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Effects on the CMB from Compactification Before Inflation

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Abstract. Many theories beyond the Standard Model include extra dimensions, though these have yet to be directly observed. In this work we consider the possibility of a compactification mechanism which both allows extra dimensions and is compatible with current observations. This compactification is predicted to leave a signature on the CMB by altering the amplitude of the low l multipoles, dependent on the amount of inflation. Recently discovered CMB anomalies at low multipoles may be evidence for this. In our model we assume the spacetime is the product of a four-dimensional spacetime and flat extra dimensions. Before the compactification, both the four-dimensional spacetime and the extra dimensions can either be expanding or contracting independently. Taking into account physical constraints, we explore the observational consequences and the plausibility of these different models.

Contents

1	Introduction	1
2	The model	3
2.1	The metric	3
2.1.1	Higher dimensional evolution.	3
2.1.2	Four-dimensional inflation, after the transition	3
2.2	Particular examples of the higher dimensional pre-inflationary stage	3
2.2.1	Higher dimensional de Sitter space	4
2.2.2	Kasner Solutions.	4
2.3	Constraints on the background evolution	4
3	Scalar Field Perturbations	6
3.1	The Compactification Transition	8
3.2	Curvature perturbations	10
3.3	The massive modes	11
4	Effects on the CMB	11
5	Conclusions	12

1 Introduction

Many of the extensions of the Standard Model involve theories which live in a higher dimensional spacetime. However, all current observational evidence points to a 4-dimensional description of the universe at large scales. A natural way to accommodate these higher dimensional theories with our observations is to allow for a compactification mechanism. This is normally achieved by the presence of a higher dimensional energy-momentum tensor that creates the conditions for a highly anisotropic evolution of the spacetime. In particular, our universe today seems to be described well in this context by a $\mathbb{R}^4 \times \mathcal{M}$ cosmological model where the extra-dimensional manifold \mathcal{M} remains stabilized. One can then assume that the compactification mechanism fixes the size of the internal space to be small enough such that the degrees of freedom associated with these extra-dimensions are effectively out of reach for the low energy 4-dimensional observers.

There are however, important open questions in this class of models. For example, it is hard to argue that our four-dimensional cosmological spacetime would be the unique vacuum solution in a higher dimensional setting. So one may wonder how our four-dimensional spacetime is dynamically selected. There are several ideas in the literature that attempt to address these questions [1, 2], usually invoking extended objects in various dimensions with dynamics that enforces compactification down to three large spatial dimensions.

Another possibility would be if the higher dimensional theory allows for many vacua with different numbers of compactified dimensions, and our universe is only one of a large number of possible compactifications. These kinds of ideas have been recently explored in the context of simple higher dimensional field theories where the compactification was obtained by the presence of fluxes [3, 4]. The properties of these vacua are typically greatly varied.

For example, the number of large dimensions can vary from one vacuum to another [5]; a transdimensional landscape. In some cases, one can identify the instanton solutions that represent quantum mechanical transitions between these vacua in a process similar to vacuum decay. These instantons show that the landscape would be populated by the formation of bubbles of different states. The result is a complex higher dimensional version of an eternal multiverse [5]. Our universe can then be within one of these bubbles.

It is interesting to explore the possibility that we can obtain an observational signature of extra dimensions due to their dynamics in our past. However, the prospect of observing these effects will be somewhat hindered by the necessity of having a period of slow roll inflation in our immediate past. This is the same situation that one faces in any model that tries to explore the pre-inflationary state. Intriguingly, there are some anomalies in the CMB data [6] that might be giving us a hint that the duration of inflation was around the minimum required to solve the cosmological problems, but not longer [7]. In such a case, we might hope to see the effects of a previous state of the universe in the power spectrum of perturbations [8–13].

There are several different possible transitions. For example, the previous universe could have been effectively lower-dimensional and the nucleation process created a four-dimensional anisotropic bubble. These kinds of transitions and their effects have been investigated in several papers [14–18], with their main focus on the anisotropic nature of the initial state of inflation.

Another possibility is that our universe was the result of dynamical compactification from a higher dimensional universe. The form of the instantons that interpolate between highly symmetric vacua were discussed in Ref. [5]. Investigating the spectrum of perturbations of such transitions is quite difficult due to the complicated geometry of the transition.

In this paper we simplify the process of the transition, and assume a global dynamical compactification of the spacetime. We assume the spacetime is divided into two parts, the (3+1)-dimensional FRW large dimensions and an internal manifold of $(d - 3)$ flat compact dimensions. Within this model, we consider the existence of an initial period of anisotropic cosmology where the dynamics are controlled by a higher dimensional fluid source. We then assume that a compactification mechanism causes the universe to quickly become effectively four-dimensional and undergoes a period of inflation of around 60 e-folds. This limited amount of inflation allows us to investigate the state of the scalar field that controls the cosmological perturbations; which is an excited state compared to the standard Bunch Davies vacuum. We compute the spectrum of perturbations from this excited state, compute the multipole moments in the CMB, and compare to data.

This is a simplified toy model since we do not provide a detailed compactification process. Our focus in this paper is to explore the effects that a rapid compactification would have on the CMB, and we hope this simple model captures some of the key effects that one can expect from more realistic situations. Similar approaches to the one presented here, albeit in a different context, include [19–22].

