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Testing for optimal monetary policy via moment inequalities*

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Summary

The specification of an optimizing model of the monetary transmission mechanism requires selecting a policy regime, commonly commitment or discretion. In this paper we propose a new procedure for testing optimal monetary policy, relying on moment inequalities that nest commitment and discretion as two special cases. The approach is based on the derivation of bounds for inflation that are consistent with optimal policy under either policy regime. We derive testable implications that allow for specification tests and discrimination between the two alternative regimes. The proposed procedure is implemented to examine the conduct of monetary policy in the United States economy.

JEL classification codes: C12; C52; E52; E58.
Keywords: *Bootstrap; GMM; Moment Inequalities; Optimal Monetary Policy.*

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1 Introduction

This paper proposes new methods for the evaluation of monetary policy within the framework set by the New Keynesian model. Since the work of Kydland and Prescott (1977), the theory of optimal monetary policy is aware of the time inconsistency problem. An optimal state-contingent plan announced ex-ante by the monetary authority may fail to steer private sector expectations because, ex-post, past commitments are ignored. The theoretical literature has considered two alternative characterizations of optimal monetary policy: commitment, whereby the optimal plan is history dependent and the time-inconsistency problem is ignored; and discretion, whereby the monetary authority re-optimizes each period. We propose a method for estimating and testing a structural model of optimal monetary policy, without requiring an explicit choice of the relevant equilibrium concept. Our procedure considers a general specification, that nests optimal policy under commitment and discretion. The approach is based on the derivation of bounds for the inflation rate that are consistent with both forms of optimal policy and yield set identification of the economy structural parameters. We derive testable implications that allow for specification tests and discrimination between the monetary authority's modes of behavior.

Under discretion, there exists a state-contingent inflation bias resulting from the fact that the monetary authority must set policy independently of the history of shocks (Svensson, 1997). The upshot of this state-contingent bias is that, when the output gap is negative, the inflation rate under discretion in the following period is higher than what it would be if the monetary authority was able to commit to history-dependent plans. This state-contingent inflationary bias allows for the derivation of an inflation lower-bound (obtained under commitment) and an upper-bound (obtained under discretion), based on the first order conditions that characterize optimal monetary policy under each policy regime.

More generally, our framework applies to the optimal linear regulator problem, and relies on state-contingent bounds for a target variable that are used to derive moment inequality conditions associated with optimal policy, and to identify the set of structural parameters for which the moment inequalities hold, i.e. the identified set. We characterize the identified set implied by optimal monetary policy using inference methods developed in Chernozhukov, Hong, and Tamer (2007). We then test whether the moment restrictions implied by a specific regime are satisfied.

Assuming a specific policy regime enables point identification. Thus, parameters can be estimated consistently and standard tests of overidentifying restrictions (Hansen, 1982) can be performed. However, if our objective is to test for discretion or commitment under the maintained assumption of optimal monetary policy, the standard Hansen's J–test does not make use of all the available information. Instead, we propose a test for discretion and a test for commitment which explore the additional information obtained from the moment inequality conditions associated with the inflation bounds implied by optimal monetary policy. Formally, the test is implemented using the criterion function approach of Chernozhukov et al. (2007) and an extension of the Generalized Moment Selection method of Andrews and Soares (2010), that takes into account the contribution of parameter estimation error on the relevant covariance matrix.

In addition, the moment inequality conditions implied by optimal monetary policy under discretion and commitment, respectively, can be used to perform a model selection test to discriminate between the two alternative policy regimes, maintaining the assumption of optimal monetary policy. Following Shi (2015), we compare the two models and select the one that is closer to the truth in terms of a pseudo-distance measure based on the Kullback-Leibler divergence measure.

We apply our testing procedure to investigate whether the time-series of inflation and output gap in the United States are consistent with the New Keynesian model of optimal monetary policy that has been widely used in recent studies of monetary policy, following the work of Rotemberg and Woodford (1997), Clarida, Galí, and Gertler (1999), and Woodford (2003). Using the sample period running from 1983Q1 until 2008Q3, we find evidence in favor of discretionary optimal monetary policy, and against commitment. In contrast, the standard J–test of overidentifying restrictions lacks power and fails to reject either policy regime.[1] Thus, by making use of the full set of implications of optimal monetary policy, we construct a more powerful model specification test, allowing the rejection of commitment but not discretion. This finding is further supported by the model selection test based on Shi (2015).

The importance of being able to discriminate between different policy regimes on the basis of the observed time-series of inflation and output is well recognized. In pioneering work, Baxter (1988) calls for the development of methods to analyze policy-making in a maximizing framework, and says that *"what is required is the derivation of appropriate econometric specifications for the models, and the use of established statistical procedures for choosing between alternative, hypothesized models of policymaking"*.[2] This paper seeks to provide such an econometric specification. Our paper is also related to work by Ireland (1999), that tests and fails to reject the hypothesis that inflation and unemployment form a cointegrating relation, as implied by the Barro and Gordon model when the natural unemployment rate is non-stationary. Ruge-Murcia (2003) estimates a model that allows for asymmetric preferences, and fails to reject the model of discretionary optimal monetary policy. Both these papers assume one equilibrium concept (discretion), and test the time-series implications of discretionary policies. Our framework instead derives a general specification of optimal policy, nesting commitment and discretion as two special cases.

[1] The lack of power of the J–test in the context of forward-looking models estimated using GMM is discussed in Mavroeidis (2005). Our test of the monetary policy regime explores a larger set of moment inequality restrictions implied by optimal monetary policy and, therefore, contributes to increasing the power of the specification test.

[2] Baxter (1988, p.145).

Using maximum-likelihood, Givens (2012) estimates a New Keynesian optimal monetary policy model for the US. The model is estimated separately under the two alternatives of commitment and discretion, using quarterly data over the Volcker–Greenspan–Bernanke era; a comparison of the log-likelihood of the two alternative models based on a Bayesian information criterion (to overcome the fact that the two models are non-nested) strongly favors discretion over commitment. A similar Bayesian approach has been used by Kirsanova and Le Roux (2013), who also find evidence in favor of discretion for monetary and fiscal policy in the UK. Debortoli and Lakdawala (2015) estimate a medium-scale DSGE model allowing for deviations from commitment plans that follow a regime switching process. They reject both the full commitment and discretion, which are nested special cases of their model.

The partial identification framework that we propose in this paper has two important advantages. First, it constitutes a general econometric specification that nests commitment and discretion as two special cases. Second, unlike full-information methods, our approach does not require strong assumptions about the nature of the forcing variables (the shock processes). The disadvantage of our method relative to the full information approach, is that it is currently limited to relatively simple models of optimal policy. Thus, our procedure may not be directly applicable to medium-scale DSGE models à la Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2005), that include several sources of endogenous persistence (such as habits, capital stock and persistence in interest rates). However, several influential models used to study optimal monetary policy yield set-ups that can be addressed using our methodology. Important examples include Clarida et al. (1999) and Svensson and Woodford (2004) canonical models of optimal policy, Giannoni and Woodford (2004) model of inflation inertia and a special (but empirically salient) case of the Erceg, Henderson, and Levin (2000) model with sticky wages as well as prices.

Simple monetary policy rules are often prescribed as guides for the conduct of monetary policy. For instance, a commitment to a Taylor rule (Taylor, 1993), according to which the short-term policy rate responds to fluctuations in inflation and some measure of the output gap, incorporates several features of an optimal monetary policy, from the standpoint of at least one simple class of optimizing models. Woodford (2001) shows the response prescribed by these rules tends to stabilize inflation and the output gap, and stabilization of both variables is an appropriate goal, as long as the output gap is properly defined.

Under certain simple conditions, a feedback rule that establishes a time-invariant relation between the path of inflation and of the output gap and the level of nominal interest rates can bring about an optimal pattern of equilibrium responses to real disturbances. Giannoni and Woodford (2017) show that it is possible to find simple target criteria that are fully optimal across a wide range of specifications of the economy stochastic disturbance processes. To the extent that the systematic behavior implied by simple rules takes into account private sector expectations, commitment-like behavior may be a good representation of monetary policy. Therefore, as McCallum (1999) forcefully argues, neither of the two modes of behavior has as yet been established empirically. Our framework develops a new procedure for testing these two alternative policy regimes.

This paper also contributes to a growing literature that proposes partial identification methods to overcome lack of information about the economic environment. For instance, Manski and Tamer (2002) examine inference on regressions with interval outcomes. Haile and Tamer (2003) use partial identification to construct bounds on valuation distributions in second price auctions. Blundell, Browning, and Crawford (2008) derive bounds that allow set-identification of predicted demand responses in the study of consumer behavior. Ciliberto and Tamer (2009) propose new methods for inference in entry games without requiring assumption about the equilibrium selection. Galichon and Henry (2011) derive set-identifying restrictions for games with multiple equilibria in pure and mixed strategies.

