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The Gibbons-Hawking radiation of gravitons in the Poincaré and static patches of de Sitter spacetime

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We discuss the quantization of linearized gravity in the background de Sitter spacetime using a gauge-invariant formalism to write the perturbed gravitational field in the static patch. This field is quantized after fixing the gauge completely. The response rate of this field to monochromatic multipole sources is then computed in the thermal equilibrium state with the well known Gibbons-Hawking temperature. We compare this response rate with the one obtained in the Bunch-Davies-like vacuum state defined in the Poincaré patch. These response rates are found to be the same as expected. This agreement serves as a verification of the infrared finite graviton two-point function in the static patch of de Sitter spacetime found previously.

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I. INTRODUCTION

Physics in de Sitter spacetime is an interesting subject in its own right but it has increased its importance because the Universe's early stage of expansion is believed to have happened in a de Sitter-like phase [1–5]. Moreover, the accelerated expansion of our Universe [6] means that de Sitter spacetime is likely to approximate its late stages of evolution as well.

It is well known that the graviton two-point function is divergent in the infrared (IR) in the synchronous-transverse-traceless gauge in the Poincaré patch, or the spatially-flat patch, of de Sitter spacetime [7]. These divergences arise because the graviton mode functions reduce to those of the massless minimally-coupled scalar field that suffers from IR divergences [8]. In fact it is known that there is no Hadamard state invariant under the de Sitter group for massless minimally-coupled scalar field in de Sitter spacetime [9]. It has been claimed that there is no de Sitter-invariant vacuum state for linearized gravity because of these and other IR divergences [see, e.g. Refs. [10–14]]. However, since the gravitational field is a gauge field unlike the scalar field, it is possible that these IR divergences can be a gauge artifact.

Indeed it has been shown that the IR-divergent part of the graviton two-point function mentioned above can be expressed in a pure-gauge form [15–17]. More recently, it was shown that the graviton mode functions can be modified by large gauge transformations corresponding to global shear transformations to make the two-point function IR finite and, hence, de Sitter invariant [18]. Some authors object by asserting that a large gauge transformation, which by definition affects spatial infinity, would change physics [19, 20]. However, as pointed out in Ref [18], a large gauge transformation is equivalent to a local one as long as one is interested only in local physics.

It is also interesting to point out that the graviton two-point function constructed in the hyperbolic patch [21], global patch [22] and static patch [23, 24] are all IR finite. These IR-finite two-point functions are consistent with the fact that the IR divergences in the two-point function constructed in the Poincaré patch can be gauged away by (large) gauge transformations.

Now, the Bunch-Davies, or Euclidean, vacuum state [25–27] is a thermal state of temperature $H/2\pi$, where H is the Hubble constant for the de Sitter expansion, with respect to the energy corresponding to the time translation in the static patch [28]. This fact, which we call the Gibbons-Hawking effect, is closely related to the Hawking radiation [29] and the Unruh effect [30, 31]. Strictly speaking, the Gibbons-Hawking effect has not been shown for the graviton field, but the two-point function of Refs. [23, 24] was found assuming this effect. That is, this two-point function is for the thermal state of gravitons with temperature $H/2\pi$ in the static patch of de Sitter spacetime.

In this paper we verify that the Bunch-Davies-like state for the graviton field in the Poincaré patch of de Sitter spacetime, which has an IR-divergent two-point function, is indeed the thermal equilibrium state with temperature $H/2\pi$ in the static patch, which has an IR-finite two-point function. We do so by showing that a conserved multipole point source responds to the graviton field in the Bunch-Davies-like state as if it was placed in a thermal bath of temperature $H/2\pi$ with respect to the energy corresponding to the time translation in the static patch. Similar calculations have been done for the scalar and vector fields in Ref. [32]. Similar comparisons between response rates of sources in Schwarzschild spacetime have also been made in the context of the Hawking and Unruh effects in Refs. [33–36].

The rest of the paper is organized as follows. In Sec. II we describe the linearized gravitational field (gravita-

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tional perturbations) in (3 + 1)-dimensional de Sitter spacetime and present the mode functions for these perturbations in spherical polar coordinates in the Poincaré patch. In Sec. III, we describe our method of quantization of the gravitation field and determine the normalization constants for the modes found in Sec. II such that the annihilation and creation operators satisfy the standard commutation relations. We also review the quantization of the linearized gravitational field in the static patch presented in Refs. [23, 24]. In Sec. IV we verify the Gibbons-Hawking effect for the gravitational field by comparing the response rates to a conserved multipole source in the Bunch-Davies-like state in the Poincaré patch and in the thermal equilibrium with temperature $H/2\pi$ in the static patch. We conclude this paper with some remarks in Sec. V. In Appendix A we present a derivation of the expansion of the gravitational plane wave in terms of the modes in spherical polar coordinates. Throughout this paper we use the metric signature $-+++$ and natural units such that $G = c = \hbar = k_B = 1$.

II. GRAVITATIONAL PERTURBATIONS IN THE POINCARÉ PATCH OF DE SITTER SPACETIME

A. Background de Sitter Spacetime

The line element covering the expanding half of de Sitter spacetime (Poincaré patch) is given by:

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + e^{2H\tau}\left(\mathrm{d}\rho^2 + \rho^2 \mathrm{d}\Omega_2^2\right), \tag{1}$$

where

$$\mathrm{d}\Omega_2^2 = \gamma_{ij}\mathrm{d}\hat{x}^i\mathrm{d}\hat{x}^j = \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2 \tag{2}$$

is the line element on the unit 2-sphere. We reserve the letters from the Latin alphabet starting from $i, j, k, \ldots$ to denote angular components. The (metric compatible) covariant derivative on the 2-sphere is denoted by $\hat{D}_i$. We also indicate any quantity on the 2-sphere with a hat over it. The line element which describes the static patch of de Sitter spacetime reads

$$\mathrm{d}s^2 = -\left(1 - H^2r^2\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - H^2r^2} + r^2\mathrm{d}\Omega_2^2, \tag{3}$$

where the coordinates t and r are given in terms of the coordinates τ and ρ as follows:

$$r = \rho e^{H\tau}, \tag{4}$$

$$t = \tau - \frac{1}{2H}\ln\left(1 - \rho^2 e^{2H\tau}\right). \tag{5}$$

The Hubble constant H is related to the cosmological constant Λ by $\Lambda = 3H^2$.

