



International Conference on the Technology of Plasticity, ICTP 2017, 17-22 September 2017,  
Cambridge, United Kingdom

## An analytical study of new material test method for tension under bending and compression in double side incremental forming

Sheng Ai<sup>a</sup>, Bin Lu<sup>a,b</sup>, Hui Long<sup>a,\*</sup>

<sup>a</sup>*Department of Mechanical Engineering, The University of Sheffield, Sheffield S1 3JD, UK*

<sup>b</sup>*Department of Plasticity Technology, Shanghai Jiao Tong University, Shanghai 200030, China*

---

### Abstract

Incremental sheet forming (ISF) has attracted considerable research interests owing to its unique advantages. Double side incremental forming (DSIF) was proposed to further improve the forming accuracy and material formability. Compared with conventional sheet forming technologies, ISF provides greater process flexibility and achieves an enhanced formability. At the same time, however, ISF has exhibited a far more complicated material deformation behavior for formability enhancement. It is now widely acknowledged that the material deformation during ISF consists of stretching, bending, and shearing with cyclic effects. Continuous bending under tension (CBT) testing method was proposed by Emmens et al. [1], which proved the cyclic stretch-bending effect for formability enhancement in single point incremental forming (SPIF). However, limited research had been reported to investigate the material deformation mechanism leading to the formability improvement in DSIF.

An analytical model of a new material test method, Tension Under Bending and Compression (TUBC), is proposed in this study to investigate the material deformation leading to the formability enhancement in DSIF. Under TUBC condition, a specimen is stretched by the pulling force on both ends, while multiple rollers, in contact with the strip on both sides, move backward and forward continuously to create both cyclic bending and compression loading at a localized area. The analytical model is used to investigate the maximum stable elongations under TUBC condition. Key test variables, bending depth and compressive force, imposed by the rollers, are introduced to consider the effects of continuous bending, compression, and contact between rollers and workpiece. From the results obtained, it is clear that bending and compression have determinant effects on the formability enhancement of DSIF. However, the results show varied degrees of sensitivity of formability to different test variables. The findings correlate well with experimental observations and help to explain the formability enhancement of DSIF.

© 2017 The Authors. Published by Elsevier Ltd.

Peer-review under responsibility of the scientific committee of the International Conference on the Technology of Plasticity.

---

\* Corresponding author. Tel.: +44(0)114 2227759.

E-mail address: [h.long@sheffield.ac.uk](mailto:h.long@sheffield.ac.uk)

*Keywords:* Formability; Tension under bending and compression; Cyclic deformation; Double side incremental forming

---

## 1. Introduction

Incremental sheet forming (ISF) is a novel sheet metal forming technology with great process flexibility. In ISF, a simple tool moves along a pre-defined path to deform the sheet material gradually. Compared with the conventional sheet metal forming processes such as stamping, the lead time for the design and manufacturing of the dies and the cost of the materials can be significantly reduced in ISF. In addition, the formability of the material by ISF is substantially enhanced. The enhanced formability has been reported extensively in the published literature. Forming limit curves (FLCs) for different materials produced by ISF have been reported by Park and Kim, Allwood et al., and Ji and Park [2-4]. However different from the typical V-shaped forming limit curves in the conventional sheet metal forming processes, forming limit lines with a negative slope on the first quadrant of the major-minor strain coordinate system were produced for ISF. The lines were above traditional FLCs, especially under plane strain condition, which showed higher forming limits than that of the traditional processes.