The rest of the paper is organized as follows. Section 2 describes the class of optimal linear regulator problems to which our framework applies. Section 3 derives the bounds for inflation implied by optimal monetary policy and outlines the inference procedure. Section 4 describes the proposed test for optimal monetary policy. Section 5 describes the model selection test. Section 6 presents Monte Carlo evidence on the small sample performance of the tests. Finally, Section 7 reports the empirical findings and Section 8 concludes.[3]

2 Optimal monetary policy

Our methodology applies to the optimal linear regulator problem obtained when the policymaker's objective function is quadratic and the structural equations describing the economy's equilibrium dynamics are linear. This framework is widely used to study optimal monetary policy in the New Keynesian model with staggered prices and monopolistic competition.[4] The objective function of the monetary authority, which in the canonical case is derived as a second order approximation to the utility of a stand-in agent around the stable equilibrium with zero inflation (Woodford, 2003), takes the form

$$\begin{aligned}\mathbf{U} &= \mathbb{E}_0\left[-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(y_t'\mathbf{W}y_t\right)\right], \\ &= \mathbb{E}_0\left[-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2 + s_t'\mathbf{Q}s_t + x_t'\mathbf{R}x_t\right)\right],\end{aligned} \tag{1}$$

where $\mathbb{E}_t$ denotes agents's expectations at date t, $y_t = [\pi_t, s_t, x_t]'$ is a $n \times 1$ vector of endogenous variables with $n \geq 2$; π_t is a scalar random variable (the inflation rate in our benchmark example),

[3] All the proofs are collected in the Supplementary Material S.1 and S.2

[4] See Woodford (2003) for a detailed description of this class of structural models.

s_t is an $m \times 1$ vector, with $m \geq 1$, and x_t is of dimension $(n-m-1) \times 1$; $\beta \in (0,1)$ is a scalar parameter representing the discount factor. The matrix $\mathbf{W}$ is a $n \times n$ symmetric positive semi-definite matrix containing the target variables' weights, with the following block diagonal structure

$$\mathbf{W} = \begin{bmatrix} 1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}, \tag{2}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are conformable square matrices.

The constraints on possible equilibrium outcomes (the structural equations) are represented by the following m-dimensional system

$$\begin{aligned} u_t &= \mathbf{A}y_{t-1} + \mathbf{B}y_t + \beta\mathbf{C}\mathbb{E}_t\left(y_{t+1}\right), \\ &= \begin{bmatrix}\mathbf{a} & \mathbf{A}_s & \mathbf{A}_x\end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ s_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix}\mathbf{b} & \mathbf{B}_s & \mathbf{B}_x\end{bmatrix} \begin{bmatrix} \pi_t \\ s_t \\ x_t \end{bmatrix} + \beta\begin{bmatrix}\mathbf{c} & \mathbf{C}_s & \mathbf{C}_x\end{bmatrix}\mathbb{E}_t \begin{bmatrix} \pi_{t+1} \\ s_{t+1} \\ x_{t+1} \end{bmatrix}, \end{aligned} \tag{3}$$

for all t, where u_t is a vector of exogenous disturbances, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are $m \times n$ matrices, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are $m \times 1$ vectors, and $\mathbf{A}_s$, $\mathbf{A}_x$, $\mathbf{B}_s$, $\mathbf{B}_x$, $\mathbf{C}_s$ and $\mathbf{C}_x$ are conformable matrices.[5] In particular, $\mathbf{B}_s$ is an $m \times m$ square matrix.

In the sequel, we restrict attention to models admitting a representation such that $\mathbf{a} = \mathbf{0}$ and $\mathbf{A}_s = \mathbf{0}$, so that the vector of target variables $[\pi_t, s_t]'$ does not include predetermined variables, with all the endogenous predetermined variables included in x_t. Moreover, we require that $\mathbf{C}_s = 0$ and the matrix $\mathbf{B}_s$ to be nonsingular. These restrictions allow the Lagrange multipliers associated with each of the m constraints to be mapped into the contemporaneous values of s_t.[6]

[5] This formulation follows Dennis (2007) and Debortoli and Lakdawala (2015).

[6] Many influential models used to study optimal monetary policy satisfy these restrictions. For example, Clarida et al. (1999), Svensson and Woodford (2004) and Giannoni and Woodford (2004) model of inflation inertia, all admit

The problem of the monetary authority under commitment is to choose bounded state-contingent sequences $\{y_t\}_{t\geq 0}$ to maximize (1) subject to (3). The Lagrangian formulation of this problem is given by

$$\mathbb{E}_0 \left\{ -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \Big[\pi_t^2 + s_t' \mathbf{Q} s_t + x_t' \mathbf{R} x_t - \lambda_t' \left(\mathbf{A} y_{t-1} + \mathbf{B} y_t + \beta \mathbf{C} y_{t+1} \right) \Big] \right\}, \tag{4}$$

where λ_t is a h-dimensional vector of Lagrange multipliers, with initial condition $\lambda_{-1} = \mathbf{0}$. The first order conditions solving the monetary authority's problem under commitment are

$$\pi_t - \mathbf{c}' \lambda_{t-1} - \mathbf{b}' \lambda_t = 0, \tag{5}$$

$$\mathbf{Q} s_t - \mathbf{B}_s' \lambda_t = \mathbf{0}, \tag{6}$$

$$\mathbf{R} x_t - \mathbf{C}_x' \lambda_{t-1} - \mathbf{B}_x' \lambda_t - \beta \mathbf{A}_x' \mathbb{E}_t \left(\lambda_{t+1} \right) = \mathbf{0}, \tag{7}$$

for all $t \geq 0$, together with the constraint (3) and the initial condition $\lambda_{-1} = \mathbf{0}$. From equation (5) the necessary conditions for optimal policy under commitment yield

$$\pi_t = \mathbf{c}' \lambda_{t-1} + \mathbf{b}' \lambda_t. \tag{8}$$

However, the commitment solution is time inconsistent in the Kydland and Prescott (1977) sense: each period t, the monetary authority is tempted to behave as if $\lambda_{t-1} = \mathbf{0}$, ignoring the impact of its current actions on the private sector expectations. Under discretion, the policymaker acts as if $\lambda_{t-1} = \mathbf{0}$, and the resulting path for target variable π_t satisfies the condition

$$\pi_t = \mathbf{b}' \lambda_t. \tag{9}$$

such representation. However, the methods proposed in this paper are not directly applicable to medium-scale DSGE models of the kind pioneered by Smets and Wouters (2003) and Christiano et al. (2005), that include sources of endogenous persistence (such as habits in consumption, capital stock and persistence in interest rates), if those sources of endogenous persistency cause the number of pre-determined target variables to exceed $n - (m + 1)$, such that $\mathbf{A}_s \neq \mathbf{0}$, or prevent a representation of the structural equations with $\mathbf{C}_s = 0$.

Both under discretion and commitment, from (6) we obtain the Lagrange multipliers as follows

$$\begin{aligned}\lambda_t &= \mathbf{B}_s'^{-1}\mathbf{Q}s_t, \\ &= \mathbf{D}s_t,\end{aligned} \tag{10}$$

In what follows, we define the sublist of structural parameters $\phi = \{\mathbf{b}, \mathbf{c}, \mathbf{D}\}$, and let $\phi_0 = \{\mathbf{b}_0, \mathbf{c}_0, \mathbf{D}_0\}$ denote the "true" value of ϕ. In addition, we define $\pi_t^c(\phi_0)$ as the inflation in period t consistent with the first order conditions for optimal policy under commitment, given knowledge of s_t and the structural parameters in ϕ_0. In the same way, $\pi_t^d(\phi_0)$ is the inflation in period t consistent with the first order conditions under discretion. Making use of (8), (9) and (10), $\pi_t^c(\phi_0)$ and $\pi_t^d(\phi_0)$ are, respectively, given by

$$\pi_t^c(\phi_0) = \mathbf{c}_0'\mathbf{D}_0 s_{t-1} + \mathbf{b}_0'\mathbf{D}_0 s_t, \tag{11}$$

$$\pi_t^d(\phi_0) = \mathbf{b}_0'\mathbf{D}_0 s_t. \tag{12}$$

To model optimal monetary policy requires a decision about whether the first order conditions of the policy maker are represented by (11) or, instead, by (12). But how does one decide whether the behavior of the monetary authority should be classified as discretion or commitment-like? We propose a general characterization of optimal monetary policy nesting both modes of behavior. The approach is based on the derivation of bounds for the inflation rate under the maintained assumption that at any point in time either (11) or (12) is satisfied.

3 Bounds for inflation

Under a specific equilibrium concept, commitment or discretion, it is in principle possible to identify ϕ_0 from observed data for inflation and the output gap using, respectively, equation (11) or (12).

A general specification for optimal monetary policy, nesting the two alternative characterizations of optimality follows from the next simple result.

Lemma 1. *Consider an economy whose structural equations can be represented by the system* (3), *with* $\mathbf{a} = \mathbf{0}$, $\mathbf{A}_s = \mathbf{0}$, $\mathbf{C}_s = \mathbf{0}$ *and* $\mathbf{B}_s$ *a nonsingular matrix. Optimal policy implies that*

$$\Pr\left(\pi_t^c(\phi_0) \le \pi_t(\phi_0) \le \pi_t^d(\phi_0) \middle| \mathbf{c}_0'\mathbf{D}_0 s_{t-1} \le 0\right) = 1,$$
$$\Pr\left(\pi_t^d(\phi_0) \le \pi_t(\phi_0) \le \pi_t^c(\phi_0) \middle| \mathbf{c}_0'\mathbf{D}_0 s_{t-1} > 0\right) = 1,$$

where $\pi_t(\phi_0)$ *is the actual inflation rate in period* t.