This paper is organized as follows. In Section 2 we present the toy model for dynamical compactification and describe the constraints on the energy momentum tensor necessary to lead to this cosmological history. In Section 3 we compute the spectrum of cosmological perturbations. In Section 4 we compute the observational predictions for the CMB using the CLASS package. Finally, in Section 5 we conclude.

2 The model

2.1 The metric

We model a transition from a higher dimensional cosmological stage where the internal dimensions were dynamical, to a purely 4d inflationary period where the degrees of freedom associated with the extra-dimensions are fixed. In order to do that we will postulate a universe (with d spatial dimensions) that is described by a $(d+1)$ -dimensional anisotropic metric with two scale factors $a(\eta)$ and $b(\eta)$ where η is the conformal time i.e. $a(t)d\eta = dt$,

$$ds^2 = a(\eta)^2 \left(-d\eta^2 + \sum_{i=1}^3 dx_i^2 \right) + b(\eta)^2 \sum_{j=1}^{d-3} dy_j^2, \quad (2.1)$$

where, for simplicity we have taken the internal space to be a flat $(d-3)$ -dimensional torus. We will also consider that the higher dimensional theory is controlled by Einstein's equations in a $(d+1)$ -dimensional spacetime. Finally, we also need to specify the matter content of the theory that will determine the dynamics of the spacetime. In the past this has been a very active area of research and several models with different sources have been presented. One can imagine that some of the fields on the higher dimensional theory play the role of the sources [23] but there could also be other ingredients present such as higher dimensional perfect fluids [24], extended objects [1], or even sources due to quantum effects [25].

Here we will take a more model independent approach and analyze a family of simple metrics of the form described above. The type of metrics that we will consider for the two stages before and after compactification will be of the following form.

2.1.1 Higher dimensional evolution.

Before the transition we have the scale factors given by,

$$a(\eta) = \frac{1}{(-H\eta_0)} \left(\frac{\eta}{\eta_0} \right)^\alpha, \quad b(\eta) = b_0 \left(\frac{\eta}{\eta_0} \right)^\beta, \quad (2.2)$$

which is valid for the range $-\infty < \eta < \eta_0$, where we take $\eta_0 < 0$ to be the time at which the compactification mechanism dominates and the universe becomes four-dimensional. Furthermore α and β control the expansion or contraction of the $(3+1)$ as well as the internal manifold. The rest of the parameters are fixed by the subsequent $4d$ evolution and the requirement that our scale factors are continuous across the transition.

2.1.2 Four-dimensional inflation, after the transition

After the compactification we assume the universe enters a pure $4d$ expansion controlled by an effective de Sitter space, so the scale factors in this case become,

$$a(\eta) = \frac{1}{(-H\eta)}, \quad b(\eta) = b_0. \quad (2.3)$$

where H is the Hubble constant associated with the $4d$ de Sitter spacetime and b_0 denotes the factor that fixes the size of the internal space.

2.2 Particular examples of the higher dimensional pre-inflationary stage

The family of solutions we described above encapsulates some interesting cases that are worth mentioning explicitly for future reference.

2.2.1 Higher dimensional de Sitter space

If we take $\alpha = \beta = -1$ and fix $b_0 = (-H\eta_0)^{-1}$ we have a symmetric situation. The universe expands isotropically in this higher dimensional stage and in fact it represents a $(d+1)$ -dimensional de Sitter space. This could represent one of the multiple vacua that exists in models of flux compactification recently discussed in the literature [3, 4]. If so, transitions from this spacetime to the 4d inflationary case can be used to estimate the effects to be expected in these kinds of transitions [4, 5].

2.2.2 Kasner Solutions.

Another simple set of solutions that are captured by our ansatz are the vacuum solutions [26] given by

$$\alpha = \frac{1}{2} \left(1 - \sqrt{3 - \frac{6}{d-1}} \right), \quad (2.4)$$

and

$$\beta = \sqrt{\frac{3}{3 - 4d + d^2}}. \quad (2.5)$$

Here we take the branch of solutions that correspond to a $4d$ expanding universe¹. The internal space is collapsing and one can show that all these solutions tend towards a singularity. These are nothing more than the higher dimensional generalizations of the familiar Kasner solutions in $4d$. On the other hand, the expectation here is that the higher dimensional energy momentum tensor needed for compactification would change the behavior of the solution before such singularity arises and the transition to an inflationary $4d$ universe will take place.

2.3 Constraints on the background evolution

As we mentioned earlier, we expect many different ingredients to possibly contribute to the effective energy-momentum tensor that compactifies the spacetime. This is the main reason to parametrize our lack of knowledge by a generic ansatz. Nevertheless we want to restrict ourselves to physical models where the total energy is positive and where the equation of state for the effective fluid is such that $-1 \leq w \leq 1$. These constraints limit significantly the range of the parameters in our set of models.

Using Einstein's equations and the metric in (2.1) we can compute the properties of the required energy momentum tensor, by first computing the Einstein tensor,

$$\begin{aligned} G_0^0 &= \frac{(d-3)ab(2(a^4+2)a'b' - a(a^4-1)b'') - 6b^2a'^2 - (d-4)(d-3)a^6b'^2}{2a^4b^2}, \\ G_i^i &= -\frac{(d-3)a^2b((a^4+1)b'' - 2a^3a'b') + b^2(6a'^2 - 4aa'') + (d-4)(d-3)a^6b'^2}{2a^4b^2}, \\ G_j^j &= -\frac{ab((d-5)a^4 + d-3)(ab'' - 2a'b') + 6b^2(2a'^2 - aa'') + (d-5)(d-4)a^6b'^2}{2a^4b^2} \end{aligned} \quad (2.6)$$

¹The exception is the $d=4$ case where the space time is a higher dimensional generalization of the $2d$ Misner spacetime where the four-dimensional part is static while the internal circle collapses. There is also the family where the large dimensions are contracting but we will not consider them here.