B. Linearized Gravity in the Poincaré patch of de Sitter spacetime

The Einstein-Hilbert action with a cosmological constant term is given by

$$S_{EH} = \frac{1}{16\pi G}\int \sqrt{-\tilde{g}}(\tilde{R} - 2\Lambda)d^4x. \tag{6}$$

The action for gravitational perturbations in a background spacetime (or linearized gravity) can be obtained by expanding the action S_{EH} about a background metric, i.e. by writing $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \sqrt{32\pi G}\, h_{\mu\nu}$, and retaining only terms of second order in $h_{\mu\nu}$[1]. In our case, the background metric $g_{\mu\nu}$ is the de Sitter metric and we obtain the following quadratic Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left[\nabla_\mu h^{\mu\lambda}\nabla^\nu h_{\nu\lambda} - \frac{1}{2}\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} + \frac{1}{2}(\nabla^\mu h - 2\nabla_\nu h^{\mu\nu})\nabla_\mu h - H^2\left(h_{\mu\nu}h^{\mu\nu} + \frac{h^2}{2}\right)\right], \tag{7}$$

where $h = g^{\mu\nu}h_{\mu\nu}$. The resulting Euler-Lagrange field equation is

$$h_{\mu\nu} - 2\nabla_{(\mu|}\nabla_\lambda h^\lambda{}_{|\nu)} + g_{\mu\nu}\nabla_\lambda\nabla_\sigma h^{\lambda\sigma} + \nabla_\mu\nabla_\nu h - g_{\mu\nu}\Box h - 2H^2\left(h_{\mu\nu} + \frac{1}{2}g_{\mu\nu}h\right) = 0. \tag{8}$$

Due to the general coordinate invariance of the full Einstein-Hilbert action, the linearized theory is invariant under the gauge transformation

$$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu, \tag{9}$$

provided that the background spacetime is a vacuum solution to the Einstein's field equation with a cosmological constant [37]. We can choose a gauge such that $h = \nabla_\mu h^{\mu\nu} = 0$, which greatly simplifies the equation of motion to

$$\left(\Box - 2H^2\right)h_{\mu\nu} = 0. \tag{10}$$

(See, e.g. Ref. [38] for justification of this gauge.) We shall find the solutions to Eq. (10) in the Poincaré patch in spherical polar coordinates. Thus, we expand the field $h_{\mu\nu}$ in terms of harmonic tensors, following Refs. [39, 40]. In $3+1$ dimensions, there will be metric perturbations (i) of the scalar type, for which the angular dependence comes from the scalar spherical harmonics and their covariant derivatives, and (ii) of the vector type, with angular dependence described by vector spherical harmonics and their covariant derivatives. Additionally, there are

[1] The zeroth order term is the Einstein-Hilbert action for the background solution and the linear term is a total derivative.

perturbations of the so-called tensor type, with angular dependence described by rank 2 tensor spherical harmonics in higher dimensions. However, as is well known, there are no rank 2 tensor spherical harmonics on the 2-sphere [41], and hence we do not need to consider them here. The scalar spherical harmonics and their derivatives are orthogonal to the vector spherical harmonics and their derivatives with respect to the integration on the unit 2-sphere.

In the Poincaré patch, it is convenient to make an additional gauge choice in which $h_{\mu\tau}=0$. This set of gauge conditions is called the synchronous-transverse-traceless (STT) gauge. (This gauge choice is possible because the $h_{\mu\tau}$ component comes from transverse-traceless solutions to Eq. (10) of the pure-gauge form $h_{\mu\nu}=\nabla_\mu\xi_\nu+\nabla_\nu\xi_\mu$.)

It follows that the non-vanishing positive-frequency[2] components of the scalar-type perturbations, satisfying the gauge constraints, read

$$h^{(S;klm)}_{\rho\rho}=\frac{A^{kl}_S}{\rho^2}\Phi^{kl}(\tau,\rho)\mathbb{S}^{(lm)}, \tag{11}$$

$$h^{(S;klm)}_{\rho i}=-\frac{A^{kl}_S\mathbb{S}^{(lm)}_i}{k_S}\left(\frac{\partial}{\partial\rho}+\frac{1}{\rho}\right)\Phi^{kl}(\tau,\rho), \tag{12}$$

$$h^{(S;klm)}_{ij}=A^{kl}_S\mathbb{S}^{(lm)}_{ij}\Psi^{kl}(\tau,\rho)-\frac{A^{kl}_S}{2}\gamma_{ij}\Phi^{kl}(\tau,\rho)\mathbb{S}^{(lm)}, \tag{13}$$

where $\mathbb{S}^{(lm)}=\mathbb{S}^{(lm)}(\theta,\phi)$ are the scalar spherical harmonics, which satisfy

$$[\hat{D}_i\hat{D}^i+k_S^2]\mathbb{S}^{(lm)}(\theta,\phi)=0. \tag{14}$$

The eigenvalues k_S^2 are

$$k_S^2=l(l+1),\quad l=0,1,2,\dots \tag{15}$$

Solutions to Eq. (14) are given by

$$\mathbb{S}^{(lm)}(\theta,\phi)=\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}P^{|m|}_l(\cos\theta)e^{im\phi}. \tag{16}$$

The tensors $\mathbb{S}^{(lm)}_i(\theta,\phi)$ and $\mathbb{S}^{(lm)}_{ij}(\theta,\phi)$ are given by

$$\mathbb{S}^{(lm)}_i(\theta,\phi)=-\frac{\hat{D}_i\mathbb{S}^{(lm)}(\theta,\phi)}{k_S} \tag{17}$$

and

$$\mathbb{S}^{(lm)}_{ij}(\theta,\phi)=\left[\frac{\hat{D}_i\hat{D}_j}{k_S^2}+\frac{1}{2}\gamma_{ij}\right]\mathbb{S}^{(lm)}(\theta,\phi). \tag{18}$$

The field $\Phi^{kl}(\tau,\rho)$ is a master variable and $\Psi^{kl}(\tau,\rho)$ reads

$$\Psi^{kl}=\frac{2\rho^2}{(l-1)(l+2)}\left[\frac{\partial^2}{\partial\rho^2}+\frac{3}{\rho}\frac{\partial}{\partial\rho}-\frac{(l-1)(l+2)}{2\rho^2}\right]\Phi^{kl}. \tag{19}$$

[2] The meaning of "positive-frequency" will be clarified later.

The normalization constants A^{kl}_S will be determined later.