The bounds for inflation in Lemma 1 follow immediately from equations (11) and (12).

In the sequel, we assume that the observed inflation rate differs from the actual inflation rate chosen by the monetary authority only through the presence of a measurement error with mean $\bar{\Pi}_0$, possibly different from zero, thus allowing for the presence of a trend in measured inflation.[7]

Assumption 1. *Let* $\pi_t(\phi_0)$ *be the actual inflation rate in period* t. *The observed inflation rate is* $\Pi_t = \pi_t(\phi_0) + v_t$, *where* v_t *has mean* $\bar{\Pi}_0$ *and variance* σ_v^2.

3.1 Moment inequalities

The upshot of Lemma 1 is that we are able to derive moment inequality conditions implied by optimal monetary policy, and nesting commitment and discretion as two special cases. From Lemma 1 it is immediate to see that

$$\begin{aligned}
&\Pr\left(\pi_t^c(\phi_0) + v_t \le \Pi_t \le \pi_t^d(\phi_0) + v_t \middle| \mathbf{c}_0'\mathbf{D}_0 s_{t-1} \le 0\right) = 1, \\
&\Pr\left(\pi_t^d(\phi_0) + v_t \le \Pi_t \le \pi_t^c(\phi_0) + v_t \middle| \mathbf{c}_0'\mathbf{D}_0 s_{t-1} > 0\right) = 1,
\end{aligned} \tag{13}$$

[7]Bernanke and Mishkin (1997) argue US measured annual inflation is overstated 0.5 to 2.0 percentage points.

which establishes a lower and upper bound for the observed inflation rate, Π_t.

We assume that enough is known about the structural parameters of the economy so that the sign of each element of ϕ_0 is known with certainty, and denote $\mathbf{S} = \mathbf{sign}\left(\mathbf{c}_0'\mathbf{D}_0\right)$ the $1 \times p$ vector which is obtained after applying the sign function to each element of $\mathbf{c}_0'\mathbf{D}_0$. Then, we define the p-dimensional vector $\mathbf{S}_t = \mathbf{S}' \circ s_t$, where $\circ$ denotes the Schur product (element by element vector multiplication). Next, we obtain $1\left(\mathbf{S}_{t-1} \leq 0\right)$ and $1\left(\mathbf{S}_{t-1} > 0\right)$, the indicator functions taking value one when, respectively, each element of $\mathbf{S}_{t-1}$ is non-positive and each element of $\mathbf{S}_{t-1}$ is positive, and zero otherwise. This yields the following moment inequalities implied by optimal policy

Proposition 1. *Under Assumption 1, the following moment inequalities*

$$\mathrm{E}\begin{bmatrix} -\left(\Pi_t - \mathbf{b}_0'\mathbf{D}_0 s_t - v_t\right) 1\left(\mathbf{S}_{t-1} \leq 0\right) \\ \left(\Pi_t - \mathbf{b}_0'\mathbf{D}_0 s_t - v_t\right) 1\left(\mathbf{S}_{t-1} > 0\right) \\ \left(\Pi_t - \mathbf{c}_0'\mathbf{D}_0 s_{t-1} - \mathbf{b}_0'\mathbf{D}_0 s_t - v_t\right) 1\left(\mathbf{S}_{t-1} \leq 0\right) \\ -\left(\Pi_t - \mathbf{c}_0'\mathbf{D}_0 s_{t-1} - \mathbf{b}_0'\mathbf{D}_0 s_t - v_t\right) 1\left(\mathbf{S}_{t-1} > 0\right) \end{bmatrix} \geq 0, \tag{14}$$

are implied by optimal monetary policy under either commitment or discretion, where $\{\mathbf{b}_0, \mathbf{c}_0, \mathbf{D}_0\}$ *denote the "true" structural parameter and* E *is the unconditional expectation operator.*

Proposition 1 follows immediately from (13) and the fact that $1\left(\mathbf{S}_{t-1} \leq 0\right) = 1$ is a sufficient condition for $\mathbf{c}_0'\mathbf{D}_0 s_{t-1} \leq 0$ and, similarly, that $1\left(\mathbf{S}_{t-1} > 0\right) = 1$ is a sufficient condition for $\mathbf{c}_0'\mathbf{D}_0 s_{t-1} > 0$.[8]

Next, we define the following set of instruments

Assumption 2. *Let Z_t denote a p-dimensional vector of instruments such that*

[8] We construct the moment functions in (14) using $1\left(\mathbf{S}_{t-1} \leq 0\right)$ instead of $1\left(\mathbf{c}_0'\mathbf{D}_0 s_{t-1} \leq 0\right)$ to obtain moment functions which are differentiable in the parameters ϕ (and in fact linear given an appropriate reparameterization), thus avoiding complications to do with non-smooth moment functions. In particular, we explore the fact that $1\left(\mathbf{S}_{t-1} \leq 0\right) = 1$ is a sufficient condition for $1\left(\mathbf{c}_0'\mathbf{D}_0 s_{t-1} \leq 0\right)$=1.

1. Z_t *has bounded support;*

2. $\mathrm{E}\left[v_t 1\left(\mathbf{S}_{t-1} \leq 0\right) Z_t\right] = \bar{\Pi}\mathrm{E}\left[1\left(\mathbf{S}_{t-1} \leq 0\right) Z_t\right]$, *and* $\mathrm{E}\left[v_t 1\left(\mathbf{S}_{t-1} > 0\right) Z_t\right] = \bar{\Pi}\mathrm{E}\left[1\left(\mathbf{S}_{t-1} > 0\right) Z_t\right]$;

3. $\mathrm{E}\left[\left(\Pi_t - \bar{\Pi}\right) 1\left(\mathbf{S}_{t-1} \leq 0\right) Z_t\right] \neq 0$, $\mathrm{E}\left[s_t 1\left(\mathbf{S}_{t-1} \leq 0\right) Z_t\right] \neq 0$, $\mathrm{E}\left[s_{t-1} 1\left(\mathbf{S}_{t-1} \leq 0\right) Z_t\right] \neq 0$.

Assumption 2.1. guarantees that, without loss of generality, the vector of instruments can be restricted to have positive support. Assumption 2.2 requires the instrumental variables to be uncorrelated with the measurement error v_t. Finally, Assumption 2.3 requires that the instruments are relevant.

3.2 The identified set

Given Assumption 2, the moment inequalities in Proposition 1 can be written as

$$\mathrm{E}\left[m_{d,t}\left(\phi_0, \bar{\Pi}_0\right)\right] \equiv \mathrm{E}\begin{bmatrix} -\left(\Pi_t - \bar{\Pi}_0 - \mathbf{b}_0' \mathbf{D}_0 s_t\right) 1\left(\mathbf{S}_{t-1} \leq 0\right) Z_t \\ \left(\Pi_t - \bar{\Pi}_0 - \mathbf{b}_0' \mathbf{D}_0 s_t\right) 1\left(\mathbf{S}_{t-1} > 0\right) Z_t \end{bmatrix} \geq 0, \tag{15}$$

$$\mathrm{E}\left[m_{c,t}\left(\phi_0, \bar{\Pi}_0\right)\right] \equiv \mathrm{E}\begin{bmatrix} \left(\Pi_t - \bar{\Pi}_0 - \mathbf{c}_0' \mathbf{D}_0 s_{t-1} - \mathbf{b}_0' \mathbf{D}_0 s_t\right) 1\left(\mathbf{S}_{t-1} \leq 0\right) Z_t \\ -\left(\Pi_t - \bar{\Pi}_0 - \mathbf{c}_0' \mathbf{D}_0 s_{t-1} - \mathbf{b}_0' \mathbf{D}_0 s_t\right) 1\left(\mathbf{S}_{t-1} > 0\right) Z_t \end{bmatrix} \geq 0. \tag{16}$$

We use $\theta = \left(\phi, \bar{\Pi}\right) \in \Theta$ to denote an ellement of the parameter space. The "true" underlying vector value of θ is denoted θ_0 and, in general, is not point identified by the conditions (15) and (16). Thus, we define the identified set consistent with optimal monetary policy as follows

Definition 1. *Let* $\theta = \left(\phi,\ \bar{\Pi}\right) \in \Theta$. *The identified set is defined as*

$$\Theta^I \equiv \left\{\theta \in \Theta : \textit{such that } \mathrm{E}\left[m_t\left(\theta\right)\right] \geq 0\right\}$$

with $m_t\left(\theta\right) \equiv \left[m_{d,t}\left(\phi, \bar{\Pi}\right),\ \ m_{c,t}\left(\phi, \bar{\Pi}\right)\right]'$.

