7	-4.7%	37.5	0.2%
	P5	48.4	-12.6%	37.3	-0.3%
0.4	P1	50.0	-9.6%	34.1	-8.7%
	P2	47.9	-13.5%	32.6	-12.8%
	P5	46.6	-15.9%	31.7	-15.2%
0.6	P1	47.6	-13.9%	34.1	-8.8%
	P2	45.6	-17.6%	33.0	-11.8%
	P5	43.1	-22.1%	31.6	-15.5%
0.8	P1	46.1	-16.8%	33.1	-11.6%
	P2	44.7	-19.2%	31.1	-16.8%
	P5	43.2	-21.9%	31.2	-16.5%
1	P1	43.5	-21.3%	31.1	-16.8%
	P2	43.6	-21.1%	31.2	-16.5%
	P5	42.5	-23.2%	31.2	-16.5%

From the analysis above, we can draw the following conclusions:

1. AVs can reduce their own fuel consumption and travel times when approaching a signalised intersection if the signal timing information is given.
2. When the penetration level is from low to moderate, the cooperation between AVs and HVs is seen to be beneficial in both fuel consumption and travel time.
3. However, this system level of cooperation requires a sacrifice from the lead AV which may be controversial to accept.
4. The level of sacrifice increases with the platoon size. As vehicles are added to the platoon of one AV then the leading vehicle has to overcompensate to affect the third and subsequent vehicle trajectories.
5. Even when the HVs are not included in the platoon, they still benefit from the preceding AVs.
6. It reduces the fuel consumption even further to provide speed advice to the following AVs in the mixed AVs and HVs compared with only controlling the leading vehicle on a green phase.
7. Larger platoon size helps to achieve a stronger reduction in fuel consumption and stabilise traffic flow.
8. The benefits of cooperation mean that the system can reach the same levels of benefit with 60% penetration rate as for 100% penetration without cooperation, which has implications for the transition towards a full penetration.
9. As the penetration rate reaches 100%, then the performance improvement resulting from cooperation is trivial and the sacrifice problem disappears.

574 These last two points taken together suggest that implementation the driving cooperation  
575 should vary over the implementation phase and that some higher levels of cooperation whilst  
576 desirable should be regulated or compensated with a promise to remove this obligation as  
577 the penetration rates increase.

578 In this paper, we assume that the future signal timing information is available to the  
579 AVs. This is fine for fixed timing control and adaptive signal control strategies that update  
580 signals every cycle (e.g. TUC (Diakaki et al., 2002, 2003)), but may not be true for other  
581 adaptive signal control systems, like SCOOT, where there is only very limited time for the  
582 AVs to respond and may reduce the performance of the proposed method. There are two  
583 solutions: (1) Use the previous signal timing as the estimation when it is not available. When  
584 the signal timing is available at some time steps ahead, it may use the current instead. As  
585 the change between two cycles is unlikely to be too strong, e.g. it is limited to +/- 4 seconds  
586 in SCOOT, the performance impact may be suppressed, but would not vanish. (2) Develop a  
587 new algorithm within the SCOOT and SCATS to consider the AVs. In the current adaptive  
588 control schemes, the information is still mainly obtained from detectors like loop detectors.  
589 The information from AVs or CVs is not considered. So we think it is an interesting topic to  
590 develop new intersection control algorithms to take the advantage of new information from  
591 AVs and CVs. This is also our ongoing research work.

592 In the current work, the signal timing is assumed to be given. In the next step, it will  
593 achieve a better performance gain to optimise the signal timing and trajectory simultaneously  
594 either for the local intersection or traffic network.