Localized deformation is considered to be the main reason of the formability enhancement in ISF. According to the mechanics of plastic deformation, the existence of contact force, friction and bending reduces the stretching force needed to induce yielding therefore the plastic deformation zone is limited to the contact area and its vicinity, which follows the continuous movement of the tool in ISF. It is now widely recognized that the deformation in ISF is a combined contribution of stretching, bending and shearing deformation [5]. A comprehensive investigation of the mechanism of material formability enhancement in ISF was conducted by Emmens et al. [6]. To confirm the combined effect of bending, tension and cyclic deformation, a continuous bending under tension (CBT) test was developed by Emmens et al. [1]. The influence of various parameters, including pulling speed, the tool movement speed, bending depth, material thickness and properties on the formability was tested. According to the results, the actual bending radius, which was controlled by both the pulling speed and the tool radius, was the most important factor. The larger the actual bending radius, the longer the sample was elongated without necking.

Despite of the advantages of SPIF, the comparatively poor forming accuracy still hinders its further industrial applications, the derivation between manufactured part and its CAD design is unacceptable for precision applications [7-9]. To address this, DSIF was introduced by Malhotra et al. [10]. Instead of using one forming tool as that in SPIF, two independently controlled tools are deployed in DSIF, one on each side of the sheet. The additional tool provides a compressive force against the master tool, which results in an altered stress distribution. As a result, in addition to maintaining all the advantages from SPIF, DSIF has improved forming accuracy [10] and process flexibility when manufacturing parts with complicated geometries [11, 12].

Apart from stretching, bending and cyclic deformation, additional loading of compression (squeezing effect) is introduced in DSIF. The effect of the compression, including the magnitude and the relative position of the two tools are proved to affect the formability in DSIF. Smith et al. compared the deformation mechanics of SPIF and DSIF by performing finite element modelling and it was found that the existence of the compression caused higher hydrostatic pressure and shear strains, which may delay the initiation and development of fracture thus improving formability [11]. However, in the experiment conducted by Lu et al., in which the compressive force produced by the supporting tool was increased from 240N to 480N, the maximum forming depth was increased considerably by about 50% at first however it was decreased when the compressive force applied was increased to 560N. It was observed that it was due to extremely high squeezing effect between the sheet and the contacting tools which caused surface damage and severe sheet stretching in the master tool movement direction [13].

In conclusion, the factors influencing DSIF process are interlinked; the superimposition of the compressive force onto the existing stresses in SPIF will lead to a significant change in both of the stress and strain fields in DSIF. It is difficult to measure and quantify individual effects of bending, stretching and compression on material deformation and fracture behaviors under the experimental test condition of DSIF. To tackle the problem, a new test method of tension under bending and compression (TUBC) is developed in this work, which extends the CBT test concept developed by Emmens et al. [1], considering the compression effect in the new method. Developing an analytical model of TUBC is an effective method to investigate the effect of the various deformation modes and parameters on material deformation in DSIF process. In this study, an analytical model is established for the TUBC test and the

stress distribution in different regions of deforming sheet is obtained. To assess the initiation of necking, the uniaxial tensile stability criterion, Considère criterion is applied. Based on the analysis, the influence of the compressive stress and bending effect on the formability in DSIF is discussed in detail.

### Nomenclature

$\sigma_{\varphi}^A(\sigma_{\varphi}^B)$	stress in the longitudinal direction in region A (B)
$r_t$	radius of the tool
$\varphi$	contact angle between bending tool and the specimen
$t_0(t)$	thickness of the specimen before (after) deformation
$\sigma_s$	flow stress of the material
$\mu$	frictional factor between the tools and the specimen
$m(k)$	ratios of contact stresses to the yielding strength; these are negative values, as shown in Fig. 2
$A^A(A^B)$	cross-sectional area in region A (B)
$K$	strength coefficient of the flow stress
$n$	strain hardening exponent of the flow stress
$\Delta\varepsilon$	difference of the strains in the longitudinal direction between region A and region B

## 2. Analytical model of the TUBC test

Compared with SPIF, the sheet material to be deformed in DSIF makes contact with tools on both sides, as shown in Fig. 1(b). To investigate the deformation mechanics of DSIF, an analytical model of TUBC testing condition is proposed in this work. A schematic of the TUBC test model is shown in Fig. 1(a). In this model, the specimen is stretched in the longitudinal direction while at the same time it is pushed against a cylindrical roller to a certain depth on one side of the sheet in the lateral direction. On the opposite side another roller is pushed against the sheet by compressing a spring. By changing the bending depth of the roller and the stiffness of the spring, the magnitude of the bending effect and compressive effect can be varied. The rollers can move cyclically along the longitudinal direction at certain speed so that localized deformation can be achieved. In this way, a complicated 3D problem is reduced to a 2D one and the relevant parameters can be individually adjusted to carry out a detailed analysis.

