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Supplementary Material: Transition from propagating polariton solitons to a standing wave condensate induced by interactions

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Numerical results on the propagation of polaritons in a semiconductor microcavity wire are presented here. Numerous references are given in the main text to link the supplementary material to the experimental results of the main text.

NUMERICAL MODEL

Equations of motion for the coherent mean field dynamics of cavity photons ψ and excitons χ can be written as (see, *e.g.*, [1]):

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m^*} + V(\mathbf{r}) + D_\psi(\mathbf{r}) + \Delta - i\frac{\hbar\Gamma}{2}\right]\psi + \frac{\hbar\Omega}{2}\chi + P_0 e^{-(x^2+y^2)/2w^2} e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})}, \tag{S1}$$

$$i\hbar\frac{\partial\chi}{\partial t} = \left[\alpha|\chi|^2 - i\frac{\hbar\Gamma_\chi}{2}\right]\chi + \frac{\hbar\Omega}{2}\psi. \tag{S2}$$

Here ∇^2 is the 2D Laplacian, $m^* = 5\times10^{-5}m_0$ is the cavity photon mass and m_0 is the free electron mass, $\Gamma = \Gamma_\chi/3 = 1/30$ ps^{-1} are average decay rates corresponding to the cavity losses and exciton dephasing (non-radiative processes), $V(\mathbf{r})$ is the optical confinement of the microwire, D_ψ is the photonic disorder [2], $\alpha = 2$ μeV μm^2 is the exciton interaction strength, $\Delta = -4.07$ meV is the photon-exciton detuning and $\hbar\Omega = 4.12$ meV is the Rabi splitting. The resonant pump is described by the amplitude P_0, energy $\hbar\omega$ and wavevector $\mathbf{k} = (2.3\ \mu\text{m}^{-1})\ \hat{\mathbf{x}} + (0.15\ \mu\text{m}^{-1})\ \hat{\mathbf{y}}$, full width half maximum (FWHM) of 20 μm. The small momentum along y is chosen to account for error in the experiment. A longer excitation time of 15 ps FWHM (as opposed to 5-7 ps of the experiment) is taken to induce a greater initial population of the microwire's excited states. In the experiment, higher order transverse modes can be also populated by Rayleigh scattering from the edges of the etched MCW, which is not accounted for in the modelling. The energy $\hbar\omega$ is chosen so that the laser is blueshifted by 0.3 meV above the lowest polariton subband.

The photon disorder potential is taken to be Gauss correlated in space defined by correlation length $\xi_\psi = 1$ μm and amplitude 25 μeV, as we estimate it to be in our sample due to imperfections in the fabrication and growth of MCW [3]. The confining potential $V(\mathbf{r})$ and an example disorder $D_\psi(\mathbf{r})$ are shown in Fig. **S1**.

In Fig. **S2** we show numerically time resolved dispersions of the microwire polaritons for $k_x = 0$. Results on polariton kinetics, analogous to experimental observations, are displayed in Figs. **S3**-**S5** at three different pump amplitudes $P_0 = 1,\ 5,\ 7$ meV μm^{-2} respectively, corresponding to Figs. 2-4 of the main text. The pseudo-colour scale for Figs. **S3**-**S5** is linear.

In the linear regime (Fig. **S3**) the polaritonic wave-packet mainly populates the $n = 0, 1, 2$ modes of the microwire. As a result of interference between the different excited modes (see Eq. S3) snaking of the polariton wavepacket is observed along the microwire.

Under an intermediate excitation (Fig. **S4**) the nonlinear focusing results in the formation of a soliton which remains focused under several reflections off the microwire ends. A strong excitation (Fig. **S5**) results in the relaxation of the polariton energy towards the ground state (with zero momenta) and formation of the standing modes within the wire. An additional feature of the strong excitation is exciton-like states with high momenta (see Fig. **S5**j) in the range of k_y from -3 to $+3$ μm^{-1} at $k_x \simeq 3$ μm^{-1}.