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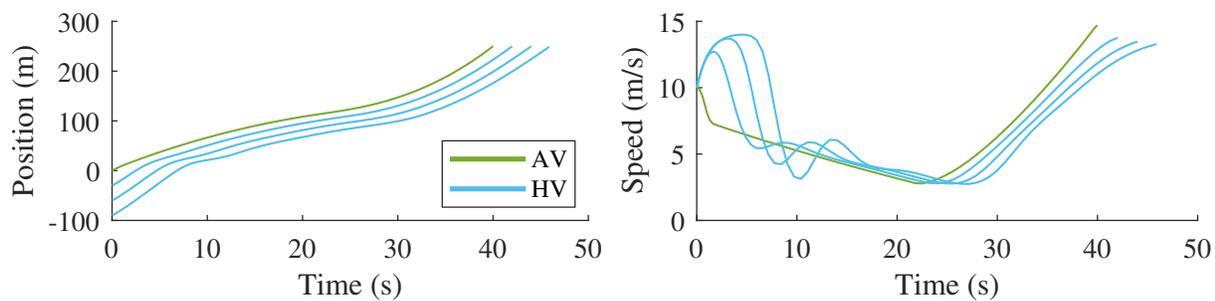
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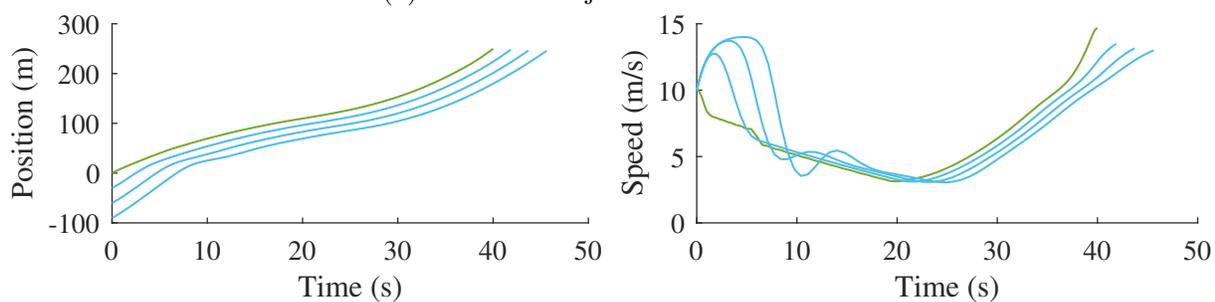
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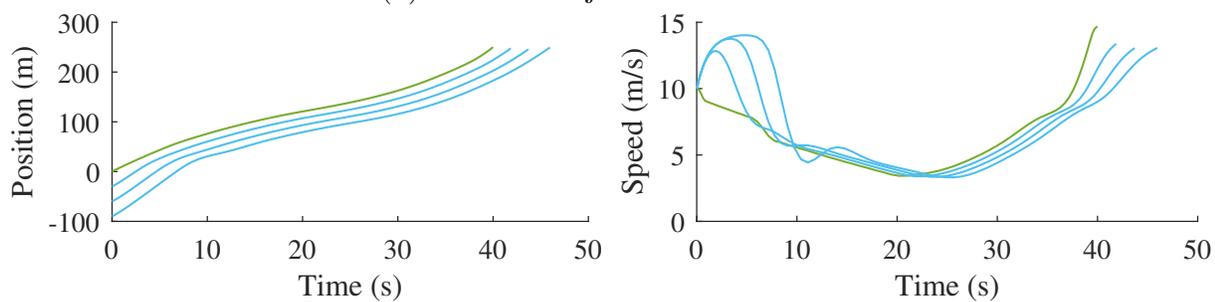
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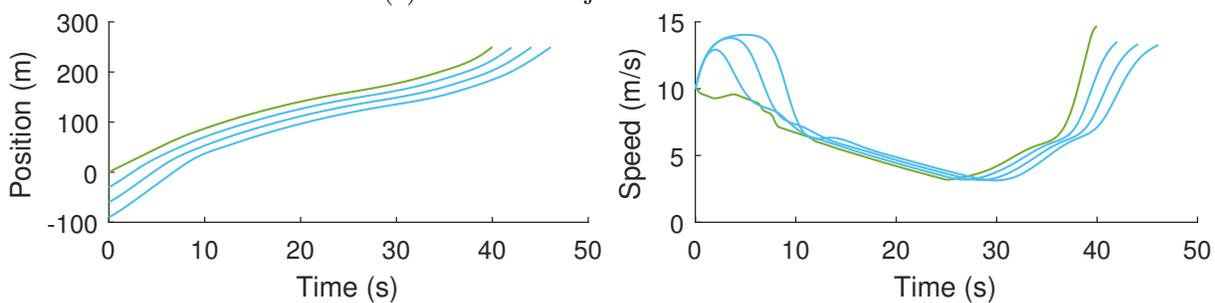
(a) Vehicles' trajectories in scenario R1