In the TUBC model, the contact zone between the specimen and the master roller is in partial contact with the rear roller. According to the contact conditions between the specimen and the rollers, the deforming area of the specimen can be divided into three regions, the double-side contact region, single-side contact region and the non-contact region. However, the single-side contact region is quite small and the contact stress in this region is much smaller than that in the double-side contact region. For the simplicity of the analysis, the single-side contact region is ignored in this study, only the non-contact area (region A) and the contact area (region B) are considered, as shown in Fig.1. In addition, the following two assumptions are made:

- The contact stresses on both sides of the specimen are evenly distributed and cover the same area of contact;
- The stress variation in the thickness direction is small enough to be ignored.

To obtain the stress distribution of the deforming sheet, a small element is taken from region B. As shown in Fig. 2, the element is subjected to three major stress components, namely tensile stresses and frictional stresses in the longitudinal direction, and contact stresses induced by the tools on both sides of the specimen in the radial direction. Based on the stress equilibrium of the element in the radial direction,

$$\left[ \sigma_{\varphi}^B \cdot t + (\sigma_{\varphi}^B + d\sigma_{\varphi}^B) \cdot t \right] \cdot \sin \frac{d\varphi}{2} + m \cdot \sigma_s \cdot r_t \cdot d\varphi = k \cdot \sigma_s \cdot (r_t + t) \cdot d\varphi \quad (1)$$

Use the Tresca yield criterion and ignore higher order terms, the relationship between  $k$  and  $m$  can be obtained,

$$m = k - \frac{1}{t} \cdot \left( \frac{t}{2} + r_t \right) \quad (2)$$

The relation between  $m$  and  $k$  shows that the stress in the thickness direction varies from inside to outside of the

specimen because of the bending and compressive effects. Furthermore, the thinner the sheet is, the smaller the difference of the contact stress between the inner and outer surfaces. In the CBT test, the length of the specimen can almost be doubled after the test without fracture showing considerable improvement of material formability in SPIF. In DSIF, it has been proved that the formability of the material can be further enhanced due to the application of compression on the sheet. Therefore, it is assumed that the thickness of the specimen is small comparing to other dimensions thus it is acceptable to ignore the stress variation in the thickness direction. A membrane model can be used to analyse the material deformation under TUBC condition.

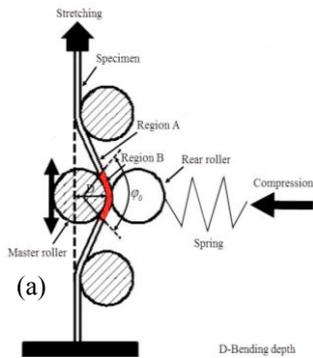


Fig. 1 Schematic of (a) TUBC model; (b) DSIF

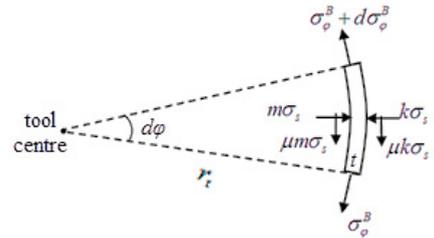


Fig. 2 Stress analysis of an element in region B

Similarly, using the stress equilibrium of the element in the longitudinal direction,

$$\sigma_\phi^B \cdot t \cdot \cos \frac{d\phi}{2} - \mu \cdot m \cdot \sigma_s \cdot r_t d\phi - \mu k \sigma_s \cdot (r_t + t) d\phi = (\sigma_\phi^B + d\sigma_\phi^B) \cdot t \cdot \cos \frac{d\phi}{2} \tag{3}$$

