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Hitting the target: Mathematical attainment in children is related to interceptive timing ability

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Abstract

Interceptive timing (IntT) is a fundamental ability underpinning numerous actions (e.g. ball catching), but its development and relationship with other cognitive functions remains poorly understood. Piaget (1955) suggested that children need to learn the physical rules that govern their environment before they can represent abstract concepts such as number and time. Thus, learning how objects move in space and time may underpin the development of related abstract representations (i.e. mathematics). To test this hypothesis, we captured objective measures of IntT in 309 primary school children (4-11 years), alongside 'general motor skill' and 'national standardized academic attainment' scores. Bayesian estimation showed that IntT (but not general motor capability) uniquely predicted mathematical ability even after controlling for age, reading and writing attainment. This finding highlights that interceptive timing is distinct from other motor skills with specificity in predicting childhood mathematical ability independent of other forms of attainment and motor capability.

Keywords: Interceptive Timing; Mathematics; Reading; Writing; Education; Posture

Introduction

Interceptive timing (IntT) is a fundamental human sensorimotor ability that underpins actions where the goal is to make contact with a target when the target and human are in relative motion (e.g hitting a baseball). These tasks require both spatial and temporal accuracy, and proficiency in these tasks appears later in a child's developmental history than skills with minimal temporal constraints such as reaching to static objects (Sugden & Wade, 2013). Neurologically intact adult humans show exquisite precision in IntT, with elite baseball batters able to swing their bat to a spatial accuracy of ±1.5cm and a temporal accuracy of ±10ms (Tresilian, 1999). The IntT skills of humans are a testimony to the incredible learning capacity of the sensorimotor system and its ability to overcome the challenges involved in controlling over 600 muscles with the inherent difficulties of nonlinearity, nonstationarity, information delays, and noise whilst operating within an uncertain world (Franklin & Wolpert, 2011). Temporal processing delays are particularly problematic when performing IntT tasks and so the individual will need to make predictions about where the object and the limb will be at the time of desired contact (Tresilian, 2012). These predictions require precise estimates of how the object will move over time, together with state estimates of the neuromuscular system.

It is widely believed that sensorimotor prediction relies on internal models within the sensorimotor system. Internal models allow for prediction of object motion through space and time (Merfeld, Zupan, & Peterka, 1999), with forward models used to estimate the sensory consequences of motor commands (Flanagan & Wing, 1997; Wolpert, Miall, & Kawato, 1998). Thus, the development of these models is central to the ontogenetic acquisition of IntT skills. The deleterious impact of developmental delays in motor prediction can be readily imagined with regard to a child's ability to engage in physical activity. But it is possible that sensorimotor impairments have consequences for a child's cognitive capabilities in a manner that is not so readily appreciated by educational authorities (Cameron et al., 2012; Grissmer, Grimm, Aiyer, Murrah, & Steele, 2010; Roebers et al., 2014; Son & Meisels, 2006). Such proposals are consistent with the view that the phylogenetic emergence of higher-order cognitive abilities were built upon the evolutionary platform provided by the motor system (Barton, 2012), particularly with respect to estimating the future state of the environment and physical body (Desmurget & Grafton, 2000).

The idea that higher-order cognitive processes emerged from sensorimotor abilities is attractive (Wilson, 2002). It has been suggested that the fundamental importance of sensorimotor substrates to cognition extends both to the individual as well as the species, with Piaget (1955) suggesting that ontogeny recapitulates phylogeny in this regard. Thus, Piaget proposed that sensorimotor interactions with the environment underpin the development of cognitive representations, including our understanding of number. This idea has received a surge of support over the last decade, with evidence that abstract representations of number are grounded in early interactions with objects and an understanding of physical space (de Hevia & Spelke, 2010; Nieder & Dehaene, 2009). There is evidence to suggest that the basic spatial processing abilities in infants (6-13 months) are related to the mathematical capabilities developed at 4 years of age (Lauer & Lourenco, 2016). It also appears that number representations become spatially orientated (Fias, van Dijck, & Gevers, 2011) with representations of number and space sharing overlapping neural circuitry (Hubbard, Piazza, Pinel, & Dehaene, 2005).

Given that there appear to be close links between spatial and temporal representations (Bueti & Walsh, 2009; Burr, Ross, Binda, & Morrone, 2011; Chang, Tzeng, Hung, & Wu, 2011; Lourenco & Longo, 2010; Srinivasan & Carey, 2010; White & Diedrichsen, 2010; Wijdenes, Brenner, & Smeets, 2014) it is no great leap to hypothesize that representations of space, time and number will all be processed by related systems. There is currently no direct evidence examining whether a child’s skill performing IntT is related to their ability in mathematics, but a robust test of this hypothesis would be to measure IntT skill and relate this to standardized school mathematical measures. A failure to find a relationship would allow us to reject the hypothesis, whilst a more general relationship between IntT skill and cognitive ability (e.g. in reading and writing) would suggest that there is no specific functional relationship between mathematics and IntT over and above general academic achievement.

Thus, we developed an IntT task with 54 moving targets to test 309 primary school children (aged 4-11 years) (see Figure 1). Three target speeds and three target widths were presented (9 trial types) with a sufficient range to challenge older children whilst allowing younger children to also succeed. The number of targets hit (IntT score) was the primary measure of interest. In a separate task the manual dexterity and postural control abilities of the children were measured to distinguish between general motor skill and IntT abilities. Mathematics ability was obtained from the children's nationally standardized mathematics attainment scores (1-14 scale; see Supplementary materials). These, along with reading and writing scores, were provided by the school.

Methods

Participants

Participants were recruited from a state primary school in Bradford, West Yorkshire, UK. There were 368 children in UK school years 1 to 6 (aged 4-11 years) at the time of testing. All children were invited to take part in the study. The children completed two test sessions in which they completed a range of motor and cognitive tasks. All motor tasks took place in the first session. Ethical approval was obtained from the University of Leeds (School of Psychology) Ethics and Research committee.

From the 368 children at the school, 309 full data sets were included in the data analysis. Eleven children were removed from the 368 because they were classed as having special education needs (SEN) by the school. Twenty-nine were excluded because the experimenter recorded that they did not complete one or more tasks. Fourteen were excluded because they did not provide data on the interception task and five did not provide data on postural control.

Measures

Interceptive Timing Task

Children completed a computer based interception task in which they hit moving targets by controlling a custom-made 1-DoF joystick (see Figure 1). The joystick was placed next to a horizontally positioned BenQ XL2720Z LCD gaming display (Resolution: 1920 × 1080, size: 598 X 336mm, brightness: 300cd / m^2, refresh rate: 144Hz). The position of the joystick was represented on screen by a black rectangular 'bat' (dimensions: 10 × 15mm) that was always in line with the joystick. All stimuli were generated using Python 2.7.9.