Under optimal monetary policy, Θ^I is never empty. From the linearity of the moment functions

$$E\left[m_t\left(\theta\right)\right] = E\left[m\left(\theta_0\right)\right] + \nabla_\theta m'\left(\theta - \theta_0\right), \tag{17}$$

where $\nabla_\theta m$ denotes the gradient of the moment functions. The first terms on the RHS of (17) is non-negative because of (15) and (16). Hence, by construction $\theta_0 \in \Theta^I$. On the other hand, Θ^I may be non-empty even if (13) does not hold. In fact, violation of (13) does not necessary imply a violation of (15) and/or (16). Thus, θ_0 may belong to the identified set even in the case of no optimal monetary policy. In this sense, a non-empty identified set, while necessary for optimal monetary policy, is not sufficient.

Although our moment inequalities are linear in the transformed parameter space $\tilde{\theta} = \left\{\phi^\dagger, \phi^\ddagger, \bar{\Pi}\right\}$, with $\phi^\dagger = \mathbf{c}'\mathbf{D}$ and $\phi^\ddagger = \mathbf{b}'\mathbf{D}$, our set-up is rather different from Bontemps, Magnac, and Maurin (2012). In their case, lack of point identification arises because one can observe only lower and upper bounds for the dependent variable. In our case, we observe Π_t, s_t and s_{t-1}, and lack of identification arises because we do not know which model generated the observed series. In particular, their characterization of the identified set relies on the boundedness of the intervals defined by the upper and lower bound of the observed variables, and thus does not necessarily apply to our set-up. Beresteanu and Molinari (2008) random set approach also applies to models which are incomplete because the dependent variable and/or the regressors are interval-valued. For this reason, in the sequel we use the criterion function of Chernozhukov et al. (2007).

Before proceeding, notice that one may be tempted to reduce the moment inequalities (15) and (16) into a single moment equality condition, given by

$$\mathrm{E}\Big[\left(\Pi_t - \bar{\Pi}_0 - \varphi_t \mathbf{c}_0'\mathbf{D}_0 s_{t-1} - \mathbf{b}_0'\mathbf{D}_0 s_t\right) Z_t\Big] = 0,$$

where $\varphi_t \in \{0,1\}$ is a random variable taking value 1 in the case of commitment and 0 in the case of discretion. If φ_t is degenerate, it may be treated as a fixed parameter φ and the model can be estimated by GMM, provided appropriate instruments are available. This is an application of the conduct parameter method sometimes used in the industrial organization literature. But, this approach is problematic since optimal monetary policy is characterized by either commitment or discretion, and the standard regularity condition for consistency are violated.[9]

3.3 Preliminaries on inference

Before describing the model specification test in Section 4, we describe some preliminary notions related to inference on the identified set Θ^I. The basic idea underlying the specification tests is to use the bounds for the observed inflation rate derived above to generate a family of moment inequality conditions that are consistent with optimal policy. These moment inequality conditions may be used to obtain a criterion function whose set of minimizers is the estimated identified set. If the estimated identified set is non-empty, we construct the corresponding confidence region.

We define the following $4p$ moment functions associated with (15) and (16)

$$m^-_{i,d,t}\left(\phi,\bar{\Pi}\right) = -\left(\Pi_t - \bar{\Pi} - \mathbf{b}'\mathbf{D}s_t\right) 1\left(\mathbf{S}_{t-1} \leq 0\right) Z^i_t,$$

$$m^+_{i,d,t}\left(\phi,\bar{\Pi}\right) = \left(\Pi_t - \bar{\Pi} - \mathbf{b}'\mathbf{D}s_t\right) 1\left(\mathbf{S}_{t-1} > 0\right) Z^i_t,$$

$$m^-_{i,c,t}\left(\phi,\bar{\Pi}\right) = \left(\Pi_t - \bar{\Pi} - \mathbf{c}'\mathbf{D}s_{t-1} - \mathbf{b}'\mathbf{D}s_t\right) 1\left(\mathbf{S}_{t-1} \leq 0\right) Z^i_t,$$

$$m^+_{i,c,t}\left(\phi,\bar{\Pi}\right) = -\left(\Pi_t - \bar{\Pi} - \mathbf{c}'\mathbf{D}s_{t-1} - \mathbf{b}'\mathbf{D}s_t\right) 1\left(\mathbf{S}_{t-1} > 0\right) Z^i_t,$$

[9] In the IO literature, conduct parameter method methods are usually applied to obtain an average estimate of market power across segmented markets with different structures (Corts, 1999, for a discussion of existing applications). Only in this context it is possible to interpret an estimator for φ with continuous support.

with Z_t^i the i^{th} element of Z_t. The corresponding sample moment functions are

$$m_{i,d,T}^{-}\left(\phi,\bar{\Pi}\right)=T^{-1}\sum_{t=1}^{T}m_{i,d,t}^{-}\left(\phi,\bar{\Pi}\right), \qquad m_{i,d,T}^{+}\left(\phi,\bar{\Pi}\right)=T^{-1}\sum_{t=1}^{T}m_{i,d,t}^{+}\left(\phi,\bar{\Pi}\right),$$
$$m_{i,c,T}^{-}\left(\phi,\bar{\Pi}\right)=T^{-1}\sum_{t=1}^{T}m_{i,c,t}^{-}\left(\phi,\bar{\Pi}\right), \qquad m_{i,c,T}^{+}\left(\phi,\bar{\Pi}\right)=T^{-1}\sum_{t=1}^{T}m_{i,c,t}^{+}\left(\phi,\bar{\Pi}\right),$$

and are collected in the $4p$-dimensional vector of sample moment functions

$$m_T\left(\theta\right)=\begin{bmatrix}\left(m_{1,d,T}^{-}\left(\phi,\bar{\Pi}\right),\ldots,m_{p,d,T}^{-}\left(\phi,\bar{\Pi}\right)\right)'\\ \left(m_{1,d,T}^{+}\left(\phi,\bar{\Pi}\right),\ldots,m_{p,d,T}^{+}\left(\phi,\bar{\Pi}\right)\right)'\\ \left(m_{1,c,T}^{-}\left(\phi,\bar{\Pi}\right),\ldots,m_{p,c,T}^{-}\left(\phi,\bar{\Pi}\right)\right)'\\ \left(m_{1,c,T}^{+}\left(\phi,\bar{\Pi}\right),\ldots,m_{p,c,T}^{+}\left(\phi,\bar{\Pi}\right)\right)'\end{bmatrix}. \tag{18}$$

We let $m_{i,T}\left(\theta\right)$ denote the i-th element of $m_T\left(\theta\right)$, and define $V\left(\theta\right)$, the asymptotic variance of $\sqrt{T}m_T\left(\theta\right)$, and $\widehat{V}_T\left(\theta\right)$ the corresponding heteroscedasticity and autocorrelation consistent (HAC) estimator.[10] Finally, we impose the following assumption

Assumption 3. *The following conditions are satisfied*

1. $W_t=\left(\Pi_t,s_t,Z_t\right)$ *is a strong mixing process with size* $-r/(r-2)$*, where* $r>2$*;*
2. $\mathrm{E}\left(\left|W_{i,t}\right|^{2r+\iota}\right)<\infty$*,* $\iota>0$ *and* $i=1,2,\ldots,p+2$*;*
3. $\mathrm{plim}_{T\to\infty}\widehat{V}_T\left(\theta\right)=V\left(\theta\right)$ *is positive definite for all* $\theta\in\Theta$*, where* Θ *is compact;*
4. $\sup_{\theta\in\Theta}\left|\nabla_\theta m_T\left(\theta\right)-D\left(\theta\right)\right|\overset{pr}{\to}0$*, where* $D\left(\theta\right)$ *is full rank.*

The criterion function we use for the inferential procedure is

$$Q_T\left(\theta\right)=\sum_{i=1}^{4p}\frac{\left[m_{i,T}\left(\theta\right)\right]_{-}^{2}}{\widehat{v}^{i,i}\left(\theta\right)}, \tag{19}$$

[10] This is obtained as $\widehat{V}_T\left(\theta\right)=\frac{1}{T}\sum_{k=-s_T}^{s_T}\sum_{t=s_T}^{T-s_T}\lambda_{k,T}\left(m_t\left(\theta\right)-m_T\left(\theta\right)\right)\left(m_{t+k}\left(\theta\right)-m_T\left(\theta\right)\right)'$, where s_T is a lag truncation parameter such that $s_T=o(T^{1/2})$ and $\lambda_{k,T}=1-k/\left(s_T+1\right)$.

where $[x]_- = x\,1\,(x \leq 0)$, and $\widehat{v}^{i,i}\,(\theta)$ is the $i-$th element on the diagonal of $\widehat{V}_T\,(\theta)$. The probability limit of $Q_T\,(\theta)$ is given by $Q\,(\theta) = \mathrm{p}\lim_{\mathrm{T}\to\infty} Q_T\,(\theta)$. The criterion function Q has the property that $Q(\theta) \geq 0$ for all $\theta \in \Theta$ and that $Q(\theta) = 0$ if and only if $\theta \in \Theta^I$, where Θ^I is as in Definition 1. Under Assumptions 1–3 a consistent estimator of the identified set $\widehat{\Theta}_T^I$ can be obtained as

$$\widehat{\Theta}_T^I = \left\{ \theta \in \Theta \text{ s.t. } TQ_T\,(\theta) \leq d_T^2 \right\}, \tag{20}$$

where d_T satisfies the conditions $\sqrt{\ln \ln T}/d_T \to 0$ and $d_T/\sqrt{T} \to 0$. In the Supplementary Material S.1 we show how to obtain an estimator for the identified set and construct a confidence region $C_T^{1-\alpha}$ that asymptotically contains the identified set Θ^I with probability $1-\alpha$.