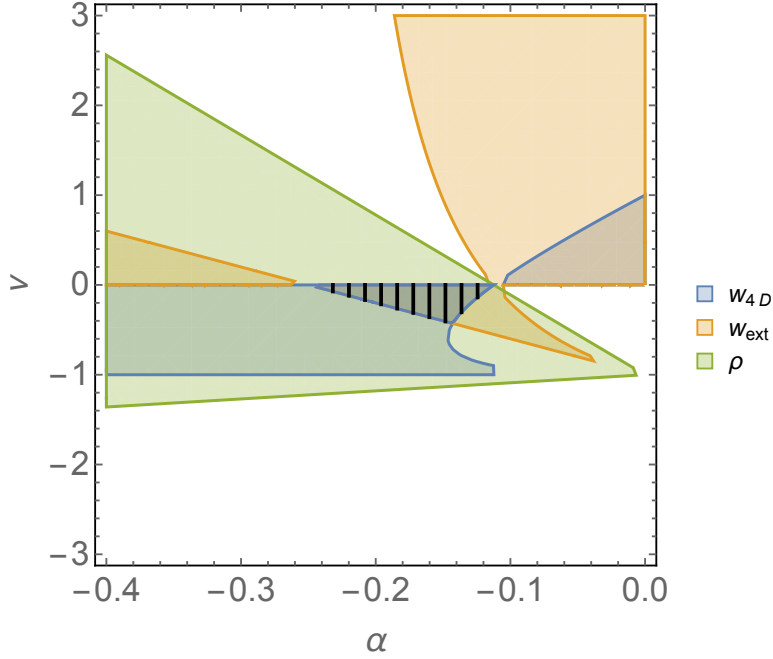


Figure 1. The shaded regions shows the allowed values for α and ν for the different constraints. The overlapping hatched region in the center of the figure represents the parameter space that respects all the constraints.

where the prime denotes differentiation with respect to conformal time. For the special case $a(\eta) = -(\eta/\eta_0)^\alpha/H\eta_0$ and $b(\eta) = b_0(\eta/\eta_0)^\beta$ becomes,

$$\begin{aligned}
 G_0^0 &= -\frac{\eta_0^2 H^2}{2\eta^2} \left(\frac{\eta}{\eta_0}\right)^{-2\alpha} (6\alpha^2 + 6\alpha\beta(d-3) + \beta^2(d-4)(d-3)) , \\
 G_i^i &= -\frac{\eta_0^2 H^2}{2\eta^2} \left(\frac{\eta}{\eta_0}\right)^{-2\alpha} (2(\alpha-2)\alpha + 2(\alpha-1)\beta(d-3) + \beta^2(d-3)(d-2)) , \\
 G_j^j &= -\frac{\eta_0^2 H^2}{2\eta^2} \left(\frac{\eta}{\eta_0}\right)^{-2\alpha} (6\alpha^2 - 6\alpha + 4\alpha\beta(d-4) + \beta(d-4)(\beta(d-3) - 2)) . \quad (2.7)
 \end{aligned}$$

The equations of state for the four-dimensional and extra dimensional fluids are then given by

$$\begin{aligned}
 w_{4D} &= -\frac{2(\alpha-2)\alpha + 2(\alpha-1)\beta(d-3) + \beta^2(d-3)(d-2)}{6\alpha^2 + 6\alpha\beta(d-3) + \beta^2(d-4)(d-3)} , \\
 w_{\text{ext}} &= -\frac{6\alpha^2 - 6\alpha + 4\alpha\beta(d-4) + \beta(d-4)(\beta(d-3) - 2)}{6\alpha^2 + 6\alpha\beta(d-3) + \beta^2(d-4)(d-3)} , \quad (2.8)
 \end{aligned}$$

where w_{4D} is the $4d$ part and w_{ext} corresponds to the analogous quantity taking into consideration the extra-dimensional part of the pressure.

Considering all these requirements the allowed region of the (d, α, β) parameter space gets reduced significantly. We show in Fig. (1) the space allowed by these constraints as the overlap of all the shaded regions for the case $d = 4$, analogous situations can be found for other dimensions. Here we have expressed β in terms of

$$\nu = -1 + 2\alpha + (d-3)\beta, \quad (2.9)$$

for later convenience. We also represent only the region with $\alpha < 0$ since these would be the cases that interest us in this paper.

