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1 **A Hybrid Stabilization Technique for Simulating Water Wave – Structure**
2 **Interaction by Incompressible Smoothed Particle Hydrodynamics (ISPH)**
3 **Method**

4
5 Ningbo Zhang^a, Xing Zheng^{a,*}, Qingwei Ma^{a,b}, Wenyang Duan^a, Abbas Khayyer^c,
6 Xipeng Lv^a, Songdong Shao^{a,d}

7
8 ^a College of Shipbuilding Engineering, Harbin Engineering University, Harbin 150001,
9 China

10 ^b School of Mathematics, Computer Science & Engineering, City, University of
11 London, London EC1V 0HB, UK

12 ^c Department of Civil and Earth Resources Engineering, Kyoto University, Kyoto
13 615-8540, Japan

14 ^d Department of Civil and Structural Engineering, University of Sheffield, Sheffield
15 S1 3JD, UK

16 * Corresponding author. E-mail address: zhengxing@hrbeu.edu.cn

17
18
19 **ABSTRACT**

20 The Smoothed Particle Hydrodynamics (SPH) method is emerging as a potential tool
21 for studying water wave related problems, especially for violent free surface flow and
22 large deformation problems. The incompressible SPH (ISPH) computations have been
23 found not to be able to maintain the stability in certain situations and there exist some
24 spurious oscillations in the pressure time history, which is similar to the weakly
25 compressible SPH (WCSPH). One main cause of this problem is related to the
26 non-uniform and clustered distribution of the moving particles. In order to improve
27 the model performance, the paper proposed an efficient hybrid numerical technique
28 aiming to correct the ill particle distributions. The correction approach is realized
29 through the combination of particle shifting and pressure gradient improvement. The
30 advantages of the proposed hybrid technique in improving ISPH calculations are
31 demonstrated through several applications that include solitary wave impact on a
32 slope or overtopping a seawall, and regular wave slamming on the subface of
33 open-piled structure.

34
35
36 **Keywords:**

37 Hybrid stabilization; ISPH; minimum pressure; particle shift; wave impact

38 **1. Introduction**

39 The Smoothed Particle Hydrodynamics (SPH) technique is a Lagrangian mesh-free
40 numerical method, which was originally introduced by Lucy (1977), and Gingold and
41 Monaghan (1977) to solve the astrophysical problems. In recent years, the SPH
42 method has been successfully used in free surface flow simulations. In an SPH
43 computation, the particles are discretized by the moving nodes and they carry field
44 variables such as the pressure, density and velocity. The smoothing kernels are used to
45 approximate a continuous flow field.

46 The incompressibility of fluid can be imposed in two different ways in an SPH
47 numerical scheme. Originally, the simulation of incompressible fluid flows was
48 through a weakly compressible SPH formulation (WCSPH), in which the water was
49 considered as slightly compressible and its pressure was related to the density through
50 an equation of state. Thus an artificially specified sound speed has to be introduced
51 (Monaghan, 1994). The WCSPH approach has quite a few advantages, such as that it
52 is easy to program and does not need to solve the pressure boundary value problem.
53 However, at least two weaknesses emerged during its application to the water wave
54 problems (Lee et al., 2008; Rafiee et al., 2012): (a) the use of very small time steps;
55 and (b) significant spurious pressure fluctuations in the spatial and temporal domains.

56 To overcome the limitation of WCSPH, a strictly incompressible SPH (ISPH)
57 approach has been proposed by Shao and Lo (2003) based on the SPH projection
58 method initiated by Cummins and Rudman (1999) to simulate the free surface flows.
59 In ISPH approach the water is considered as truly incompressible with a constant
60 density. The method projects the intermediate velocity field to a divergence-free space
61 by solving a Poisson equation of pressure (PPE). It employs a strictly incompressible
62 SPH formulation, and thus the CFL condition is based on the fluid velocity rather than
63 the speed of the sound. Therefore, the pressure is not an explicit thermodynamic
64 variable obtained through an equation of the state such like in WCSPH, but obtained
65 through a hydrodynamic equation. For the ISPH modeling techniques, there are
66 mainly two types of the formulation, i.e. the density-invariant ISPH (Shao and Lo,
67 2003) and velocity divergence-free ISPH (Lee et al., 2008). The ISPH has also been
68 widely applied in the field of water wave dynamics (Khayyer et al., 2008; Lind et al.,
69 2012). According to the comparative studies carried out by Lee et al. (2008) and
70 Violeau and Leroy (2015), the time step used for the ISPH can be five times larger. In
71 addition, the computational results from ISPH could be much more stable and
72 accurate than those from the WCSPH without extra smoothing techniques (Zheng et

73 al., 2014). However, Shadloo et al. (2011; 2012) and Hughes and Graham (2010)
74 noted that the inclusion of certain numerical treatments could significantly enhance
75 the performance of WCSPH. On the other hand, we should also realize that the
76 turbulent flows involve more complex particle convections and free surface
77 deformations, which has more stringent requirement on the pressure solution schemes.
78 In addition, as indicated by Gotoh and Khayyer (2016), one distinct advantage of
79 ISPH corresponds to its superior volume conservation properties. It should be realized
80 that the SPH approaches have been recently expanded to solve the shallow-water
81 equations (SWEs) where the flow is over large domain and the vertical variation of
82 parameters of interest is not demanding (Chang et al., 2016; Chang et al. 2017).