It is not possible to find $h_{\mu\nu}$ satisfying the STT gauge conditions in this form if $l=0$ or 1. There are solutions with $l=0,1$ which are not in this form, but they are either singular at the origin or of pure-gauge form. Thus, we only need to consider the values of l larger than or equal to 2. To emphasize this point we have outlined, in Appendix A, the expansion of the gravitational plane wave in terms of the modes in spherical polar coordinates, where only the modes with $l\geq 2$ are present.

The non-vanishing components of the vector-type metric perturbations can be written as

$$h^{(V;klm)}_{\rho i}=A^{kl}_V\Phi^{kl}(\tau,\rho)\mathbb{V}^{(lm)}_i, \tag{20}$$

$$h^{(V;klm)}_{ij}=-\frac{2k_VA^{kl}_V\rho^2\mathbb{V}^{(lm)}_{ij}}{(l-1)(l+2)}\left(\frac{\partial}{\partial\rho}+\frac{2}{\rho}\right)\Phi^{kl}(\tau,\rho). \tag{21}$$

The vector spherical harmonics satisfy

$$(\hat{D}_j\hat{D}^j+k_V^2)\mathbb{V}^{(lm)}_i=0,\quad \hat{D}^i\mathbb{V}^{(lm)}_i=0, \tag{22}$$

with

$$k_V^2=l(l+1)-1,\quad l=1,2,3,\dots. \tag{23}$$

The tensor $\mathbb{V}^{(lm)}_{ij}$ is written as

$$\mathbb{V}^{(lm)}_{ij}=-\frac{1}{2k_V}(\hat{D}_i\mathbb{V}_j+\hat{D}_j\mathbb{V}_i). \tag{24}$$

On the unit 2-sphere, one can write solutions to Eqs. (22) as

$$\mathbb{V}^{(lm)}_i(\theta,\phi)=\frac{\epsilon_{ij}}{\sqrt{l(l+1)}}\hat{D}^j\mathbb{S}^{(lm)}(\theta,\phi), \tag{25}$$

where ϵ_{ij} is the Levi-Civita tensor on S^2, defined by

$$\epsilon_{\theta\theta}=\epsilon_{\phi\phi}=0, \tag{26}$$

$$\epsilon_{\theta\phi}=-\epsilon_{\phi\theta}=\sin\theta. \tag{27}$$

As in the scalar-type case there are no solutions to the gauge conditions of this form if $l=1$. (There are no vector spherical harmonics for $l=0$ as can be seen from the definition 25.) For the same reason as for the scalar-type case, we only need to consider the case with $l\geq 2$. The normalization constants A^{kl}_V will be chosen later.

For the scalar- and vector-type perturbations to solve the equations of motion given by Eq. (10), the master variable $\Phi^{kl}(\tau,\rho)$ takes the following form (or its complex conjugate or a linear combination of the two):

$$\Phi^{kl}(\tau,\rho)=\frac{ke^{\frac{H\tau}{2}}}{\sqrt{2H}}\mathrm{H}^{(1)}_{\frac{3}{2}}\left(\frac{k}{H}e^{-H\tau}\right)j_l(k\rho), \tag{28}$$

where $\mathrm{H}^{(1)}_{\frac{3}{2}}\left(\frac{k}{H}e^{-H\tau}\right)$ is the Hankel's function of the first kind, $j_l(k\rho)$ is the spherical Bessel function of the first kind, k is a positive constant, and the overall constant factor has been chosen for later convenience. The time-dependence of $\Phi^{kl}(\tau,\rho)$ is the same as that for the plane-wave modes.

We now give a criterion to specify positive-frequency solutions in this setting. We require that Φ^{kl} for the positive-frequency solutions of the gravitational perturbations to satisfy

$$\frac{\partial}{\partial\tau}\Phi^{kl} \approx -ike^{-H\tau}\Phi^{kl}, \qquad (29)$$

in the limit $k\to\infty$. In other words, it should approach the positive-frequency solution in flat spacetime in the short wavelength limit[3]. Note that Φ^{kl} given in Eq. (28) satisfies this requirement. Now, one of the de Sitter boosts, $\tau\to\tau+\alpha$, $\rho\to e^{-\alpha}\rho$, transforms the solution Φ^{kl} to $\Phi^{ke^{-\alpha},l}$. Thus, once we choose the solutions (28) as the positive-frequency solutions for large k, we need to choose them as such for arbitrary k to preserve the de Sitter invariance of the set of positive-frequency solutions, which leads to the de Sitter invariance of the vacuum state (see, e.g. [16]). This choice of positive-frequency solutions corresponds to the Bunch-Davies-like state, which is the standard choice of the vacuum [42]. From now on, we also set the Hubble constant to unity, i.e. $H=1$.

III. QUANTIZATION OF METRIC PERTURBATIONS

To quantize the field $h_{\mu\nu}$, we follow a standard procedure outlined, for example, in Refs. [23, 43], which follow the general framework given in Ref. [44]. We first define the symplectic product between two solutions of the equations of motion, given by Eq. (8), to be

$$\Omega(h,h') \equiv \int_\Sigma \mathrm{d}\Sigma n_\alpha(h_{\mu\nu}p'^{\alpha\mu\nu} - p^{\alpha\mu\nu}h'_{\mu\nu}), \qquad (30)$$

where Σ is a Cauchy surface of a given patch of the spacetime with future-directed unit normal n^α and $p^{\alpha\mu\nu}$ is the conjugate momentum current defined by

$$p^{\alpha\mu\nu} \equiv \frac{1}{\sqrt{-g}}\frac{\partial\mathcal{L}}{\partial(\nabla_\alpha h_{\mu\nu})}. \qquad (31)$$

This symplectic product is independent of the choice of the Cauchy surface [45].

We choose the set of positive-frequency solutions given in Sec. II, together with their complex conjugates, as a basis for the solutions to the free field equations (8) in the STT gauge. Then we define the inner product

$$\langle h,h'\rangle = -i\Omega(\overline{h},h'), \qquad (32)$$

where $\overline{h}_{\mu\nu}$ is the complex conjugate of $h_{\mu\nu}$. A positive- and a negative-frequency solutions are mutually orthogonal with respect to this inner product. Moreover, the inner product (32) is positive definite on the space of positive-frequency solutions. Note that, since the STT gauge fixes the gauge completely, the symplectic product is non-degenerate. In other words, there are no solutions $h^{(\mathrm{null})}_{\mu\nu}$ in the STT gauge satisfying $\Omega(h^{(\mathrm{null})},h)=0$, for all solutions $h_{\mu\nu}$. (In our case it can readily be verified that all such solutions to Eq. (8) are pure-gauge solutions of the form $\nabla_\mu\xi_\nu+\nabla_\nu\xi_\mu$.) Thus, we are considering only the space of physical solutions, i.e. all gauge degrees of freedom are eliminated, and the inner product (32) is positive definite in the space of positive-frequency solutions.