3 Scalar Field Perturbations

Here we will consider the cosmological perturbations generated in the spacetime described before. In order to do that we will first consider the perturbations for a massless scalar field in this background with action,

$$S = -\frac{1}{2} \int d\eta dx^3 dy^{d-3} \sqrt{-g} \partial_\mu \phi \partial^\mu \phi. \quad (3.1)$$

The equation of motion for this field in a metric given by Eq. (2.1) is

$$\square \phi = \phi'' + \left(\frac{2a'}{a} + \frac{(d-3)b'}{b} \right) \phi' - \sum_{i=1}^d \partial_i \partial_i \phi - \frac{a^2}{b^2} \sum_{j=d+1}^{d-3} \partial_j \partial_j \phi = 0. \quad (3.2)$$

Expanding the field in terms of Fourier modes,

$$\phi(\eta) = \int \frac{d^3 k}{(2\pi)^{3/2}} \frac{d^{d-3} k_y}{(2\pi)^{(d-3)/2}} \left[a_{(k, k_y)} \varphi_{(k, k_y)}(\eta) e^{ik \cdot x} e^{ik_y \cdot y} + \text{cc} \right], \quad (3.3)$$

we arrive to the equations for the mode functions,

$$\varphi'' + \left(\frac{2a'}{a} + \frac{(d-3)b'}{b} \right) \varphi' + \left(k^2 + \frac{a^2}{b^2} k_y^2 \right) \varphi = 0. \quad (3.4)$$

The field in (3.1) does not have a canonical kinetic term. We can define a canonically normalized field $v(\eta)$ by the field redefinition,

$$\varphi(\eta) = a(\eta)^{-1} b(\eta)^{-(d-3)/2} v(\eta), \quad (3.5)$$

which leads to

$$v'' - \left(\frac{a''}{a} + \frac{(d-3)a'b'}{ab} + \frac{(d-3)b''}{2b} + \frac{(d-3)(d-5)(b')^2}{4b^2} - \frac{a^2}{b^2} k_y^2 - k^2 \right) v = 0. \quad (3.6)$$

Taking the general ansatz for the scale factors in Eq. (2.2), one arrives at the equation

$$v'' - \left[\frac{\alpha(\alpha-1)}{\eta^2} + \frac{d-3}{2\eta^2} \left[2\alpha\beta + \beta(\beta-1) + \frac{(d-5)\beta^2}{2} \right] - \frac{1}{(-H\eta_0 b_0)^2} \left(\frac{\eta}{\eta_0} \right)^{2(\alpha-\beta)} k_y^2 - k^2 \right] v = 0.$$

Let us first consider the evolution of the purely four-dimensional modes², those with $k_y = 0$. In this case, one can get a general solution of this equation of the form,

$$v_k(\eta) = \sqrt{|k\eta|} \left(A_k J_{\nu/2}(|k\eta|) + B_k Y_{\nu/2}(|k\eta|) \right), \quad (3.7)$$

where ν is defined in Eq. (2.9). Fixing the coefficients A_k and B_k one identifies the vacuum state for these mode functions. The usual approach in these situations, in particular in an

²We will later comment on the relevance of the Kaluza-Klein massive modes.

inflationary stage, is to identify the mode functions with the ones given by the so-called Bunch-Davies vacuum. Hence, at very early times, where the modes are deep inside the horizon, they should match to the Minkowski modes, therefore

$$\lim_{\eta \rightarrow -\infty} v_k(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}. \quad (3.8)$$

This is a legitimate assumption if the $4d$ comoving Hubble radius decreases with time so that four-dimensional perturbations start being sub horizon and then leave the horizon as time passes. In our ansatz this is the case when $\alpha < 0$. In the following we will only consider this possibility.

Using the asymptotic form of the Bessel functions of type J and Y for $k|\eta| \gg 1$

$$\begin{aligned} J_{\nu/2}(z) &\approx \sqrt{\frac{2}{\pi|z|}} \cos\left(|z| - \frac{\nu\pi}{4} - \frac{\pi}{4}\right), \\ Y_{\nu/2}(z) &\approx \sqrt{\frac{2}{\pi|z|}} \sin\left(|z| - \frac{\nu\pi}{4} - \frac{\pi}{4}\right), \end{aligned} \quad (3.9)$$

we can compute the coefficients A_k and B_k that match correctly at early times.

$$A_k = -iB_k = \sqrt{\frac{\pi}{4k}} e^{i\pi(1+\nu)/4}. \quad (3.10)$$

Therefore the modes are given by

$$v_k(\eta) = \frac{\sqrt{\pi|\eta|}}{2} e^{i\pi(1+\nu)/4} [J_{\nu/2}(k|\eta|) + iY_{\nu/2}(k|\eta|)] = \frac{\sqrt{\pi|\eta|}}{2} e^{i\pi(1+\nu)/4} H_{\nu/2}^{(1)}(k|\eta|). \quad (3.11)$$

Here $H_{\nu/2}^{(1)}$ are Hankel functions of the first kind. All the information about the background evolution is therefore encoded in the index of the Hankel function, ν . In particular this also gives us the information about the spectral index of the power spectrum of the perturbations that leave the horizon during this period. Using their limiting form

$$H_{\nu/2}^{(1)}(x) = \frac{i2^{d/2}x^{-\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)}{\pi}, \quad (3.12)$$

we get the power-spectrum:

$$P(k) = \lim_{k\eta \rightarrow 0} a^{1-\nu} |v_k|^2 = \lim_{k|\eta| \rightarrow 0} (H\eta)^{\nu-1} |v_k|^2 = H^{\nu-1} 2^{\nu-2} \pi^{-1} k^{-\nu} \Gamma\left(\frac{\nu}{2}\right)^2. \quad (3.13)$$

For future reference we note that for our particular examples we have,

$$\nu_{dS_D} = -d \quad \text{for a } (d+1) \text{ de Sitter spacetime, so the spectrum goes to } k^{-d} \quad (3.14)$$

and

$$\nu_{\text{Kasner}} = 0 \quad \text{for the vacuum solutions for any } d \text{ so the spectrum is } k \text{ independent.} \quad (3.15)$$

3.1 The Compactification Transition

As we mentioned in the previous section, we would like to understand the effect on the spectrum of perturbations of a transition from a higher dimensional universe to a purely $4d$ de Sitter space. We model this transition as a quick change in scale factors at a particular time $\eta = \eta_0$.