83 The wave impact loadings on structure constitute an important practical problem
84 with highly distorted free-surface motion. For the SPH application in this field,
85 considerable progress has been made in the investigation of wave-structure
86 interactions, such as documented by Khayyer and Gotoh (2011), Rudman and Cleary
87 (2016) and Ren et al. (2016). According to the extensive computations in engineering
88 practice, it has been found that the homogeneity of particle distributions plays an
89 important role in the accuracy and robustness of the SPH models. The formation of ill
90 particle distributions could significantly degrade the SPH numerical accuracy and lead
91 to the failure of correct solutions.

92 There have been some remedies which were proposed to address this issue. For
93 example, Monaghan (2000) introduced an additional set of stress node at the points
94 other than the SPH particle locations to address the tensile instability, which was
95 mainly proposed for WCSPH. As for ISPH, Khayyer and Gotoh (2011) and Gotoh et
96 al. (2014) proposed an error compensating scheme to minimize such numerical errors.
97 Following the similar concept, to maintain a more uniform particle distribution,
98 Sriram and Ma (2012) proposed that the pressure of reference particle should be
99 replaced by the minimum pressure of all neighboring particles when calculating the
100 pressure gradient, based on the original idea of Koshizuka and Oka (1996) and
101 improved by Khayyer and Gotoh (2013) in the Moving Particle Semi-implicit (MPS)
102 method.

103 Another numerical scheme to improve the particle distribution is through the
104 shifting of particle positions directly. Xu et al. (2009) initially used this idea to correct
105 the non-uniformity of particle distributions. Recently a more efficient method based
106 on the Fick's law for adjusting the particle distributions has been introduced by Lind
107 et al. (2012) and Skillen et al. (2013). Besides, Shadloo et al. (2012) also proposed

108 a particle fracture repairing procedure and a corrected SPH discretization scheme to
109 eliminate the instability induced by the particle clustering. The improved model
110 performance has been demonstrated in the benchmark water wave propagations and
111 wave-body interactions. However, we have found from various tests of violent water
112 wave impact on fixed structures, especially those involve longer simulation time, the
113 above-mentioned approaches could face some challenges at the free surface because
114 the shifting scheme is a function of the gradient of concentration field. This challenge
115 is highlighted by Khayyer et al.(2017a), where a correction for elimination of shifting
116 normal to the free-surface is proposed. Despite that the particle shifting algorithm may
117 partially violates the principle of volume conservation for free-surface flows (Nair and
118 Tomar, 2015; Pahar and Dhar, 2016), the issues of particle non-homogeneity have
119 been well resolved.

120 To make full use of the potentials of available practice, this paper introduces a
121 hybrid ISPH model by combining the particle shifting algorithm of Xu et al. (2009)
122 and minimum pressure idea of Sriram and Ma (2012). The improved numerical
123 scheme would be expected to effectively eliminate the particle clustering/stretching
124 issues and make the particle/pressure distributions more stabilized in wave impact
125 simulations.

126 **2. Review of ISPH Methodology**

127 The governing equations used to solve the fluid problems in an ISPH method are
128 the mass and momentum conservation equations. As there is no major improvement in
129 the fundamental ISPH theory in present paper, Tab.1 briefly summarizes the ISPH
130 solution algorithms, spatial derivative approximations and boundary treatments.
131

132 **3. Hybrid Particle Stabilization Scheme**

133 This section first reviews the available stabilization approaches, followed by the
134 proposal of a hybrid technique. Then a benchmark test is done to validate the accuracy
135 of this new method.

136 **3.1. Existing stabilization techniques**

137 Among a variety of the particle stabilization algorithms reported in the literature, we
138 have found the minimum pressure (MP) approach of Sriram and Ma (2012) provided
139 an effective solution. When computing the pressure gradient, the minimum pressure
140 P_{\min} as illustrated in Fig. 1 in the influence domain of reference particle i is used
141 instead of P_i , which is shown in Eq. (1). Here this approach is named as ISPH_MP.

$$\nabla P_i = \sum_{j=1, j \neq i}^N \frac{n_{i,x_m} B_{ij,x_m} - n_{i,xy} B_{ij,x_k}}{n_{i,x} n_{i,y} - n_{i,xy}^2} (P_j - P_{\min}) \quad (1)$$

143 Nevertheless, we should realize that the force exerted on particle i by particle j ,
 144 and on particle j by particle i , would not be the same, and thus the momentum is
 145 not exactly conserved even if the number of particles in the sub-domain is identical
 146 and also whether it is uniformly or irregularly distributed.

