Using perceptual cues for brake response to a lead vehicle:

–Comparing threshold and accumulator models of visual looming

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Abstract: Previous studies have shown the effect of a lead vehicle's speed, deceleration rate and headway distance on drivers' brake response times. However, how drivers perceive this information and use it to determine when to apply braking is still not quite clear. To better understand the underlying mechanisms, a driving simulator experiment was performed where each participant experienced nine deceleration scenarios. Previously reported effects of the lead vehicle's speed, deceleration rate and headway distance on brake response time were firstly verified in this paper, using a multilevel model. Then, as an alternative to measures of speed, deceleration rate and distance, two visual looming-based metrics (angular expansion rate $\dot{\theta}$ of the lead vehicle on the driver's retina, and inverse tau τ^{-1}, the ratio between $\dot{\theta}$ and the optical size θ), considered to be more in line with typical human psycho-perceptual responses, were adopted to quantify situation urgency. These metrics were used in two previously proposed mechanistic models predicting brake onset: either when looming surpasses a threshold, or when the accumulated evidence (looming and other cues) reaches a threshold. Results showed that the looming threshold model did not capture the distribution of brake response time. However, regardless of looming metric, the accumulator models fitted the distribution of brake response times better than the pure threshold models. Accumulator models, including brake lights, provided a better model fit than looming-only versions. For all versions of the mechanistic models, models using τ^{-1} as the measure of looming fitted better than those using $\dot{\theta}$, indicating that the visual cues drivers used during rear-end collision avoidance may be more close to τ^{-1}.

Keywords: rear-end collision, brake response time, multilevel model, visual looming, threshold model, accumulator model

1. Introduction

According to statistics provided by the World Health Organization, about 1.25 million people die each year as a result of road traffic crashes (WHO, 2015). Among all the collisions types, rear-end crashes account for about 20% of all crashes in Shanghai, China (Wang et al., 2016) and 32% approximately in the US (National Highway Traffic Safety Administration, 2014).

To avoid rear-end collisions, the initiation of a brake response, when required, is of great importance. Total brake response time is defined as the time from stimulus appearance to the reaction of the driver, plus the movement time to hit the brake pedal (Schweitzer et al., 1995). It is a measurement which has been widely used and analysed in crash-related investigations. Previous studies have reported that brake response time values vary in a large range under different conditions (Johansson et al., 1971; Sohn and Stepleman, 1998; Green, 2000). Summala (2010) suggested that urgency of a situation was one of the factors which may affect drivers' brake response time. Situation urgency can be described by the behaviour of the lead vehicle (e.g. lead vehicle's deceleration rate) and the driving state when the lead vehicle's brake onset (e.g. headway distance and time to collision). Liebermann et al (1995), Schweitzer et al. (1995) and Summala et al. (1998) tested the effects of speed and following distance on reaction time, finding that drivers reacted faster at a shorter following distance, whereas the driving speed did not show any significant effects both in Liebermann et al (1995) and Schweitzer et al. (1995) studies. Hulst (1999) tested the effects of a lead vehicle's deceleration rate on response time, and showed that this was longer for slow deceleration rates. The combined effect of a lead vehicle's deceleration rate and driving distance on response time has also been studied by Lee et al. (2002) and Wang et al. (2016), who showed that drivers responded faster when the lead vehicle's deceleration increased or when the initial headway decreased. Li et al (2016) tested the effect of driving speed, headway distance, gender and cell phone use on drivers' brake response time and showed that drivers reacted faster with faster speed and reduced headway distance. Therefore, although the overall behavioural pattern emerging from previous studies suggest that brake response time decreases with increasing situation urgency, the effect of a lead vehicle's speed, lead vehicle deceleration, and initial headway to the lead vehicle, on drivers' brake response times has not yet been considered. Thus, the first goal of this paper was to test the overall effect of the above mentioned variables on drivers' brake response time.

According to previous studies, during a rear-end collision avoidance process, drivers control braking on the basis of their assessment of the situation urgency. However, the extent to which drivers can perceive the lead vehicle's distance, speed and deceleration information is not clear. Since brake lights do not indicate how hard the lead vehicle is braking, drivers have to rely on other visual information to determine how rapidly they are closing in on the lead vehicle (Lee, 1976). One much-studied form of such information is visual looming, which is produced by an object moving towards the subject, and may indicate an impending collision (Terry et al., 2008). The angular projection of an object on the subject's retina is defined as θ, with $\dot{\theta}$ being the angular expansion rate (Lee, 1976). Liebermann et al. (1995) pointed out that changes in angular velocity during optical expansion of the lead vehicle may be used as a cue to modulate braking movement, and Yilmaz and Warren (1995) provided empirical support for this idea.

Previous authors have often assumed that there is a threshold at which drivers realize that they are approaching the lead vehicle in such a way that they must take some action to avoid a rear-end collision (Lamble et al., 1999; Muttart, 2005; Olson et al., 2010; Maddox and Kiefer, 2012). One version of this threshold, which has often been discussed in the literature, is looming detection threshold, which is the minimum threshold at which drivers start perceiving the threat, and is generally assessed using $\dot{\theta}$. These threshold models assume that drivers respond within 0.75-2 s after reaching the detection threshold (Plotkin, 1976; Mortimer, 1990). Maddox and Kiefer (2012) assumed three candidate values of perception-reaction time, and examined real-world accident data to obtain an estimate of the detection threshold, but found that the data could be described by a range of possible combinations of detection thresholds and reaction times. Another type of threshold model just assumes a single response threshold, at which drivers start directly responding to the threat. There have been a number of studies investigating response threshold models (Lee, 1976; Kiefer et al., 2003; Flach et al., 2004) but all assuming slightly different looming cues. Lee (1976) suggested that braking performance might be contingent on the optical parameter τ and its derivative $\dot{\tau}$. τ is the ratio of θ and $\dot{\theta}$. τ has units of time and is an approximation of time-to-contact. Drivers are assumed to start their braking actions when τ reaches a certain margin value τ_m. The inverse of τ, τ^{-1} has also been considered as a cue in near-accident control. Kiefer et al. (2003) developed a model which is based on a τ^{-1} threshold that decreases linearly with own driving speed. Kondoh et al. (2014) demonstrated the tight connection between drivers' perception of risk and τ^{-1}, following a driving simulator experiment.

However, it seems reasonable to assume that in real traffic, drivers' response behaviour is not only based on responding to perceptual quantities such as τ^{-1}. The stimulus in the threshold models mentioned above has been limited to visual looming, while various other stimuli were ignored (e.g., brake light onset). An alternative to the threshold model, which has been proposed by Markkula and colleagues (Markkula et al., 2014; Markkula et al., 2016) is the accumulator model, suggesting that visual looming might be used as one source of evidence for the need to brake, combined with other sources of evidence in noisy accumulation (i.e., integration), to a decision threshold at which brake onset occurs. Markkula et al. (2016) showed that qualitative patterns of brake timing in naturalistic near-crashes and crashes aligned better with this type of account than with a threshold-based account.