4 Specification tests

The next step in our analysis is to test for the null hypothesis of discretion (commitment), taking into account the lower (upper) bound imposed by optimal monetary policy. Heuristically, this implies testing whether there is a θ in the identified set for which the moment inequality conditions associated with either discretion or commitment hold as equalities. If there is such θ, then we have evidence in favor of discretion (commitment). The test consists of a two-step procedure: in the first step the structural parameters are estimated under either discretion or commitment; in the second step we test if the estimated parameters are in the identified set implied by optimal monetary policy under either discretion or commitment.

In the sequel we consider our benchmark application, the New Keynesian model with staggered prices and monopolistic competition that has become widely used to study optimal monetary policy.[11] The optimizing model of staggered price-setting proposed by Calvo (1983) results in the

[11] See Appendix A for a more detailed description of the structural model.

following equation relating the inflation rate to the economy-wide real marginal cost and expected inflation

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \psi s_t + u_t, \tag{21}$$

where ψ and β are positive parameters related to technology and preferences, π_t is the inflation rate, s_t the real marginal cost in deviation from the flexible-price steady state, and u_t is an exogenous stochastic shock resulting from time-varying markups and other distortions.

The objective function of the monetary authority is derived as a second order approximation to the utility of a stand-in agent around the stable equilibrium associated with zero inflation, and takes the form

$$\mathbf{U} = \mathbb{E}_0 \left[-\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \zeta s_t^2 \right) \right], \tag{22}$$

with ζ a positive parameter that relates to technology and preferences. Thus, in the benchmark model we obtain $\mathbf{b} = 1$, $\mathbf{B}_s = -\psi$, $\mathbf{c} = -1$, $\mathbf{Q} = \zeta$, and $\mathbf{D} = -(\zeta/\psi)$, and the moment inequality conditions that characterize optimal monetary policy corresponding to (15) and (16) specialize as follows

$$\mathrm{E}\left[m_{d,t}\left(\mathbf{D}, \bar{\Pi}\right)\right] \equiv \mathrm{E}\begin{bmatrix} -\left(\Pi_t - \bar{\Pi} - \mathbf{D}s_t\right) 1\left(s_{t-1} \leq 0\right) Z_t \\ \left(\Pi_t - \bar{\Pi} - \mathbf{D}s_t\right) 1\left(s_{t-1} > 0\right) Z_t \end{bmatrix} \geq 0, \tag{23}$$

$$\mathrm{E}\left[m_{c,t}\left(\mathbf{D}, \bar{\Pi}\right)\right] \equiv \mathrm{E}\begin{bmatrix} \left(\Pi_t - \bar{\Pi} - \mathbf{D}\Delta s_t\right) 1\left(s_{t-1} \leq 0\right) Z_t \\ -\left(\Pi_t - \bar{\Pi} - \mathbf{D}\Delta s_t\right) 1\left(s_{t-1} > 0\right) Z_t \end{bmatrix} \geq 0, \tag{24}$$

with $\theta = \left\{\mathbf{D}, \bar{\Pi}\right\}$, the parameter space. In more general applications, the parameter vectors in $\mathbf{b}$ and $\mathbf{c}$ may be unknown, and $\theta = \left\{\mathbf{b}, \mathbf{c}, \mathbf{D}, \bar{\Pi}\right\}$. In such cases, $\mathbf{b}$ and $\mathbf{c}$ may be pre-estimated from the system (3) as they are invariant across policy regimes, and the covariance estimator $\widehat{V}_T(\theta)$ needs to capture the estimation error due to the estimators $\widehat{\mathbf{b}}$ and $\widehat{\mathbf{c}}$.

4.1 Testing for discretion

If the monetary authority implements optimal policy under discretion the joint path of actual inflation and the economy-wide real marginal cost satisfies the moment conditions

$$\mathrm{E}\left[m_{d,t}^{0}\left(\theta_0\right)\right] = \mathrm{E}\left[\left(\Pi_t - \bar{\Pi}_0 - \mathbf{D}_0 s_t\right) Z_t\right] = 0, \tag{25}$$

$$\mathrm{E}\left[m_{c,t}\left(\theta_0\right)\right] = \mathrm{E}\left[\begin{array}{c} \left(\Pi_t - \bar{\Pi}_0 - \mathbf{D}_0 \Delta s_t\right) 1\left(s_{t-1} \leq 0\right) Z_t \\ -\left(\Pi_t - \bar{\Pi}_0 - \mathbf{D}_0 \Delta s_t\right) 1\left(s_{t-1} > 0\right) Z_t \end{array}\right] \geq 0, \tag{26}$$

with m^0 denoting the moment functions that do not include the indicator on s_{t-1}. The moment equality conditions in (25) follow from the assumption of discretion and the moment inequality conditions (26) impose a lower bound to the observed inflation rate as implied by optimal monetary policy. As already mentioned, conditions (25) point identify θ_0, provided we can find at least one instrument, in addition to the intercept, satisfying Assumption 2. We define the following test for optimal monetary policy under discretion.

Definition 2. *Let $\theta_0 \equiv \left(\mathbf{D}_0,\, \bar{\Pi}_0\right) \in \Theta$. We define the null hypothesis of discretion and optimal monetary policy as,*

$$H_0^d: \ \theta_0 \textit{ satisfies conditions } (25)\text{–}(26),$$

against the alternative $H_1^d: \ \theta_0$ does not satisfy conditions (25)–(26).

To test the null hypothesis of discretion we follow a two-step procedure. Under the null hypothesis, the structural parameter vector θ_0 is point-identified and it can be consistently estimated via the optimal GMM estimator using the moment conditions (25).[12] Thus, to test the null hypothesis of

[12] If we assume that θ_0 satisfies (25)–(26), then it is possible to obtain an estimator using the approach of Moon and Schorfheide (2009), who consider the case in which the set of moment equalities point identify the parameters of interest, and use the additional information provided by the set of moment inequalities to improve efficiency. However, our objective is to test whether there exists θ_0 satisfying (25)–(26).

discretion we first obtain an estimate for the structural parameter vector using the optimal GMM estimator, denoted $\widehat{\theta}_d$. In the second step, we construct the following test statistic

$$TQ_T^d\left(\widehat{\theta}_d\right) = T\left[\sum_{i=1}^{p} \frac{m_{i,d,T}^0\left(\widehat{\theta}_d\right)^2}{\widehat{v}^{i,i}\left(\widehat{\theta}_d\right)} + \sum_{i=1}^{2p} \frac{\left[m_{i,c,T}\left(\widehat{\theta}_d\right)\right]_-^2}{\widehat{v}^{i,i}\left(\widehat{\theta}_d\right)}\right], \tag{27}$$

where $\widehat{v}^{i,i}\left(\widehat{\theta}_d\right)$ is the i-th diagonal element of $\widehat{V}_T\left(\widehat{\theta}_d\right)$, the HAC estimator of the asymptotic variance of $\sqrt{T}\left[m_{d,T}^0\left(\widehat{\theta}_d\right), m_{c,T}\left(\widehat{\theta}_d\right)\right]$, which takes into account the estimation error in $\widehat{\theta}_d$.[13]

Notice that since the first p moment conditions hold with equality, they all contribute to the asymptotic distribution of $TQ_T^d\left(\widehat{\theta}_d\right)$. Thus, we apply the Generalized Moment Selection procedure introduced by Andrews and Soares (2010) only to the inequality conditions.[14] Andrews and Soares (2010) study the limiting distribution of the statistic in (27) evaluated at a fixed θ. In our case, due to the two-step testing procedure, we need to take into account the contribution of the estimation error to the asymptotic variance of the moment conditions, and compute bootstrap critical values that properly mimic the contribution of parameter estimation error. The first order validity of the bootstrap percentiles is established in the following Proposition.

Proposition 2. *Let Assumptions 1, 2 and 3 hold. Let $c_{B,\alpha}^d$ be the $(1-\alpha)$ percentile of the empirical distribution of $TQ_T^{*d}\left(\widehat{\theta}_d^*\right)$, the bootstrap counterpart of $TQ_T^d\left(\widehat{\theta}_d\right)$. Then, as $T\to\infty$, $B\to\infty$, $l\to\infty$, and $l^2/T\to 0$, we have that:*

(i) under H_0^d, $\limsup_{T,B\to\infty} \Pr\left(TQ_T^d\left(\widehat{\theta}_d\right) > c_{B,\alpha}^d\right) = \alpha$,

(ii) under H_1^d, $\lim_{T,B\to\infty} \Pr\left(TQ_T^d\left(\widehat{\theta}_d\right) > c_{B,\alpha}^d\right) = 1$,

where B denotes the number of bootstrap replications.