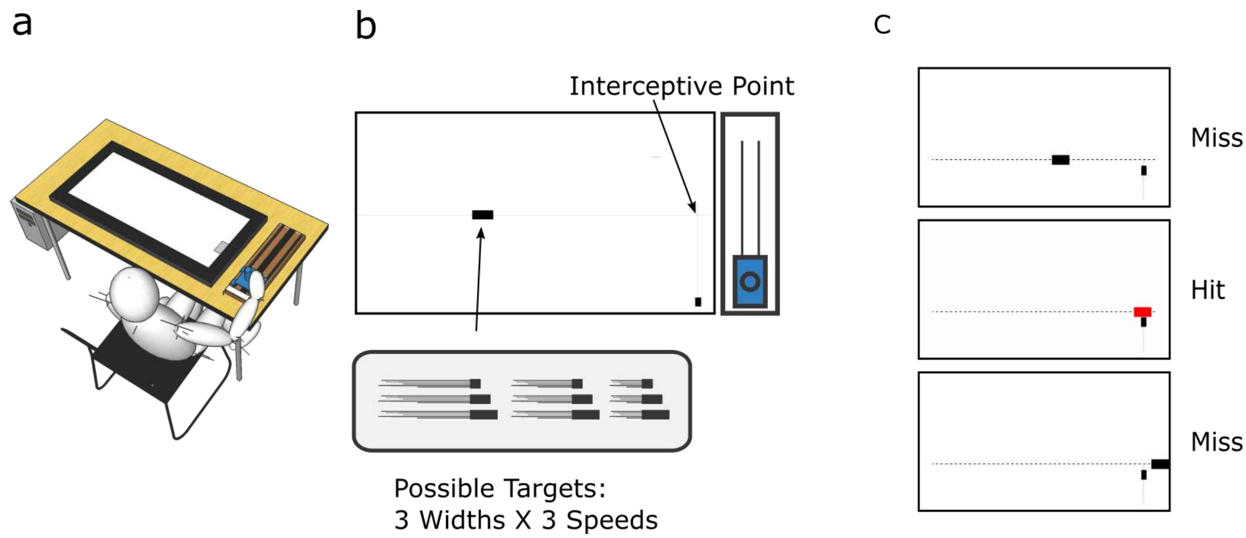

Figure 1. *a) The experimental setup for a right handed child: children viewed a horizontally oriented monitor while controlling an onscreen 'bat' via a 1-DoF manipulandum (placed on the left of the display for left handed participants with stimuli reversed). b) Schematic of the target and bat on the experimental display, and manipulandum to the right of the display. Targets moved from left to right across the screen. Participants were instructed to hit the target from beneath with the bat. c) Possible outcomes: in the upper panel the bat has arrived too early and missed the target. In the middle panel the bat successfully hits the target on the underside. In the lower panel the bat was too late and missed the target.*

A 'start box' appeared onscreen at the beginning of every trial and the participant was instructed to place the bat within it (coordinates [570mm, 20mm]; coordinate origin at bottom left of screen). A black target (height: 15mm) then appeared at the left hand side of the screen (coordinates [0mm,150mm] (for left handed participants the apparatus and stimuli were reversed, with the manipulandum placed on the left side of the screen). After a delay drawn from a uniform distribution U(0.25, 3.0 sec) the target moved from left to right at a constant speed. The center of the target passed in front of the center of the bat after moving 570mm. The children were instructed to hit the target with the bat. The target was successfully hit if the upper edge of the bat collided with the lower edge of the target (see Figure 1c). The target then stopped moving, turned red and span before disappearing, thereby providing motivating animated feedback for the children. If the bat passed in front of the target's horizontal path the target immediately stopped moving and then remained on screen for 1 second. Thus, participants could not simply move the bat in front of the target's path and wait for the target. If the bat crossed the target's path after the target had moved too far to be struck then the target stopped and remained visible for 1 second. The position of the bat and target was timestamped and saved to computer memory at 144Hz. The bat's positional data were filtered using a low pass second order zero-lag Butterworth filter with a cut off frequency of 10Hz. Spline interpolation was used to estimate the time at which the bat reached the interception point. The total number of targets hit by each participant provided our measure of interceptive timing ability.

Children performed 54 trials in which the target speed (250mm\s, 400mm\s, 550mm\s) and target width (30mm, 40m, 50mm) varied (9 trial types x 6). Each target type was presented in a block of 3 trials, with 2 blocks for each trial type. The blocks were pseudorandomly ordered with the constraint that two blocks of the same kind could not occur sequentially. All participants experienced an identical pseudorandom sequence of blocks.

Manual Dexterity

To distinguish between general motor skills and IntT ability we took measures of manual dexterity and postural ability. Manual dexterity was measured using the Kinematic Assesment Tool (Flatters, Hill, Williams, Barber, & Mon-Williams, 2014) which consists of three sensorimotor tasks that are presented on a tablet computer screen (Toshiba Portege M700-13p tablet, screen: 260x163 mm, 1200x800 pixels, 60 Hz refresh rate) and completed using a hand-held stylus. The planar position of the stylus was recorded at 120Hz and smoothed using a 10Hz dual-pass Butterworth filter at the end of each testing session.

Figure 2. a) Steering task: Participants traced a spatial path (oriented in different ways) from the open to the closed black dot using the stylus, while staying within a moving box. b) Aiming Task: Participants made movements to sequentially appearing targets (indicated by the numbers – invisible to participant) with a stylus. Open circles were not visible when moving between dots two and three. c) Tracking task: Participants followed a dot with the stylus. In the first trial the dot followed the dashed (invisible) path. In the second trial the guide track was visible. In each trial the dot made three revolutions of the figure of eight pattern at each speed: fast, medium and slow.

Steering Task

The steering task required participants to trace a path displayed on the tablet (Figure 2a). A box moved along the path every 5 seconds. Participants were told to trace the path as accurately as possible while ensuring they stayed within the moving box at all times. At each time point (120Hz) the minimum two-dimensional distance between a reference path and the stylus was calculated. The arithmetic mean was calculated for these values across each trial, giving a measure of path accuracy (PA). The ideal trial time if the participant remained within the moving box was 36 seconds. To normalise PA for task time, PA was adjusted by the percentage that participant's actual MT deviated from the ideal 36 seconds value (adjusted PA). Adjusted PA, a measure that incorporated both timing and accuracy components, was used to determine performance on the steering task (with larger values indicating worse performance).

Aiming Task

The aiming task (Figure 2b) required participants to make 75 aiming movements to sequentially appearing circular targets (5mm diameter). Once the participant successfully moved the stylus to the target dot then that target disappeared and the next target appeared (see Flatters, Hill et al., (2014) for details). Movement time (*MT*) was the measure of interest and was defined as the time between arriving at one target location and arriving at the next. The mean MT over the first 50 trials provided our measure of 'aiming' performance (with longer trials indicating worse performance). The last 25 trials contained 'jump' trials in which the target dot moved position during the aiming movement and were not of interest in this experiment.

Tracking Task (with and without spatial guide)

Participants completed two types of trial in the tracking task (Figure 2c). In the first trial, they placed the stylus on a static dot (10 mm diameter) displayed on the center of the screen. After one second the dot began to move across the screen in a 'figure-of-8' pattern. Participants were instructed to keep the tip of the stylus as close as possible to the dot's center for the duration of the trial. The dot completed nine revolutions of the 'figure-of-8' pattern. The dot moved at a 'slow' pace during the first three revolutions. In the next three revolutions the dot moved at a 'medium' pace and in the last three the dot moved at a 'fast pace' (see Flatters, Hill et al., (2014) for details). Participants then completed a second trial which was identical to the first except that a black 3mm wide 'guide' line was displayed on the screen, indicating the path which the dot would follow.

The root mean square error (*RMSE*) provided a measure of the participant's spatio-temporal accuracy, where the error was the straight line distance in mm between the center of the target dot and the stylus. A separate *RMSE* score was calculated for each target speed within each trial. The median value of these was taken to provide an overall measure of performance on the tracking task (with larger values indicating worse performance).

Postural control Task

Postural movements were measured using a custom rig (Flatters, Culmer, Holt, Wilkie, & Mon-Williams, 2014). Children stood with their feet shoulder width apart on a Nintendo Wii Fit board, which recorded the participant's center of pressure (COP) at 60Hz. The data were filtered using a wavelet filter as described in (Flatters, Culmer, et al., 2014). The two-dimensional path length subtended by the COP (in mm) provided a measure of balance, first with eyes open and then with eyes closed. Larger values therefore indicated worse performance.