[3] Note that the proper wave number is given by $ke^{-H\tau}$ in this case.

A. Quantization in the Poincaré patch

The quantum field $h_{\mu\nu}$ (in the STT gauge) can be expanded as

$$h_{\mu\nu} = \sum_{P,l,m}\int \mathrm{d}k\left[a^P_{lm}(k)h^{(P;klm)}_{\mu\nu} + a^P_{lm}(k)^\dagger\overline{h^{(P;klm)}_{\mu\nu}}\right], \qquad (33)$$

where the label $P=S,V$ stands for scalar-type or vector-type perturbations, respectively, and the classical solutions $h^{(P;klm)}_{\mu\nu}$ are the positive-frequency solutions given by Eqs. (11)- (13) and Eqs. (20) and (21). The canonical equal-time commutation relations are equivalent to

$$[a^P_{lm}(k), a^{P'}_{l'm'}(k')^\dagger] = \delta^{PP'}\delta^{ll'}\delta^{mm'}\delta(k-k') \qquad (34)$$

and

$$[a^P_{lm}(k), a^{P'}_{l'm'}(k')] = [a^P_{lm}(k)^\dagger, a^{P'}_{l'm'}(k')^\dagger] = 0, \qquad (35)$$

provided the complete set of positive-frequency solutions are normalized with respect to the inner product (32), i.e. if

$$\langle h^{(P;klm)}, h^{(P';k'l'm')}\rangle = \delta^{PP'}\delta^{ll'}\delta^{mm'}\delta(k-k'). \qquad (36)$$

Then, the vacuum $|0\rangle$, defined to be the state annihilated by all $a^P_{lm}(k)$, is the standard Bunch-Davies-like state.

In the STT gauge, the conjugate momentum current is simply $p^{\alpha\mu\nu}=-\nabla^\alpha h^{\mu\nu}$, so that the inner product can be written as

$$\langle h,h'\rangle = -i\int_\Sigma \mathrm{d}\Sigma\left(h'_{\mu\nu}\partial_\tau\overline{h^{\mu\nu}} - \overline{h_{\mu\nu}}\partial_\tau h'^{\mu\nu}\right), \qquad (37)$$

where, in this case, Σ is a $\tau=$ constant hypersurface. Using Eq. (37) and the identities

$$\overline{\mathrm{H}^{(1)}_\nu(x)}\partial_x\mathrm{H}^{(1)}_\nu(x) - \mathrm{H}^{(1)}_\nu(x)\partial_x\overline{\mathrm{H}^{(1)}_\nu(x)} = \frac{4ie^{\pi\mathrm{Im}\nu}}{\pi x} \qquad (38)$$

and

$$\int_0^\infty \mathrm{d}\rho\,\rho^2\,j_l(k\rho)j_l(k'\rho) = \frac{2}{\pi k^2}\delta(k-k'), \qquad (39)$$

one can readily compute the normalization constants for $h^{(P;klm)}_{\mu\nu}$, with $P=S$ and V, defined by Eqs. (11)-(13)

and Eqs. (20) and (21), respectively. After some cumbersome but straightforward computations, we obtain

$$A_V^{kl} = \frac{1}{k}\sqrt{\frac{(l-1)(l+2)}{2}} \tag{40}$$

and

$$A_S^{kl} = \frac{1}{k^2}\sqrt{\frac{(l-1)l(l+1)(l+2)}{2}}. \tag{41}$$

B. Quantization in the static patch

In Refs. [23, 24], the quantization procedure outlined in the previous subsection was used to quantize the metric perturbations in the static patch of de Sitter spacetime. We review it here for completeness.

One can write the non-vanishing components of the (positive-frequency) scalar-type metric perturbations as

$$h_{ab}^{(S;\omega lm)} = \mathbb{S}^{(lm)}\left(D_a D_b - \frac{1}{2}g_{ab}\Box\right)\left(r\psi_S^{\omega l}\right), \tag{42}$$

$$h_{ij}^{(S;\omega lm)} = \frac{r^2}{2}\gamma_{ij}\mathbb{S}^{(lm)}(\Box+2)\left(r\psi_S^{\omega l}\right), \tag{43}$$

where $\psi_S^{\omega l}$ is the master field for this case (see Ref. [23] for the details). The first letters of the Latin alphabet $(a, b, c, \ldots)$ are used to denote components in the orbit spacetime spanned by the t and r coordinates, with metric

$$\mathrm{d}s^2_{\text{orbit}} = -\left(1-r^2\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-r^2}. \tag{44}$$

The derivative operator D_a is the covariant derivative on this spacetime. The positive-frequency vector-type perturbations read

$$h_{ai}^{(V;\omega lm)} = \epsilon_{ab}D^b\left(r\psi_V^{\omega l}\right)\mathbb{V}_i^{(lm)}, \tag{45}$$

with all other components vanishing, where ϵ_{ab} is the Levi-Civita tensor in the orbit spacetime.

The master fields $\psi_S^{\omega l}$ and $\psi_V^{\omega l}$ are given by

$$\psi_P^{\omega l}(t,r) = A_{\text{static}}^{P;\omega l}e^{-i\omega t}r^{l+1}(1-r^2)^{i\omega/2} \times F\left(\frac{1}{2}(i\omega+l+1), \frac{1}{2}(i\omega+l+2); l+\frac{3}{2}; r^2\right), \tag{46}$$

where $A_{\text{static}}^{P;\omega l}$ are normalization constants. Since we are in the static patch, the positive-frequency property is manifest with the factor $e^{-i\omega t}$. One then expands the quantum field in the same manner as in Eq. (33). That is,

$$h_{\mu\nu} = \sum_{P,l,m}\int \mathrm{d}\omega\left[b_{lm}^P(\omega)h_{\mu\nu}^{(P;\omega lm)} + b_{lm}^P(\omega)^\dagger\overline{h_{\mu\nu}^{(P;\omega lm)}}\right]. \tag{47}$$