147 On the other hand, Xu et al. (2009) introduced an artificial particle displacement
 148 (APD) method to prevent the particle clustering, which is named as ISPH-APD in this
 149 paper. In this approach the trajectory of particles is re-distributed by adding a small
 150 artificial displacement $\delta \mathbf{r}_i^\zeta$ to the advection of the particles as

$$\delta \mathbf{r}_i^\zeta = \beta \sum_{j=1}^N \frac{r_{ij}^\zeta}{r_{ij}^3} r_0^2 V_{\max} \Delta t \quad (2)$$

152 where β is a problem-dependent parameter; ζ is the direction
 153 component; $r_0 = \sum_{j=1}^N r_{ij} / N$ is the cut-off distance; and V_{\max} is the largest particle
 154 velocity in the computational system. Here, N is the number of neighbours for
 155 particle i in its support domain. The problem-dependent parameter β was
 156 recommended to be 0.01 ~ 0.1 by Xu et al. (2009). It should be noted that β must be
 157 selected carefully such that it should be small enough not to affect the physics of the
 158 flow, but large enough to prevent the occurrence of particle clustering and fracture in
 159 SPH simulation. The artificial particle displacement approach has also been used by
 160 Shadloo et al. (2011), where β was kept constant as 0.01. Fig. 2(a) gives the
 161 comparison between the experimental data and ISPH-APD results for the pressure
 162 time history of a solitary wave impacting on the vertical wall (detailed in Section
 163 4.2.1) with the parameter $\beta = 0.01$. From the stability in the pressure results and
 164 reasonable agreement with the experimental data, we could fix this value in other
 165 simulations as well.

166 Moreover, Lind et al. (2012) proposed another approach based on the Fick's law for
 167 adjusting the particle distribution. This was further improved by Skillen et al. (2013),
 168 in which a particle displacement vector $\delta \mathbf{r}_s$ was used to update the particle position

$$\delta \mathbf{r}_s = -A h \|\mathbf{U}\|_i \Delta t \nabla C \quad (3)$$

170 where a value of $A=2$ has been found to provide good compromise in Lind et al.
 171 (2012), $\|\mathbf{U}\|_i$ is the velocity amplitude of particle i , and $\nabla C = \sum_{j=1}^N V_j \nabla_i W(\mathbf{r}_{ij})$ is
 172 defined, in which V_j is the volume of particle.

173 Fig. 2(b) gives the comparison between experimental data and SPH results for the

174 same case as Fig. 2(a) but using the particle shifting method of the Fick's law. It is
 175 shown that this approach still generates some spurious oscillations in the pressure
 176 time history. As mentioned before, the reason could be attributed to that the shifting
 177 scheme is based on the function of the concentration gradient, which cannot be
 178 accurately calculated near the free surface. Therefore, we would use ISPH_APD as a
 179 viable approach in this work.

180 3.2. A hybrid stabilization scheme

181 In order to further improve the ISPH modelling capacity, here we introduce a hybrid
 182 particle stabilization technique to improve the numerical stability through correcting
 183 the irregular particle distributions, by combining the ISPH_MP and ISPH_APD in
 184 Section 3.1. In principle it uses the minimum pressure in the influence domain of
 185 reference particle i to replace the actual pressure of this particle for calculating the
 186 pressure gradient, and meanwhile adds a small artificial displacement $\delta \mathbf{r}_i^\zeta$ to the
 187 advection of the particle. This hybrid approach is named as ISPH_MPAPD in the
 188 paper. After some numerical trials, it has been found that a value of $\beta = 0.001 \sim$
 189 0.01 for $\delta \mathbf{r}_i^\zeta$ would be appropriate for modelling the violent water wave impact. It
 190 has also been noted that since the physical velocity of a particle is different from the
 191 velocity with which the particle position is shifted with $\delta \mathbf{r}_i^\zeta$, we should interpolate
 192 the physical velocity to the new position of the particles in the next computational
 193 cycle. The same interpolation technique as used by Xu et al. (2009) is also adopted
 194 here as

$$195 \quad \mathbf{u}_{i'} = \delta \mathbf{r}_{i'} \mathbf{u}_i \quad (4)$$

196 where i and i' refer to the old and new values, respectively; and $\delta \mathbf{r}_{i'}$ is the
 197 distance vector between the two particles.

198 To examine whether or not Eq. (4) still satisfies the pressure Poisson equation PPE,
 199 Fig. 3(a) and (b) give the time history of the averaged velocity divergence and the
 200 impact pressure, computed with and without the SPH interpolation technique.
 201 Meanwhile, the analytical solutions and experimental data (Zheng et al., 2015) are
 202 also provided for the validation purpose. The numerical test is for the solitary wave
 203 propagation which will be detailed in Section 4.2.1. It can be seen that there is almost
 204 no difference observed between the two ISPH results. So we could judge that this
 205 interpolated velocity field should still satisfy the PPE.

206 3.3. Model test on vortex spin-down

207 To validate the proposed hybrid method, a vortex spin-down simulation following
 208 Xu et al. (2009) is conducted. In this study a vortex is bounded by the four walls and

209 placed in the middle of the domain, as shown in Fig. 4. The initial velocity field is
 210 given by $u = U_0(y - 0.5)$ and $v = U_0(0.5 - x)$ inside a unit square, where $D = 1.0$
 211 m is the width of the square and $U_0 = 1$ m/s is the velocity scale. The kinematic
 212 viscosity ν is taken 0.001 m²/s and the vortex spin-down process is simulated for
 213 the Reynolds number $Re = 1000$.