Academic Attainment

Nationally standardized academic attainment scores for mathematics, reading and writing were provided by the school (https://www.gov.uk/national-curriculum/overview). Children were graded on a scale from 1 to 15 which map to UK standardized scores (see Supplementary information).

Data Analysis

Ordered-probit regression was employed to model the data. This is appropriate when the dependent variable is ordinal, as is the case for the academic attainment metrics. The model linearly combines predictor variables (IntT, manual dexterity, posture and age) to generate a latent academic attainment score for the i_{th} data point (y_i^*). This is done in exactly the same way as in linear regression,

$$y_i^* = N(\mu_i, \sigma) \qquad \text{(Equation 1)}$$

$$\mu_i = X_i^T \beta \qquad \text{(Equation 2)}$$

where X_i^T is a vector of predictors, β is a vector of regression coefficients and μ_i is the expected latent attainment outcome for the i_{th} participant (Eqn 2). The latent attainment score (y_i^*) is then drawn from a normal distribution with mean μ_i and standard deviation σ (Eqn 1). However, unlike in standard regression, y_i^* is a latent score which is then mapped to the ordinal attainment variable (y_i). This is done by slicing through the latent outcome scale with ordered thresholds $C, \ldots C_{K-1}$, where K is the number of possible categorical outcomes. The ordered outcome y is then defined by which thresholds y^* falls between (as illustrated in Figure 3). This is known as the probit link function.

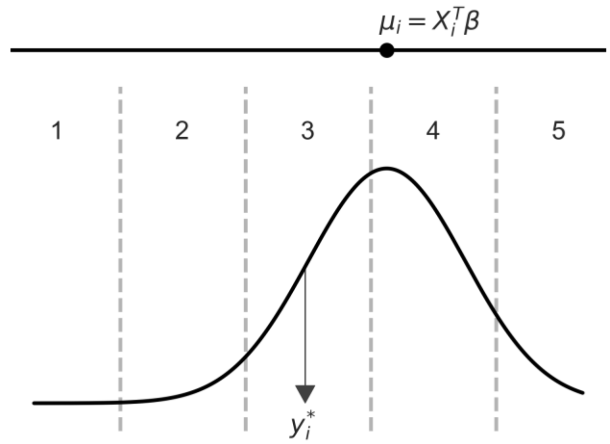

Figure 3. Illustration of an ordered probit model. The upper line represents a continuous latent attainment score. The expected latent attainment score for the i^th participant is given by $\mu_i = X_i^T\beta$, and is represented by the position of the black dot on the upper line. A latent attainment score y_i^* is then sampled from a normal distribution (curved black line) with mean μ_i, and standard deviation σ. The observed attainment score then depends on which of the thresholds $C, \ldots C_{K-1}$ (grey dotted lines) y_i^* falls between. Here y_i^* falls between the 2^nd and 3^rd thresholds, giving an observed attainment score of 3. Note that the threshold parameters will not necessarily be equally spaced.

As in standard regression we wish to fit the model parameters (the regression coefficients and standard deviations; β and σ) to the data. In addition we also wish to simultaneously fit the threshold parameters ($C_{1 \ldots K-1}$). While methods such as maximum likelihood can be used to fit the model, we employed Bayesian estimation techniques to yield a joint posterior distribution over all model parameters. Formally, we estimated the posterior distribution $P(\beta, \sigma, C_{1 \ldots K-1}|y)$ using the No-U-Turn algorithm (Hoffman & Gelman, 2011) implemented in RStan 2.16.2. The posterior distribution was summarized using 95% highest density intervals (HDI) which provide an upper and lower bound for an interval which, according to the posterior, has a 95% probability of containing the true model parameter value, given the data, likelihood and priors. The width of the HDI provides information about the estimate's precision.

A model was fit separately for each of the attainment outcomes (mathematics, reading and writing). For each model a representative sample was taken from the posterior distribution. Four chains of 10,000 samples were started at random locations of the joint posterior parameter space. Each chain first took 5000 warm up samples that were then discarded. Convergence was assessed by visually inspecting the chains and examining the gelman-rubin statistic ($\hat{R}$) (Gelman, 2014) and effective sample size of all parameters. All $\hat{R}$ values were close to 1 and the effective sample size was >6000 for all parameters.

Results

We were primarily interested in whether IntT would be predictive of mathematics attainment after controlling for age and other motor skills. Figure 4a indicates that there is a relationship between mathematics attainment and IntT but also between these variables and age (Figure 4b, c). Figure 4d plots the correlation between interceptive timing and mathematics attainment after controlling for age ($r = 0.208$).

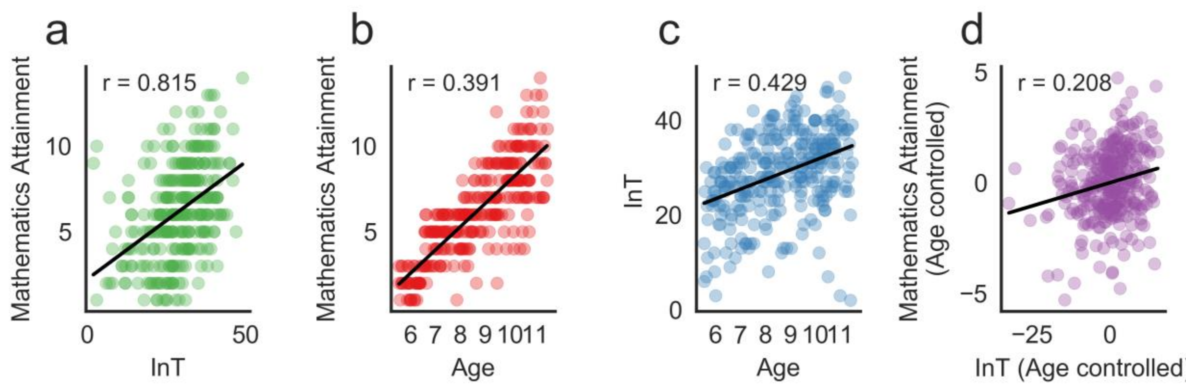

Figure 4. a,b,c) Correlations between Mathematics Attainment, Interceptive Timing (IntT) and Age. d) Partial correlation between IntT and Mathematics Attainment after controlling for Age. The fitted black lines are the least squared regression lines. Note: Pearson's correlation coefficients are given but these values should be treated with caution due to ordinal nature of attainment scores (hence reporting of the ordinal probit model elsewhere).

Whilst Figure 4 provides a useful illustration of the range of performance of children in the interceptive timing task, the primary question of interest was whether IntT would be predictive of mathematics attainment even after controlling for age and general motor skills. Linear regression is not the most appropriate model for these data given that the attainment metrics used were ordinal in nature (thus the Pearson's correlation coefficients given in Figure 4 should be interpreted with caution). In order to fully capture the relationships between the variables of interest, we utilized an ordered probit model to make inferences from the data. First we fitted the model separately for each educational attainment outcome (mathematics, reading and writing). We then examined the 95% highest density interval (HDI; thick horizontal black lines in Figure 5) for each β parameter, to determine the region where the true parameter was likely to fall (with 95% confidence, given the likelihood, priors and the data). The β parameters determine the amount by which a 1 unit change in the predictor variable will change the latent academic attainment score (see Figure 3).