(b) Vehicles' trajectories in scenario R2

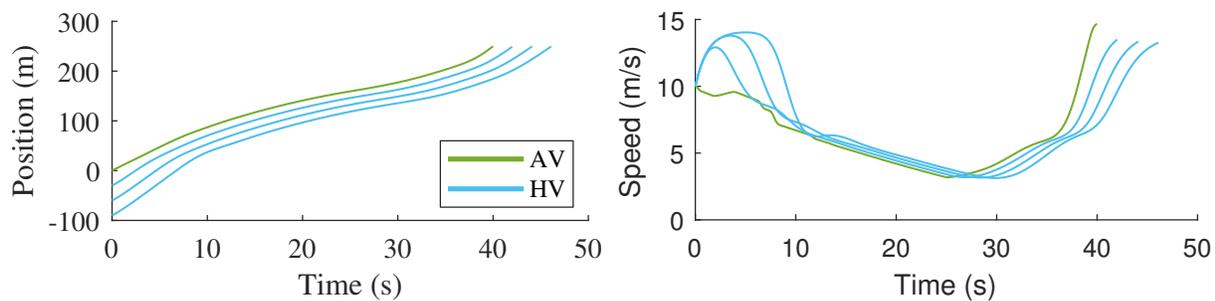


(c) Vehicles' trajectories in scenario R3

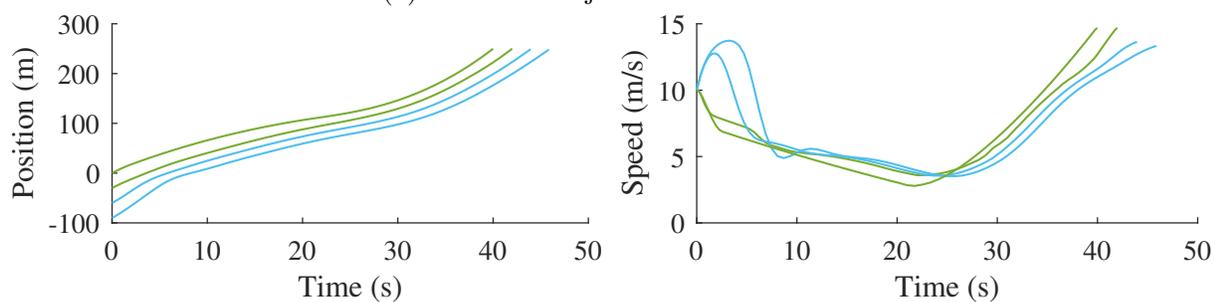


(d) Vehicles' trajectories in scenario R4

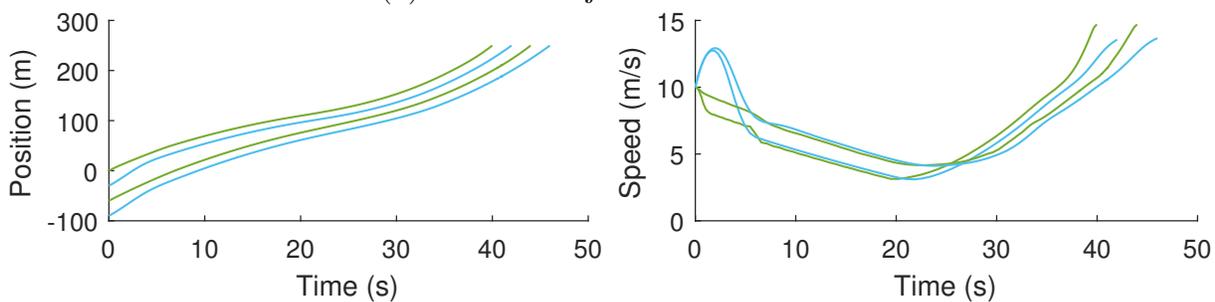
Figure 8: State trajectories of all vehicles under four scenarios in case 1