Accumulator-type models have been studied extensively in perceptual decision tasks in the laboratory, often using Ratcliff's (1978) drift diffusion model. The underlying assumption is that the brain extracts, per time unit, a piece of evidence from the stimulus (drift) which is disturbed by noise (diffusion) and subsequently accumulates these over time, until a decision criterion is hit, at which point a response is initiated (Ratcliff and Smith, 2004; Ratcliff and Van Dongen, 2011; Bitzer et al., 2014). These models have been applied in a variety of domains such as psychology and neuroscience (Gold & Shadlen, 2001; Ratcliff et al., 2003; Schall et al., 2011, Roe, Busemeyer & Townsend, 2001; Krajbich & Rangel, 2011). Ratcliff and Strayer (2014), successfully fitted this type of model to a distribution of reaction times to the lead vehicle's brake lights, in a simulated driving task, but did not consider the possible influence of situation urgency, e.g., in terms of visual looming on response.

Although the role of visual looming in driver brake action has been investigated in previous studies, the threshold and accumulator types of model have not been stringently compared, and especially not in their ability to model distributions of brake response times. Therefore, both types of model, referred to here as mechanistic models (since they propose specific mechanisms for what determines brake onset), were tested here, with the aim of investigating which of the two hypothesised mechanisms better explains human brake timing distributions. For different versions of the visual looming-based mechanistic models, perceptual cues were quantified both as $\dot{\theta}$ and τ^{-1}; the comparison of these two cues was another aim of this study. Finally, as the multilevel model is a linear model, based on values such as speed, deceleration and distance. A comparison between multilevel model fitting and accumulator model fitting was conducted, to see whether the accumulator model can be an alternative to the regression analysis, when considering the effects of scenario urgency on drivers' response time.

2. Methodology

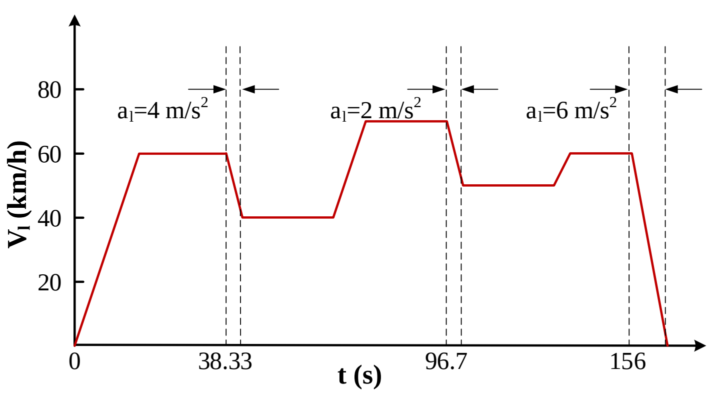
2.1 Equipment

The equipment used in this experiment was the Beijing Jiaotong University driving simulator (as shown in Figure 1). The simulator was produced by Real-time Technologies. Inc in U.S. It is composed of a cabin of a Ford Focus with automatic gearbox, gas/brake pedal and other components, which are in full accordance with the real vehicle. The simulator has a linear motion base, capable of operating with a single degree of freedom (the rotation of pitch). The driving scenarios were designed using SimVista (Real-time Technologies. Inc, U.S) and projected on five screens to realize a 300-degree field of front view, with each of the screens having a resolution of 1400×1050 pixels. The core simulator and visual subsystems operate at a 60 Hz update rate, supporting smooth graphics presentation and rapid system response in complex driving environments. The simulator used in this study was very similar to the one used by Western Transportation Institute (WTI), where both its physical and behavioural fidelity were demonstrated in a summary report by Philips and Morton (2013). In addition, the visual system used in this study was very similar to the simulator used in studies of McGehee et al. (2000) and Hoffman et al. (2002) (details in Kuhl et al., 1995). They compared drivers' braking response between a driving simulator study and on a test track, and no statistically significant difference was found for the reaction time between simulator studies and test track. The relative validity of the driving simulator, which can be used for brake initiation timing problems with reduced risk of harm on participants, can thus be supported.

Fig. 1 Illustration of the driving simulator system.

2.2 Scenario design

In this study, each participant experienced three experimental drives, each on a bidirectional straight rural road with a speed limit of 60 km/h. At the beginning of each drive, the lead vehicle was stationary 1500 m ahead of the start point of the driver, with its brake lights on. When the participant was 50 m behind the lead vehicle, the lead vehicle began to accelerate and then followed one of three predefined speed profiles, shown in Figure 2. In each drive, the lead vehicle reduced its speed (with brake lights on) three times, at one of three deceleration rates: 2 m/s^2, 4 m/s^2 or 6 m/s^2 (the order of deceleration rates in the three experimental drives is shown in Figure 2). The time that the lead vehicle needed to reach the first designated deceleration point was about 40 s. The order of the three drives was counterbalanced among participants. Drivers in this experiment thus experienced 3×3 deceleration scenarios in total, and the time gap between two deceleration scenarios in one drive was around one minute. To collect drivers' natural driving behaviour, headway distances were not controlled in this experiment. The basic road scenario and pattern of travel for the other vehicles in this experiment is shown in Figure 3. The double yellow solid lines in the middle of the road indicate that drivers were not allowed to take over the lead vehicle, as ruled by the Chinese traffic laws. The duration of one drive was about 5 min, so that participants had a total driving time of about 15 min.



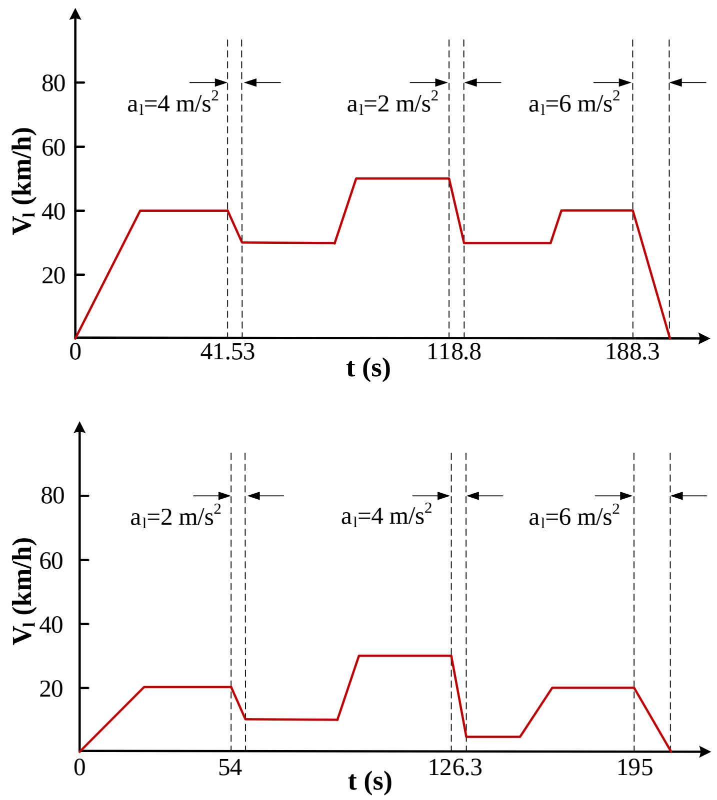


Fig. 2 Lead vehicle's speed and deceleration rate in three drives (dashed vertical lines show start and end points of lead vehicle's deceleration).