The β coefficient for IntT (Figure 5, green curves, second column) was clearly non-zero for the mathematics attainment model (Figure 5, top row; 95% HDI excluded zero for IntT), with a mean estimate of 0.03 (95% HDI = [0.01, 0.05]). This suggests that for every five additional targets hit, the model estimates an average increase of 0.15 on *latent mathematics* score for that individual. The link between IntT and mathematics attainment can be contrasted with the reading and writing models (Figure 5, second column, middle and bottom row) where the 95% HDI of the IntT slopes contained zero and concentrated around comparatively smaller values, suggesting little or no relationship. Thus it appears that IntT may have a specific relationship with mathematics, but not educational attainment in general. This pattern contrasts with the other motor measures, none of which showed the same specificity for mathematics. Fine motor skills (Figure 5, Purple) showed a more general relationship with attainment measures: Steering had clear non-zero relationships with all three attainment scores, while Aiming also showed a possible relationships with mathematics, reading and writing. Tracking only showed a non-zero relationship with reading, while smaller coefficient values were more likely for mathematics and writing.

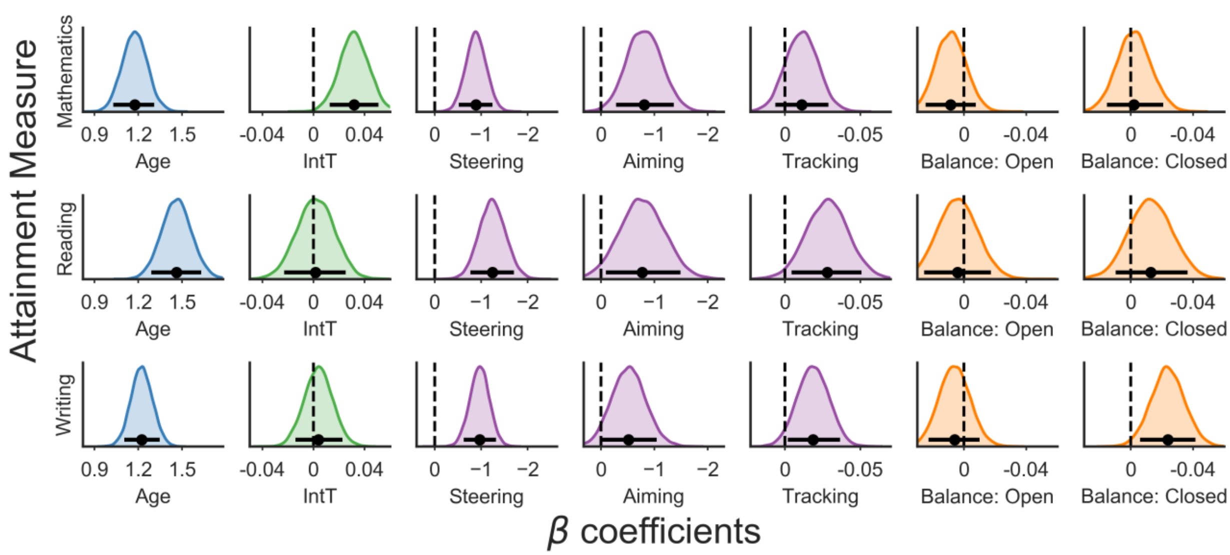

Figure 5. Marginal posterior distributions over β coefficients (i.e. regression slopes) for the Mathematics, Reading and Writing models. For clarity the x-axes for Steering, Aiming, Tracking and Balance have been reversed since for these measures negative values indicate an increase in the latent attainment score. The x-axis scales are consistent within columns to allow comparisons between Mathematics, Reading and Writing models. The black vertical dashed lines highlight the zero point where there would be no clear relationship, and the filled black circles represent the means and horizontal bars the 95% HDI.

Balance measures of gross motor skills showed no clear relationship with mathematical or reading attainment scores, though there did seem to be a relationship between balance with eyes closed and writing attainment (Figure 5, Orange). This pattern highlights the importance of having a stable base when performing fine motor tasks such as writing (Flatters, Mushtaq, et al., 2014).

Effect size

The modelling performed in the previous section provides a method for describing the association between particular variables. However the β coefficients are scale specific and the observed coefficients may reflect small effects with little real-world significance. To allow for a meaningful examination of the size of these effects we estimated how many months of age the typical range of scores on each sensorimotor task was worth, with respect to the associated increase in academic attainment. To perform this calculation the typical range was defined as two times the standard deviation (SD) for each sensorimotor task after controlling for age (see Supplementary materials for further details).

The effect size was calculated as follows,

$$Equivilant\ age\ change = \frac{2 \times SD_j \times \beta_j}{\beta_{age}} \times 12$$

where SD_j is the estimated standard deviation for the jth sensorimotor measure (after controlling for age), β_j is the corresponding model coefficient and β_{age} is the coefficient for age. We multiplied SD_j by 2 to give the typical range of scores, and by 12 to convert the units from years to months. A detailed example of the effect size calculation, and how SD_j was calculated is provided in the Supplementary materials.

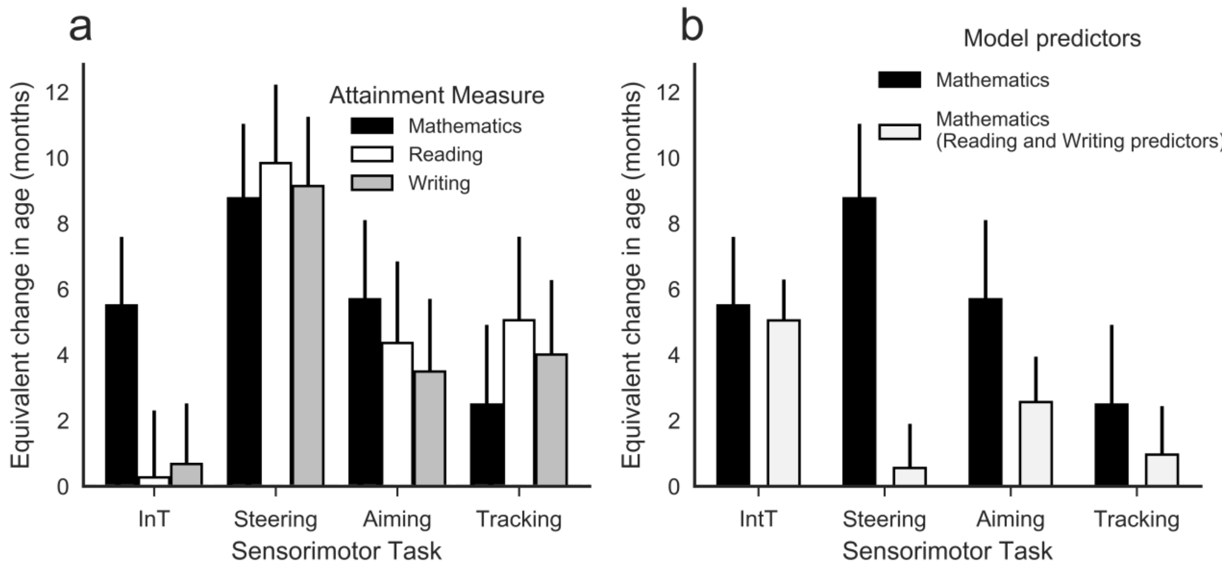

Figure 6. a) Equivalent change in age (months) explained by change in performance in IntT and fine motor skills (Steering, Aiming and Tracking) for Mathematics (Dark bars), Reading (White bars) and Writing Attainment (Grey bars). b) Equivalent change in age for the Mathematics attainment motor task predictors both with (light bars) and without (dark bars) Reading and Writing included as predictors. Adding Reading and Writing had little effect on the beta value for IntT, but it did change beta values for Steering, Aiming and Tracking. The vertical error bars indicate the Standard Deviation of the posterior (SD).