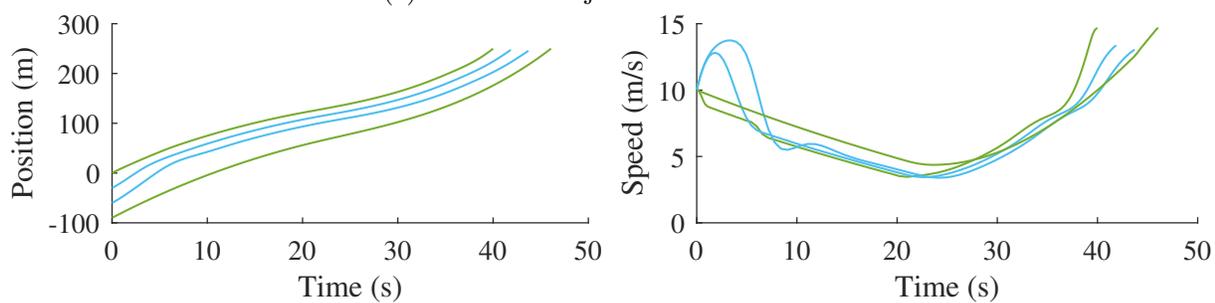
(a) Vehicles' trajectories in scenario R5



(b) Vehicles' trajectories in scenario R6

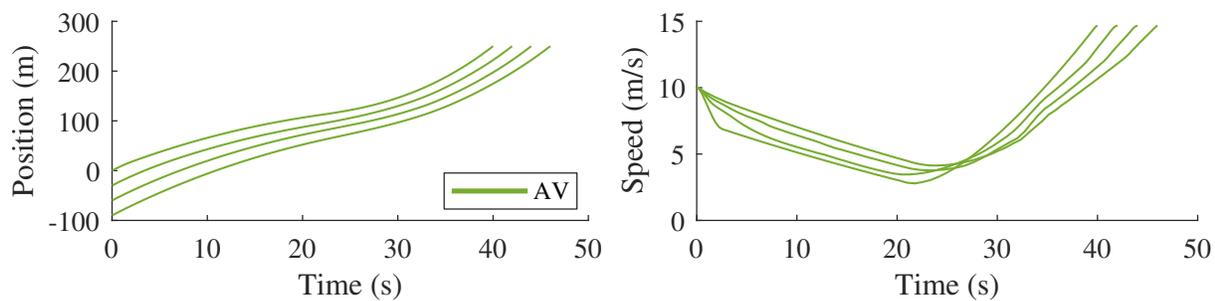


(c) Vehicles' trajectories in scenario R7

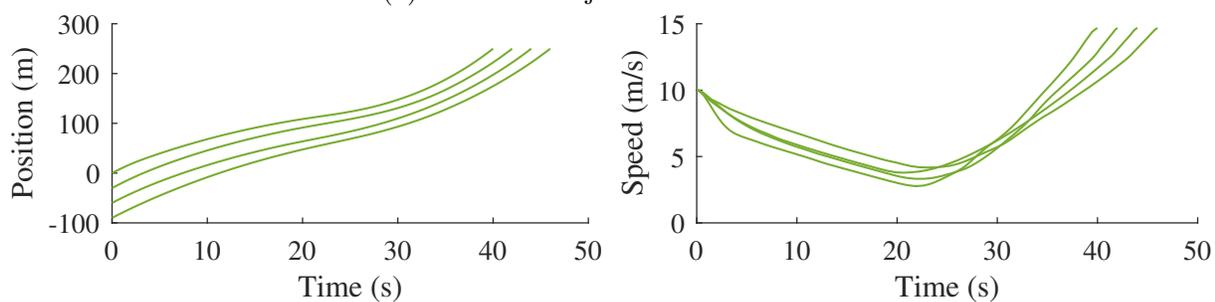


