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Experimental Study of Anisotropy and Non-Coaxialty of Granular Solids

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Abstract

Previous experimental studies of anisotropy and non-coaxiality by using the hollow cylindrical apparatus are mainly focused on granular soils, which feature irregular particle shapes and non-uniform particle size distributions. This paper experimentally investigates these two characteristics on assemblages of particulate materials with regular particle shapes and uniform particle sizes. These assemblages are made from spherical, cylindrical and cubical particles, with an increasing order of particle angularity. Two types of loading paths in the hollow cylindrical apparatus are applied. One is the monotonic loading path with a range of fixed angles of major principal stress with respect to the horizontal bedding plan, used to investigate the anisotropy of materials. The other is the path of pure principal stress rotations, used to study the non-coaxiality. The experimental results indicate that these three materials exhibit a strong anisotropy and non-coaxiality. Their stress-strain responses are dependent on the orientation of major principal stress. The non-coaxiality is a function of stress ratio. In addition, there is a noticeable trend that these two characteristics are dependent on the angularity of particles. The more angular the particles are, the greater anisotropy and non-coaxiality take place.

Introduction

Anisotropy and non-coaxiality are two important characteristics of particulate materials, and they significantly influence their mechanical behavior (Arthur & Menzies, 1972; Oda, 1972; Yamada & Ishihara, 1979; Arthur et al, 1980; Ishihara & Towhata, 1983, Ishihara, 1993). The anisotropy is mainly attributed to non-uniform distributions of particle contacts and particle interaction forces. There generally exists a bedding plane in assemblage of granular solids and different loading directions with respect to the bedding plane lead to different stress-strain responses (Nakata et al, 1998; Yoshimine et al, 1998; Lade & Abelev, 2003). Studies indicate that even the assemblage of spherical particles exhibit anisotropy. The non-coaxiality refers to the non-coincidence of principal stress directions and plastic strain rate directions when an assemblage of granular solids is subjected to loading paths involving principal stress rotations. Numerous test results indicate that the change of principal stress orientations without the change of their magnitudes can lead to plastic deformations, and the orientation of principal plastic strain rates is different from that for principal stresses (Towhata & Ishihara, 1985; Miura et al, 1986; Gutierrez et al, 1991).

Numerous studies have been undertaken to study the anisotropy and non-coaxiality of granular solids on theoretical, numerical and experimental perspectives. Typical approaches include micromechanics, the discrete element method and testing by the hollow cylindrical apparatus (Miur et al, 1986; Oda & Nakayama, 1988; Calvetti et al, 1997; Nemat-Nasser, 2000; Jiang et al, 2005; Yu & Yuan, 2005; Yang & Yu, 2006; Yang & Yu, 2010). Important findings have been obtained on these two characteristics. For instance, the anisotropy depends on many factors, such as material densities, particle shapes, particle roughness and depositional methods. Among these influencing factors, the particle shape plays an important role, and non-spherical particles generally lead to a stronger anisotropy than spherical particles (Shinohara et al, 2000; Cho et al, 2006; Cai et al, 2013). It is well established that the non-coaxial behavior is dependent on mobilized friction angles of granular materials, in which a larger mobilized friction angle leads to a smaller non-coaxiality (Miura et al, 1986; Gutierrez et al, 1991). The non-coaxiality is also a function of anisotropy, in which a stronger anisotropy leads to a larger non-coaxiality (Zdravkovic & Jardine, 2001; Cai et al, 2013)