214 Fig. 5(a) - (d) show the comparisons of particle distribution computed by using the
 215 standard ISPH, ISPH_MP, ISPH_APD and ISPH_MPAPD, respectively, at time $t =$
 216 1.0 s. The particle number in the x direction is $N_x = 60$. The traditional ISPH model
 217 cannot achieve the converged result and the computation breaks at $t = 0.53$ s. From
 218 the comparisons between three particle stabilization methods, the result of ISPH_APD
 219 and ISPH_MP still demonstrates particle clustering and stretching patterns near the
 220 corner region, as clearly demonstrated by the enlarged portion of the particle
 221 distributions at $0 < x < 0.25$ and $0 < y < 0.25$. In contrast, the hybrid ISPH_MPAPD
 222 computation has obtained the most satisfactory particle distributions.

223 In order to quantify the accuracy of different particle stabilization methods, Fig. 6(a)
 224 gives the comparison of horizontal velocity components at $x = 0.5$ m and $t = 1.0$ s.
 225 Here the particle number in the x direction is $N_x = 200$. The reference value of the
 226 velocity component was provided by Xu et al. (2009) using the STAR-CD. It shows
 227 that all ISPH computations achieved good agreement with the STAR-CD results.
 228 Besides, Fig. 6(b) gives the convergence test on the horizontal velocity component,
 229 where N_t is the total particle number at different values of 3600, 6400, 10000 and
 230 40000, respectively. The relative error Err is defined as

$$231 \quad Err = \frac{1}{N_y} \sum_{j=1}^{N_y} \sqrt{(u_j - u_{j,s})^2} \quad (5)$$

232 where u_j and $u_{j,s}$ are the horizontal velocity components computed by ISPH and
 233 STAR-CD, respectively, N_y is the particle number in the y direction. It is shown
 234 that the hybrid ISPH_MPAPD computation achieved the smallest errors as compared
 235 with either ISPH_MP or ISPH_APD results. However, we should also realize that all
 236 three ISPH numerical schemes are below first-order accurate in the convergence
 237 behaviour when the particle distribution becomes disordered, in spite of the use of
 238 various correction techniques.

239 To demonstrate the time history of velocity variations, Fig. 7(a) gives the maximum
 240 velocity computed by different ISPH particle stabilization methods with $N_x = 200$,
 241 in which $u_{\max} = \max(|U_i|)$ is defined and i is the index of particle. It shows that the
 242 ISPH_MP computations demonstrate some kinds of oscillation in the velocity time
 243 histories, while both the ISPH_APD and ISPH_MPAPD results are quite stable and
 244 smooth. To further investigate the convergence behaviour of ISPH_MPAPD, Fig. 7(b)

245 gives the comparison of maximum velocity time histories for different particle
246 numbers at $N_x = 60, 80, 100$ and 200 , respectively. Again the close overlap of four
247 computational curves and the noise-free velocity profiles indicate the convergence of
248 the model.

249 Since pressure field is the most sensitive one to the particle disorder and instability,
250 Fig. 8(a) - (c) give the comparisons of pressure distribution computed by
251 ISPH_MPAPD at time $t = 1.0$ s with different total particle numbers of $N_t = 3600,$
252 10000 and 40000 , respectively. It shows that with an increase in the particle number,
253 the pressure distributions become much more reasonable. This is further supported by
254 the enlarged portion near the corner regions. Besides, Fig. 9 gives the comparison of
255 pressure profiles at $x = 0.0$ m between different ISPH results with $N_x = 200$ and the
256 STAR-CD computation made by Xu et al. (2009). From this it is shown that
257 ISPH_MPAPD can get the best agreement with STAR-CD, while ISPH_MP and
258 ISPH_APD significantly underestimate the pressure values in the centre domain.

259 To study the computational efficiency, Fig. 10 gives the comparisons of CPU time
260 versus total particle number N_t for different particle stabilization schemes, where
261 T is the CPU time measured in seconds. It demonstrates that ISPH_MP consumes
262 the longest CPU time especially at high particle numbers, since it requires more
263 iterations to solve the pressure Poisson equation under particle clustering or stretching.
264 On the other hand, the irregular particle distributions have less influence on the
265 numerical iterations in an ISPH-APD scheme, which takes similar CPU expenses as
266 the ISPH_MPAPD.

267 **4. Model Applications in Wave Impact**

268 In this section, to test the effectiveness of the hybrid ISPH_MPAPD on modelling
269 the violent water wave impact, we consider five practical applications. These include
270 a dam break flow, solitary wave impact on the vertical and inclined walls, wave
271 overtopping of an impermeable structure, and wave slamming on subface of an
272 open-piled structure. The enhanced performance of ISPH_MPAPD will be
273 demonstrated through the quantitative comparisons with standard techniques such as
274 ISPH_MP and ISPH_APD, as well as the experimental data.