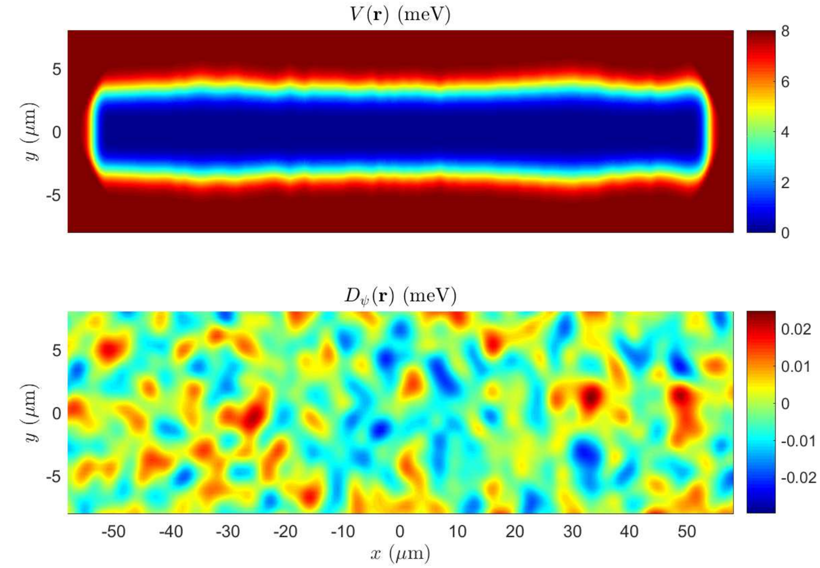

Figure **S1**. (a) Confining potential $V(\mathbf{r})$. (b) Photon disorder calculated with 1 μm Gaussian correlation length.

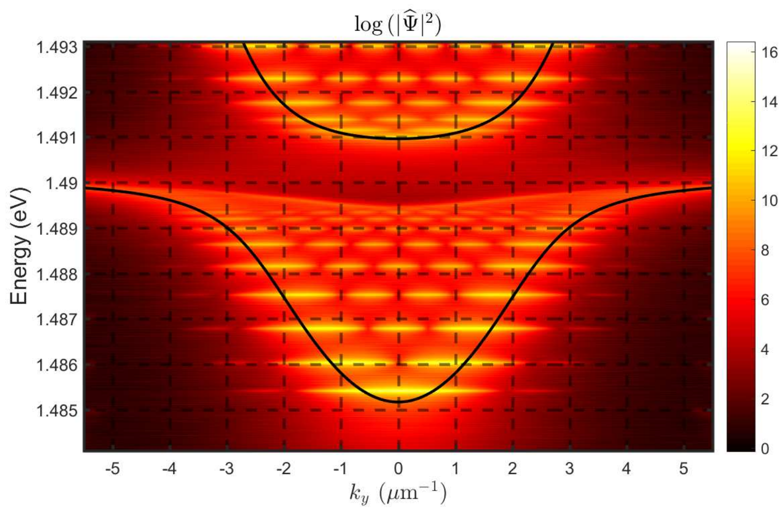

Figure **S2**. Time resolved dispersion along the y-coordinate at $k_x = 0$. Black solid lines are the analytical upper and lower polariton dispersion curves. Black dashed lines are guide for the eye. Colour scale is logarithmic.

APPEARANCE OF SNAKING

The appearance of *snaking* in the lower polariton macroscopic wavefunction within the microwire at low excitation strengths can be derived from a simple superposition of excited modes within the system,

$$\Psi(\mathbf{r}, t) = c_0 X_0(x, t) Y_0(y) + c_1 X_1(x, t) Y_1(y),$$

where $N = |c_0|^2 + |c_1|^2$ is the total polariton particle number. Time independent quantised states along the y-coordinate are taken as the infinite quantum well eigensolutions,

$$Y_n(y) = \sqrt{\frac{2}{L}} \sin\left[(n+1)\pi\left(\frac{y}{L} - \frac{1}{2}\right)\right], \quad n = 0, 1, 2\ldots, \quad y \in [-L/2, L/2].$$