On the aspect of experimental studies, the well-established equipment is the hollow cylindrical apparatus (HCA). This equipment can apply well controlled stress paths involving principal stress rotations to investigate the non-coaxiality. In addition, it can apply monotonic stress paths making any angles to the bedding plane to examine the anisotropy. In the experimental study of anisotropy and non-coaxiality, granular soils are widely employed (Gutierrez et al, 1991; Naka et al, 1998; Yoshimine et al, 1998). Granular soils feature irregular particle shapes and non-uniform distribution of particle sizes. In addition, the particles are susceptible to abrasion and spalling, which can change their particle shapes and sizes during the process of loading. However, there are many other types of granular solids different from granular soils on the aspects of particle shapes and particle size distributions. A typical example is agricultural products, such as barleys and beans stored in silos. Their particles are of regular shapes and uniform sizes. The impact of anisotropy and non-coaxiality on behaviors of these granular solids in silos are as important as that on geotechnical engineering structures (Nielsen, 1998; Rotter et al, 1998; Rotter, 2001). Various filling methods of granular solids in silos can create strong anisotropy. Different filling structures and anisotropy can influence silo wall pressure distributions and flow patterns. Discharge of granular solids in silos leads to considerable principal stress rotations, which are even greater than those in geotechnical engineering problems. Although some experiments were conducted to study the stress-strain responses of granular solids with regular particle shapes and uniform particle sizes, those tests were mainly conducted in biaxial, triaxial or direct shear apparatuses by using spherical glass beads, steel balls or at most oval rods (Konishi et al, 1983; Li & Puri, 1996; O'Sullivan et al, 2004; Haertl & Ooi, 2011). Those relatively simple testing apparatuses can't investigate the anisotropy and non-coaxiality as effectively as the advanced HCA, and the role of non-spherical particle shapes has not been thoroughly studied. For instance, in biaxial and triaxial tests, the principal stress direction is fixed. In direct shear tests, the failure plane is pre-determined, and the stress distribution in a sample is highly non-uniform and can't be accurately measured.

This paper aims to experimentally investigate the anisotropy and non-coaxiality of assemblages of granular particles with regular particle shapes and uniform particle sizes by using the HCA. This will provide further insights to the influences of particle properties on those two important characteristics. Granular solids with different particle shapes are used, including spherical glass beads, cylindrical and cubical particles made from stiff polymer.

Experimental Methodology

Three types of granular solids are tested. The first material, denoted as M1, is glass beads with a diameter of 0.7mm. The other two materials M2 and M3 are made from stiff polymers. M2 is of a cylindrical shape, which is 0.75mm long and 0.7mm across. M3 is of a cubical shape, which is 0.75mm long and 0.7mm wide. All these three types of granular solids are of regular shapes and uniform sizes, shown in Figure 1. Although quantitative measurement of particle angularity is not undertaken, it is evident that M3 is the most angular, followed by M2 and M1 in the order of angularity. All the specimens are prepared by using the air pluviation method. The grains are rained in a dry state from a three column-mounted container and passed through three ASTM sieves in order to achieve uniform flow. Since both the height of fall and the rate of pluviation can influence the uniformity of specimens, care is taken to keep both of the parameters approximately constant during pluviation in the HCA. The preparation of a sample is carried out in several layers, and it is carefully compacted in each layer. Dense samples with an approximate relative density of 75% are prepared for these three materials, which gives a unit weight of γ_{M1}=15.66 kN/m^3, γ_{M2}=7.66 kN/m^3, γ_{M3}=7.26 kN/m^3. The specimen is 200 mm tall, and its inner and outer diameters are 60 mm and 100 mm, respectively. The air pluviation and compaction give the sample a horizontal bedding plane, where particle contacts are the strongest along the vertical direction. This can be categorized as inherent anisotropy.

The HCA can independently apply and control a torque M_T, axial load W, outer pressure P_o, inner pressure P_i and back pressure to a specimen, shown in Figure 2. P_o and P_i are pressures applied on membrane of a specimen. The axial load and torque are generated by two servomotors. The outer, inner and back pressures are controlled by three pressure/volume controllers. The pore pressure can be measured by using an external pore pressure transducer connected to the base pedestal. The axial displacement and rotation of a specimen are measured by using high resolution digital encoders mounted in the actuator unit. The radial strain and circumferential strain are inferred from the change of inner and outer radii of the specimen, which are measured by digital pressure volume controllers. In this paper, the axial stress is denoted by σ_z, radial stress by σ_r, circumferential stress by σ_θ and shear stress by $\tau_{z\theta}$. The axial strain is denoted by ε_z, radial strain by ε_r, circumferential strain by ε_θ, and shear strain by $\varepsilon_{z\theta}$. In all the tests, the inner and outer pressure of specimens remain equal, which makes σ_r equal to σ_θ. The idealized applied loads, stress and strain components are shown in Figure 2. The figure shows that the combination of σ_z, σ_θ and $\tau_{z\theta}$ make the principal stress direction vary. As all the tests are conducted under drained conditions, the total stresses are the same as the effective stresses. These stress components give three principal stresses as,