[13] See Supplementary Material S.2 for the definition of $\widehat{V}_T\left(\widehat{\theta}_d\right)$.

[14] See Supplementary Material S.1 for details on the Generalized Moment Selection method.

4.2 Testing for commitment

If the monetary authority implements optimal policy under commitment, the joint path of actual inflation and the economy-wide real marginal cost is given by

$$\mathrm{E}\left[m_{d,t}\left(\theta_0\right)\right]=\mathrm{E}\begin{bmatrix}-\left(\Pi_t-\bar{\Pi}_0-\mathbf{D}_0 s_t\right)1\left(s_{t-1}\leq 0\right)Z_t\\ \left(\Pi_t-\bar{\Pi}_0-\mathbf{D}_0 s_t\right)1\left(s_{t-1}>0\right)Z_t\end{bmatrix}\geq 0, \tag{28}$$

$$\mathrm{E}\left[m^0_{c,t}\left(\theta_0\right)\right]=\mathrm{E}\left[\left(\Pi_t-\bar{\Pi}_0-\mathbf{D}_0\Delta s_t\right)Z_t\right]=0, \tag{29}$$

where the moment equality condition (29) follows from the assumption of commitment and the moment inequality condition (28) imposes an upper bound to the observed inflation rate, as implied by optimal monetary policy. We define the following test for optimal policy under commitment.

Definition 3. *Let $\theta_0\equiv\left(\mathbf{D}_0,\ \bar{\Pi}_0\right)\in\Theta$. We define the null hypothesis of commitment and optimal monetary policy as,*

$$H^c_0:\ \theta_0 \textit{ satisfies conditions } (28)\text{–}(29).$$

against the alternative $H^c_1:\ \theta_0$ does not satisfy conditions (28)–(29).

The test of optimal monetary policy under commitment has the same structure as the test under discretion, with an analogous test statistic, given by

$$TQ^c_T\left(\widehat{\theta}_c\right)=T\left[\sum_{i=1}^{2p}\frac{\left[m_{i,d,T}\left(\widehat{\theta}_c\right)\right]^2_-}{\widehat{v}^{i,i}\left(\widehat{\theta}_c\right)}+\sum_{i=1}^{p}\frac{m^0_{i,c,T}\left(\widehat{\theta}_c\right)^2}{\widehat{v}^{i,i}\left(\widehat{\theta}_c\right)}\right], \tag{30}$$

with $\widehat{\theta}_c$ the optimal GMM estimator under commitment. We establish the following Proposition.

Proposition 3. *Let Assumptions 1, 2 and 3 hold. Let $c^c_{\alpha,B}$ be the $(1-\alpha)$ percentile of the empirical distribution of $TQ^{*c}_T\left(\widehat{\theta}^*_c\right)$, the bootstrap counterpart of $TQ^c_T\left(\widehat{\theta}_c\right)$. Then, as $T\to\infty$, $B\to\infty$, $l\to\infty$, and $l^2/T\to 0$, we have that:*

(i) under H_0^c, $\limsup_{T,B\to\infty} \Pr\left(TQ_T^c\left(\widehat{\theta}_c\right) > c_{\alpha,B}^c\right) = \alpha$,

(ii) under H_1^c, $\lim_{T,B\to\infty} \Pr\left(TQ_T^c\left(\widehat{\theta}_c\right) > c_{\alpha,B}^c\right) = 1$,

where B *denotes the number of bootstrap replications.*

In the Supplementary Material S.3 we provide an alternative formulation of the specification test, and in the Supplementary Material S.4 we show how to adapt our framework to provide an interpretation of the specification test based on the set-up developed by Bontemps et al. (2012).

5 Model selection

The moment conditions (25), (26), (28) and (29) can be used to construct a model selection test that discriminates between discretion and commitment, maintaining the assumption of optimal monetary policy. Following Shi (2015), we construct a quasi-likelihood ratio test for the null hypothesis that both models are equally close to the true data. If the null hypothesis is rejected, we select the one closer to the true model in terms of a pseudo-distance measure. The null hypothesis is

$$H_0 : d(\mathcal{D}, \mu) = d(\mathcal{C}, \mu), \tag{31}$$

against the alternative $H_1 : d(\mathcal{D}, \mu) < d(\mathcal{C}, \mu)$, where $\mathcal{D}$ is the model for discretion and optimal policy in (25) and (26), $\mathcal{C}$ is the model for commitment and optimal policy in (28) and (29), and μ is the true model. To test the null hypothesis in (31), we construct the test statistic

$$QLR_T = \max_{\theta\in\Theta} \frac{1}{T} \sum_{t=1}^{T} \mathcal{M}_t^d\left(\theta, \widehat{\gamma}_d\left(\theta\right)\right) - \max_{\theta\in\Theta} \frac{1}{T} \sum_{t=1}^{T} \mathcal{M}_t^c\left(\theta, \widehat{\gamma}_c\left(\theta\right)\right), \tag{32}$$

where $\mathcal{M}_t^d(\theta,\gamma(\theta)) = \exp\left(\gamma(\theta)'\begin{bmatrix} m_{d,t}^0(\theta) \\ m_{c,t}(\theta)\end{bmatrix}\right)$, $\mathcal{M}_t^c(\theta,\gamma(\theta)) = \exp\left(\gamma(\theta)'\begin{bmatrix} m_{c,t}^0(\theta) \\ m_{d,t}(\theta)\end{bmatrix}\right)$, and

$$\widehat{\gamma}_i(\theta_i) = \underset{\gamma\in\mathbb{R}^p\times\mathbb{R}_+^{2p}}{\arg\min}\; T^{-1}\sum_{t=1}^{T}\mathcal{M}_t^i(\theta_i,\gamma), \tag{33}$$

with $i \in \{d,c\}$. In turn, the pseudo true set of parameters can be estimated as

$$\widehat{\Theta}_T^i = \underset{\theta\in\Theta}{\arg\max}\; T^{-1}\sum_{t=1}^{T}\mathcal{M}_t^i(\theta,\widehat{\gamma}_i(\theta)). \tag{34}$$

Under discretion and optimal policy, conditions (25) and (26) point identify $\widehat{\theta}_T^d$ and, similarly, under commitment and optimal policy, conditions (28) and (29) point-identify $\widehat{\theta}_T^c$.

Using the estimated parameters, we define

$$T\widehat{\omega}_T^2\left(\widehat{\theta}_d,\widehat{\theta}_c\right) = \sum_{k=-s_T}^{s_T}\sum_{t=s_T}^{T-s_T}\lambda_{k,T}\left(\Delta_t-\bar{\Delta}\right)\left(\Delta_{t+k}-\bar{\Delta}\right)', \tag{35}$$

with $\Delta_t = \mathcal{M}_t^d\left(\widehat{\theta}_d,\widehat{\gamma}_d\left(\widehat{\theta}_d\right)\right) - \mathcal{M}_t^c\left(\widehat{\theta}_c,\widehat{\gamma}_c\left(\widehat{\theta}_c\right)\right)$, $\bar{\Delta} = T^{-1}\sum_{t=1}^{T}\Delta_t$, and where s_T and $\lambda_{k,T}$ are as defined in footnote 10. Shi (2015) shows that under H_0 we have

$$\sqrt{T}\frac{QLR_T}{\widehat{\omega}_T\left(\widehat{\theta}_d,\widehat{\theta}_c\right)} \to_d N(0,1), \tag{36}$$

and, therefore, we reject the null hypothesis (31) in favor of the alternative at the α level if $\sqrt{T}QLR_T/\widehat{\omega}_T\left(\widehat{\theta}_d,\widehat{\theta}_c\right) > z_\alpha$, where z_α is the $1-\alpha$ quantile of the standard normal distribution.

6 Monte Carlo experiments

In this section, we perform Monte Carlo simulations to analyze the small sample properties of the model specification test presented in Section 4. The data generating process (DGP) used in the

Table 1: Monte Carlo experiments: rejection rates (nominal level $\alpha = 0.10$)

	DGP: Discretion ($T = 250$)		
Instrument lags:	$t-1\dots t-3$	$t-2\dots t-4$	$t-3\dots t-5$
H_0: discretion	0.118	0.112	0.126
H_0: commitment	1.000	1.000	0.992
	DGP: Commitment ($T = 250$)		
Instrument lags:	$t-1\dots t-3$	$t-2\dots t-4$	$t-3\dots t-5$
H_0: discretion	0.980	0.954	0.926
H_0: commitment	0.148	0.192	0.174
	DGP: Discretion ($T = 500$)		
Instrument lags:	$t-1\dots t-3$	$t-2\dots t-4$	$t-3\dots t-5$
H_0: discretion	0.086	0.092	0.122
H_0: commitment	1.000	1.000	1.000
	DGP: Commitment ($T = 500$)		
Instrument lags:	$t-1\dots t-3$	$t-2\dots t-4$	$t-3\dots t-5$
H_0: discretion	1.000	0.994	1.000
H_0: commitment	0.126	0.160	0.152

The table reports the rejection rates of the test statistics TQ_T, in (27) and (30), with 10% nominal level. Each Monte Carlo simulation has T observations and “burn-in” sample of size 1,000. The critical values $c^d_{\alpha,B}$ and $c^c_{\alpha,B}$ are based on 500 block-bootstrap replications of block size 4.