By normalizing the classical fields $h_{\mu\nu}^{(P;\omega lm)}$ with respect to the inner product (32), i.e. by letting

$$\langle h^{(P;\omega lm)}, h^{(P';\omega' l' m')}\rangle = \delta^{PP'}\delta^{ll'}\delta^{mm'}\delta(\omega-\omega'), \tag{48}$$

one obtains the usual commutation relations between the operators $b_{lm}^P(\omega)$ and $b_{lm}^P(\omega)^\dagger$, i.e.

$$\left[b_{lm}^P(\omega), b_{l'm'}^{P'}(\omega')^\dagger\right] = \delta^{PP'}\delta^{ll'}\delta^{mm'}\delta(\omega-\omega'), \tag{49}$$

with all other commutators vanishing. The static vacuum $|0S\rangle$ is defined by requiring that it should be annihilated by all the annihilation operators $b_{lm}^P(\omega)$. By computing the inner product (32) with the metric perturbations given in Eqs. (42)-(43) and Eq. (45), the normalization constants are determined as follows [23]:

$$|A_{\text{static}}^{S;\omega l}|^2 = \frac{\sinh\pi\omega\left|\Gamma\left(\frac{i\omega+l+2}{2}\right)\Gamma\left(\frac{i\omega+l+1}{2}\right)\right|^2}{2\pi^2(l-1)l(l+1)(l+2)\left|\Gamma\left(l+\frac{3}{2}\right)\right|^2} \tag{50}$$

and

$$|A_{\text{static}}^{V;\omega l}|^2 = \frac{\sinh\pi\omega\left|\Gamma\left(\frac{i\omega+l+1}{2}\right)\Gamma\left(\frac{i\omega+l+2}{2}\right)\right|^2}{8\pi^2(l-1)(l+2)\left|\Gamma(l+\frac{3}{2})\right|^2}. \tag{51}$$

IV. RESPONSE RATE TO A MULTIPOLE EXTERNAL SOURCE

A. Response rate in the Poincaré patch

Having obtained the normalized graviton modes, we introduce a multipole source term that couples to the field $h_{\mu\nu}$ in the Lagrangian density (7) as follows:

$$\frac{\mathcal{L}_{\text{int}}}{\sqrt{-g}} = \frac{\sqrt{32\pi}}{2}T^{\mu\nu}(x)h_{\mu\nu}(x), \tag{52}$$

where $T^{\mu\nu}$ is the energy-momentum tensor of the source. We note that, since $T^{\mu\nu}$ is a symmetric second rank tensor, one can expand it in the same way as the metric perturbations. Moreover, the coupling in the interaction term implies that products of scalar- and vector-type parts vanish when integrated on the whole spacetime. Thus, we can consider separately each type of energy-momentum tensor which couples to the same type of graviton modes. Moreover, the energy-momentum tensor has to be conserved in the background spacetime, in order for the interaction Lagrangian given by Eq. (52) to be gauge invariant. We construct the conserved scalar-type energy-momentum tensor $T^{\mu\nu}_{(S;Elm)}$ with the condition that $T^{\tau\mu}_{(S;Elm)} = 0$. Then the conservation equation $\nabla_\mu T^{\mu\nu}_{(S;Elm)} = 0$ leads to the following nonzero components:

$$T^{\rho\rho}_{(S;Elm)} = \frac{j^{El}_S}{\rho^2}\mathbb{S}^{(lm)}, \tag{53}$$

$$T^{i\rho}_{(S;Elm)} = -\frac{1}{k_S\rho^2}\left(\frac{\partial}{\partial\rho}+\frac{1}{\rho}\right) j^{El}_S \mathbb{S}^{i(lm)}, \tag{54}$$

$$T^{ij}_{(S;Elm)} = g^{El}_S \mathbb{S}^{ij(lm)} - \frac{\gamma^{ij} j^{El}_S}{2\rho^4}\mathbb{S}^{(lm)}, \tag{55}$$

where

$$g^{El}_S \equiv \left\{ \frac{2\rho^{-2}}{(l+2)(l-1)}\left[\frac{\partial^2}{\partial\rho^2}+\frac{3}{\rho}\frac{\partial}{\partial\rho}\right] - \frac{1}{\rho^4}\right\} j^{El}_S. \tag{56}$$

The function $j^{El}_S(\tau,\rho)$ is arbitrary and we will choose its form later.

The conserved vector-type energy-momentum tensor can be found under the same condition $T^{\tau\mu}_{(V;Elm)} = 0$ as

$$T^{\rho i}_{(V;Elm)} = \frac{j^{El}_V}{\rho^2}\mathbb{V}^{i(lm)}, \tag{57}$$

$$T^{ij}_{(V;Elm)} = -\frac{2k_V g^{El}_V}{\rho^2}\mathbb{V}^{ij(lm)}, \tag{58}$$

with all other components vanishing, where

$$g^{El}_V \equiv \frac{1}{(l+2)(l-1)}\left(\frac{\partial}{\partial\rho}+\frac{2}{\rho}\right) j^{El}_V. \tag{59}$$

Note that this energy-momentum tensor satisfies the conservation condition because the $\mathbb{V}^{(lm)}_{ij}$ are traceless. The function $j^{El}_V(\tau,\rho)$ is arbitrary, as in the scalar-type case.

We now let

$$j^{El}_P(\tau,\rho) = \lim_{\rho_0\to 0}\lambda\frac{e^{-(l+5+n_P)\tau}}{(l+2)!}\left(-\frac{\partial}{\partial\rho}\right)^{l+2}\delta(\rho-\rho_0)e^{iE\tau}, \tag{60}$$

where $n_S = 2$, $n_V = 1$, and λ is a small coupling constant. The number of ρ-derivatives has been chosen so that there is a nonzero but finite response rate for given angular momentum l. The exponential factor $e^{-(l+5+n_P)\tau}$ has been chosen so that the response rate does not vary with τ.