(d) Vehicles' trajectories in scenario R8

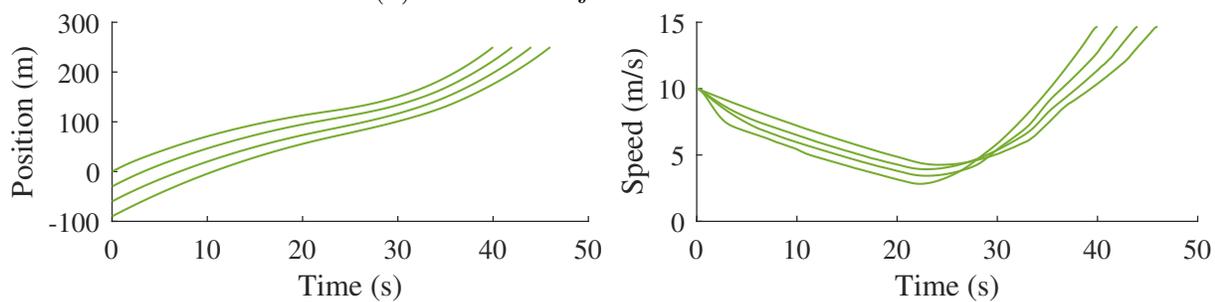
Figure 9: State trajectories of all vehicles under four scenarios in case 2



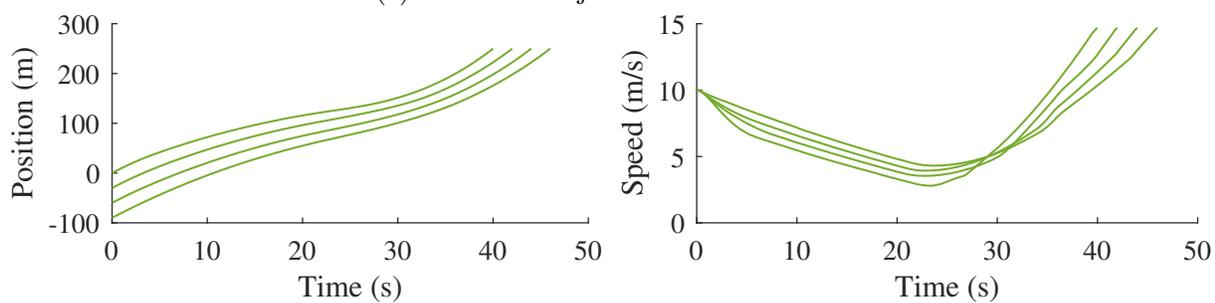
(a) Vehicles' trajectories in scenario R9