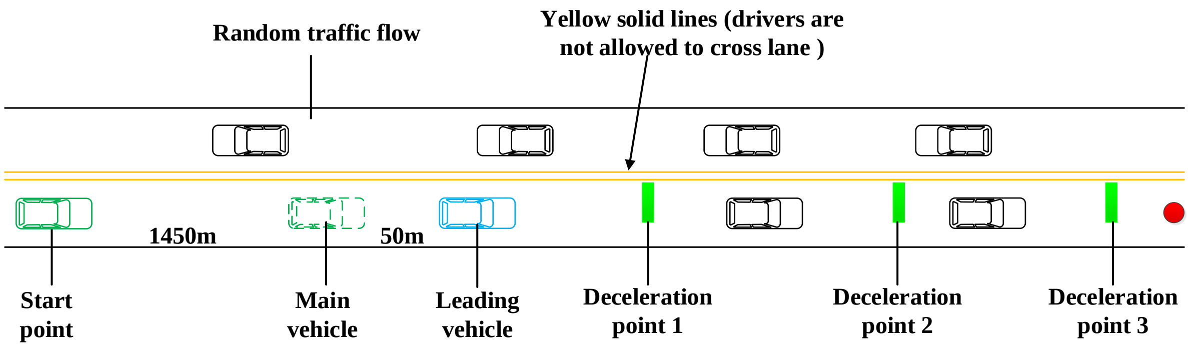


Fig.3 Basic road scenario.

2.3 Participants and procedure

In this experiment, a total of 46 participants (24 males and 22 females) aging from 30-40 years (M =34.33, SD =2.99) were recruited. Each participant held a valid Chinese driving license and had at least one year's driving experience and 30,000 km driving mileage per year. After arrival, each participant was briefed on the requirements of the experiment. The basic road scenario was explained and they were told there would be a vehicle driving in front, and that they should follow this lead vehicle as they normally would, and that they were not allowed to overtake. The participants were not informed beforehand that the lead vehicle would be braking and they signed an informed consent form. Before the formal experiment, the participants were given at least 10 minutes of training, to familiarize them with the driving simulator operation. For the training session, participants were asked to drive on a straight section of road, instructed to accelerate or decelerate to a designated speed, so that they could adapt to the acceleration and braking operation. For the formal experiment, participants had to drive three times and would rest for at least 5 min between the drives. All participants received 100 RMB (around 15 USD) for their participation in the study.

2.4 Testing the effects of situation urgency on brake response times

Multilevel regression models, also known as random coefficient models, hierarchical linear models or mixed-effects or mixed models (Tso and Guan, 2014), form a class of models that incorporate multilevel hierarchies in data (Nakagawa and Schielzeth, 2013), including longitudinal designs, where one variable is sampled repeatedly from the same set of individuals at different time points (Gelman and Hill, 2007; Buxton, 2008; Snijders and Bosker, 2011). In this study, each driver experienced nine deceleration scenarios, with the lead vehicle's speed and deceleration in different controlled combinations, but with self-paced speed and time headway. The adopted multilevel model, considering individual differences, can be written as:

$$Y_{ij} = \beta_0 + \sum_{h=1}^{p} \beta_h X_{hij} + \alpha_j + \varepsilon_{ij}; \quad (1)$$

$$\alpha_j \sim N(0, \sigma_\alpha^2); \quad (2)$$

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2), \quad (3)$$

where Y_{ij} is the ith brake response time of the jth individual, X_{hij} is the ith value of the jth individual for the hth predictor, β_0 is the overall intercept, β_h is the slope (regression coefficient) of the hth predictor, α_j is the individual-specific effects with mean of zero and variance of σ_α^2 and ε_{ij} is the residual associated with the ith value of the jth individual from a normal distribution of residuals with mean of zero and variance of σ_ε^2. This multilevel model was applied using the Matlab function fitlme with default settings (R2016a), with the lead vehicle's driving speed, deceleration rate and headway distance input as predictors. All of these predictor variables were measured at the lead vehicle's brake onset. As the headway distance was not controlled in the experimental design, 78 recordings with time headway larger than 3.5 s were excluded. A total of 336 samples, including both the lead and own vehicle's driving behaviour variables were obtained from the driving simulator experiment.

2.5 Mechanistic models of brake response time

2.5.1 Threshold models

According to looming response threshold models of brake onset (Kiefer et al., 2003; Maddox and Kiefer, 2012), once the looming exceeds the driver's threshold, a brake action will be taken to avoid hitting the lead vehicle. The optical variables θ and $\dot{\theta}$ can be calculated by the following formulas (Lee, 1976):

$$\theta = 2 \cdot \arctan(W/2d); \quad (4)$$

$$\dot{\theta} = -\mathrm{W}v_{rel}/(d^2 + W^2/4); \quad (5)$$

$$\tau^{-1} = \dot{\theta}/\,\theta. \quad (6)$$

In which W is the width of the lead vehicle, d is the distance from the driver's eyes to the tail of the lead vehicle and v_{rel} is the relative speed of the two vehicles. Threshold model can be described as:

$$L(t) + \varepsilon(t) \geq L_0(t). \tag{7}$$

Here, $L(t)$ is either $\dot{\theta}(t)$ or $\tau^{-1}(t)$, $\varepsilon(t)$ is noise, $\varepsilon(t) \sim N(0, \sigma_a)$ and $L_0(t)$ is the looming threshold. Essentially the noise term can be considered as sensory noise, and the model initiates braking as soon as the noisy signal exceeds the threshold. Typical looming threshold models are deterministic; Eq. (7) is a generalization to a stochastic formulation. In this study, threshold models are formulated as:

$$K \cdot (L(t) + \varepsilon(t)) \geq 1, \tag{8}$$

where K is the model parameter. The threshold here can thus be described as 1/K.

2.5.2 Accumulator models

In accumulator models, instead of simply continuously comparing a sensory input to a threshold, there is instead a gradual process of accumulation of evidence over time. The accumulator models used in this paper were based on the evidence accumulation framework developed by Markkula (2014), here considering two types of sensory evidence, visual looming and the lead vehicle's brake light onset.

A simple, looming-only accumulator was defined as:

$$dA(t)/dt = K \cdot L(t) - M + \varepsilon(t), \tag{9}$$

where K and M are model parameters, $\varepsilon(t)$ is noise, and a braking response is generated when $A(t) \geq A_0 = 1$. $L(t)$ is one piece of looming evidence and represented by $\dot{\theta}(t)$ or $\tau^{-1}(t)$. The –M can be interpreted as the sum of negative gating together with all the other available evidence for and against the drivers' brake action (Markkula, 2014). In addition to looming evidence, some drivers may also react to the lead vehicle's brake light. Considering this possibility, another version of the accumulator model including drivers' reaction to brake light was also defined:

$$dA(t)/dt = K \cdot L(t) - M + a_{BL} + \varepsilon(t). \quad (10)$$

Here a_{BL} is the evidence supported by the lead vehicle's brake light that can help drivers to take brake action. In this version of the accumulator model, there is a p_{BL} probability that drivers will also consider the brake lights as extra evidence for the need to brake, e.g. the term a_{BL} was added in Eq. (10) with a probability of p_{BL}. Eq. (10) is in practice a direct summation of the previous looming-only model by Markkula (2014), i.e., Eq. (9), and the brake lights-only model of Ratcliff and Strayer (2014).