The 'equivalent change in age' metrics (Figure 6a) highlight that the *typical range* of IntT scores for mathematics attainment is equivalent to approximately 5.5 months of age (i.e. for children of the same age with interceptive timing scores differing by the typical range we should expect a difference in latent mathematics attainment equivalent to 5.5 months). Steering actually has a larger effect size for mathematics attainment than IntT (8.8 months) but Steering also has similar large effects for reading and writing attainment (9.8 and 9.1 months respectively) whereas IntT has very little effect on these other attainment scores (0.3 and 0.7 months respectively). The 'equivalent change in age' metric for Aiming suggests that for mathematics attainment, Aiming has a similar effect size to IntT (5.7 months), but with values of 4.4 months and 3.4 months for reading and writing respectively. Tracking had a value of 5 months for reading attainment, and smaller values for mathematics and writing attainment (2.5 and 4 months).

As with any observational study, there is always the possibility that omitted variables (e.g. general intelligence, or hand writing ability) may be mediating the relationship between the sensorimotor measures and academic attainment (see discussion). A reviewer noted that controlling for reading and writing scores (by including them as predictors in the mathematics model), may reduce the chances of an omitted variable bias, and also provide a useful test of whether the relationship between IntT and mathematics could be explained by a more general relationship between sensorimotor performance and academic ability. Thus, we carried out further (exploratory) analyses of the data by adding reading and writing to the mathematics model (see Figure 6b). Adding the additional educational attainment scores resulted in a substantial drop in the estimated 'equivalent age' effect size estimate for general fine motor measures (Steering, Aiming and Tracking), but the effect size of IntT was left largely unchanged.

Discussion

This study demonstrates for the first time that interceptive timing ability can predict mathematical performance in primary school children. This finding is consistent with human sensorimotor systems and cognitive abilities being intrinsically linked. Correlational studies always raise questions about the direction of causality, but in this case it is difficult to see how enhanced mathematics ability could have improved performance on the IntT task given that the task involved sub-second sensorimotor processes (mean movement time = 340ms, SD = 266). We probed the relationship in a variety of ways to determine whether it could be simply explained by generalized links between motor performance and educational attainment. We did indeed observe that some measures of fine motor skill had a general relationship with academic attainment: notably manual 'Steering' predicted academic attainment on reading, writing and mathematics. However IntT reflected a more specialized relationship independent of general motor ability, and also independent of academic attainment scores for reading and writing.

It is worth considering whether there is an obvious unmeasured mediating variable that could explain this relationship. For example, imagine that the children who are better at mathematics are also those that spend longer playing computer games and it is this exposure that leads to improved interceptive timing (rather than mathematics ability per se). Whilst it is impossible to completely rule-out such mediating variables, the specificity of the observed relationship makes it seem unlikely. In the computer game example, the games played would have to have no effect on general fine motor skills (Steering, Tracking and Aiming), nor on academic attainment for reading or writing. As such this explanation cannot rely on general exposure to computer games, rather it would require specific training to ensure that those who are better at mathematics are selected to improve their interceptive timing abilities (whilst leaving other general fine motor control unchanged). There was no evidence that games of such specificity were being deployed in this way within the school that took part in this study.

When considering why there is a relationship between sensorimotor IntT capability and the cognitive development of a child, one must also allow for the possibility that sensorimotor performance is a proxy measure of psychopathology, especially as populations with clinical motor control deficits sometimes exhibit poor mathematics ability (Pieters, Desoete, Van Waelvelde, Vanderswalmen, & Roeyers, 2012; Tinelli et al., 2015; Van Rooijen, Verhoeven, & Steenbergen, 2011). Indeed, 'fine motor skills' can predict measures of mathematics ability in healthy children (Carlson, Rowe, & Curby, 2013; Grissmer et al., 2010; Luo, Jose, Huntsinger, & Pigott, 2007; Pagani, Fitzpatrick, Archambault, & Janosz, 2010; Son & Meisels, 2006). Whilst our data confirm these findings by showing a relationship between fine motor tasks (Steering and Aiming) and mathematics attainment, the relationship seemed to generalize to all the educational attainment measures (mathematics, reading and writing). Furthermore when we controlled for fine motor skills (Steering, Aiming and Tracing) we still found IntT score was predictive of mathematics attainment (but not reading or writing attainment). These controls would seem to rule out simplistic explanations based on IntT skills acting as a proxy measure for psychopathology, and also other potential mediating variables such as differences in parental involvement, access to technology, or social economic status (Ritchie & Bates, 2013).

These findings are consistent with the idea that number representations are linked with concepts of time and space, perhaps through a common representation of magnitude (Walsh, 2003). It is possible that children must first learn the physical rules that govern how objects move before they can form related abstract representations (Piaget, 1955). The ability to learn these physical rules is likely to vary between individuals, and our findings may reflect variance in the development of the neural structures that underpin predictive learning regarding how objects move in space and time. In this regard, our results are consistent with recent findings showing that basic spatial processing abilities in infants relate to later mathematical ability (Lauer & Lourenco, 2016).

We should emphasize that we believe the relationship between IntT ability and mathematics is likely to be complex, since it is a matter of common observation that not all elite sports people are excellent mathematicians, whilst many people with physical disability excel in mathematics. When evaluating the observed relationships between motor control performance and educational attainment outcomes it is worth considering the magnitude of the observed effects. Once the change in attainment scores are transformed into 'equivalent change in age' units (Figure S1 and Figure 6) it can be seen that the fine motor measure 'Steering' accounts for approximately 9 months difference in reading, writing and mathematics attainment. Whilst this finding is noteworthy, it is likely that the relationship between Steering and mathematics is fairly general since it disappears once reading and writing attainment have been taken into account, possibly relating to general executive function (Roebers et al., 2014). In contrast to the Steering measure, IntT has a smaller relationship with mathematics attainment (approximately 5.5 months) but this is independent of reading and writing attainment (Figure 6). An important point to consider is whether an ‘equivalent change in age’ value of 5.5 months is actually important. From the perspective of a child with reduced academic attainment this would be considered a substantial difference. However, because the mathematics attainment scores themselves are fairly coarse it actually takes quite a large change in mathematical ability to move between attainment brackets. It would, therefore, be unwise to use effects of this magnitude to try to persuade school teachers to redirect precious resources away from mathematics teaching in order to target training of interceptive timing. However, these effects do suggest that we should not neglect the importance of sensorimotor development in young children (given that the environment – broadly construed – is known to exert a large influence on sensorimotor ability). Indeed, the present work complements reports that physical activity can exert positive benefits on cognitive processing, even if the mechanisms remain opaque (Hill, Williams, Aucott, Thomson, & Mon-Williams, 2011). Thus, the quality of early sensorimotor interactions with the environment may have important implications for children’s education.

Author contributions:

Oscar T. Giles: Designed study, developed experimental hardware and software, collected data, conducted all statistical analysis, created figures. Co-wrote manuscript.

Richard M. Willkie: Designed study, advised on statistical analysis, guided project. Co-wrote manuscript.

Amanda Waterman: Designed study, organized data collection. Co-wrote manuscript.

Katy Shire: Designed study, collected data. Co-wrote manuscript.