$$\sigma_1 = \frac{\sigma_z + \sigma_\theta}{2} + \sqrt{\left(\frac{\sigma_z - \sigma_\theta}{2}\right)^2 + \tau_{z\theta}^2} \tag{1}$$

$$\sigma_2 = \sigma_r \tag{2}$$

$$\sigma_3 = \frac{\sigma_z + \sigma_\theta}{2} - \sqrt{\left(\frac{\sigma_z - \sigma_\theta}{2}\right)^2 + \tau_{z\theta}^2} \tag{3}$$

The mean normal stress (confining pressure) and the deviatoric stress are given as,

$$p = (\sigma_1 + \sigma_2 + \sigma_3)/3 \tag{4}$$

$$q = \sigma_1 - \sigma_3 \tag{5}$$

The inclination angle of principal stresses is defined as,

$$\tan 2\alpha = \frac{2\tau_{z\theta}}{\sigma_z - \sigma_\theta} \tag{6}$$

Similarly, the principal strains are defined as,

$$\varepsilon_1 = \frac{\varepsilon_z + \varepsilon_\theta}{2} + \sqrt{\left(\frac{\varepsilon_z - \varepsilon_\theta}{2}\right)^2 + \varepsilon_{z\theta}^2} \tag{7}$$

$$\varepsilon_2 = \varepsilon_r \tag{8}$$

$$\varepsilon_3 = \frac{\varepsilon_z + \varepsilon_\theta}{2} - \sqrt{\left(\frac{\varepsilon_z - \varepsilon_\theta}{2}\right)^2 + \varepsilon_{z\theta}^2} \tag{9}$$

The volumetric strain and deviatoric strain are defined as,

$$\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{10}$$

$$\varepsilon_q = \varepsilon_1 - \varepsilon_3 \tag{11}$$

All the specimens are saturated with a back pressure of 150 kPa, inner and outer cell pressures of 170 kPa. In this first type of stress paths to examine the anisotropy, the specimen is first isotropically consolidated to a confining pressure of 100 kPa, followed by a monotonic loading path at a fixed angle of principal stresses until the failure of specimens. Along the loading path, the confining pressure remains constant at 100 kPa, making $\Delta\sigma_r = \Delta\sigma_\theta = -0.5\Delta\sigma_z$. The loading rate is at the order of 0.02 kPa/second. A total of seven loading paths with an interval of 15^0 of major principal stress orientation are applied, shown in Figure 3(a). The loading paths of 0^0 and 90^0 correspond to the triaxial compression and extension loading paths, respectively. All these seven stress paths are applied on materials M1 and M2, and M3 is subjected to the loading paths of 0^0 and 90^0, which serves to further verify the results on M1 and M2. The second type of stress paths is of pure principal stress rotations used to examine the non-coaxiality of three materials. Similar to the first type of stress paths, the specimen is first consolidated to a confining pressure of 100 kPa. Two deviatoric stresses are considered. While the confining pressure remains constant, σ_z increases and σ_θ decreases until the deviatoric stress q reaches a certain value of 30 kPa and 60 kPa, respectively. After that, the magnitude of deviatoric stress q and principal stresses remain constant, and the major principal stress orientation α continuously changes from 0^0 to 90^0 shown in Figure 3(b). The orientations of major principal stress and plastic strain rate are compared during the process of principal stress rotations. The strain rate orientation is determined by the difference of two consecutive readings of strains at an interval of 10 seconds. Given the elastic strain can be negligible, the orientation of principal plastic strain rate is defined as,

$$\tan 2\beta = \frac{2\Delta\varepsilon_{z\theta}}{\Delta\varepsilon_z - \Delta\varepsilon_\theta} \tag{12}$$