Let us now compute the response rate (probability of emission/absorption per unit time) of the graviton field in the vacuum to the multipole sources $T^{\mu\nu}_{(P:Elm)}$. If the initial state is the vacuum, there is only the possibility of emission, to lowest order in λ. Due to the form of the sources given by Eqs. (53)-(55), in the scalar-type case, and by Eqs. (57)-(58), in the vector-type case, the only non-vanishing amplitudes (to lowest order in perturbation theory) are the ones for the emission of a P-type graviton (when the initial state is the vacuum $|0\rangle$) with quantum numbers k, l and m. These amplitudes are given by

$$\begin{aligned}\mathcal{A}^P_{klm} &= i\langle 0|a^P_{lm}(k)\int \mathrm{d}^4x\mathcal{L}_{\rm int}|0\rangle \\ &= i\int \mathrm{d}\tau\int \mathrm{d}\rho\mathrm{d}\Omega_2 e^{3\tau}\rho^2\overline{h^{(P;klm)}_{\mu\nu}}T^{\mu\nu}_{(P;Elm)}.\end{aligned} \tag{61}$$

The response rate from the vacuum $|0\rangle$ is then [32]

$$\mathcal{R}^{P;E}_{\text{Poincaré}} = \int_0^\infty \mathrm{d}k\frac{|\mathcal{A}^P_{klm}|^2}{T_{\rm tot}}, \tag{62}$$

where

$$T_{\rm tot} = 2\pi\delta(0) = \int_{-\infty}^{\infty}\mathrm{d}\tau \tag{63}$$

is the total time as measured by the comoving observer (cf. Refs. [34, 35, 46–48] and references therein). The source is nonzero only at $\rho = 0$. Therefore, we can use the following expansion around $\rho = 0$ for the master field:

$$\begin{aligned}\phi^{kl}(\tau,\rho) \approx \frac{\sqrt{\pi}ke^{\frac{\tau}{2}}\mathrm{H}^{(1)}_{\frac{3}{2}}(ke^{-\tau})}{2^{\frac{3}{2}}\Gamma\left(l+\frac{3}{2}\right)}&\left[\left(\frac{k\rho}{2}\right)^l\right.\\ &\left.-\frac{\left(\frac{k\rho}{2}\right)^{l+2}}{\left(l+\frac{3}{2}\right)}+\frac{\left(\frac{k\rho}{2}\right)^{l+4}}{2\left(l+\frac{3}{2}\right)\left(l+\frac{5}{2}\right)}\right].\end{aligned} \tag{64}$$

Using this expansion and Eq. (61), we find that the squared transition amplitude, integrated over k, can be written as

$$\int \mathrm{d}k|\mathcal{A}^P_{klm}|^2 = \frac{\pi\lambda^2|k^{n_P}A^{kl}_P|^{-2}}{2^{2l+3}|\Gamma\left(l+\frac{3}{2}\right)|^2}\int\frac{\mathrm{d}k}{k}\int_{-\infty}^{\infty}\mathrm{d}\tau\int_{-\infty}^{\infty}\mathrm{d}\tau'(ke^{-\tau})^{l+n_P+\frac{3}{2}}(ke^{-\tau'})^{l+n_P+\frac{3}{2}}H^{(1)}_{\frac{3}{2}}\left(ke^{-\tau}\right)\overline{H^{(1)}_{\frac{3}{2}}}\left(ke^{-\tau'}\right)e^{iE(\tau-\tau')}. \tag{65}$$

Note that the factor $|k^{n_P}A^{kl}_P|^{-2}$ does not depend on k and, hence, it can be moved outside the integral.[4]

[4] The normalization factor squared, $|A^{kl}_P|^2$, appears in the numerator, but a factor proportional to $|A^{kl}_P|^4$ appears in the denominator. This explains the factor $|A^{kl}_P|^{-2}$ in Eq. (65).

Now, we make the following change of variables

$$T = \frac{\tau + \tau'}{2}, \tag{66}$$

$$\tau_r = \tau - \tau', \tag{67}$$

$$K = k e^{-\frac{\tau+\tau'}{2}}, \tag{68}$$

so that the integrand does not depend on T and the integral over this variable can be factored out. It will be cancelled by the total time [see Eq. (63)] when we compute the response rate. Thus, we find

$$\mathcal{R}^{P;E}_{\text{Poincaré}} = \frac{\pi\lambda^2 |k^{n_P} A^{kl}_P|^{-2}}{2^{2l+3}|\Gamma\left(l+\frac{3}{2}\right)|^2} \int \frac{\mathrm{d}K}{K} \int_{-\infty}^{\infty} \mathrm{d}\tau_r K^{2l+2n_P+3} \times H^{(1)}_{\frac{n+1}{2}}\left(Ke^{-\tau_r/2}\right) \overline{H^{(1)}_{\frac{n+1}{2}}}\left(Ke^{\tau_r/2}\right) e^{iE\tau_r}. \tag{69}$$

We perform a further change of variables given by

$$x = Ke^{-\tau_r/2}, \tag{70}$$

$$y = Ke^{\tau_r/2}. \tag{71}$$

We thus obtain

$$\mathcal{R}^{P;E}_{\text{Poincaré}} = \frac{\pi\lambda^2 |k^{n_P} A^{kl}_P|^{-2}}{2^{2l+3}|\Gamma\left(l+\frac{3}{2}\right)|^2} \times \left| \int_0^\infty \mathrm{d}x x^{l+n_P+\frac{1}{2}+iE} H^{(1)}_{\frac{3}{2}}(x) \right|^2. \tag{72}$$

Using Eq. (A6) of Ref. [32], namely

$$\int_0^{\infty+i\epsilon} z^\mu H^{(1)}_\nu(z)\mathrm{d}z = \frac{2^\mu}{\pi}\exp\left[\frac{1}{2}i(\mu-\nu)\pi\right] \times\Gamma\left(\frac{\mu+\nu+1}{2}\right)\Gamma\left(\frac{\mu-\nu+1}{2}\right), \tag{73}$$

for $\mathrm{Re}\,\mu - |\mathrm{Re}\,\nu| + 1 > 0$, we find the following result:

$$\mathcal{R}^{P;El}_{\text{Poincaré}} = \frac{\lambda^2 e^{-\pi E}\left|\Gamma\left(\frac{l+iE+n_P+3}{2}\right)\Gamma\left(\frac{l+iE+n_P}{2}\right)\right|^2}{4^{1-n_P}\pi|k^{n_P}A^{kl}_P|^2\left|\Gamma\left(l+\frac{3}{2}\right)\right|^2}. \tag{74}$$

B. Response rate in the static patch

We now compare the response rate in the Poincaré patch, Eq. (74), to the one obtained in the static patch from the same source in thermal equilibrium with temperature $1/2\pi$, the Gibbons-Hawking temperature for de Sitter spacetime (with $H = 1$).