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References

Barton, R. A. (2012). Embodied cognitive evolution and the cerebellum. *Philosophical Transactions of the Royal Society B: Biological Sciences*, *367*(1599), 2097–2107. http://doi.org/10.1098/rstb.2012.0112

Bueti, D., & Walsh, V. (2009). The parietal cortex and the representation of time, space, number and other magnitudes. *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences*, *364*(1525), 1831–1840. http://doi.org/10.1098/rstb.2009.0028

Burr, D. C., Ross, J., Binda, P., & Morrone, M. C. (2011). Saccades Compress Space, Time, and Number. In *Space, Time and Number in the Brain* (Vol. 14, pp. 175–186). Elsevier. http://doi.org/10.1016/B978-0-12-385948-8.00012-8

Cameron, C. E., Brock, L. L., Murrah, W. M., Bell, L. H., Worzalla, S. L., Grissmer, D., & Morrison, F. J. (2012). Fine Motor Skills and Executive Function Both Contribute to Kindergarten Achievement. *Child Development*, *83*(4), 1229–1244. http://doi.org/10.1111/j.1467-8624.2012.01768.x

Carlson, A. G., Rowe, E., & Curby, T. W. (2013). Disentangling Fine Motor Skills' Relations to Academic Achievement: The Relative Contributions of Visual-Spatial Integration and Visual-Motor Coordination. *The Journal of Genetic Psychology*, *174*(5), 514–533. http://doi.org/10.1080/00221325.2012.717122

Chang, a. Y.-C., Tzeng, O. J. L., Hung, D. L., & Wu, D. H. (2011). Big Time Is Not Always Long: Numerical Magnitude Automatically Affects Time Reproduction. *Psychological Science*, *22*(12), 1567–1573. http://doi.org/10.1177/0956797611418837

de Hevia, M. D., & Spelke, E. S. (2010). Number-Space Mapping in Human Infants. *Psychological Science*, *21*(5), 653–660. http://doi.org/10.1177/0956797610366091

Desmurget, M., & Grafton, S. (2000). Forward modeling allows feedback control for fast reaching movements. *Trends in Cognitive Sciences*, *4*(11), 423–431. http://doi.org/10.1016/S1364-6613(00)01537-0

Fias, W., van Dijck, J. P., & Gevers, W. (2011). How is Number Associated with Space? The Role of Working Memory. In *Space, Time and Number in the Brain* (pp. 133–148). http://doi.org/10.1016/B978-0-12-385948-8.00010-4

Flanagan, J. R., & Wing, A. M. (1997). The role of internal models in motion planning and control: evidence from grip force adjustments during movements of hand-held loads. *The Journal of Neuroscience : The Official Journal of the Society for Neuroscience*, *17*(4), 1519–1528. http://doi.org/10.1007/s00221-008-1691-3

Flatters, I., Culmer, P., Holt, R. J., Wilkie, R. M., & Mon-Williams, M. (2014). A new tool for assessing head movements and postural sway in children. *Behavior Research Methods*, *46*(4), 950–959.

http://doi.org/10.3758/s13428-013-0419-x

Flatters, I., Hill, L. J. B., Williams, J. H. G., Barber, S. E., & Mon-Williams, M. (2014). Manual control age and sex differences in 4 to 11 year old children. *PloS One*, *9*(2), e88692. http://doi.org/10.1371/journal.pone.0088692

Flatters, I., Mushtaq, F., Hill, L. J. B., Holt, R. J., Wilkie, R. M., & Mon-Williams, M. (2014). The relationship between a child's postural stability and manual dexterity. *Experimental Brain Research*, *232*(9), 2907–17. http://doi.org/10.1007/s00221-014-3947-4

Franklin, D. W., & Wolpert, D. M. (2011). Computational mechanisms of sensorimotor control. *Neuron*, *72*(3), 425–42. http://doi.org/10.1016/j.neuron.2011.10.006

Gelman, A. (2014). *Bayesian Data Analysis* (3rd ed.). Boca Raton: Taylor & Francis Group.

Grissmer, D., Grimm, K. J., Aiyer, S. M., Murrah, W. M., & Steele, J. S. (2010). Fine motor skills and early comprehension of the world: two new school readiness indicators. *Developmental Psychology*, *46*(5), 1008–1017. http://doi.org/10.1037/a0020104

Hill, L. J. B., Williams, J. H. G., Aucott, L., Thomson, J., & Mon-Williams, M. (2011). How does exercise benefit performance on cognitive tests in primary-school pupils? *Developmental Medicine and Child Neurology*, *53*(7), 630–635. http://doi.org/10.1111/j.1469-8749.2011.03954.x

Hoffman, M. D., & Gelman, A. (2011). The No-U-Turn Sampler: Adaptively Setting Path Lengths in Hamiltonian Monte Carlo. *Journal of Machine Learning Research*, *15*, 1351–1381.

Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews. Neuroscience*, *6*(6), 435–448. http://doi.org/10.1038/nrn1684

Lauer, J. E., & Lourenco, S. F. (2016). Spatial Processing in Infancy Predicts Both Spatial and Mathematical Aptitude in Childhood. *Psychological Science*, *27*(10), 1291–1298. http://doi.org/10.1177/0956797616655977

Lourenco, S. F., & Longo, M. R. (2010). General magnitude representation in human infants. *Psychological Science : A Journal of the American Psychological Society / APS*, *21*(6), 873–81. http://doi.org/10.1177/0956797610370158

Luo, Z., Jose, P. E., Huntsinger, C. S., & Pigott, T. D. (2007). Fine motor skills and mathematics achievement in East Asian American and European American kindergartners and first graders. *British Journal of Developmental Psychology*, *25*(4), 595–614. http://doi.org/10.1348/026151007X185329

Merfeld, D. M., Zupan, L., & Peterka, R. J. (1999). Humans use internal models to estimate gravity and linear acceleration. *Nature*, *398*(6728), 615–618. http://doi.org/10.1038/19303

Nieder, A., & Dehaene, S. (2009). Representation of number in the brain. *Annual Review of Neuroscience*, *32*, 185–208. http://doi.org/10.1146/annurev.neuro.051508.135550

Pagani, L. S., Fitzpatrick, C., Archambault, I., & Janosz, M. (2010). School readiness and later achievement: a French Canadian replication and extension. *Developmental Psychology*, *46*(5), 984–994. http://doi.org/10.1037/a0018881

Piaget, J. (1955). *The construction of reality in the child*. Routledge & K. Paul.

Pieters, S., Desoete, A., Van Waelvelde, H., Vanderswalmen, R., & Roeyers, H. (2012). Mathematical problems in children with developmental coordination disorder. *Research in Developmental Disabilities*, *33*(4), 1128–1135. http://doi.org/10.1016/j.ridd.2012.02.007

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science*, *24*(7), 1301–8. http://doi.org/10.1177/0956797612466268

Roebers, C. M., Röthlisberger, M., Neuenschwander, R., Cimeli, P., Michel, E., & Jäger, K. (2014). The relation between cognitive and motor performance and their relevance for children's transition to school: A latent variable approach. *Human Movement Science*, *33*(1), 284–297. http://doi.org/10.1016/j.humov.2013.08.011

Son, S.-H., & Meisels, S. J. (2006). The Relationship of Young Children's Motor Skills to Later School Achievement. *Merrill-Palmer Quarterly*, *52*(4), 755–778. http://doi.org/10.1353/mpq.2006.0033

Srinivasan, M., & Carey, S. (2010). The long and the short of it: On the nature and origin of functional overlap between representations of space and time. *Cognition*, *116*(2), 217–241. http://doi.org/10.1016/j.cognition.2010.05.005

Tinelli, F., Anobile, G., Gori, M., Aagten-Murphy, D., Bartoli, M., Burr, D. C., … Concetta Morrone, M. (2015). Time, number and attention in very low birth weight children. *Neuropsychologia*, *73*, 60–69. http://doi.org/10.1016/j.neuropsychologia.2015.04.016

Tresilian, J. R. (1999). Visually timed action: Time-out for `tau'? *Trends in Cognitive Sciences*, *3*(8), 301–310. http://doi.org/10.1016/S1364-6613(99)01352-2

Tresilian, J. R. (2012). Sensorimotor Control and Learning. In *Sensorimotor Control and Learning*. London: Palgrave Macmillan.