(b) Vehicles' trajectories in scenario R10

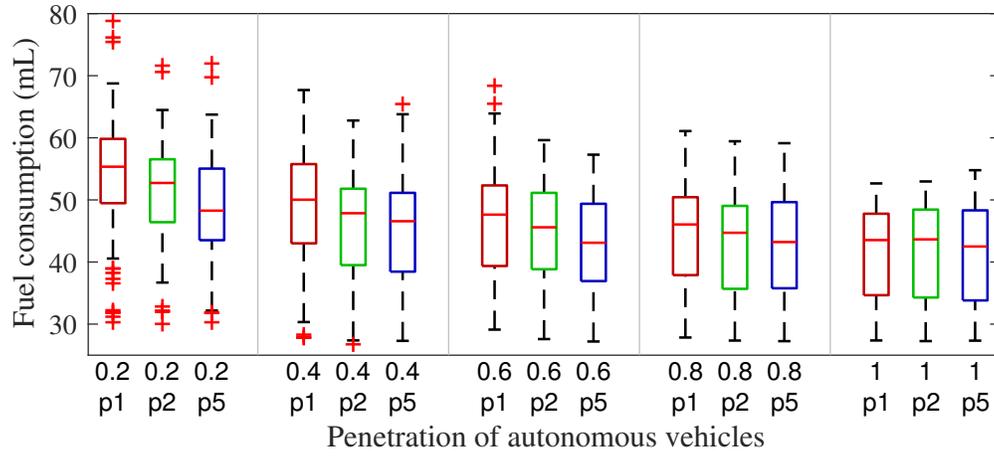


(c) Vehicles' trajectories in scenario R11

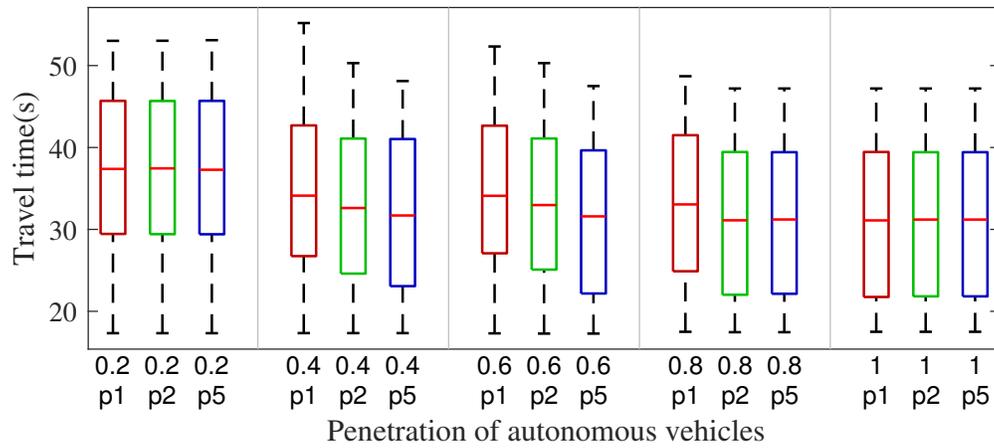


(d) Vehicles' trajectories in scenario R12

Figure 10: State trajectories of all vehicles under four scenarios in case 3

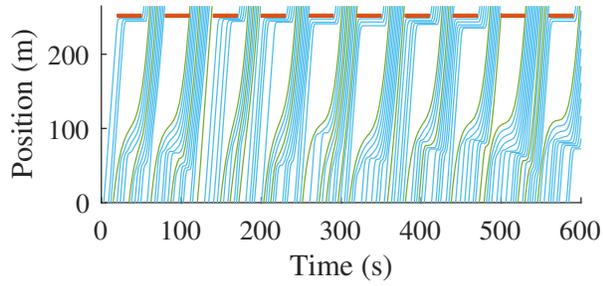


(a)

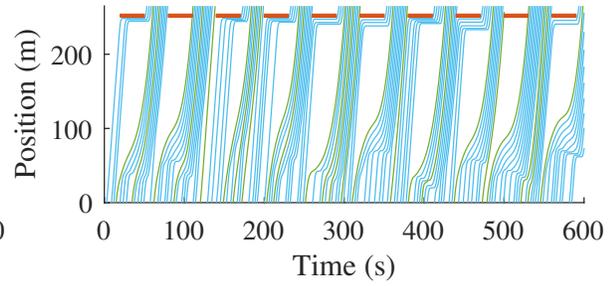


(b)