We first assume $E > 0$. Then

$$\int \mathrm{d}^4x\sqrt{-g}\, T^{\mu\nu}_{(S;Elm)} h_{\mu\nu} = 2\pi\lambda(l+1-iE)(l+3-iE) \times A^{S;\omega l}_{\text{static}} b^S_{l,-m}(E) \tag{75}$$

and

$$\int \mathrm{d}^4x\sqrt{-g}\, T^{\mu\nu}_{(V;Elm)} h_{\mu\nu} = 4\pi\lambda(iE-l-2) \times A^{V;\omega l}_{\text{static}} b^V_{l,-m}(E). \tag{76}$$

If the initial state is given by a one-particle state $b^P_{l,-m}(\omega)^\dagger|0S\rangle$, $P = S$ or V, in the static patch, we find that the absorption probability per unit time is

$$\mathcal{P}^{S;\omega l,-m}_{\text{static}} = 2\pi\lambda^2|l+1+iE|^2|l+3+iE|^2 \times|A^{S;\omega l}_{\text{static}}|^2\delta(\omega - E), \tag{77}$$

in the scalar-type case, and

$$\mathcal{P}^{V;\omega l,-m}_{\text{static}} = 2\pi\lambda^2|l+2+iE|^2|A^{V;\omega l}_{\text{static}}|^2\delta(\omega - E), \tag{78}$$

in the vector-type case. Hence, in the scalar-type case the absorption rate in thermal equilibrium with temperature $1/2\pi$ is

$$\mathcal{R}^{S;El}_{\text{static}} = \int \mathcal{P}^{S;\omega l,-m}_{\text{static}} \frac{\mathrm{d}\omega}{e^{2\pi\omega}-1} = \frac{8\lambda^2 e^{-\pi E}\left|\Gamma\left(\frac{l+iE+5}{2}\right)\Gamma\left(\frac{l+iE+2}{2}\right)\right|^2}{\pi(l-1)l(l+1)(l+2)\left|\Gamma\left(l+\frac{3}{2}\right)\right|^2}, \tag{79}$$

and the absorption rate in the vector-type case reads

$$\mathcal{R}^{V;El}_{\text{static}} = \frac{2\lambda^2 e^{-\pi E}\left|\Gamma\left(\frac{l+iE+4}{2}\right)\Gamma\left(\frac{l+iE+1}{2}\right)\right|^2}{\pi(l-1)(l+2)\left|\Gamma\left(l+\frac{3}{2}\right)\right|^2}. \tag{80}$$

If $E < 0$, there is emission of a graviton by the source. The emission probabilities per unit time are again given by Eqs. (77) and (78) with the change $E \to |E|$. However, in this case, we have to take into account both spontaneous and induced emissions. Hence, the emission rates are

$$\mathcal{R}^{P;El}_{\text{static}} = \int \mathcal{P}^{P;\omega l,-m}_{\text{static}} \mathrm{d}\omega\left(\frac{1}{e^{2\pi\omega}-1}+1\right). \tag{81}$$

Thus, we find that the emission rates are again given by Eqs. (79) and (80) (without the change $E \to |E|$). By comparing these results with Eq. (74), where $n_S = 2$ and $n_V = 1$, and where A^{kl}_P, with $P = S$ and V, are given by Eqs. (41) and (40), respectively, we find $\mathcal{R}^{P;El}_{\text{static}} = \mathcal{R}^{P;El}_{\text{Poinacaré}}$ for both $P = S$ and V.

Thus, we have shown that the response rate of the vacuum $|0\rangle$ to the conserved external multipole sources $T^{\mu\nu}_{(P;Elm)}$, $P = S, V$, is identical to the response rate of the heat bath with temperature $1/2\pi$ in the static patch.

V. CONCLUDING REMARKS

In this paper we verified the Gibbons-Hawking effect, i.e. the fact that the standard vacuum state for quantum field theory in de Sitter spacetime is a thermal equilibrium state with temperature $H/2\pi$, where H is the

Hubble constant, for the gravitational perturbations. Although this was an expected result, it is reassuring to verify it explicitly. Strictly speaking, derivations of this and other related effects in general spacetimes with bifurcate Killing horizons [28, 49, 50] have been given only for non-gauge fields. It would be interesting to close this gap and find a general derivation of this and other related effects applicable also to gauge fields including perturbative gravity.

Our result also serves as a check of the IR-finite graviton two-point function in the static patch found in Refs. [23, 24]. That is, we have verified explicitly that the standard vacuum state for the gravitational pertubations in the Poincaré patch, correponding to an IR-divergent two-point function, and the thermal state in the static patch, corresponding to an IR-finite two-point function, have the same response to conserved external energy-momentum sources. The conservation of the energy-momentum tensor also ensures gauge invariance of the response rates. This is an interesting first step for examining physics in de Sitter spacetime using the static patch, where the IR properties of the gravitational perturbations are better controlled. Since there have been disagreement about the physical significance of the IR divergences in the Poincaré patch, it would be interesting to develop gravitational perturbation theory in the static patch, now that the thermal state studied in Refs. [23, 24] has been shown to produce the correct physics when probed by an external source.

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Appendix A: Expansion of the gravitational plane wave in flat space in terms of the modes in spherical polar coordinates

In this Appendix we review the expansion of the gravitational plane wave in spatially-flat spacetime, including the Poincaré patch of de Sitter spacetime, in terms of the modes in spherical polar coordinates. This Appendix is included in order to emphasize that only the modes with $l \geq 2$ are present in the expansion of the gravitational plane waves in the Poincaré patch. We note that both the plane-wave modes and the modes in spherical polar coordinates have the time-dependence given by $e^{\frac{H\tau}{2}}\mathrm{H}^{(1)}_{\frac{3}{2}}(ke^{-H\tau}/H)$ [see Eq. (28)]. Hence, it is sufficient to consider the space-dependence of the plane waves and the vector- and scalar-type modes. Thus, we extract the space-dependent part of the scalar-type modes given by Eqs. (11)-(13) as

$$H_{\rho\rho}^{(S;klm)} = \frac{A_S^{kl}}{\rho^2} j_l(k\rho)\mathbb{S}^{(lm)}, \tag{A1}$$

$$H_{\rho i}^{(S;klm)} = -\frac{A_S^{kl}\mathbb{S}_i^{(lm)}}{k_S}\left(\frac{\partial}{\partial\rho}+\frac{1}{\rho}\right) j_l(k\rho), \tag{A2}$$

$$H_{ij}^{(S;klm)} = A_S^{kl}\mathbb{S}_{ij}^{(lm)}\psi^{kl}(\rho) - \frac{A_S^{kl}}{2}\gamma_{ij} j_l(k\rho)\mathbb{S}^{(lm)}, \tag{A3}$$

where

$$\psi^{kl}(\rho) = \frac{2\rho^2}{(l-1)(l+2)} \times \left[\frac{\partial^2}{\partial\rho^2} + \frac{3}{\rho}\frac{\partial}{\partial\rho} - \frac{(l-1)(l+2)}{2\rho^2}\right] j_l(k\rho). \tag{A4}$$