On the lower boundary of region B, when  $\phi = 0$ , no friction effect is considered. Therefore Eq. (3) can be solved and the stress component in region B in the longitudinal direction is,

$$\sigma_\phi^B = \left[ -\mu \frac{r_t}{t} (m + k) \cdot \phi + \frac{m+k}{2} + 1 \right] \sigma_s \tag{4}$$

It can be seen from Eq. (4) that the tensile stress in the longitudinal direction increases with the contact angle  $\phi$ . Therefore,  $\sigma_\phi^B$  reaches its maximum value when  $\phi = \phi_0$  on the boundary of region B.

Due to the existence of the contact stresses, the strain state under TUBC condition is different from that in the uniaxial tensile state. According to Levi-Mises equation, the width reduction of the specimen under TUBC condition with respect to the elongation of the specimen in the longitudinal direction is smaller than that under the uniaxial tension state. In this work, it is assumed that upon the occurrence of deformation instability, the strain ratio between the strain components can be described by using a constant,  $p$ , its value varies between  $-1$  and  $-0.5$ ,

$$\epsilon_t = p \epsilon_\phi \quad \epsilon_b = -(p + 1) \epsilon_\phi$$

Accordingly, the flow stress of the material can be described by the power hardening model  $\sigma_s = K \bar{\epsilon}^n$ ,

$$\sigma_s = K \cdot \left( 2\sqrt{p^2 + p + 1} \cdot \epsilon_\phi \right)^n \tag{5}$$

Region A is under uniaxial tensile force resulting from the material deformation in region B. Consider force equilibrium, the tensile force which leads to the deformation in region A should be equal to that of region B,

$$F_B = \sigma_\phi^B A^B = F_A = \sigma_\phi^A A^A$$

Consequently, the tensile stress in region A can be obtained by,

$$\sigma_\phi^A = \frac{F_B}{A^A} = \frac{\sigma_{\phi_0}^B \cdot A^B}{A^A} = \sigma_{\phi_0}^B \cdot e^{(1+\Delta\epsilon)} \tag{6}$$

Combine Eq. (2), (4), (5) and (6), a direct mathematical relationship between  $\sigma_\phi^A$  and other parameters can be

obtained. According to Considère criterion, under uniaxial tensile condition, when the increasing rate of the resistance in the deformed area cannot counterbalance that of the forming force, necking happens. In this case, if

$$\frac{d\sigma_{\phi}^A}{d\varepsilon_{\phi}} = \sigma_{\phi}^A \quad (7)$$

the deformation instability in the region A occurs. Substitute the commonly used values for the parameters in DISF in analysis TUBC,  $\mu = 0.1$ ,  $r_t = 5\text{mm}$ ,  $t = 0.8\text{mm}$ ,  $n = 0.1$ , Eq.(7) can be solved as a relationship between the maximum strain in the longitudinal direction and the contact angle and the compressive stress, as shown in Fig.3.

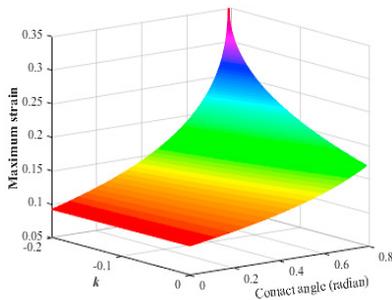


Fig. 3 Effects of key variables in TUBC

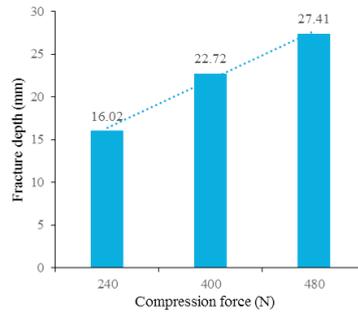


Fig. 4 Effect of compression force in DSIF [13]