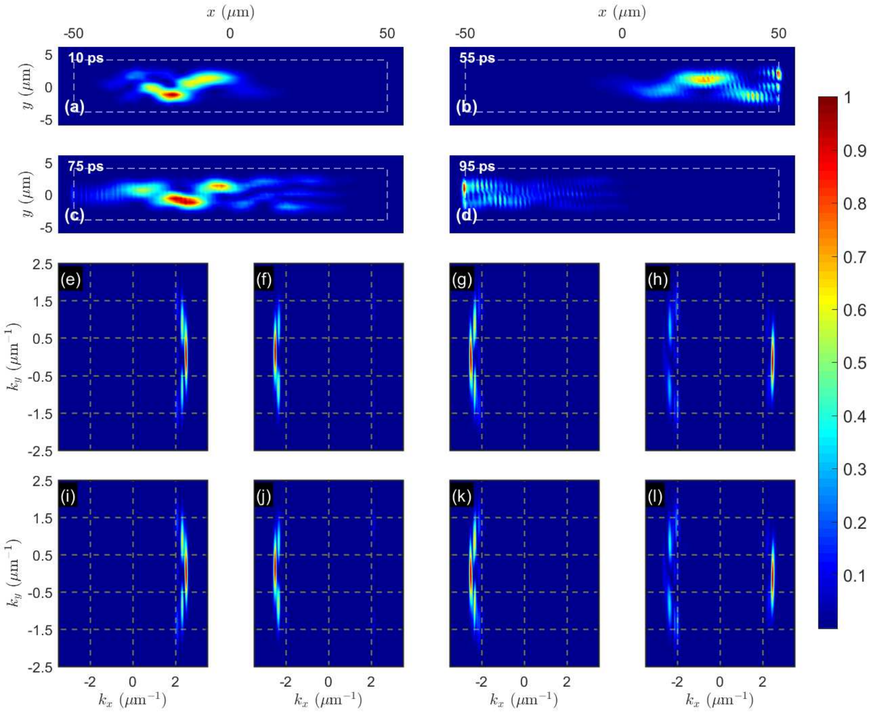

Figure **S3**. Low amplitude excitation, $P_0 = 1$ meV μm^{-2}. (a-d) Real- and (e-h) reciprocal space intensity of the cavity photon wavefunction ψ at different times. (i-l) Exciton intensity in reciprocal space. Yellow dashed lines are guide for the eye.

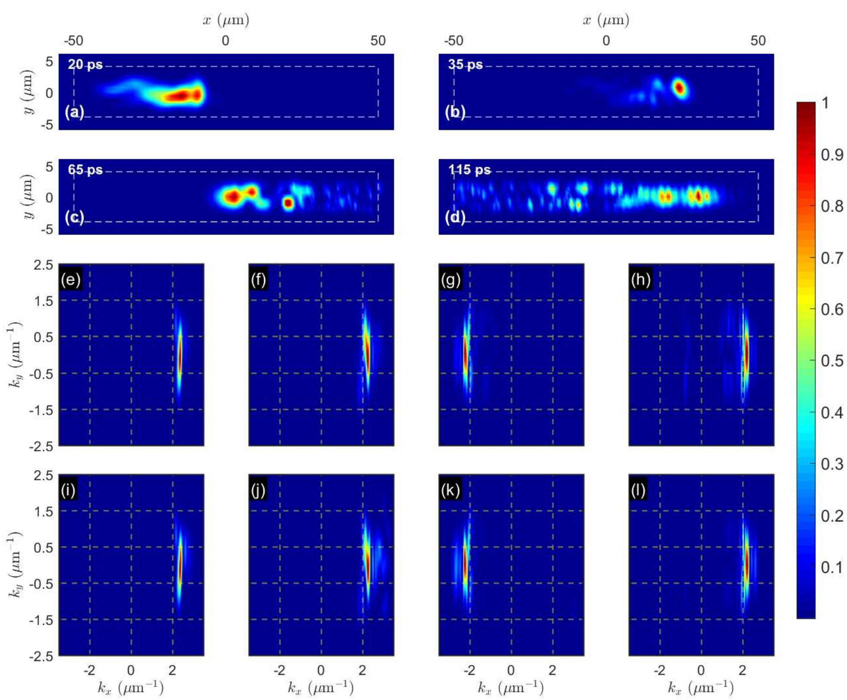

Figure **S4**. Intermediate amplitude excitation, $P_0 = 5$ meV μm^{-2}. (a-d) Real- and (e-h) reciprocal space intensity of the cavity photon wavefunction ψ at different times. (i-l) Exciton intensity in reciprocal space. Yellow dashed lines are guide for the eye.