2.5.3 Data for model fitting

For both the threshold and accumulator models, the aim of model fitting is to find the parameters for the model that allow it to produce predicted brake response times, that are as close as possible to the observed data. To predict response times, the looming traces $L(t)$, i.e., time histories of $\dot{\theta}$ or τ^{-1}, were simulated from the lead vehicle's brake onset, to full stop, by adopting Eq.(5) and Eq.(6). A total number of 336 samples were used as the full dataset for multilevel model in section 2.4. Thus 336 unique looming traces can be generated from the collected data. The original intention for the mechanistic model fitting was to fit to each such looming trace individually, but the computational requirements turned out to be excessive. Instead, nine unique looming traces were generated to represent average visual looming conditions in the nine deceleration scenarios. This allows an approximate, but less computationally intensive fitting. The average looming traces for the nine deceleration scenarios, using $\dot{\theta}$ and τ^{-1}, are shown in Figure 4. For the same driving speed, e.g., 60 km/h, the harder braking makes both the speed difference and the decreased distance between the two vehicles per time unit become larger. Visual looming thus grows faster when the lead vehicle brakes harder. Also, for the same deceleration rate, visual looming grows faster when drivers are driving at a slower speed. Drivers usually keep a closer headway distance when driving at a slower speed (Taieb-Maimon et al., 2001; Duan et al., 2013), so for a given deceleration, the relative distance change in a slow driving condition can be more significant than in a fast driving condition. As a result, the strongest (fastest increasing) looming occurred in the V20A6 deceleration scenario, while the weakest looming occurred in the V60A2 and V40A2 deceleration scenarios. Note that the looming trace for the V20A6 scenario has a 'knee' at around 0.9 s, this is because the lead vehicle comes to a complete stop at that point, which makes the looming grow more slowly.

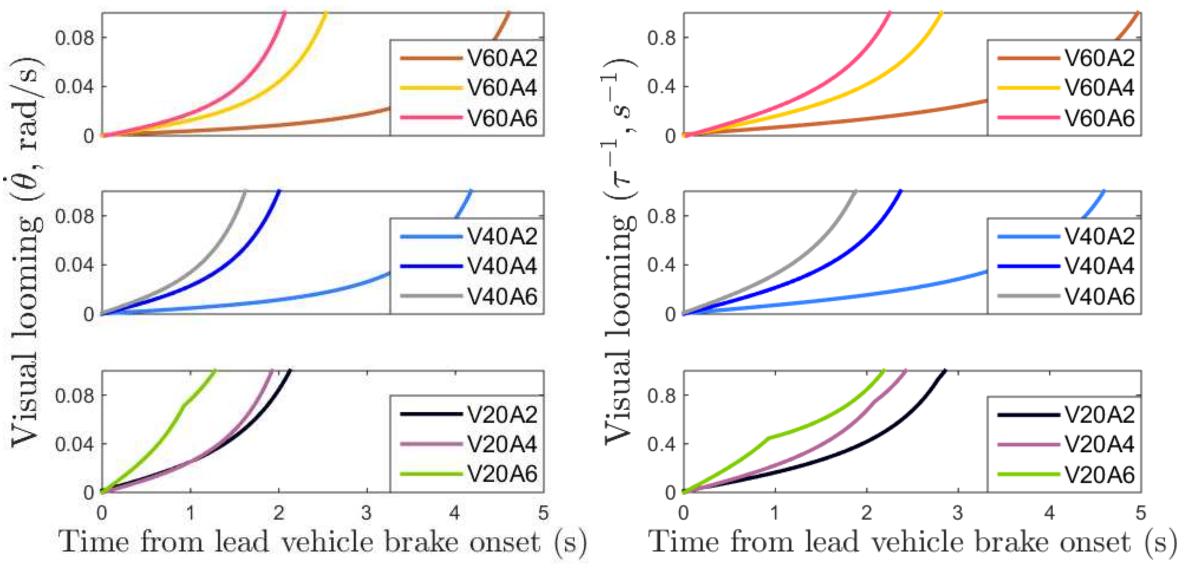


Fig.4 Looming traces for the nine deceleration scenarios. V60, V40 and V20 indicates that the lead vehicle's speed is 60 km/h, 40 km/h and 20 km/h, respectively. And A2, A4 and A6 indicates that the lead vehicle's deceleration rate is 2 m/s^2, 4 m/s^2 and 6 m/s^2, respectively.

When fitting based on these nine unique looming traces, the dataset had to be narrowed to exclude recordings that deviated too much from the average self-selected following speed and time headway in the scenario in question. First, only recordings where the following vehicle's (i.e., the participant's) speed fell within the range of lead vehicle's speed ±1 m/s were included. Then, mean time headways within the remaining recordings were calculated per deceleration scenario, and only recordings with time headway within ±0.5 s were retained. As can be seen in Table 1, between 11 and 19 data points remained per deceleration scenario after these two steps. The nine unique looming traces shown in Figure 4 were generated using the average following speeds and headways shown in Table 1. Further below, it will be described how the mechanistic model that performed best on this constrained dataset, was also tested on the full dataset.

Table 1 The narrowed-down dataset.

Deceleration scenario	LV's speed (m/s)	FV's speed range (m/s)	THW range (s)	FV's mean speed (m/s)	Mean THW (s)	Number of subjects
V60A2	19.44	19.44±1	1.77±0.5	19.85	1.86	11
V60A4	16.67	16.67±1	1.65±0.5	16.64	1.62	14
V60A6	16.67	16.67±1	1.71±0.5	16.47	1.70	19
V40A2	13.89	13.89±1	2.21±0.5	14.08	2.22	12
V40A4	11.11	11.11±1	1.94±0.5	11.14	1.86	14
V40A6	11.11	11.11±1	2.06±0.5	11.35	1.97	17
V20A2	5.56	5.56±1	2.47±0.5	5.71	2.48	14
V20A4	8.33	8.33±1	2.38±0.5	8.29	2.37	18
V20A6	5.56	5.56±1	2.75±0.5	5.57	2.70	14

LV: lead vehicle; FV: following vehicle; THW: time headway

2.5.4 Model fitting

To perform maximum-likelihood fitting of the mechanistic models on the dataset, all model parameters were searched on a uniformly spaced grid. The search range for each parameter is listed in Table 2. For each combination of parameters, 200 simulations were run for each of the nine deceleration scenarios. A numerical distribution of predicted brake response time was thus generated per scenario, for each combination of parameters, and the maximum likelihood parameterisation, i.e., the one which yielded probability distributions under which the observed data were maximally probable, was retained.

Table 2 Parameters search range.

Parameter	Searched values	
	Threshold model	Accumulator model
K	τ^{-1} {1, 1.25, 1.5, …, 6} $\dot{\theta}$ {15, 15.25, 15.5, …, 30}	τ^{-1} {1, 2.25, 2.5, …, 6} $\dot{\theta}$ {20, 20.25, 20.5, …, 30}
M	--	{-0.7, -0.675, -0.65, …, 0}
σ_a	{0, 0.005, 0.0.01, …, 0.4}	{0.1, 0.15, 0.2, …, 0.4}
a_{BL}	--	{0, 0.25, 0.5, …, 2}

p_{BL}	--	{0, 0.025, 0.05, …, 1}

The Akaike Information Criterion (AIC), which has been widely used in model selection (Burnham and Anderson, 2002; Washington et al., 2011; Haque and Washington, 2014), was used to evaluate the model fitting of different versions of mechanistic model on drivers' brake response time and model fitting of multilevel model and best version of mechanistic models on the full dataset. The general form for calculating AIC is (Akaike, 1973):

$$AIC = 2k - 2\ln(\hat{L}), \quad (11)$$

where ln is the natural logarithm, k is the number of parameters in the model and $\hat{L}$ is the value of the likelihood. According to the AIC selection criterion, for a given dataset, smaller AIC values indicate preferable models.