Monte Carlo experiment is described in the Supplementary Material S.5. We simulate 500 vectors of time-series, each with 1,000 + T observations and we discard the first 1,000 observations to eliminate the influence of the initial values. The resulting time-series length is T, for which we consider two possible values: $T = 250$ and $T = 500$. This way, we are able to study the influence of sample size on the properties of our test.

We consider both discretion and commitment, and we seek to analyze the size and power properties of the tests described in Propositions 2 and 3. We also examine how the performance of the proposed tests varies with the strength of the instruments, by varying the length of the lags used as instruments. In particular, the instrumental variables used in the Monte Carlo are lagged values of inflation and the labor income share, and we look at the performance of the test when the instrument list includes the lags: $(t-1, t-2, t-3)$; $(t-2, t-3, t-4)$; and $(t-3, t-4, t-5)$.

For each sample, we obtain the critical values $c^d_{\alpha,B}$ and $c^c_{\alpha,B}$ following the bootstrap procedure described in Propositions 2 and 3. Table 1 reports the percentage of times the null hypothesis is rejected, obtained from the critical values based on the nominal level $\alpha = 0.10$. The results show that our test performs well in small samples. The power properties of the test are good, and the test is correctly sized with the empirical level close to the nominal level for both $T = 250$ and $T = 500$. As expected, the empirical level of the test departs from the 10% nominal level with weaker instruments. But, the test is found to still perform well when long lags are used.

7 Empirical application

In this section, we apply the specification and selection tests proposed above to study the monetary policy in the United States since the start of the 1980s. The sample spans a period in which monetary policy has been perceived as good (Clarida, Gali, and Gertler, 2000).[15]

7.1 Data and sample

We use quarterly time-series for the US economy over the sample period 1983Q1 to 2008Q3. Following Galí and Gertler (1999) and Sbordone (2002), we use the labor income share in the non-farm business sector, detrended using a quadratic polynomial, to measure s_t. The measure of inflation is the percentage change in the GDP deflator.

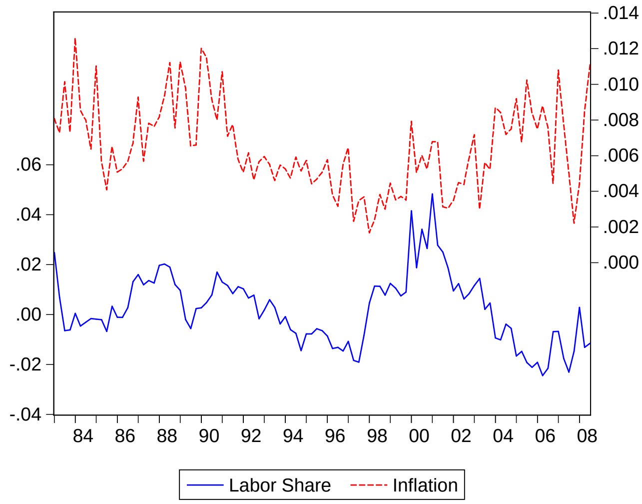
[15] The term "good" is used loosely to describe a period in which monetary policy is consistent with achieving stable and low inflation. Clarida et al. (2000) argue that this is due to a stronger systematic reaction of monetary policy to changes in expected inflation.

The econometric framework developed in this paper is for stationarity data (see Assumption 3). Halunga, Osborn, and Sensier (2009) show that there is a change in inflation persistence from $I(1)$ to $I(0)$ dated at June 1982. This result is related to the study of Lubik and Schorfheide (2004) who estimate a structural model of monetary policy for the US using full-information methods, and find that only after 1982 the estimated interest-rate feedback rule that characterizes monetary policy is consistent with equilibrium determinacy. Moreover, following the analysis in Clarida et al. (2000), we study the sample starting from 1983Q1, that removes the first three years of the Volcker era. Clarida et al. (2000) offer two reasons for doing this. First, this period was characterized by a sudden and permanent disinflation episode bringing inflation down from about 10 percent to 4 percent. Second, over the period 1979Q4 – 1982Q4, the operating procedures of the Federal Reserve involved targeting non-borrowed reserves as opposed to the Federal Funds rate. Thus, our empirical analysis focuses on the sample period 1983Q1 to 2008Q3, which spans the period starting after the disinflation and monetary policy shifts that occurred in the early 1980s and extends until the period when the interest rate zero lower bound becomes a binding constraint.[16] Figure 1 plots the time-series of the US labor income share and inflation.

Following standard practice in the literature (see, for example, Galí and Gertler, 1999), we include in the instrument set lagged values of the labor income share and inflation, assumed orthogonal to the measurement error in inflation (Assumption 2.2). The instrument set used comprises the first, second and third-order lags of the labor income share and inflation. These instrumental variables are adjusted using the transformation $Z_+ = Z - \min(Z)$, guaranteeing positiveness. Notice that Assumption 2.1 guarantees this transformation always exists. The complete instrument set also includes the unit vector, yielding $p = 7$ instruments and 28 moment conditions overall. Of course,

[16]After 2008Q3, the federal funds rate rapidly fell toward the lower bound, signaling a period of unconventional monetary policy for which our econometric specification may be inadequate.

Figure 1: labor share and inflation in the US, 1983Q1–2008Q3.



weak instruments are a potential problem given that the first step in our test requires a consistent estimator of the structural parameter vector θ_0. For example, when we are testing discretion we require that

$$\mathrm{E}\left[m_{d,t}^{0}\left(\theta_0\right)\right] = \mathrm{E}\left[\left(\Pi_t - \bar{\Pi}_0 - \mathbf{D}_0 s_t\right) Z_t\right] = 0, \tag{37}$$

holds at the "true" value $\theta_0 = \left(\mathbf{D}_0, \bar{\Pi}_0\right)'$ and no other value of θ. If the instrument is irrelevant, in the sense that the correlation between Π_t and Z_t is zero (or weakly different from zero), then $\theta_0 = \left(\mathbf{D}_0, \bar{\Pi}_0\right)'$ is not identified as, given $\bar{\Pi}_0$, any value of $\mathbf{D}$ satisfies the moment condition. Instrument relevance requires strong correlation between Π_t and Z_t, as indicated by Assumption 2.3.

Reassuringly, the instruments used (which include lags of inflation and of the real marginal cost) pass the standard tests of weak instruments. In particular, the Kleibergen and Paap (2006) Wald statistic (the robust counterpart of the Cragg-Donald Wald statistic) is 14.079, which suggests that weak identification should not be considered a problem. Underidentification is clearly rejected based on the Kleibergen and Paap (2006) rank test, that yields a p-value of 0.001.

7.2 Baseline empirical results

We first examine the formal test statistics developed in sections 4.1 and 4.2 to test for discretion and commitment, under the maintained assumption of optimal monetary policy. The tests are based on a two-step procedure. In particular, to test discretion we first estimate the parameter vector θ_d via optimal GMM from condition (25). Next, using the estimated vector of parameters $\widehat{\theta}_d$ we construct the test statistic for discretion $TQ_T^d\left(\widehat{\theta}_d\right)$ and compute the bootstrap critical value. To test commitment, we proceed in an analogous way, making use of condition (29) to obtain $\widehat{\theta}_c$.

Results are reported in the Panel A of Table 2. Since we use enough instrumental variables for overidentification, we start by obtaining results from the standard Hansen J–test statistic for overidentifying restrictions. The table reports the J–tests and the corresponding p–values, for the null hypotheses of discretion (first column) and commitment (second column) based, respectively, on the moment conditions in (25) and (29). The p–value of the J–test for discretion is 16% and that for commitment is 20%. Thus, the standard J–test fails to reject either model.

By not making use of the full set of implications of optimal monetary policy, we are unable to reject either policy regimes. However, using the additional information implied by the maintained assumption of optimal monetary policy, we can test the composite null hypothesis of optimal monetary policy and a specific policy regime, discretion or commitment, by constructing the test statistic TQ_T. The test statistic is based on equation (27) for the case of discretion and equation (30) for commitment. For discretion, the p–value associated with the test statistic is 41%. Instead, for commitment, the p–value is 3%, allowing for rejection at the 5% level. Thus, we reject commitment but fail to reject discretion at all conventional levels.

Finally, Panel B of Table 2 reports the results from the Shi (2015) model selection test presented

Table 2: model specification and model selection tests

	Panel A: model specification tests	
	H_0: discretion	H_0: commitment
J–test	11.73	11.63
p–val	(0.16)	(0.20)
TQ_T	16.90	21.82
p–val	(0.41)	(0.03)

	Panel B: model selection test
	$H_0 : d(\mathcal{D}, \mu) = d(\mathcal{C}, \mu)$
QLR_T	4.999
p–val	(0.00)

The p–values for the J test and for TQ_T are obtained from 1,000 block-bootstrap replications with blocks of size 4. The J test is based on the moment condition (25) for discretion, and (29) for commitment. The test statistics TQ_T correspond to (27) and (30). The test statistic QLR_T is given by (32). The instrument list includes $\Pi_{t-1}, \Pi_{t-2}, \Pi_{t-3}$, and $s_{t-1}, s_{t-2}, s_{t-3}$.

in Section 5. We consider the null hypothesis that the distance between the commitment and discretion model is zero, against the two-sided alternative, and construct the test statistic so that a positive realization of QLR_T constitutes a rejection of the null hypothesis in favor of discretion. We reject the null hypothesis, with $QLR_T = 4.999$ and p-value less than 1%.