Van Rooijen, M., Verhoeven, L., & Steenbergen, B. (2011). Early numeracy in cerebral palsy: Review and future research. *Developmental Medicine and Child Neurology*, *53*(3), 202–209. http://doi.org/10.1111/j.1469-8749.2010.03834.x

Walsh, V. (2003). A theory of magnitude: Common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences*, *7*(11), 483–488. http://doi.org/10.1016/j.tics.2003.09.002

White, O., & Diedrichsen, J. (2010). Responsibility assignment in redundant systems. *Current Biology*, *20*(14), 1290–1295. http://doi.org/10.1016/j.cub.2010.05.069

Wijdenes, L. O., Brenner, E., & Smeets, J. B. J. (2014). Online manual movement adjustments in response to target position changes and apparent target motion. *Motor Control*, *18*(1), 44–54. http://doi.org/10.1123/mc.2012-0116

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review*, *9*(4), 625–636. http://doi.org/10.3758/BF03196322

Wolpert, D. M., Miall, R. C., & Kawato, M. (1998). Internal models in the cerebellum. *Trends in Cognitive Sciences*, *2*(9), 338–347. http://doi.org/10.1016/S1364-6613(98)01221-2

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Supplementary Information

Ordered Probit Model

The full ordered probit model and priors are specified below with Interceptive Timing (IntT), age, steering, aiming, tracking and postural balance (eyes open and eyes closed) scores entered as predictors. The model was based on Kruschke (2015) and the model code is available online at https://github.com/OscartGiles/Ordered-Probit-Stan.

$$\boldsymbol{\beta} \sim N(0, K)$$

$$\boldsymbol{\mu} = \boldsymbol{X\beta}$$

$$\boldsymbol{C}_1 \equiv 1.5$$

$$\boldsymbol{C}_{t=2\ldots K-1} \sim N(t + 0.5, K)$$

$$\boldsymbol{C}_{K-1} \equiv K - 0.5$$

$$\sigma \sim Cauchy^{+}(0, 100)$$

$$\boldsymbol{\theta}_{i,k} = \begin{cases} 1 - \phi\left(\frac{\mu_i - \boldsymbol{C}_1}{\sigma}\right), & k = 1 \\ \phi\left(\frac{\mu_i - \boldsymbol{C}_{k-1}}{\sigma'}\right) - \phi\left(\frac{\mu_i - \boldsymbol{C}_k}{\sigma}\right), & 1 < k < K \\ \phi\left(\frac{\mu_i - \boldsymbol{C}_{k-1}}{\sigma}\right), & k = K \end{cases}$$

$$\boldsymbol{y_i} \sim \textbf{Categorical}(\boldsymbol{\theta_i})$$

Where N is the number of data points, K is number of levels in the attainment outcome, $i = 1 \ldots N$, $k = 1 \ldots K$, and $t = 1 \ldots K - 1$. $\boldsymbol{X}$ is an $N \times 7$ matrix of predictor variables where the first column is equal to 1. $\boldsymbol{\theta}$ is an $N \times K$ matrix, specifying the probabilities of obtaining each observed academic attainment score for the ith participant. ϕ is the cumulative normal function. $\boldsymbol{\mu}$ represents a continuous latent attainment outcome, and **y** is the observed attainment scores.

The first and last threshold value $\boldsymbol{C}_1$and $\boldsymbol{C}_{K-1}$ were fixed in order to identify the model. Thus all other model parameters must be interpreted with regards to this constraint. In addition, each threshold parameter was constrained to be greater than the last ($\boldsymbol{C}_k < \boldsymbol{C}_{k+1}$).

Effect size calculations

In the main text we provide an estimate of the effect size for each predictor in the model in terms of the equivalent change in age that would be required to produce the same change on the latent attainment score as the *typical range* of each of the sensorimotor measures (where the typical range was defined as 2 times the standard deviation of the motor measure of interest). The effect size can be formally defined as,

$$Equivilant\ age\ change = \frac{2 \times SD_j \times \beta_j}{\beta_{age}} \times 12$$

where SD_j is the estimated standard deviation for the jth sensorimotor measure (after controlling for age), β_j is the corresponding model coefficient and β_{age} is the coefficient for age. For clarity we illustrate this graphically in Figure S1 (see caption for details).

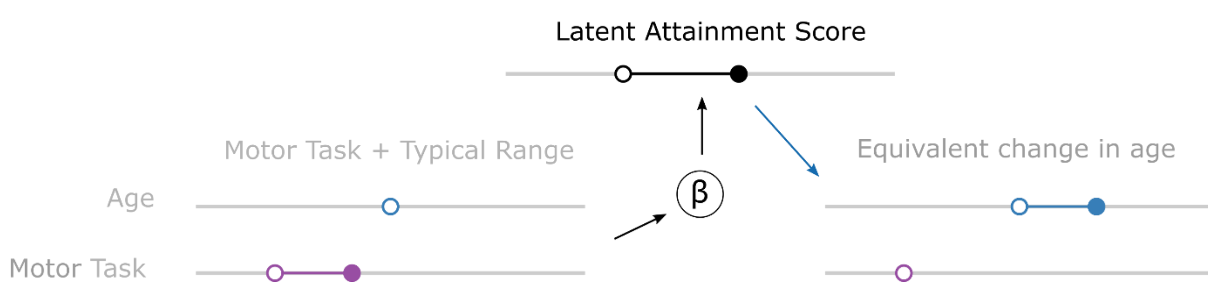

Figure S1: Illustration of how the effect size metric was calculated. The top line shows the latent Mathematics attainment score (μ_i) on a continuous scale. The model states that $\mu_i = X_i^T \beta$, where X is a design matrix specifying the predictor scores for each participant. As we change the values of the predictor variables, the predicted latent attainment score will change. Changing a motor task score by the *typical range* (left side; open to filled purple circle) results in a change in the predicted latent attainment score (open to filled black circle). Our effect size measure defines how much we would need to change the age predictor (right side; open to filled blue circle) in order to achieve the same change in the latent attainment score. In other words, how many months the typical range of the sensorimotor task predictor is worth.

Typical range of sensorimotor measures after controlling for age

We chose the *typical range* to be $2 \times SD$ as this is the difference between a score one SD above and below the mean. We therefore needed to estimate the SD for each motor task. However, we know that a substantial proportion in the variance in each motor task is explained by age. Thus we calculated the SD after controlling for age. For a single motor task we could calculate this by fitting a simple regression with age as a predictor and the motor task as the outcome variable. The SD then provides a measure of the variance not explained by age. Here we used a "seemingly unrelated regression" model which allowed for all the motor tasks to be modelled as output variables simultaneously. This is essentially the same as fitting multiple simple regressions between age and each motor task, except that the covariance between motor tasks is also estimated. The full model code is provided at https://github.com/OscartGiles/Hitting-the-target.

Understanding how the latent attainment score maps to the observed score

The latent attainment score is mapped to the observed data by a probit link function. For a given predicted latent attainment score (μ) the model provides a vector of probabilities for each possible ordered attainment outcome. For illustrative purposes, Figure S2a shows the probability distribution when $\mu = 5$, which we refer to here as μ_1 (orange bars) and when μ increases as a result of IntT increasing by the typical range, referred to as μ_2 (blue bars). We can see that in both cases an attainment score of 5 is most probable, but in the latter case higher scores have become more probable overall, while the probability of lower scores has decreased. Figure S2b shows the logarithm of the ratio between the two probability distributions shown in Figure S2a. Again, this shows that observed attainment scores above 5 are more probable when the latent attainment score is increased (positive values), while lower scores are less probable (negative values).