After the transition the Eq. (3.7) becomes

$$v'' + \left(k^2 - \left(2 - \frac{m_y^2}{H^2} \right) \frac{1}{\eta^2} \right) v = 0 , \quad (3.16)$$

where we have denoted by m_y the masses of the Kaluza-Klein states given by

$$m_y = \frac{k_y}{b_0} . \quad (3.17)$$

The general solution is w instead of v

$$w_{k,k_y} = \sqrt{\eta} \left[C_{k,k_y} J_{\mu/2}(k|\eta|) + D_{k,k_y} Y_{\mu/2}(k|\eta|) \right] , \quad (3.18)$$

where

$$\mu = \sqrt{9 - 4 \left(\frac{m_y}{H} \right)^2} . \quad (3.19)$$

Considering the mode functions for the zero modes, we should take $k_y = 0$, so $\mu = 3$ and the solutions become

$$w_k(\eta) = \sqrt{\frac{2}{\pi k}} \left[C_k \left(\frac{\cos k\eta}{k\eta} + \sin k\eta \right) + D_k \left(\cos k\eta - \frac{\sin k\eta}{k\eta} \right) \right] , \quad (3.20)$$

which is the usual four-dimensional result for a massless scalar during inflation.

We now have the expression for the mode functions before and after the transition so the only thing left to do is to match these solutions across the surface $\eta = \eta_0$. The mode functions must be continuous across this boundary, namely,

$$w(\eta_0) = v(\eta_0) .$$

On the other hand we can not impose the same continuity for the derivative of the mode functions. The reason for this can be found by looking at Eq. (3.6). In our model the transition between the two stages occurs by a sharp variation of the scale factor. This means that the derivative of $a(\eta)$ and $b(\eta)$ are discontinuous across the transition and therefore its second derivative would have a delta function contribution. For our particular case we have,

$$\frac{a''}{a} \supset - \left(\frac{1 + \alpha}{\eta_0} \right) \delta(\eta - \eta_0) , \quad (3.21)$$

and

$$\frac{b''}{b} \supset - \left(\frac{\beta}{\eta_0} \right) \delta(\eta - \eta_0) . \quad (3.22)$$

Therefore integrating Eq. (3.6) across the surface of matching on obtains that the derivative of mode functions w and v acquires a jump given by,

$$w'(\eta_0) - v'(\eta_0) = - \frac{(3 + \nu)}{2\eta_0} v(\eta_0) . \quad (3.23)$$

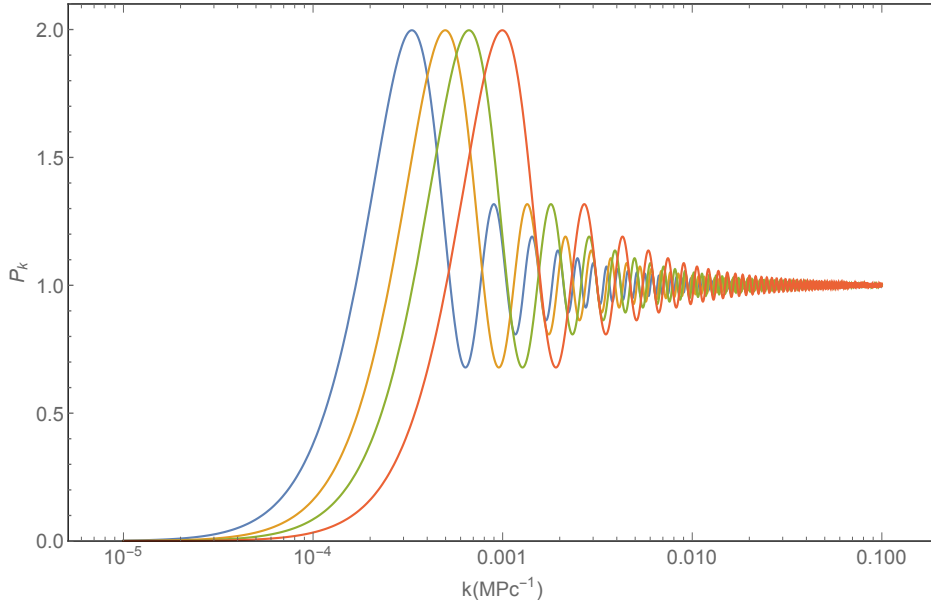


Figure 2. Normalized power spectrum for different values of compactification time η_0 . For later compactification times the power spectrum is shifted to the right.