3. Results

3.1 Multilevel regression analysis

Figure 5 shows observed brake response time from the full dataset. The black square and whiskers show average brake response time and its 95% confidence interval, respectively. Table 3 shows the multilevel model regression results. All of the three predictors have a significant effect on drivers' brake response time (with $p< 0.01$). Generally, drivers' brake response time increases with the increase of distance gap, while it decreases with the increase of lead vehicle's deceleration rate and speed, in line with previous literature as referenced in the Introduction.

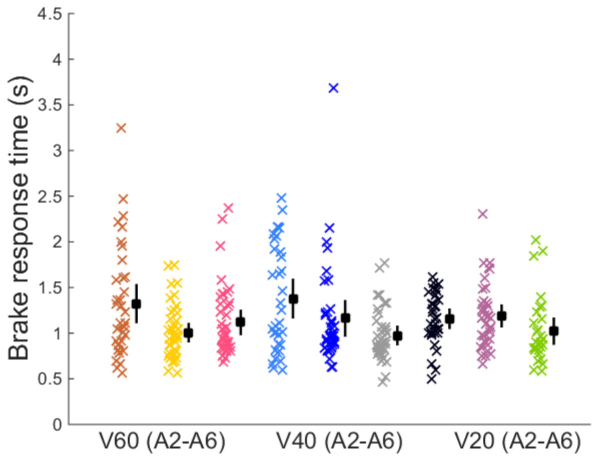


Fig.5 Drivers' brake response time for nine deceleration scenarios (Whiskers indicate the variance of brake response time on 95% CI).

Table 3 Multilevel regression results of brake response time, all with $p < 0.01$.

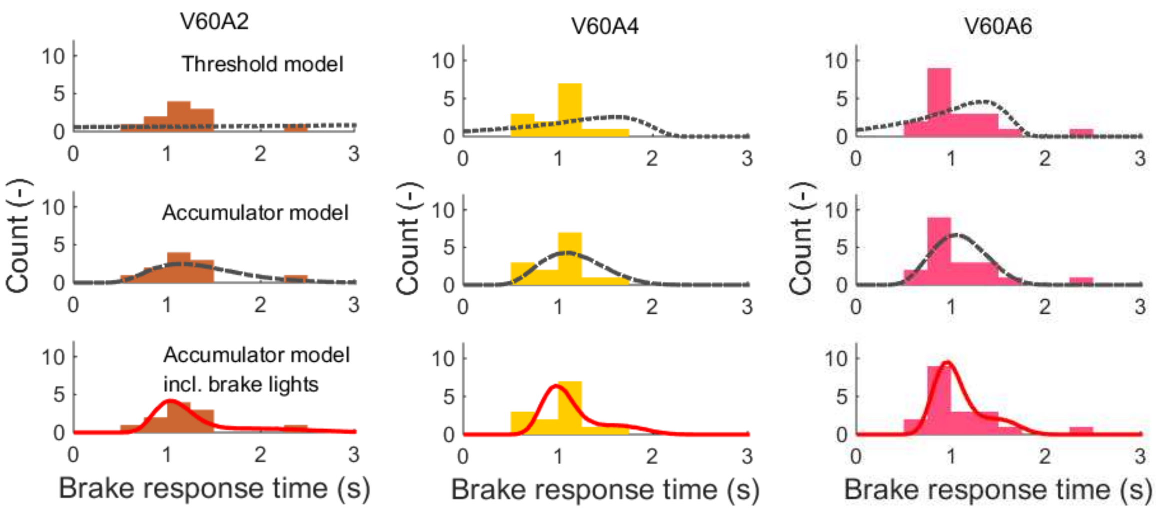
	Multilevel Model	
	Estimate (95% CI)	T test
Intercept	1.20 [1.03, 1.37]	13.86
LV's deceleration rate	-0.05 [-0.07, -0.02]	-3.61
LV's speed	-0.02 [-0.03, -0.01]	-4.13
Distance gap	0.02 [0.015, 0.025]	8.15
σ_ϵ	0.36 [0.33, 0.39]	--
σ_α	0.16 [0.11, 0.23]	--
R^2	0.326	

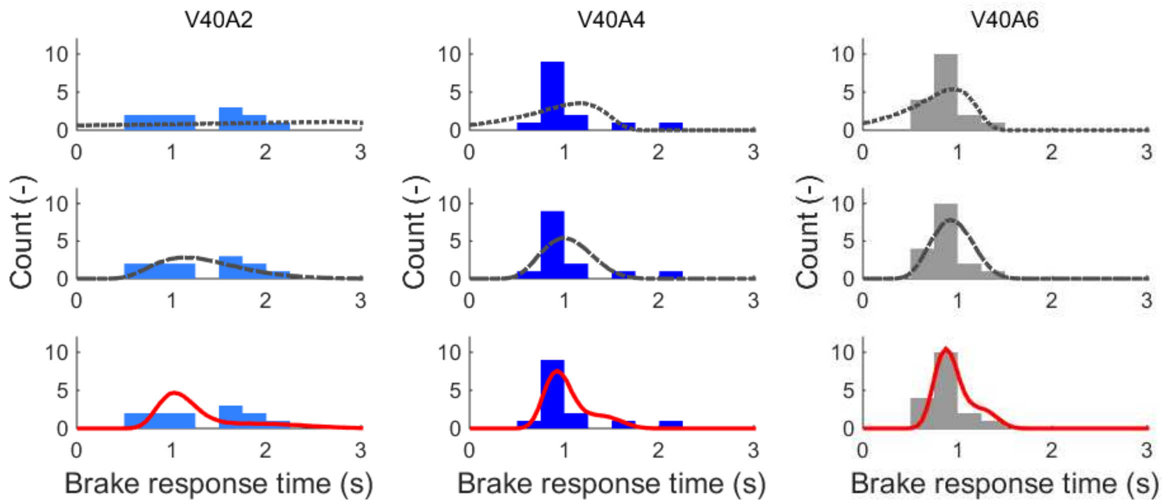
3.2 Mechanistic model fitting (constrained dataset)

Table 4 shows the best parameterisation obtained for the constrained dataset using grid search with maximum likelihood estimation. The model fitting results of the threshold model and two versions of the accumulator model, using $\dot{\theta}$ and τ^{-1} are shown in Figure 6 and Figure 7, respectively. For the threshold model, the obtained response thresholds (cf. Eq. (8)) were $\dot{\theta}$ = 1/15.5 = 0.06 rad/s and τ^{-1} = 1/1.75 = 0.57 s^{-1}. The threshold model was not able to capture the observed brake response time distributions very well. For instance, in the V60A2 and V40A2 (weak looming) scenarios, the human drivers reacted faster than the model, while the converse was true in the V20A6 (strong looming) scenario; see Figure 6 and Figure 7. For both the $\dot{\theta}$ and τ^{-1} accumulator models including brake lights, the best-fitting parameterisation had a 77.5% probability of adding an extra a_{BL}= 0.5 from brake light onset. These combined brake light-looming models fitted better than the looming-only versions; in practice by providing (a hint of) an extra peak in the distribution of drivers' brake response time (again see Figure 6 and Figure 7). The AIC values in Table 4 suggest that the τ^{-1} accumulator model including braking lights can better model drivers' brake response time than the second-best, corresponding $\dot{\theta}$ model (424.07 vs. 427.06). In the terminology of the AIC, the τ^{-1} model is exp((427.06-424.07)/2) = 4.5 times more probable to minimize the information loss; i.e., to be the better model (Burnham and Anderson, 2002).