275 **4.1. Dam-break flow impact on a vertical wall**

276 In this test a rectangular column of water is confined between the two vertical walls
277 as shown in Fig. 11. The width of water column is L and the height is H . At
278 beginning the dam is instantaneously removed and water is allowed to flow out along
279 the dry horizontal bed. D is the length of horizontal section of water tank and a

280 pressure sensor P_1 is located on the right wall at a vertical distance of h_1 from the
281 bottom. In the interpretation of numerical result, all variables and parameters are
282 non-dimensionalised by the characteristic dam height H and gravitational
283 acceleration g .

284 The following parameters are studied here: $L = 0.5$ m, $H/L = 2.0$ and $D = 4L$.
285 To show the convergence of ISPH_MPAPD model results, the time history of impact
286 pressures at P_1 computed by using different time steps and particle numbers are
287 presented in Fig. 12(a) and (b), respectively. Here it should be mentioned that the
288 computed pressures are obtained by the particle nearest to the measuring location
289 which does not involve the samplings from neighbouring particle. It is shown from
290 Fig. 12 that as the time step or particle spacing becomes smaller (i.e. when the particle
291 number becomes larger), the difference between two adjacent numerical results
292 becomes smaller. Also the numerical results become smoother and less fluctuating,
293 following the refinement in spatial and temporal resolutions. These have clearly
294 evidenced the convergence of numerical results in the temporal and spatial domains.

295 Besides, Fig. 13 gives the comparisons of wave front and water column height of
296 dam break flow computed by three alternative ISPH methods. The numerical results
297 are compared with the experimental data of Martin and Moyce (1952). It seems that
298 very minor differences are found between them, which may imply that the water
299 surface profiles are not very sensitive to the particular choice of particle stabilization
300 schemes as compared with the impact pressure.

301 In order to further quantify the accuracy of different particle stabilization schemes,
302 another benchmark dam break flow as documented by Colagrossi and Landrini (2003)
303 is considered, where the dimensions $L = 2.0$ m, $H = 0.5L$ and $D = 5.3667L$
304 are used in Fig. 11. On the right wall, there is also a pressure sensor point P_1 with
305 height $h_1 = 0.14H$ to record the impact pressure time history. For all controlled SPH
306 simulations in this case, the particle numbers keep the same at 120×60
307 corresponding to a particle size of 0.0167 m. The time step is taken to be constant Δt
308 $= 0.003$ s. Fig. 14 illustrates the particle distributions by using different ISPH
309 stabilization methods and the snapshots were extracted at time $t = 2.775$ s. We could
310 observe that there is a slight particle strip distribution in the ISPH_MP results as
311 shown in Fig. 14(a), and the particle distribution becomes disordered in the
312 ISPH_APD results as shown in Fig. 14(b). Overall speaking, the particle distributions
313 computed by ISPH_MPAPD seem to be most satisfactory as shown in Fig. 14(c).

314 The time histories of pressure at P_1 computed by using different ISPH particle
315 correction methods (with total particle number $N_t = 7200$) are compared with the
316 experimental data of Zhou et al. (1999) in Fig. 15. It shows that the pressure obtained
317 by ISPH_MPAPD is much better than that from the other two methods, i.e. ISPH_MP
318 or ISPH_APD. The ISPH_MP result exhibits a more obvious phase shift in the second

319 pressure peak, while the ISPH_APD result demonstrates a much larger pressure
320 oscillation. For the three ISPH results, their major differences appear after the second
321 pressure peak. One reason could be due to the lack of two-phase water-air modelling,
322 since the influence of air becomes increasingly significant during the second violent
323 wave impact when the water column plunges down onto the surface and forms a
324 cavity region. It has been recorded that the CPU expense (Intel i7 3.4 GHz with RAM
325 8 GB) of present simulation is 324 s by using ISPH_MP, 332 s by ISPH_MP and 326
326 s by ISPH_MPAPD, respectively.

327 4.2. Solitary wave impact on a vertical wall

328 In order to further evidence the effectiveness of improved particle stabilization
329 technique, the analysis of numerical results of solitary wave impact on a vertical wall
330 is provided below. The experiment of solitary wave propagation and its impact on a
331 vertical wall was carried out by Zheng et al. (2015) in a 3-D wave flume with piston
332 wave maker in Harbin Engineering University (HEU). The schematic diagram of the
333 wave tank is shown in Fig. 16. The wave tank is 10 m long and the water depth is d
334 $= 0.25$ m. The solitary wave height is $h = 0.15$ m, thus the wave nonlinearity is
335 $\varepsilon = h/d = 0.6$. A measurement point P_1 is located on the right wall at a distance of
336 0.05 m from the tank bottom to monitor the pressure time history. In ISPH
337 computation the initial particle spacing is 0.01 m and the time step is 0.001 s.