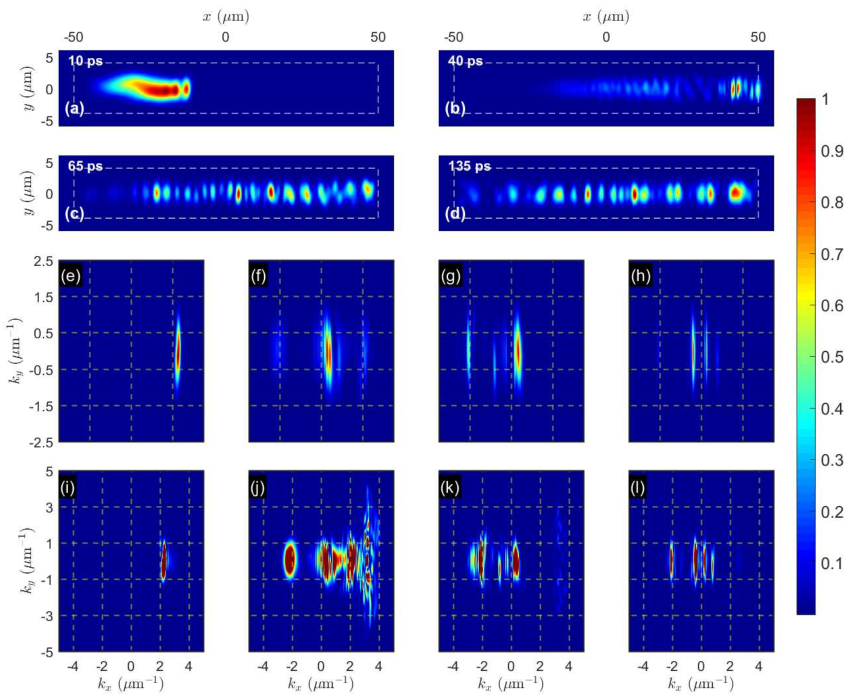

Figure **S5**. High amplitude excitation, $P_0 = 7$ meV μm^{-2}. (a-d) Real- and (e-h) reciprocal space intensity of the cavity photon wavefunction ψ at different times. (i-l) Exciton intensity in reciprocal space with a colorscale saturated at 20% max intensity to bring out the high momenta states more clearly. Yellow dashed lines are guide for the eye.

with energies $E_n = (n+1)^2\pi^2\hbar^2/2mL^2$. We assume a parabolic dispersion in k_x for each state n. The group velocity of the n-th mode evolving as a time dependent diffusive wavepacket along the x-coordinate is written,

$$v_{n,g} = \frac{\partial \omega_n(k_n)}{\partial k_n} = \frac{\hbar k_n}{m},$$

where

$$\hbar\omega_n(k_n) = E_n + \frac{\hbar^2 k_n^2}{2m}.$$

The normalised diffusive wavepacket travelling along the microwire (x-coordinate) is then written [4],

$$X_n(x,t) = \sqrt{\frac{1}{\sqrt{\pi}\sigma(1 + i\hbar t/m\sigma^2)}} \exp\left[-\frac{(x - \hbar k_n t/m)^2}{2(\sigma^2 + i\hbar t/m)} + i(k_n x - \omega_n t)\right].$$

Here we focus on the case of the quasi-resonant laser beam with frequency $\hbar\omega$ exciting the ground and first excited state dispersions corresponding to momenta k_0 and k_1 along the x-axis. It can then be shown that the mean wavepacket position along the y-axis evolves according to

$$\begin{aligned} \langle y(t) \rangle &= \int \Psi(\mathbf{r},t)^* y \Psi(\mathbf{r},t) d\mathbf{r} \\ &= 2\Re\left\{ c_0^* c_1 \int_{-\infty}^{\infty} y Y_0(y) Y_1(y) dy \int_{-\infty}^{\infty} X_0^*(x,t) X_1(x,t) dx \right\} \\ &= 2|c_0 c_1| L \left(\frac{4}{3\pi}\right)^2 \cos\left(\frac{\hbar(k_0^2 - k_1^2)}{2m} t - \phi\right) \exp\left(-\frac{(k_0 - k_1)^2\sigma^2}{4}\right), \end{aligned} \tag{S3}$$

where $\phi = \phi_1 - \phi_0$ is the phase difference between c_0 and c_1. The observed *snaking* of the excited polariton wavepacket can therefore be regarded as interference between the two excited modes resulting in oscillatory movement of the mean polariton wavefunction location along the y-coordinate with frequency $\omega_s = \hbar(k_0^2 - k_1^2)/2m$. The generalisation to any number of N excited modes is straightforward and the mean wavepacket position along the y-axis is given by:

$$\langle y(t)\rangle = 2 \sum_{l<n<N} |c_l c_n| \xi_{ln} \cos\left(\frac{\hbar(k_l^2 - k_n^2)}{2m} t - \phi_{ln}\right) \exp\left(-\frac{(k_l - k_n)^2 \sigma^2}{4}\right), \tag{S4}$$

where

$$\xi_{ln} = \int_{-\infty}^{\infty} y Y_l(y) Y_n(y) dy. \tag{S5}$$

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