Results and Discussion

Figure 4 shows the stress-strain responses for material M1 under those seven monotonic loading paths. Figure 4(a) shows the relationship between the stress ratio q/p and the deviatoric strain ε_q. Figure 4(b) shows the relationship between the volumetric strain ε_v (expansion is negative) and ε_q in which the volumetric expansion is positive. Figure 5 shows the responses for material M2. These figures indicate that loading orientations play an important role, and a smaller angle of major principal stress leads to a higher strength and a greater tendency of volumetric expansion. These two figures both show that the response is the strongest along the stress path of 15^0 and the weakest along the stress path of 90^0. This is slightly different from test results of sands in which the strongest response generally occurs along the loading path of 0^0 and the weakest along the path of 75^0 (Cai et al, 2013). Further, comparisons of the responses between M1 and M2 indicate that these two materials exhibit different levels of anisotropy. It is evident that the distinction between the highest and lowest strengths for material M2 is greater than that for material M1. The anisotropy can be attributed to the specimen preparation process. The air pluviation process leads to uneven spatial distribution of particle contacts and contact forces. The preferable contacts and contact forces are along the vertical direction, which gives a horizontal bedding plane. The closer the major principal stress orientation is to the preferable vertical direction, the stronger the response becomes. Because material M2 is more angular than M1, the former anisotropy is greater than the latter anisotropy. The anisotropy ratio RA is defined as,

$$RA = \frac{\eta_{\max} - \eta_{\min}}{\eta_{\min}} \tag{13}$$

Where $\eta_{\max}$ and $\eta_{\min}$ represent the highest and lowest strengths (q/p), respectively. RA is 34% for M1 and 62% for M2. To further corroborate the argument, the most angular material M3 is tested along the stress paths of 0^0 and 90^0. M3 and M2 are made from the same material, and their responses are shown in Figure 6. It shows that the more angular material M3 has a higher strength than less angular M2 along the stress path of 0^0, and a slightly lower strength along the path of 90^0. The anisotropy for M3 is greater than M2, indicated by RA of 88%. It indicates that the angularity of material has a great impact on its anisotropy.

Figure 7 shows the orientation of major principal strain rates for these three materials, under the loading paths of pure principal stress rotations with q=30 and 60 kPa, respectively. The orientation of major principal stress is represented with a straight line of 45^0 in the figure. The distance between the symbols and the line represents the angle of non-coaxiality. There is a little scatter in the experimental data, especially under the smaller deviatoric stress. This is because a smaller deviatoric stress leads to smaller strain rates, which causes some errors for computations of strain rate orientations. It should be noted that this scatter takes place in many other experimental studies to measure the non-coaxiality in sands (Gutierrez et al, 1991; Cai et al 2013). However, some meaningful trends can still be observed. The figure shows that the major principal stress is always behind the strain rate for all these materials, indicating the non-coaxiality. Comparison of results in Figure 7(a) and 7(b) also indicates that the non-coaxiailty is smaller for a higher deviatoric stress for all the materials. Further, this figure indicates that a greater angularity of particles leads to a larger non-coaxiality. This trend is more noticeable under the larger deviatroic stress which generates larger strain rates with smaller errors for computation of strain rate orientations. It shows that the most angular material M3 has the largest non-coaxiality, followed by M2, and the least angular material M1 has the smallest non-coaxiality. This can be attributed to the anisotropy created by both the pluviation process in specimen preparations and further loading to the specified deviatoric stresses. The pluviation process initially generates non-uniform spatial distribution of particle contacts and their contact forces. The preferable contacts are on the vertical direction, which forms a horizontal bedding plane. This part of anisotropy is the inherent anisotropy. Further, imposing the deviatoric stress of 30 kP and 60 kPa increases the anisotropy, and this part of anisotropy is the induced anisotropy. It is evident that the material with greater angularity has a larger anisotropy, which leads to a larger non-coaxiality.