From these matching conditions we can calculate the coefficients C_k and D_k in mode function $w(\eta)$ which are given by

$$\begin{aligned}
 C_k &= -\frac{\pi e^{\frac{1}{4}i\pi(\nu+1)}}{2\sqrt{2}(-\eta_0 k)^{1/2}} \left\{ \eta_0 k \sin(\eta_0 k) H_{\frac{\nu}{2}}^{(1)}(-k\eta_0) + H_{\frac{\nu+2}{2}}^{(1)}(-k\eta_0) [\eta_0 k \cos(\eta_0 k) - \sin(\eta_0 k)] \right\}, \\
 D_k &= \frac{\pi e^{\frac{1}{4}i\pi(\nu+1)}}{2\sqrt{2}(-\eta_0 k)^{1/2}} \left\{ -\eta_0 k \cos(\eta_0 k) H_{\frac{\nu}{2}}^{(1)}(-k\eta_0) + H_{\frac{\nu+2}{2}}^{(1)}(-k\eta_0) [\eta_0 k \sin(\eta_0 k) + \cos(\eta_0 k)] \right\}.
 \end{aligned} \tag{3.24}$$

These determine the mode function $w(k)$. From those functions we can get the power spectrum for the scalar field after the transition as,

$$P_\phi(k) = \lim_{\eta \rightarrow 0} (H\eta)^2 |w_k(\eta)|^2 \tag{3.25}$$

$$= \frac{\pi H^2}{4|\eta_0|k^4} \left| \eta_0 k H_{\frac{\nu}{2}}^{(1)}(-k\eta_0) \sin k\eta_0 + H_{\frac{\nu+2}{2}}^{(1)}(-k\eta_0) (\eta_0 k \cos k\eta_0 - \sin k\eta_0) \right|^2. \tag{3.26}$$

We can normalize this power spectrum to obtain,

$$\mathcal{P}_\phi(k) = \frac{2k^3}{H^2} P_\phi(k). \tag{3.27}$$

This normalized power spectrum has the property that goes to one in the usual Bunch-Davies vacuum and deviates from there otherwise, so it is a good way to measure the deviation or excitation of our power spectrum.

This power spectrum is parametrized by two quantities, η_0 and ν . We show in Figs. 2 and 3 the normalized power spectrum for different values of η_0 and ν . The role of η_0 is clear

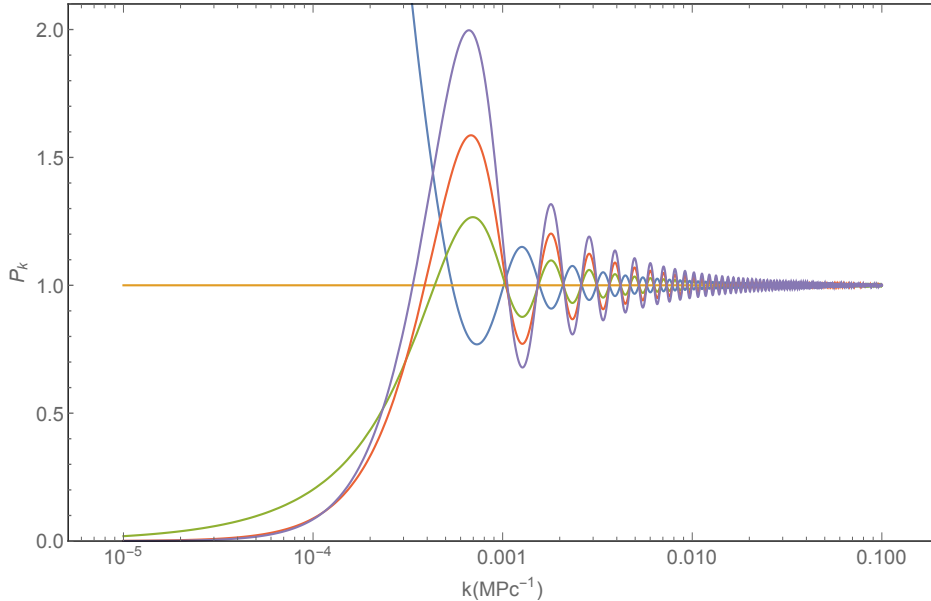


Figure 3. Normalized power spectrum for different values of the parameter ν . For $\nu = -4$ (blue) we have enhancement of power for low ℓ 's. For $\nu = -2$ (green), $\nu = -1$ (orange) and $\nu = 0$ (purple) we have suppression of power for low ℓ 's. For $\nu = -3$ we return to the four-dimensional case.

physically; it controls the moment at which the transition takes place. Changing this number shifts the power spectrum along the k -axis so its value fixes the transition point on the power spectrum graph. Modes that leave the horizon well after the transition evolve most of their lives in a pure $4d$ de Sitter stage and are therefore driven towards a flat spectrum. On the other hand, modes that left the horizon during the pre-inflationary stage were affected by different dynamics. The transition from one kind of spectrum to the other occurs at the scale of the horizon at time η_0 .

The effect of the parameter ν is two fold. On one hand, it controls the spectral index of the “initial power spectrum” of the mode functions; the one they would have if only the first stage of the evolution existed. This is easy to see by realizing that ν appears on the index of the Hankel functions and therefore on their asymptotic form. On the other hand, ν appears also in the expression for the jump of the first derivative of the mode functions. This has as its most dramatic effect a change on the height of the power spectrum at the transition region, in particular on its first peak. This is an effect that is clearly specific of our model and in principle it could set it apart from other similar models.