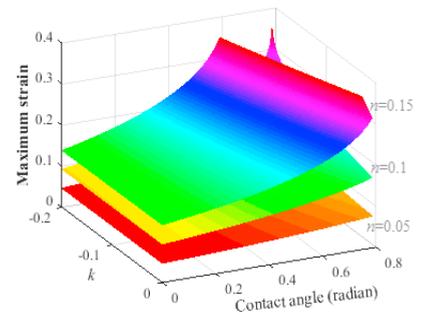


Fig. 5 Effect of strain hardening in TUBC

### 3. Results and discussion

It can be seen in Fig. 3 that the maximum forming strain in the longitudinal direction generally increases with the compressive stress ( $k\sigma_s$ ) and the contact angle, in the considered ranges of variations. The maximum strain increases sharply from 0.1 to 0.35 when the contact angle increases from 0 to  $\pi/4$  and the compressive stress increases from 0 to  $-0.2\sigma_s$ . The same trend in DSIF was observed experimentally by Lu et al. [13], as shown in Fig. 4, when the compression force was increased from 240N to 480N, the fracture depth of the formed conic parts with varied wall angle increased substantially. The increase of the maximum stable deformation of the specimen in TUBC can be explained by the decrease of the tensile stress required for material yielding due to the compressive contact stress supplied by the rear tool. As can be observed from Eq.(4), if the magnitude of compression factor  $k$  (negative value) becomes bigger, the tensile stress  $\sigma_{\phi}^B$  will decrease. The small tensile stress cannot lead to plastic deformation in region A because of the material strain hardening in the beginning of deformation so that the stable deformation continues. However, the gradient of the tensile stress increases while the deformation continues. At one point the increase of the tensile stress will overcome the strain hardening effect and deformation instability occurs. In general, the existence of the compression slows down the growth of the tensile stress in the beginning of deformation so that an extended stable deformation of the specimen can be obtained under TUBC condition. In the meantime, the bending effect is introduced by the contact angle in TUBC, the existence of bending will also reduce the tensile stress thus promoting the stable deformation before the deformation instability happens. When the contact angle is zero, there is no bending thus the TUBC condition is changed to uniaxial tension condition. According to Considère criterion, the maximum strain is material-related, it remains the same without being affected by changes of  $k$  value.

Considering it has already been proved that DSIF achieves higher formability than that by SPIF, it is acceptable to ascertain that the compression effect has further improved the formability in DSIF. However, different variation trends can be seen when the key testing variables are varied over different ranges, as shown in Fig.3. When the contact angle is small, the changes of compression is minimal, the maximum elongation of the specimen almost stays unchanged. An obvious increase of the maximum elongation can be observed when the contact angle is increased, even though the compression is still small. Furthermore, the larger the compression is, the more sensitive the maximum elongation to the contact angle will be, or vice versa. In addition, if the material strain hardening,  $n$ , is increased from 0.05 to 0.15, different trends of the maximum elongation variation appear, as shown in Fig.5. The formability improvement is more obvious when the strain hardening of a material is greater. This analysis correlates to the experimental observation that the hard-to-deform materials gain more obvious formability enhancement in

DSIF process. However, when both of the test variables, i.e. compression and contact angle, are increased to certain values, the formability decreases, which is not shown in Fig.5. It means the stability criterion becomes invalid, new stability criteria may be required to enable further analysis and explanations of these observations in the future.

Regarding to the assumptions made in the analysis, ignoring the single-side contact area and assuming the even distribution of the contact stresses lead to a smaller tensile force prediction in region A in the longitudinal direction. On the contrary, the increased double-side contact area increases the frictional effect, which results in a larger tensile force prediction. Considering that the first effect is more obvious thus the maximum elongation of the specimen may be overestimated in this analytical model. The analytical results will be compared with the results obtained from the TUBC experiment and finite element simulation in the future studies.