338 Fig. 17 illustrates the particle distributions with pressure contour by using the
339 original ISPH (Shao and Lo, 2003) and improved ISPH with different particle
340 stabilization methods. The snapshots were extracted at time $t = 1.2$ s after the wave
341 is initiated. Under such a high wave-to-depth ratio, it would be very easy to generate
342 the particle clustering in standard ISPH computation, which is illustrated in Fig. 17(a).
343 On the other hand, it can be seen that these abnormal particle distributions can be
344 corrected effectively by using the different stabilization techniques as shown in Fig.
345 17(b) - (d). However, we could still find that there is a slight particle strip distribution
346 in ISPH_MP result as shown in Fig. 17(b). Besides, the particle distribution is slightly
347 disordered in ISPH_APD result as shown in Fig. 17(c). Overall speaking, the
348 distribution of particles in ISPH_MPAPD result is the most desirable, as shown in Fig.
349 17(d), which demonstrates its superiority in predicting the pressure fields.

350 To investigate the conservation of volume for all ISPH models, Fig. 18 shows the
351 time history of water particle volume variations during the wave propagation. It can
352 be seen that ISPH_MP and ISPH_APD cannot satisfy the strict volume conservation,
353 namely the mass conservation, while the proposed ISPH_MPAPD has the best
354 conservation performance. By analysis it was found that the relative volume errors are
355 about 1.45% for ISPH_MP, 1.24% for ISPH_APD and only 0.71% for ISPH_MPAPD
356 in Fig. 18. Besides, the comparisons of wave surface profile at two time instants of t

357 = 2.0 s and 3.1 s are shown in Fig. 19(a) and (b), respectively, which shows that all
358 ISPH simulated free surfaces have an overall agreement with the analytical solution,
359 although there are some differences in the wave crest. Here the relative errors in wave
360 height are about 1.013% for ISPH_MP, 5.153% for ISPH_APD and 0.433% for
361 ISPH_MPAPD in Fig. 19(a), while they are 5.31% for ISPH_MP, 2.5% for
362 ISPH_APD and 2.86% for ISPH_MPAPD in Fig. 19(b). Generally speaking,
363 ISPH_MPAPD computation also shows the best accuracy and stability in the wave
364 surface profiles.

365 Furthermore, the comparisons of wave impact pressure at sensor point P_1 between
366 the experimental data (Zheng et al., 2015) and numerical results by using different
367 ISPH particle stabilization methods, are illustrated in Fig. 20(a) - (d). It should be
368 mentioned that Fig. 20(a) is the superposition of all the data, while Fig. 20(b) - (d)
369 is the comparison with each individual ISPH correction scheme. It is shown that in Fig.
370 20(b) there appear spurious oscillations around the ISPH_MP pressure peak. In Fig.
371 20(c) the pressure peaks computed by ISPH_APD are larger than the experimental
372 data. Again the proposed ISPH_MPAPD achieves the best agreement in both the
373 pressure peak and its evolutions, as shown in Fig. 20(d). Comparing Fig. 20 with Figs.
374 17-19, it can be understood that the impact pressure simulations can best demonstrate
375 the superiority of ISPH_MPAPD than the other illustrations, such as the particle
376 snapshot and volume and free surface profile.

377 4.3. Solitary wave impact on a slope wall

378 In this section, the ISPH method with improved particle stabilization technique is
379 used to the simulation of solitary wave impacting on a slope with angle of 150° . The
380 computational domain is the same as that used in the laboratory experiment of Zheng
381 et al. (2015), so a direct comparison can be made. Four pressure sensors, labelled as
382 $P_1 - P_4$, are placed along the slope at a distance of 0.05 m from the bed and
383 subsequent intervals of 0.1 m upward. The schematic diagram of the domain is shown
384 in Fig. 21.

385 As shown in Fig. 21 a solitary wave with wave amplitude $h/d = 0.6$ is studied. The
386 water depth is $d = 0.25$ m and the length of horizontal section is $L = 10.0$ m. The
387 initial particle spacing is 0.01 m and approximately 25000 particles are involved in
388 the ISPH computations.

389 Fig. 22 illustrates the process of solitary wave running up and down the slope at
390 different times computed by ISPH_MPAPD, whose particle snapshots coincide well
391 with the laboratory photographs. It can be seen from Fig. 22(a) that the wave front
392 reaches its maximum climbing point at time $t = 6.5$ s. Then the run-down process
393 starts and the main flow retreats from the slope. It is shown in Fig. 22(b) that a violent
394 backflow occurs near the original shoreline at $t = 7.0$ s, which explains the abrupt

395 pressure drop in its time history (as shown in later Fig. 24). Generally the agreement
396 between numerical and experimental free surfaces is quite satisfactory.

397 Fig. 23 illustrates the particle distributions with pressure field computed by using
398 different particle stabilization methods. The snapshots were extracted at time $t = 7.1$ s
399 and $t = 7.25$ s after the model was run. It can be seen from Fig. 23(a1) and (a2) that
400 there exist particle clustering and disorders in the pressure field, which was computed
401 by using ISPH_MP. In Fig. 23(b1) and (b2), the pressure fields computed by
402 ISPH_APD displayed obvious local chaos, especially at later stage of the wave impact.
403 On the other hand, the distribution of particles and their pressure fields in
404 ISPH_MPAPD result shows much more stable and uniform patterns, as indicated in
405 Fig. 23(c1) and (c2).