Table 4 Best parameterisation for threshold model and accumulator model of brake response time using $\dot{\theta}$ and τ^{-1} as measures of kinematical urgency.

Parameter	$\dot{\theta}$			τ^{-1}		
	Threshold model	Accumulator model	Accumulator model incl. braking light	Threshold model	Accumulator model	Accumulator model incl. braking light
K	15.5	28.75	20.5	1.75	3.25	1.75
M	--	-0.675	-0.4	--	-0.625	-0.425
σ_a	0.025	0.35	0.2	0.22	0.35	0.2
a_{BL}	--	--	0.5	--	--	0.5
p_{BL}	--	--	0.775	--	--	0.775
AIC	520.3	436.1208	427.0574	493.68	427.3708	424.0698





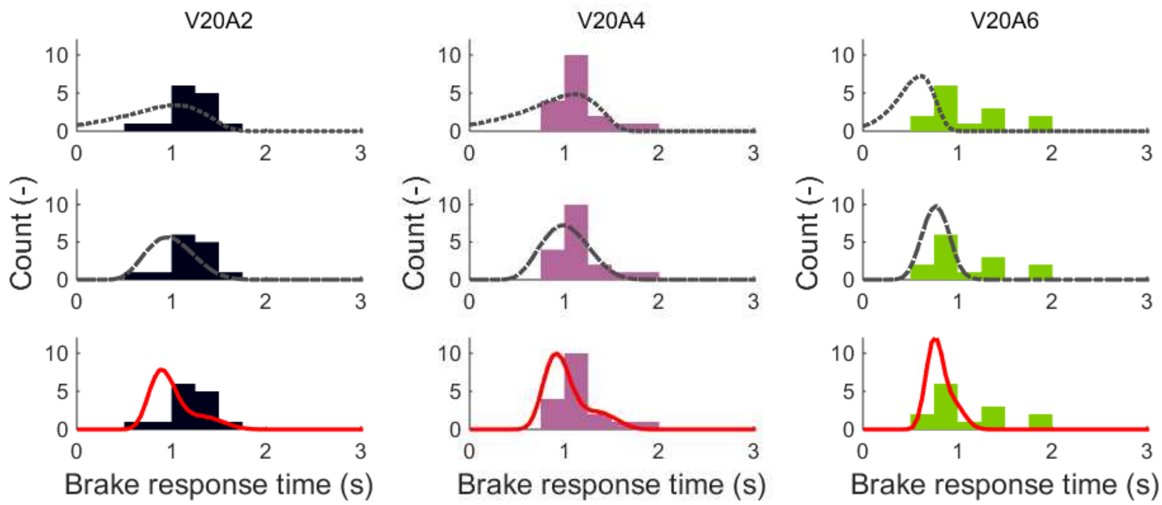


Fig.6 Mechanistic model fitting for the nine deceleration scenarios on the constrained dataset using $\dot{\theta}$. The bar histograms represent the observed brake response times and different line types show the fitted distribution of the mechanistic models. From row 1 to row 3, the lead vehicle's speed is 60 km/h, 40 km/h and 20 km/h. The lead vehicle's deceleration rate is 2 m/s^2, 4 m/s^2 and 6 m/s^2, from the left panel to the right panel.

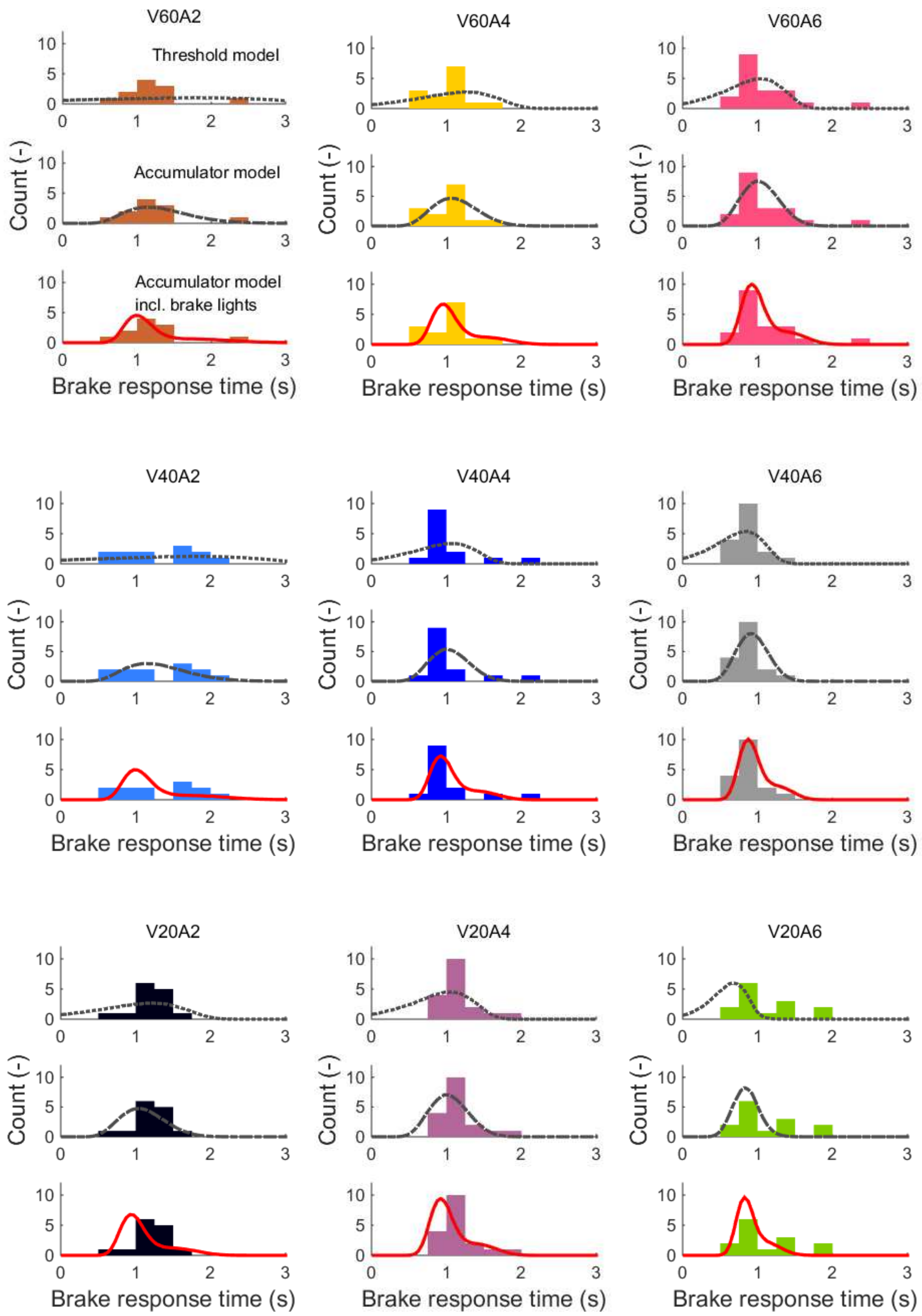
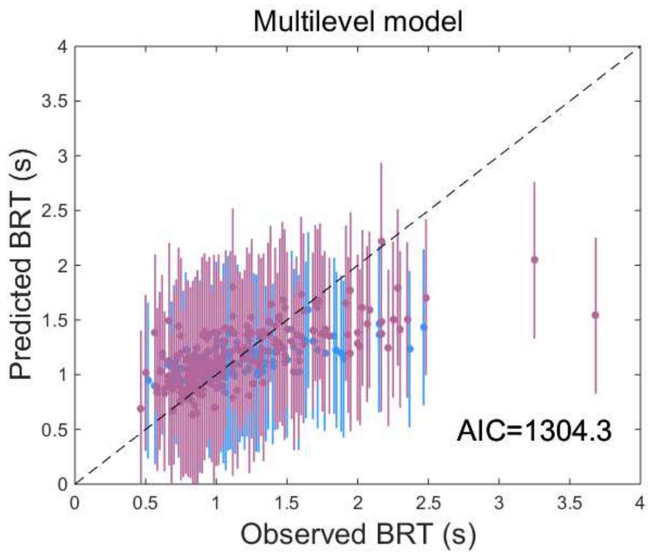