#### 4. Conclusion

In order to investigate the formability enhancement in DSIF, an analytical model of a new material test method for tension under bending and compression (TUBC) is proposed. The maximum elongation of the specimen can be related to two key variables: the contact angle, representing bending effect, and the compressive stress, with the consideration of effects of the mechanical properties of the material. The following conclusions can be drawn:

- The maximum elongation of the material increases with increased compressive stress and contact angle, within the tested ranges of variations of the two variables in this study;
- The effect of the two key variables on the maximum elongation varies, in the case studied, the maximum elongation is more sensitive to the contact angle than to the compressive stress. The maximum elongation is more sensitive to the changes of the two key variables when their magnitudes become greater;
- The greater the material strain hardening is, the more sensitive the formability is to the change of the two variables.

#### Acknowledgements

The authors would like to acknowledge funding supports from EU FP7 Marie Curie Actions: FLEXFORM Project (FP7-PEOPLE-2013 IIF 628055 & 913055) and MatProFuture Project (FP7-PEOPLE-2012 IRSES 318968).

#### References

- [1] Emmens, W. and A. Van den Boogaard, Incremental forming by continuous bending under tension—an experimental investigation. *Journal of Materials Processing Technology*, 2009. 209(14): p. 5456-5463.
- [2] Park, J. J. and Y. H. Kim, Fundamental studies on the incremental sheet metal forming technique. *Journal of Materials Processing Technology*, 2003. 140(1): p. 447-453.
- [3] Allwood, J., D. Shouler, and A.E. Tekkaya. The increased forming limits of incremental sheet forming processes. *Key Engineering Materials*. 2007. Trans Tech Publ.
- [4] Ji, Y. and J. Park, Formability of magnesium AZ31 sheet in the incremental forming at warm temperature. *Journal of materials processing technology*, 2008. 201(1): p. 354-358.
- [5] Jackson, K. and J. Allwood, The mechanics of incremental sheet forming. *Journal of materials processing technology*, 2009. 209(3): p. 1158-1174.
- [6] Emmens, W. and A. Van den Boogaard, An overview of stabilizing deformation mechanisms in incremental sheet forming. *Journal of Materials Processing Technology*, 2009. 209(8): p. 3688-3695.
- [7] Allwood, J.M., D. Braun, and O. Music, The effect of partially cut-out blanks on geometric accuracy in incremental sheet forming. *Journal of Materials Processing Technology*, 2010. 210(11): p. 1501-1510.
- [8] Dufloy, J., et al., Laser assisted incremental forming: formability and accuracy improvement. *CIRP Annals-Manufacturing Technology*, 2007. 56(1): p. 273-276.
- [9] Micari, F., G. Ambrogio, and L. Filice, Shape and dimensional accuracy in single point incremental forming: state of the art and future trends. *Journal of Materials Processing Technology*, 2007. 191(1): p. 390-395.
- [10] Malhotra, R., et al., Improvement of geometric accuracy in incremental forming by using a squeezing toolpath strategy with two forming tools. *Journal of manufacturing science and engineering*, 2011. 133(6): p. 061019.
- [11] Smith, J., et al., Deformation mechanics in single-point and accumulative double-sided incremental forming. *The International Journal of Advanced Manufacturing Technology*, 2013. 69(5-8): p. 1185-1201.
- [12] Wang, Y., et al. Experimental study on a new method of double side incremental forming. *ASME 2008 International Manufacturing Science and Engineering Conference collocated with the 3rd JSME/ASME International Conference on Materials and Processing*. 2008. American Society of Mechanical Engineers.
- [13] Lu, B., et al., Investigation of material deformation mechanism in double side incremental sheet forming. *International Journal of Machine Tools and Manufacture*, 2015. 93: p. 37-48.