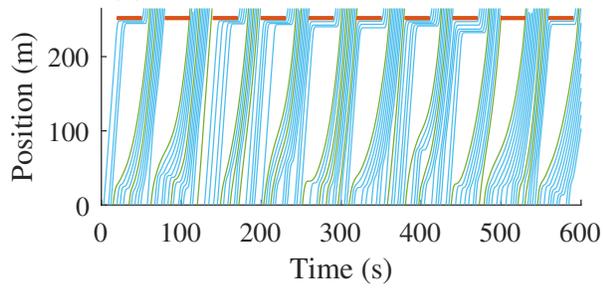
Figure 11: Simulation results in different penetrations of AVs. (a) fuel consumption, (b) travel time



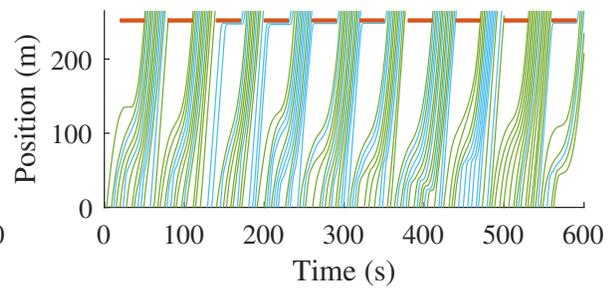
(a) p1 with penetration of 20%



(b) p2 with penetration of 20%



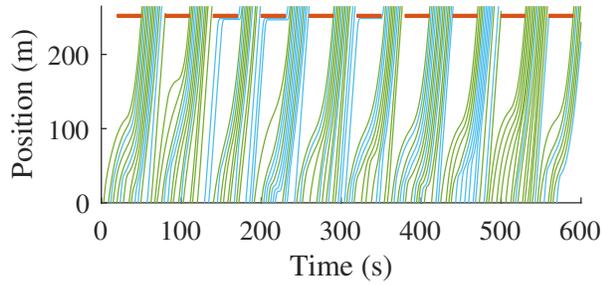
(c) p5 with penetration of 20%



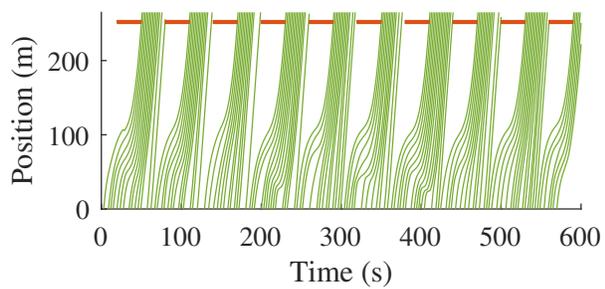
(d) p1 with penetration of 60%



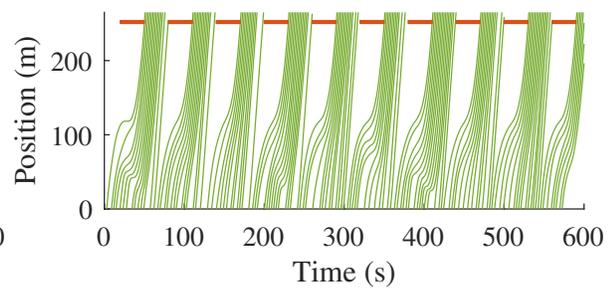
(e) p2 with penetration of 60%



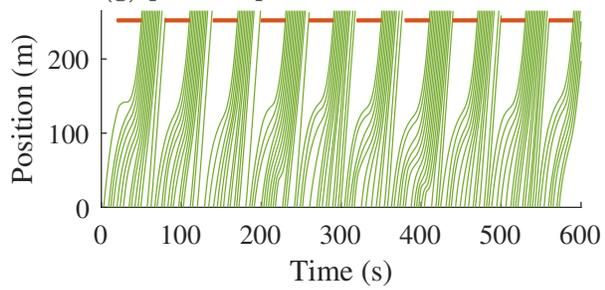
(f) p5 with penetration of 60%



(g) p1 with penetration of 100%



(h) p2 with penetration of 100%



(i) p5 with penetration of 100%

Figure 12: Some examples of trajectories in different penetrations of AVs