It is worth mentioning that numerous experiments of this type have been conducted on sands in geomechanics, and similar findings have been obtained. The strength of sands is a function of the orientation of major principal stress to the horizontal bedding plane of specimens or vertical deposition direction. A smaller angle of major principal stress to the vertical deposition direction leads to a stronger response. The principal stress rotation can incur plastic deformations and the non-coaxiality. The non-coaxiality decreases with increasing stress ratios. Further, the greater the anisotropy of sand is, the larger the non-coaxiality becomes. The impact of particle shapes was also investigated on these behaviors. For example, Cho et al (2006) studied the influences of particle shapes including sphericity and roundness on mechanical properties of 36 types of sands. They found that the strength of a sand increases with decreasing regularity of particles. In more recent study, Cai et al (2013) comprehensively compared behaviors of two types of sands, Portaway sand and Leighton Buzzard sand. The former is more angular than the latter. The extensive study indicates that the more angular Portaway sand exhibits stronger anisotropy than the less angular Leighton Buzzard sand. Further, Portaway sand has a greater non-coaxiality than Leighton Buzzard sand. Those findings are similar to these from the experiments in the paper. On the other hand, the experimental study in this paper is unique in that the materials used are different from those in previous studies. Sand particles feature non-uniform and irregular shapes, and the measurement of their sphericity and roundness is made on an average basis. Their particle sizes are not uniform as well. In addition, the abrasion in the process of loading can change particle shapes and sizes. All of these play their roles in sand stress-strain responses. In this paper, the granular solids used feature regular particle shapes and uniform particle sizes. It is merely regular particle shapes that influence the behaviors of granular solids. This study has important implications. For example, many types of granular solids in agricultural and chemical engineering are characterized with regular particle shapes and uniform particle sizes. Various filling methods of granular solids into a silo can incur strong anisotropy, and discharge of granular solids out of a silo can lead to considerable principal stress rotations.

Conclusion

This paper experimentally investigates the anisotropy and non-coaxiality of three types of granular solids by using the hollow cylindrical apparatus. They feature regular particle shapes and uniform particle sizes. The first material M1 is of glass beads, M2 is of cylindrical polymer particles, and M3 is of cubical polymer particles. The angularity of particles is at an increasing order from M1 to M3. The test results along seven monotonic loading paths with various angles of major principal stress with the vertical depositional direction indicate that these three materials exhibit strong anisotropy. A smaller angle leads to a stronger response. On the aspect of particle shape impact, the most angular material M3 has the greatest anisotropy, followed by M2, and M1 has the smallest anisotropy. The experimental results along the loading path of pure principal stress rotations at different stress ratios indicate that all these materials exhibit non-coaxial characteristics, and a larger stress ratio leads to a smaller non-coaxiality. In addition, a greater anisotropy leads to a larger non-coaxiality.

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Figures

Figure 1: Granular solids used in the experiments (M1: glass beads, M2: cylindrical polymer, M3: cubic polymer)

Figure 2: Idealized stress and strain components within the HCA subjected to axial load W, torque M_T, internal pressure P_i and outer pressure P_o; (a) hollow cylinder coordinates and loads; (b) element component stresses; (c) element component strains; (d) element principal stresses (after Zdravkovic and Jardine, 2001)

Figure 3: Schematic illustrations of loading paths (a: monotonic loading with various angles of major principal stress at an interval of 15^0; b: pure principal stress rotation at a constant deviatoric stress of 30 and 60 kPa)

Figure 4: Stress-strain responses of glass beads M1 under monotonic loading paths with various angles of major principal stress (a: deviatoric strain – deviatoric stress/confining pressure; b: deviatoric strain – volumetric strain)

Figure 5: Strenss-strain responses of cylindrical solids M2 under monotonic loading paths with various angles of major principal stress (a: deviatoric strain – deviatoric stress/confining pressure; b: deviatoric strain – volumetric strain)

Figure 6: Stress-strain responses of particulate assemblage of cubical solids M3 and cylindrical solids M2 under the loading paths of 0^0 and 90^0

Figure 7: Orientations of principal strain rates for different materials (M1: glass beads; M2: cylindrical polymer; M3: cubical polymer at deviatoric stress of 30 and 60 kPa (a: 30kPa; b: 60 kPa)