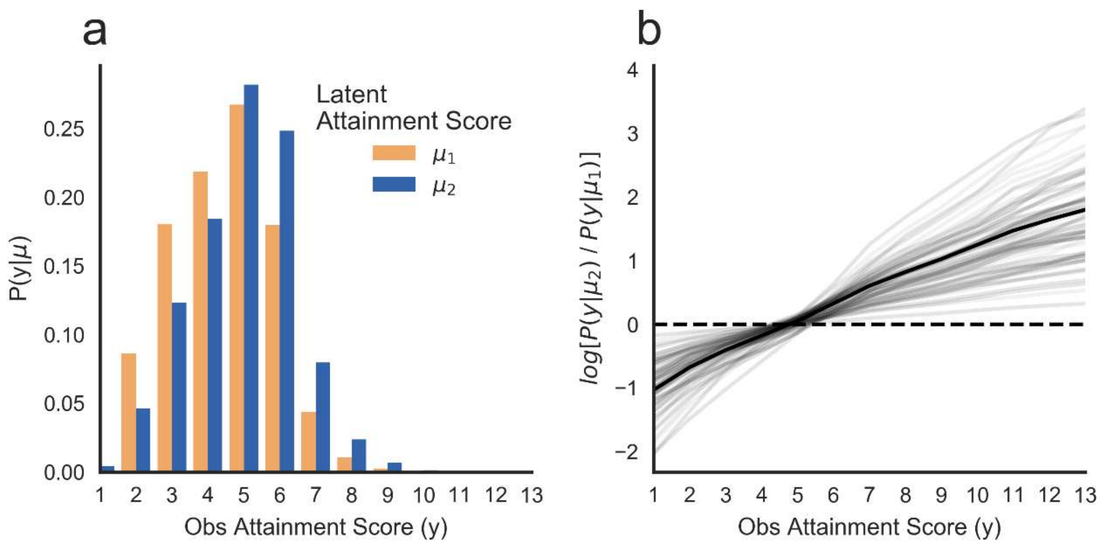

Figure S2: a) The probability of obtaining each possible observed Mathematics attainment outcome (y) when the latent Mathematics score is equal to 5 (μ_1; orange bars) and when the latent Mathematics score increases by the amount induced by the typical range of the interceptive timing metric (μ_2; blue bars). b) Log ratio of probability of each observed Mathematics attainment score given μ_1 and μ_2. Dark line shows the posterior mean. Grey lines show 100 random samples from the posterior.

Graphical probes of model fit – Posterior predictive checks

To assess how well the model captures the data we simulated 16,000 data sets from the posterior (y_{rep}) and calculated the mean and standard deviation for each. The distribution of these test statistics are shown in Figure S3a and S3b respectively. The true mean and SD of the observed data is clearly plausible under the model simulations, suggesting this model captures these statistics well. We also calculated the mean score for each data point across all the expected score for each data point, $E(y_{rep})$. This is plotted again IntT in figure S4 (red dots) while the true Mathematics attainment scores are also plotted against IntT (blue dots). It's clear that the model captures the general pattern of observed relationship between interceptive timing and Mathematics attainment well.

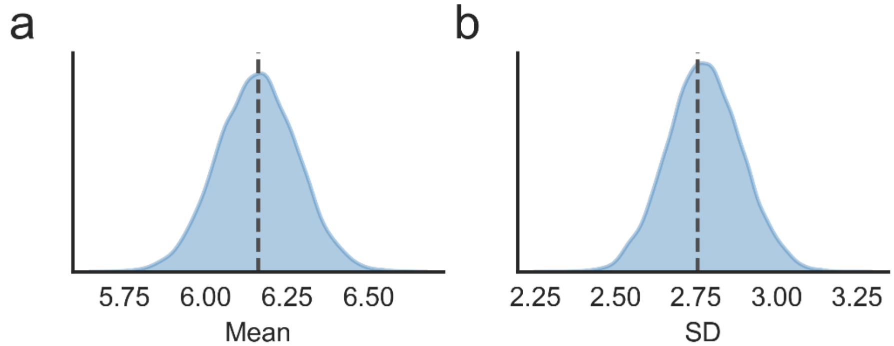

Figure S3: Distribution of the (a) mean and (b) standard deviation of test statistics for 16,000 simulated data sets (blue kernel density plots) alongside the true data sets (vertical black dashed line).

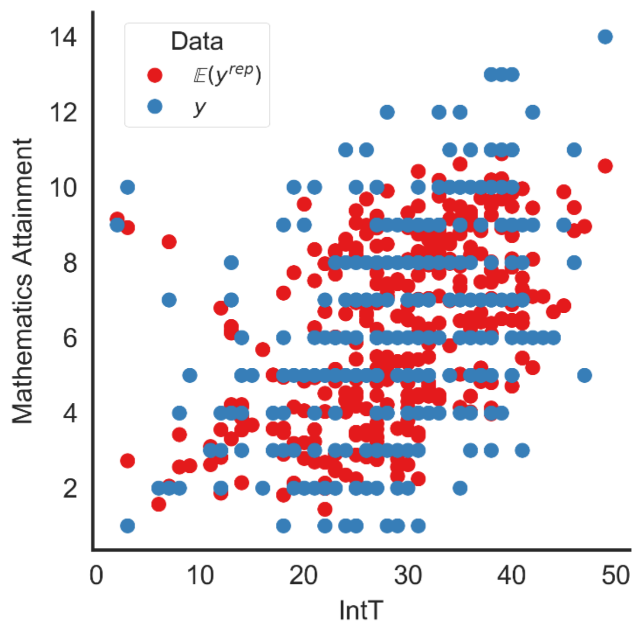

Figure S4: The expected value of the simulated data (y_{rep}) as a function of IntT score (blue dots). The observed data is also shown as a function of IntT score (red dots).

School Attainment Metrics:

Table S1 shows how the educational attainment code maps to the original code used by schools, as well as the school year and age at which children are expected to reach key attainment levels.

Table S1. Attainment score conversion table. A scale of 1 to K (where K was the highest observed score in the data) was used for the Bayesian Attainment Model. This scale maps to the UK nationally standardized scores. The school year and age at which children are expected to achieve these scores is shown.

Attainment Score	Government Code	Expected Year Group	Expected Age
1	1c		
2	1b		
3	1a		
4	2c		
5	2b	2	6-7
6	2a		
7	3c		
8	3b		
9	3a		
10	4c		
11	4b	6	10-11
12	4a		
13	5c		
14	5b	9	13-14
15	5a		

Table S2. In UK primary schools, mathematics is taught and assessed in two stages – Key stage 1 (years 1 and 2 when the children are 4-6 years) and Key stage 2 (years 3 to 6 when the children are 7-11 years). The table below is an extracted from: https://www.gov.uk/government/collections/national-curriculum-assessments-test-frameworks

<table>
<tr><th></th><th>Year</th><th></th></tr>
<tr><td rowspan="2">Key Stage 1
The mathematics taught is very practical and related to everyday experiences. A variety of resources, such as coins, dice, dominoes, playing cards, beads and plastic bricks for counting.</td><td>1</td><td>number bonds, early skills for multiplication and solving simple problems; very practical mathematic related to everyday experiences.</td></tr>
<tr><td>2</td><td>working on numbers through rehearsal and using addition and subtraction facts regularly; using number lines, tracks and 100 squares.</td></tr>
<tr><td rowspan="4">Key Stage 2
Shape, space, data handling, money and measures in addition to numeracy.

Children are expected to read, write and order numbers on a number line (and place value cards, beads on a string etc).</td><td>3</td><td>puzzles, problems and investigations to practice, consolidate and extend understanding with an emphasis on real world situations.</td></tr>
<tr><td>4</td><td>decimals (particularly with money and measurement); equivalent fractions introduced via diagrams and number lines used to teach fractions.</td></tr>
<tr><td>5</td><td>Fractions, decimals and percentages; comparing, ordering and converting and solving problems in a meaningful context</td></tr>
<tr><td>6</td><td>more complicated problems, including those that have decimals, fractions and percentages; expectation of working systematically, using the correct symbols and to check their results. They also learn about positive and negative numbers.</td></tr>
</table>