406 To quantify the accuracy of ISPH_MPAPD, Fig. 24(a) - (d) show the comparisons
407 of wave impact pressure at four measurement point ($P_1 - P_4$) between the experimental
408 data and different ISPH correction results. It is shown that good agreement has been
409 found in spite of some discrepancies, due to that the pressure fields are always
410 difficult to predict by any numerical model. Similar to experimental data, the
411 computed pressures at P_1 and P_2 which are located below the surface of water, share
412 similar evolution features. That is to say, the impact pressure first reaches its
413 maximum value when the wave runs up to the maximum point, and then it gradually
414 decreases to negative pressure as the wave runs down freely, until to the minimum
415 pressure point. However, all ISPH computations exhibit much larger pressure
416 oscillations than the experimental observations. It is also promising to note
417 ISPH_MPAPD computation demonstrates much less pressure noise and shows better
418 agreement with the experiment. This conclusion has been further strengthened by the
419 zoomed sub-figures of Fig. 24(a1 - a3) and (b1 - b3) with separate comparison with
420 each ISPH model, which shows that ISPH_MPAPD is superior to either ISPH_MP or
421 ISPH_APD in obtaining the stable and accurate pressure predictions.

422 On the other hand, as shown in Fig. 24(c) and (d), the computed pressures at sensor
423 point P_3 and P_4 , which is on and above the still-water shoreline, exhibit much more
424 stable pressure patterns as compared with those at P_1 and P_2 . Both pressures increase
425 rapidly to the maximum value when the solitary wave impacts on the slope and then
426 fall to zero without generating the negative pressures. Again the numerical results of
427 ISPH_MPAPD show an overall better agreement with the experiment.

428 Since maximum pressure generated during the wave impact is quite important for
429 the safety and reliability of marine structures, we carry out an error analysis and find
430 out that the relative errors are around 10.25% for ISPH_MP, 10.69% for ISPH_APD,
431 and only 0.3% for ISPH_MPAPD, as compared with the experimental peak pressure
432 in Fig. 24(a). In contrast these errors are about 11.83%, 9.24% and 5.6%, respectively,
433 in Fig. 24(b).

434 4.4. Solitary wave overtopping on an impermeable seawall

435 Here another robust test is carried out to investigate the tsunami-like solitary wave
436 impinging and overtopping on an impermeable trapezoidal seawall located on a 1:20
437 sloping beach. The numerical computation was based on the benchmark physical
438 experiment documented by Hsiao and Lin (2010). In the study, the wave nonlinearity
439 $\varepsilon = h/d$ is 0.35 and other relevant parameters are shown in Fig. 25(a) inside the
440 wave tank. For analysis, the relative time $t' = t - t_{MR}$ is used, where t_{MR} is the time
441 of maximum wave run-up against the wall.

442 The ISPH computation used a particle spacing of 0.01 m and constant time step of
443 0.001 s, involving 21360 particles. The solitary wave was generated by pushing a
444 solid wave paddle on the offshore boundary. The numerical simulations were carried
445 out to 10.0 seconds of the wave propagation. The experimental data of water surface
446 profile and wave impact pressure are used to validate the ISPH results and evaluate
447 the accuracy of different particle stabilization schemes. The measurement points of
448 water surface “G” and impact pressure “P” are shown in Fig. 25(b). It should be noted
449 that only selected results from the experiment of Hsiao and Lin (2010) are used here
450 for the model comparisons.

451 Fig. 26 shows the particle snapshots with pressure field during the wave impinging
452 and overtopping on the trapezoidal caisson at $t' = 3.19$ s, computed by all ISPH
453 particle correction schemes. It is shown that as the wave overtops over the seawall an
454 overtopping tongue develops on the crown. In addition, the experimental photo and
455 measured free surface profiles (Hsiao and Lin, 2010) indicated by the black dots are
456 superimposed on the ISPH particle snapshots, quantifying the good accuracy of
457 numerical simulations. From the enlarged portion of the sub-figures, we could observe
458 that there is a slight particle strip distribution near the run-up boundary in ISPH_MP
459 results as shown in Fig. 26(a). On the other hand, the particle distribution seems to be
460 noisy in ISPH_APD results as shown in Fig. 26(b). In comparison, the distribution of
461 particles and pressure patterns in ISPH_MPAPD results are still the most satisfactory
462 as shown in Fig. 26(c).

463 Fig. 27(a) - (d) show the time histories of free surface variation compared between
464 experimental data (Hsiao and Lin, 2010) and numerical results at four wave gauging
465 points (see Fig. 25(a)). Although the computed free surface elevations seem to be
466 generally higher than the experimental values, the overall good agreement is quite
467 promising. For Fig. 27(a) - (b) the ISPH_APD gives a slight overestimation of the
468 peak elevation as compared with the ISPH_MP and ISPH_MPAPD, while the time
469 histories of ISPH_MPAPD computation are much more stable than the ISPH_APD
470 and ISPH_MP results as shown in Fig. 27(c) - (d). Besides, the small and narrow
471 spread of free surface profile in Fig. 27(d) indicates that only a small portion of water
472 overtops on the impermeable seawall, thus explaining the oscillation in numerical free

473 surfaces at G_{37} and the slightly larger discrepancy in predicting the maximum wave
474 height, in contrast to the situations at G_3 , G_{10} and G_{28} .