3.2 Curvature perturbations

To convert from scalar field modes to physical curvature perturbations, we introduce $\mathcal{R} = -\delta\phi/(\sqrt{2\epsilon} M_{Pl})$. Moreover, to convert from modes in pure de Sitter to modes in a slow-roll background, we replace $H \rightarrow H_*(k/k_*)^{-\epsilon_*}$ in (35) and replace $\epsilon \rightarrow \epsilon_*(k/k_*)^{4\epsilon_*-2\eta_*}$ in the conversion factor to \mathcal{R} , as different modes see a different Hubble parameter. This gives [27]

$$P_{\mathcal{R}}(k) = \mathcal{P}_{\phi}(k) \times \left(\mathcal{A}_{\mathcal{R}} \left(\frac{k}{k_*} \right)^{n_s-1} \right), \quad (3.28)$$

where $n_s - 1 = -6\epsilon_* + 2\eta_*$ and the amplitude is $\mathcal{A}_{\mathcal{R}} = H_*^2/(4M_{Pl}^2\epsilon_*)$ where the subscript $*$ refers to a pivot scale. Here $\mathcal{P}_\phi(k)$ is the normalized version of the power spectrum computed in Eq. (3.27), n_s is the spectral index which depends on the slow roll parameters and $\mathcal{A}_{\mathcal{R}}$ is the amplitude of the power spectrum at pivot scale k_* . We take the values of $\mathcal{A}_{\mathcal{R}}, k_*$ and n_s from the latest results from PLANCK [6].

3.3 The massive modes

As we mentioned in the previous sections the pre-inflationary phase will also excite the massive modes and not only the zero modes. However, any successful compactification mechanism must be the source of the dominant energy momentum tensor at the time of the transition. This means that an important assumption in our model is that the energy stored in these modes at the onset of inflation must be subleading. On the other hand, these modes are by definition more massive than the Hubble parameter during the second stage of evolution, during the $4d$ inflationary part and therefore their energy density will be further diluted by the expansion of the universe during that time. It is therefore likely that this modes do not play a significant role in the kind of models we are studying here.

4 Effects on the CMB

Using the CLASS code ([28]) and the cosmological parameters recently published by the PLANCK collaboration [6] we propagated the power spectrum of Eq. (3.28) to look at the observational effects of dimensional compactifications on the CMB temperature data.

As we described earlier the power spectrum is parametrized by setting the time of compactification η_0 and ν that describes the type of pre-inflationary dynamics. It is clear that using a very early compactification time would eliminate any possibility of observing any effect of this transition since it will push the scale of the transition outside of the CMB window of scales. On the other hand, it is also clear that this transition can not occur too late since this would ruin the agreement of the power spectrum with the CMB data currently well fitted by a simple power law. These considerations force us to limit the range of the time of the transition to be around the scale of $k = 0.001 \text{ Mpc}^{-1}$.

Next we investigate the effect of varying the value of ν keeping fixed the transition time. We show in Fig. (4) the corresponding power spectrum for the temperature fluctuations computed using different values of ν and compare the results to the PLANCK data. We show in Fig. (5) a close up of the low- l region for the same set of values.

We notice that the power is significantly enhanced for values of $|\nu| > 3$. This includes all the values that correspond to a higher dimensional de Sitter space prior to the slow roll period. This seems to indicate that if such transdimensional transition occurred it was too far in the past and its observational evidence is out of our reach.

For $|\nu| < 3$ there is a suppression on the power at small ℓ as it seems to be required by the PLANCK data. However, the details of the transition create a high bump on the power spectrum that makes the fit more problematic. This is particularly clear in the case of $\nu = 0$ where the suppression is present but the effects of the sharp transition undo this effect in the region of interest.

We have not attempted to make a systematic treatment of the data to see what would be the best fit value but things appear to get better for $-3 < \nu < -2$. On the other hand, these values seem to be at odds with the constraints based on the equation of state of the fluid required to create such dynamics. See the discussion in Section 2.