Fig.7 Mechanistic model fitting for the nine deceleration scenarios on the constrained dataset, using τ^{-1}. Bar histograms, line types, and the scenarios order as in Figure 6.

3.3 Model evaluation (full dataset)

The best-fitting version of the accumulator model generated from the constrained dataset (section 3.2), was then tested on the full dataset. Instead of using the nine average looming traces, the full set of 336 unique looming traces were adopted here, to obtain a model-predicted response time distribution per specific recorded event. For both the multilevel model and the accumulator model, AIC were calculated on this full dataset. Results for both of the two models are illustrated in Figure 8. The blue dots refer to the data already in the constrained dataset, while the purple dots refer to the remaining data in the full dataset. The vertical lines indicate the width of the model-predicted distribution. It can be noted that the multilevel model produces distributions of predicted brake response time, which are scenario-independent (vertical lines are all of the same length), symmetric (dot in middle of each line) and wider (longer lines) than for the accumulator model, which exhibits scenario-dependent, asymmetric, more narrow distributions. At first glance, it is easy to interpret Figure 8 solely in terms of the average predicted response times (the dots), in which case one notes that small observed values near 0.5 s seem overestimated by the model, and vice versa for large values. It should be noted however that this is not an indication that the models are incorrect or insufficient in some way; it is instead a direct consequence of the models being not only urgency-dependent but also probabilistic. Consider for example a situation where a single normal distribution was the exactly correct model; this would look even 'worse' in Figure 8, appearing as a completely horizontal stretch of dots and equal-length vertical lines. The fact that Figure 8 here shows non-horizontal, slightly slanted configurations of averages is because there is indeed a certain scenario-dependence in the observed response times, the lack of exact alignment of means with the y = x diagonal is because of the probabilistic nature of the phenomenon. A more appropriate way of reading Figure 8 is to focus on the vertical lines, and to note that these lines envelop the y = x diagonal.

However, for the accumulator model, three observations around 0.8 s with small variance stand out; these are for recordings with very close following, at around 0.3-0.8 s time headway. Also note that there are two data points with long observed brake response time, where the accumulator model performs notably worse than the multilevel model; both of these two data points are for recordings with long time headway, e.g. at around 3 s. Note that all of these data points are plotted in purple, i.e., none of these short or long headway recordings were part of the constrained data set to which the accumulator model was fitted. And as shown in Figure 8, the AIC value of the accumulator model is considerably larger than the multilevel model here (1386.3 vs. 1304.3). Besides AIC, the mean squared error (MSE) of the two models on the full dataset are also calculated, to examine the predictive ability of the two models. In line with the AIC results, the predicted brake response time of multilevel model is closer to the observed values than accumulator model with a smaller MSE (0.12 vs 0.18).



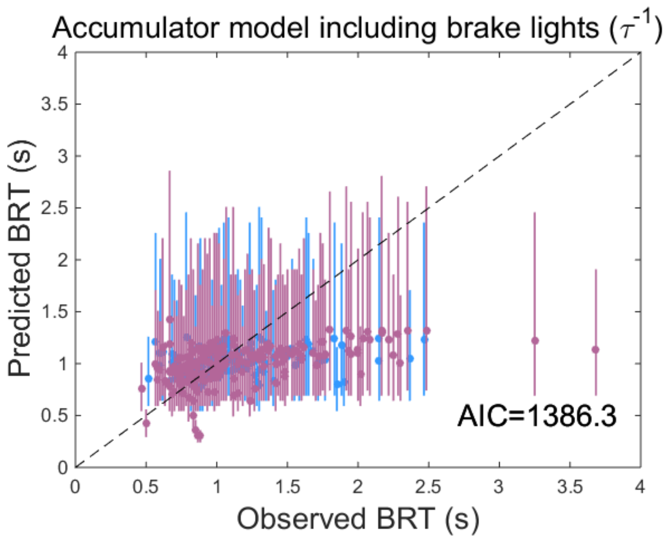


Fig. 8 Model fitting for multilevel model and accumulator model on the full dataset. Blue dots indicate data from the constrained dataset while purple dots indicate remaining data in the full dataset. The line extending around each average indicates the central 95% of the model-predicted response time distribution.

4. Discussion

Drivers' brake response time plays an important role in avoiding rear-end collisions. Factors which may affect brake response time have been investigated by numerous studies. The multilevel model adopted in this paper corroborated the significant effect of lead vehicle's speed, deceleration rate and headway on drivers' brake response time. Generally, drivers' brake response time decreases when the lead vehicle decelerates at a larger deceleration rate, drives at a higher speed, or keeps a shorter distance from the lead vehicle (Lee et al., 2002; Wang et al., 2016; Li et al., 2016). Here, we show for the first time, the combined effect of the lead vehicle's speed, deceleration rate and headway distance in a single study, which provides a strong replication and synthesis of effects previously reported separately. However, the potential effects of repeated exposures on drivers' brake response time have not been examined directly in this study. As the lead vehicle in this experiment was stationary 1500 m ahead of the start point, drivers braked once they drove close to the stationary lead vehicle. Strictly speaking, it was the first brake drivers applied in this experiment. But the brake response time included in this paper was measured from the lead vehicle's brake to the driver's brake. That is why drivers' average brake response time for each first deceleration scenario was not the largest, as shown in Figure 5. Future studies should consider the effect of repeated exposures effects.

Though the combined effect of the lead vehicle's speed, deceleration rate and headway distance was tested, it is generally assumed that drivers do not perceive speed, distance and deceleration information directly, but rather cues like visual looming affect response time. For a potential rear-end collision, measures used to quantify visual looming, e.g., τ^{-1} and $\dot{\theta}$, both increase as the threat draws nearer. Although these two measures have been adopted in many studies to quantify situation urgency (Lamble et al., 1999; Maddox and Kiefer, 2012; Markkula et al., 2016), stringent comparisons between the two measures have not been conducted. Among all three model types in this study (threshold, accumulator and accumulator including brake lights model), models based on visual looming measured by τ^{-1} always fit the data better than models based on $\dot{\theta}$ (AIC values listed in Table 4). This could be taken to suggest that drivers make use of visual cues which are more similar to τ^{-1} than $\dot{\theta}$.