We extract the space-dependent part of the vector-type modes given by Eqs. (20) and (21) as

$$H_{\rho i}^{(V;klm)} = A_V^{kl} j_l(k\rho)\mathbb{V}_i^{(lm)}, \tag{A5}$$

$$H_{ij}^{(V;klm)} = -\frac{2k_V A_V^{kl}\rho^2\mathbb{V}_{ij}^{(lm)}}{(l-1)(l+2)}\left(\frac{\partial}{\partial\rho}+\frac{2}{\rho}\right) j_l(k\rho). \tag{A6}$$

The scalar plane wave propagating in the z-direction can be expanded as follows:

$$e^{ikz} = \sum_{l=0}^{\infty}(2l+1)i^l j_l(k\rho)\mathrm{P}_l(\cos\theta). \tag{A7}$$

The space-dependent part of a circularly polarized gravitational plane wave propagating in the z-direction can be given as

$$H_{xx}^{\mathrm{pl}} = -H_{yy}^{\mathrm{pl}} = \frac{1}{2}e^{ikz}, \tag{A8}$$

$$H_{xy}^{\mathrm{pl}} = \pm\frac{i}{2}e^{ikz}. \tag{A9}$$

By the standard coordinate transformation of a tensor, we find

$$\begin{aligned} H_{\rho\rho}^{\mathrm{pl}} &= \frac{1}{2}\sin^2\theta\, e^{\pm 2i\phi}e^{ikz} \\ &= \frac{1}{2}\sum_{l=0}^{\infty}(2l+1)i^l j_l(k\rho)\sin^2\theta \mathrm{P}_l(\cos\theta)e^{\pm 2i\phi}. \end{aligned} \tag{A10}$$

Now, by repeated use of the formula

$$\sqrt{1-x^2}\mathrm{P}_l^m(x) = \frac{1}{2l+1}\left[-\mathrm{P}_{l+1}^{m+1}(x) + \mathrm{P}_{l-1}^{m+1}(x)\right], \tag{A11}$$

where we let $\mathrm{P}_l^m(x) = 0$ if $|m| > l$, we obtain

$$H^{\rm pl}_{\rho\rho} = \frac{1}{2}\sum_{l=0}^{\infty} i^l j_l(k\rho) e^{\pm 2i\phi} \times \left\{ \frac{1}{2l+3}\left[\mathrm{P}^2_{l+2}(\cos\theta) - \mathrm{P}^2_l(\cos\theta)\right] - \frac{1}{2l-1}\left[\mathrm{P}^2_l(\cos\theta) - \mathrm{P}^2_{l-2}(\cos\theta)\right]\right\}$$
$$= -\frac{1}{2}\sum_{l=2}^{\infty} i^l \mathrm{P}^2_l(\cos\theta) e^{\pm 2i\phi} \times \left\{ \frac{1}{2l-1}\left[j_{l-2}(k\rho) + j_l(k\rho)\right] + \frac{1}{2l+3}\left[j_l(k\rho) + j_{l+2}(k\rho)\right]\right\}. \tag{A12}$$

Then, by using

$$j_{l-1}(x) + j_{l+1}(x) = \frac{2l+1}{x} j_l(x), \tag{A13}$$

we find

$$H^{\rm pl}_{\rho\rho} = -\frac{1}{2k^2\rho^2}\sum_{l=2}^{\infty} i^l (2l+1) j_l(k\rho) \mathrm{P}^2_l(\cos\theta) e^{\pm 2i\phi}$$
$$= -\frac{1}{\rho^2}\sum_{l=2}^{\infty} i^l \sqrt{2\pi(2l+1)}\, j_l(k\rho) A^{kl}_S \mathbb{S}^{(l,\pm 2)}(\theta,\phi), \tag{A14}$$

where $\mathbb{S}^{(lm)}(\theta,\phi)$ is defined by Eq. (16) with the constant A^{kl}_S defined by Eq. (41). By comparing this expression with Eq. (A1) we find

$$H^{\rm pl}_{\rho\rho} = -\sum_{l=2}^{\infty} i^l \sqrt{2\pi(2l+1)}\, H^{(S;kl,\pm 2)}_{\rho\rho}. \tag{A15}$$

To find the vector-type contribution to the plane wave, we note that

$$H^{\rm pl}_{\rho\theta} = \frac{\rho}{2}\sin\theta\cos\theta e^{\pm 2i\phi} e^{ik\rho\cos\theta}, \tag{A16}$$
$$H^{\rm pl}_{\rho\phi} = \pm\frac{i\rho}{2}\sin^2\theta e^{\pm 2i\phi} e^{ik\rho\cos\theta}. \tag{A17}$$

Hence

$$\epsilon^{ij}\hat{D}_i H^{\rm pl}_{\rho j} = \pm k\rho^2 H^{\rm pl}_{\rho\rho}$$
$$= \mp\sum_{l=2}^{\infty} i^l \sqrt{2\pi(2l+1)l(l+1)}\, A^{kl}_V j_l(k\rho) \mathbb{S}^{(l,\pm 2)}, \tag{A18}$$

where the constant A^{kl}_V is given by Eq. (40). On the other hand

$$\epsilon^{ij}\hat{D}_i H^{(V;klm)}_{\rho j} = \sqrt{l(l+1)}\, A^{kl}_V j_l(k\rho)\mathbb{S}^{(lm)}. \tag{A19}$$

By comparing this equation with Eq. (A18) we conclude that

$$\epsilon^{ij}\hat{D}_i H^{\rm pl}_{\rho j} = \mp\sum_{l=2}^{\infty} i^l \sqrt{2\pi(2l+1)}\, \epsilon^{ij}\hat{D}_i H^{(V;kl,\pm 2)}_{\rho j}. \tag{A20}$$

From this equation and Eq. (A15) we find

$$H^{\rm pl}_{\mu\nu} = -\sum_{l=2}^{\infty} i^l \sqrt{2\pi(2l+1)}\left[H^{(S;kl,\pm 2)}_{\mu\nu} \pm H^{(V;kl,\pm 2)}_{\mu\nu}\right]. \tag{A21}$$

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