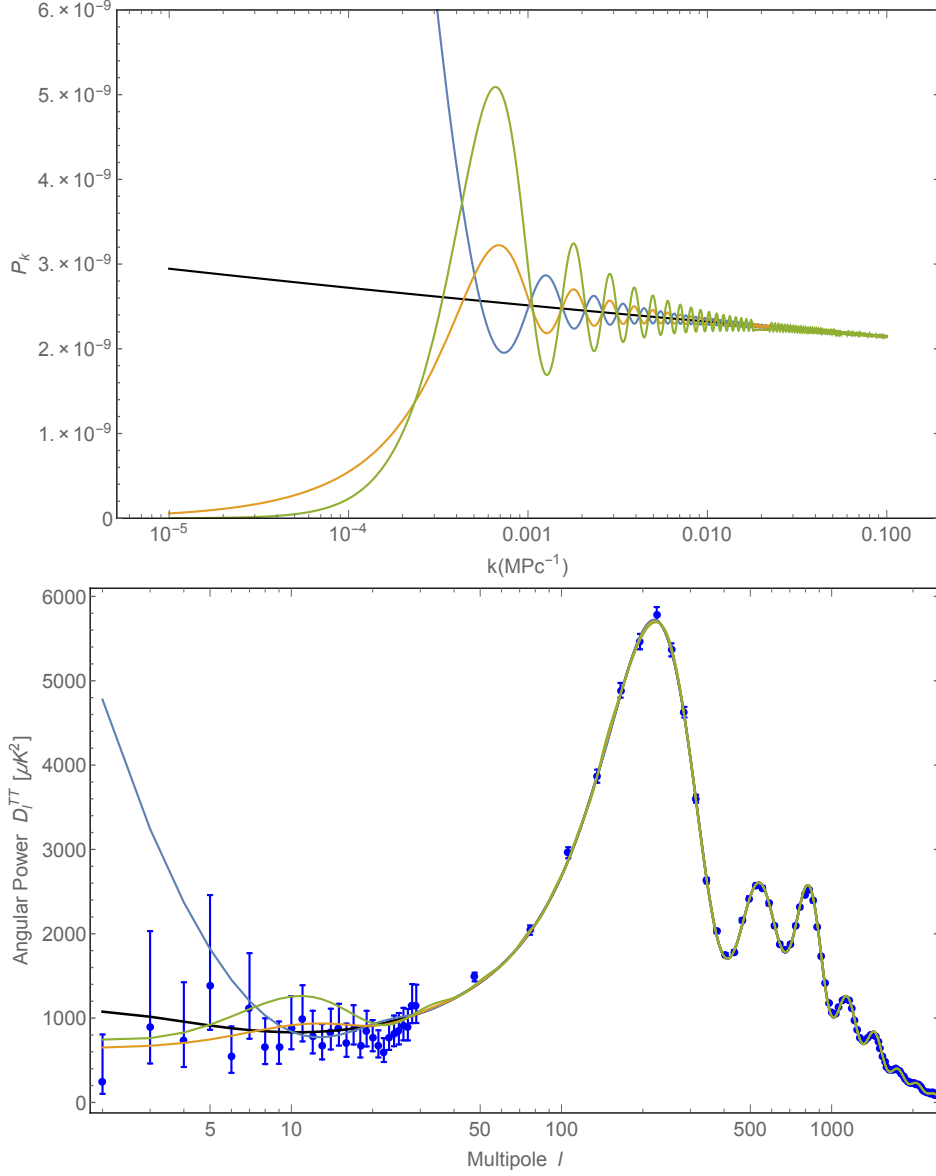


Figure 4. Comparison of the PLANCK data points (blue) with the standard Λ CDM model (black) and our model for: $\nu = -4$ (blue), $\nu = -2$ (orange) and $\nu = 0$ (green).

5 Conclusions

Observing evidence of the existence of extra dimensions would potentially be revolutionary. In this paper we showed if extra dimensions exist and they undergo a dynamical compactification process in the early universe, this could leave observable imprints on the CMB. This is possible if the subsequent $4d$ inflationary period did not last too long. Using a simple model for the higher dimensional evolution we investigated how the different parameters of this scenario would affect the spectrum of perturbations.

We found that this mechanism could lead to an enhancement or suppression of the fluctuations in the CMB on large scales. Comparison with the Planck data put important

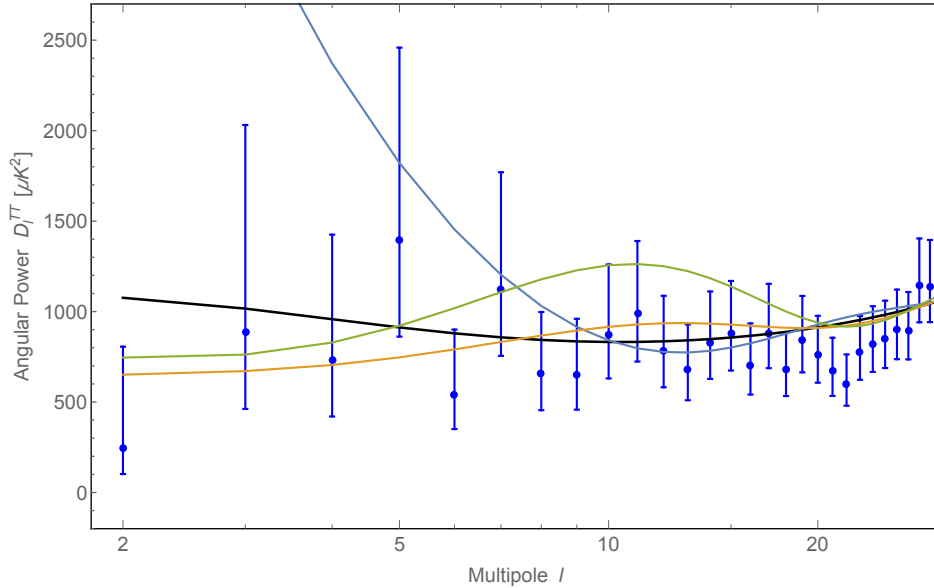


Figure 5. Low- l region of the temperature power spectrum for the same parameters as in Fig. 4.

constraints on this kind of transition occurring right before the beginning of inflation. In particular we show that a rapid transition from a higher dimensional de Sitter space to our $4d$ inflation period is ruled out unless there is a large number of e-folds after the transition.

One point of concern is that the data is better fit in a region of parameter space that requires a higher dimensional fluid with an equation of state outside of the range $-1 \leq w \leq 1$. Moreover, there are a number of other issues that remain to be investigated. Most importantly, this work does not provide a detailed mechanism of compactification but assumes a rapid transition. While we think this captures the basic effects of a dimensional compactification transition, a more realistic model is needed to verify the details of our results. In particular, one could argue that a detailed study of realistic compactifications will imply a slow transition. This will change the details of the power spectrum and likely making some of the models a better fit to the data.

Finally, it is important to investigate the necessary matter ingredients that are required to produce the transition itself. In particular, one should understand whether a transition of the kind discussed here would imply a violation of any energy conditions. In some cases this is obviously true. For example, a transition from a collapsing higher dimensional universe to an expanding effectively $4d$ case would violate the null energy condition at the bounce. It would be interesting to see if this is in fact a generic situation for the cases of interest.

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