For the looming threshold model, previous studies have suggested that they can describe rear-end collision avoidance behaviour in both routine driving and surprise emergencies (Fajen, 2005; Markkula et al., 2016) and that most drivers brake within a second after $\tau^{-1} = 0.2\ \mathrm{s}^{-1}$ or $\dot{\theta} = 0.02\ \mathrm{rad/s}$ (Markkula et al., 2016). Indeed, the 0.57 s^{-1} and 0.06 rad/s response thresholds obtained here are in line with the Markkula et al. (2016) findings, i.e. $\tau^{-1} = 0.57\ s^{-1}$ and $\dot{\theta} = 0.06\ \mathrm{rad/s}$ happened within 1 s of $\tau^{-1} = 0.2\ s^{-1}$ and $\dot{\theta} = 0.02\ \mathrm{rad/s}$, respectively, for all deceleration scenarios except the two least critical ones (see Figure 4). However, despite this relative success at the level of average brake response times, as seen in Figures 6 and 7, the threshold model was not able to capture the observed variability of brake reaction time very well, and especially not in the scenarios with the weakest looming (e.g., V60A2) and strongest looming (e.g., V20A6). This implies that the decision-making process behind drivers' brake onset is more likely based on evidence accumulation than a particular threshold.

In fact, as illustrated in Figure 9, the way in which the threshold model fails to capture behaviour in the weakest and strongest looming scenarios provides further support for the looming accumulation hypothesis. The coloured areas in Figure 9 are equal; integration of a small quantity over a long time is equivalent to integration of a large quantity over a short time. Therefore, if brake onset timing is determined by evidence accumulation, i.e., an integration, then an average looming threshold fitted to a mix of weak and strong looming scenarios will predict a brake response time that is too short in strong looming conditions, and too long in weak looming conditions. As can be seen in Figures 6 and 7, this is exactly the pattern observed for the threshold model here.

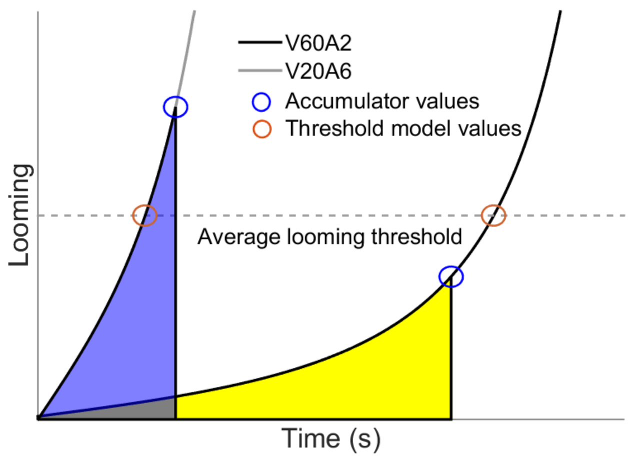


Fig. 9. Schematic illustration of the errors in a threshold model predicted by evidence accumulation. Growth of looming over time for V60A2 and V20A6 deceleration scenarios, with the two coloured areas under these curves of equal size. The accumulator model responds at the blue circles, and a threshold model fitted to these values could do no better than the average threshold, responding too quickly in the strong looming V20A6 scenario, and too slowly in the weak looming V60A2 scenario.

Previously, many experiments on basic decision-making tasks in the psychology or neuroscience laboratory have supported the idea that stimulus-driven action timing is determined by noisy accumulation of sensory evidence (Gold and Shadlen, 2007; Purcell et al., 2010; Ratcliff and Van Dongen, 2011), and qualitative analyses have pointed toward the possibility that visual looming can be accumulated in this way to guide braking (Markkula et al., 2016). However, this has not been quantitatively demonstrated. Thus, the accumulator model was included in this study to test this possibility, and explore the mechanism behind when drivers decide to brake. The results here indicate that sensory stimuli (both visual looming and brake lights onset) can be accumulated and with just three or four parameters, the accumulator model is relatively successful in reproducing the varying distributions of brake response time across the nine deceleration scenarios.

When comparing the accumulator model including brake lights with the version excluding brake lights, the former model, with 77.5% probability of adding an extra a_{BL}=0.5 from brake light onset, provided a better fit of the observed data[1]. The accumulator model including brake lights effectively provide an extra, kinematics-independent, peak in the response time distribution, which proved an improvement in general. However, for weak looming conditions (e.g. V60A2 and V40A2 deceleration scenarios in Figures 6 and 7), the accumulator model without brake lights seems fitted slightly better. It might thus be that drivers' tendencies to react to brake lights decrease with increasing headway distances.

When the accumulator model was tested on the full dataset in Section 3.3, the multilevel model worked better than the accumulator model, with a smaller AIC value. A possible reason is that

[1] In this paper, we did not look into whether some drivers always seemed to be using brake lights or whether all or some drivers had mixed strategies, i.e. we did not look into whether p_{BL} is to be interpreted as a fraction of drivers in a population or a fraction of responses within one individual.

the parameters of the accumulator model were generated from the constrained dataset with nine average looming traces, due to the computational challenges of fitting to the full dataset. Simulation on the full dataset with all observations' individual looming traces may provide a better model fitting of the observed brake response time.

5. Conclusion

Car following occurs very frequently for drivers in their daily driving, thus an appropriate rear-end collision avoidance measure is necessary for all drivers. To better understand the mechanisms behind such avoidance, this study investigated drivers' brake response time under different lead vehicle behaviours, in a high fidelity driving simulator. A multilevel model was first applied to investigate the combined effect of the lead vehicle's driving speed, deceleration rate and distance gap on brake response time. Although all the factors had significant effects on drivers' brake response time, there was still large variance between the 336 observations and modelled values. In addition, the predictors in the multilevel model (distances, speeds, accelerations) are not easy for drivers to perceive during their driving. Threshold models of visual looming were tested, but were not able to capture the variability of observed brake response timing well. Therefore, models based on the neurobiologically established mechanism of evidence accumulation were also tested, and were found to provide a better account of response times than the threshold models. The accumulator models assume that the driver's brake response is initiated when evidence for and against the brake action has accumulated to a decision threshold. Accumulator models assuming a certain probability of responding to brake lights, in addition to visual looming, fitted better than looming-only accumulator models. Models using τ^{-1} as the visual looming cue were found to be preferable over models using $\dot{\theta}$. The best-fitting accumulator model did not generalise well enough to the full dataset to outperform the multilevel model (fitted on the full dataset), but the accumulator provides an advantage in that it helps provide a better understanding of drivers' brake response behaviour. The accumulator model tested here provides, not only a powerful means of predicting drivers' brake response time, but also a plausible account of the mechanisms underlying drivers' use of looming cues for deciding on brake activation. The evidence accumulation mechanism tested in the paper can be further adopted for the design of driving assistance systems, e.g., Forward Collision Warning (FCW) systems, and for self-driving vehicles. The FCW can be designed on the basis of calculating real-time visual looming related measures, a warning could, for example, be issued when most typical drivers have reacted. The effects of warnings on brake timing also merit further investigation within the evidence accumulation framework, since they might be regarded, and thus possibly modelled, as another piece of evidence for the need to apply braking (Markkula, 2014). The results presented here could also provide support for the design of brake timing in a self-driving vehicle involved in a potential rear-end collision situation, or for the self-driving vehicle to predict likely behaviour of surrounding road users.

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