7.3 Inflation indexation

The baseline model considered is the simplest New Keynesian model. However, it is possible to apply our method to more general versions that include sources of endogenous persistence found to be empirically relevant. To illustrate this, we now consider a version of the model including inflation inertia.[17] We incorporate inflation inertia by considering partial inflation indexation, as in Giannoni and Woodford (2004) and Christiano et al. (2005). Partial indexation results in the following equation relating inflation to real marginal costs, lagged inflation and expected inflation

$$\pi_t - \gamma\pi_{t-1} = \beta\mathbb{E}_t\left(\pi_{t+1} - \gamma\pi_t\right) + \psi s_t + u_t, \tag{38}$$

where $\gamma \in [0,1]$ indicates the degree of indexation. This hybrid version of the Phillips Curve is widely used in empirical work. Following Giannoni and Woodford (2004) the objective function is

$$\mathbf{U} = -\mathbb{E}_0\left[\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\widetilde{\pi}_t^2 + \zeta s_t^2\right)\right], \tag{39}$$

with $\widetilde{\pi}_t = \pi_t - \gamma\pi_{t-1}$, subject to (38).

This model is analogous to our baseline model, except that π_t is everywhere replaced by the quasi-differenced inflation rate $\widetilde{\pi}_t$. The moment inequality conditions that characterize optimal monetary policy are given by the moment conditions analogous to (15) and (16), but with $\widetilde{\Pi}_t = \Pi_t - \gamma_0\Pi_{t-1}$ in place of Π_t and $(1-\gamma_0)\,\bar{\Pi}_0$ in place of $\bar{\Pi}_0$, as follows

$$\mathrm{E}\left[m_{d,t}\left(\mathbf{D}_0,\gamma_0,\bar{\Pi}_0\right)\right] \equiv \mathrm{E}\left[\begin{array}{c} -\left(\widetilde{\Pi}_t - (1-\gamma_0)\,\bar{\Pi}_0 - \mathbf{D}_0 s_t\right)1\left(s_{t-1}\leq 0\right)Z_t \\ \left(\widetilde{\Pi}_t - (1-\gamma_0)\,\bar{\Pi}_0 - \mathbf{D}_0 s_t\right)1\left(s_{t-1}> 0\right)Z_t \end{array}\right] \geq 0, \tag{40}$$

$$\mathrm{E}\left[m_{c,t}\left(\mathbf{D}_0,\gamma_0,\bar{\Pi}_0\right)\right] \equiv \mathrm{E}\left[\begin{array}{c} \left(\widetilde{\Pi}_t - (1-\gamma_0)\,\bar{\Pi}_0 - \mathbf{D}_0 \Delta s_t\right)1\left(s_{t-1}\leq 0\right)Z_t \\ -\left(\widetilde{\Pi}_t - (1-\gamma_0)\,\bar{\Pi}_0 - \mathbf{D}_0 \Delta s_t\right)1\left(s_{t-1}> 0\right)Z_t \end{array}\right] \geq 0. \tag{41}$$

Given Assumption 1, the measurement error in quasi-differenced inflation is given by the first-order moving average $\widetilde{v}_t = v_t - \gamma v_{t-1}$, and has mean $(1-\gamma)\,\Pi_0$ and variance $(1+\gamma^2)\,\sigma_v^2$. Thus, the instruments in Z_t should not include the first lag of inflation, to be independent from v_{t-1}.

In Table 3 we show results for the standard J-test, our model specification test, and the model

[17]In Supplementary Material S.6, we show how our methodology can be applied to a special (but empirically salient) case of the Erceg et al. (2000) model with sticky wages as well as prices.

Table 3: specification test (model with inflation indexation)

	Panel A: H_0 is Discretion			
indexation:	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
J-test (p–val)	0.220	0.172	0.124	0.096
TQ_T (p–val)	0.286	0.260	0.265	0.393
	Panel B: H_0 is Commitment			
indexation:	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
J-test (p–val)	0.383	0.397	0.396	0.289
TQ_T (p–val)	0.072	0.089	0.160	0.128
	Panel C: model selection test			
indexation:	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
QLR_T	2.404	2.082	1.937	2.534
p–val	0.008	0.019	0.026	0.006

The p–values for the J test and for TQ_T are obtained from 1,000 block-bootstrap replications with blocks of size 4. The instrument list includes $\Pi_{t-2}, \Pi_{t-3}, \Pi_{t-4}$, and $s_{t-2}, s_{t-3}, s_{t-4}$.

selection test of Shi (2015) based on (40) and (41). The results are shown for different levels of inflation indexation, including $\gamma = 0.2, 0.4, 0.6$ and 0.8, for discretion (Panel A) and commitment (Panel B). We notice first that the J-test fails to reject at conventional levels any of the 8 models considered. Instead, our test fails to reject discretion for each level of indexation considered but rejects commitment for $\gamma = 0.2$ and $\gamma = 0.4$.

Another important finding is that it is harder to reject either discretion or commitment as the degree of indexation increases. This result has a natural interpretation. Without indexation, optimal policy yields a process for inflation with low persistency. This counterfactual feature leads to the empirical rejection of the model. Instead, with higher degrees of inflation indexation, optimal policy is consistent with some persistency in the level of inflation, and rejecting either discretion or commitment is harder.

In these circumstances, the model selection test presented in Section 5 is specially relevant. Results are reported in Panel C of Table 3. Positive realizations of QLR_T constitute a rejection of the null hypothesis in favor of discretion. For all four levels of indexation, the p-value is less than 5%. Regardless of the level indexation, the null hypothesis is rejected decisively in favor of discretion.

8 Conclusion

This paper develops a method for testing for optimal monetary policy without requiring an explicit choice of the relevant equilibrium concept. The procedure considers a general specification of optimal policy that nests discretion and commitment as two special cases. It is obtained by deriving bounds for inflation that are consistent with both forms of optimal policy. This allows for the construction of a test statistic based on the combination of moment equality and inequality conditions that incorporate a wider set of implications of optimal monetary policy and provides a more powerful specification test. Unlike full-information methods, our approach does not require strong assumptions about the forcing variables.

We investigate if the behavior of the US monetary authority is consistent with the simple baseline New Keynesian model of optimal monetary policy (Clarida et al., 1999). We fail to reject the null hypothesis of discretion but reject that of commitment, a result consistent with findings in previous studies using full-information methods. In contrast, the standard J–test of overidentifying restrictions fails to reject either model. By making use of the full set of implications of optimal monetary policy, we discriminate across policy regimes, rejecting commitment but not discretion.

Our two-step testing procedure can be used more generally to test the validity of models that combine moment equality and inequality conditions, when the parameters of the model can be consistently estimated under the null hypothesis. Currently, the method proposed in this paper may not be directly applicable to medium-scale DSGE models of the kind pioneered by Smets and Wouters (2003) and Christiano et al. (2005), that include sources of endogenous persistence (such as habits in consumption, capital stock and persistence in interest rates). This is an interesting avenue for future work.

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Appendix

A Benchmark structural model

The framework is the New Keynesian model with monopolistic competition and Calvo pricing described in Clarida et al. (1999) and Woodford (2010). In log-linear form, inflation π_t, average real marginal costs s_t, and the output gap $z_t = \ln(Y_t/Y_t^n)$, satisfy the following relationships

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \psi s_t + u_t, \tag{A.1}$$

$$s_t = \left(\varsigma + \sigma^{-1}\right) z_t, \tag{A.2}$$

$$z_t = \mathbb{E}_t z_{t+1} - \sigma \left(i_t - \mathbb{E}_t \pi_{t+1}\right) + \nu_t, \tag{A.3}$$

where $i_t \geq -i^\star$ denotes the nominal interest rate in deviation from its steady state $i^\star$, u_t is an exogenous shock capturing time-varying desired markups, ν_t is a shock to the natural real rate, β is the discount factor, $\sigma > 0$ is the elasticity of intertemporal substitution and ψ is given by

$$\psi = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\vartheta\varsigma)}, \tag{A.4}$$

with $\alpha \in (0,1)$ the fraction of prices that are not reset optimally each period, $\vartheta > 1$ the elasticity of substitution across goods, and $\varsigma > 0$ the output elasticity of real marginal cost.

Finally, the second order approximation to the utility of a stand-in agent around the steady state equilibrium associated with zero inflation takes the form

$$\mathbf{U} = \mathbb{E}_0 \left[-\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \zeta s_t^2\right) \right], \tag{A.5}$$

with $\zeta = \psi / \left[\left(\varsigma + \sigma^{-1}\right) \vartheta\right]$ the relative target weight on the log average real marginal cost.