475 Furthermore, Fig. 28(a) - (d) shows the time histories of experimental (Hsiao and
476 Lin, 2010) and numerical impact pressures computed by using different ISPH particle
477 correction schemes, at pressure gauge of P_1 , P_4 , P_7 and P_8 on the weather side of
478 trapezoidal structure (see Fig. 25(b)). It is shown that the general trend of impact
479 pressures computed by all ISPH models follows good consistency with the
480 experimental measurement, in spite of unavoidable discrepancies due to the
481 complication of the physical problem. The pressure time history of ISPH_MP and
482 ISPH_MPAPD is much more stable than that of ISPH_APD, in which larger pressure
483 oscillations are observed. Also it is found that ISPH_MP computation generates more
484 pressure noises than the ISPH_MPAPD, especially in Fig. 28(a) at the first pressure
485 measuring point.

486 Although all ISPH computations underestimate/overestimate the peak pressures to
487 some extent, the relative errors are about 34.77% for ISPH_MP, 43.6% for
488 ISPH_APD and 32.55% for ISPH_MPAPD in Fig. 28(a). On the other hand, these
489 errors are around 20.2% for ISPH_MP, 41.9% for ISPH_APD and 5.6% for
490 ISPH_MPAPD, respectively, in Fig. 28(c). Overall speaking, the present wave
491 overtopping simulation further provides the indication that the hybrid ISPH_MPAPD
492 stabilization technique is superior to existing ones in accurately predicting the wave
493 impinging and overtopping process.

494 4.5. Regular wave slamming on subface of an open-piled structure

495 To finally validate the computational accuracy and stability of the hybrid
496 ISPH_MPAPD model again, the simulation of a regular wave slamming on the
497 subface of an open-piled structure is investigated in this section. The schematic setup
498 of computational domain is shown in Fig. 29(a), where the wave flume is 14.0 m long
499 with a wavemaker being located at $x = 0.5$ m. The incident wave is a regular wave
500 with a wave height $H = 0.15$ m and wave period $T = 1.2$ s. A horizontal platform
501 is fixed at $0.1H$ above the still water surface and 8.0 m away from the left-hand-side
502 of the flume. Eleven pressure measuring points ($P_1 - P_{11}$) on the subface of the
503 horizontal structure are shown in Fig. 29(b). The detailed information on the physical
504 experiment is illustrated in Ren and Wang (2005) and Gao et al. (2012). Similar
505 problems have also been addressed in the benchmark work of Gomez-Gesteira et al.
506 (2005).

507 By using a particle spacing of 0.015 m and totally 36000 particles, the ISPH
508 simulations are carried out. The particle distributions with pressure field computed by
509 different particle stabilization methods are shown in Fig. 30 at time $t = 11.67$ s. It can
510 be seen from Fig. 30(a) that there is a slight particle strip distribution in the ISPH_MP
511 results, such that a small blank area around the left corner of the platform is observed.

512 By examining Fig. 30(b), the particle distributions under the platform demonstrate
513 irregularity and there also exists an obvious separation zone with the structure in the
514 ISPH_APD results. On the other hand, the distribution of particles in the
515 ISPH_MPAPD results is again much more stable and uniform than the other two
516 results, as shown in Fig. 30(c). In addition, the comparisons of experimental (Gao et
517 al., 2012) and ISPH wave profiles are also shown in Fig. 30 and the general
518 agreement is acceptable, since there are unavoidable discrepancies found especially in
519 the upper region of the platform.

520 Fig. 31(a) and (b) shows the time histories of experimental and ISPH impact
521 pressures computed by different correction methods at pressure gauges P_2 and P_8 (see
522 Fig. 29(b)), respectively. The numerical pressure at each measuring point is obtained
523 by the spatial averaging of the pressures of neighboring fluid particles within a radius
524 of three-time particle spacing. It can be seen that the computed impact pressures by all
525 ISPH models reasonably coincide with the experimental data of Gao et al. (2012), in
526 spite of the unavoidable discrepancies. Besides, the pressure history of ISPH_MPAPD
527 is much more promising than that of ISPH_APD, which shows larger pressure
528 oscillations, also more reliable than that of ISPH_MP, which demonstrates severe
529 pressure noises, especially in Fig. 31(a) at the measuring point P_2 . The present regular
530 wave slamming simulations once again evidence that the improved ISPH_MPAPD
531 stabilization technique has great potentials in wider wave application fields.

532 **5. Conclusions**

533 In this paper an improved hybrid particle stabilization scheme of ISPH is proposed
534 to simulate violent wave impact with coastal structure. The method adopts an
535 ISPH_MPAPD approach, which combines the ISPH_MP and artificial particle
536 displacement ISPH_APD algorithms to reduce particle clustering and instability so as
537 to improve the ISPH modeling capacity. To validate the accuracy and stability of the
538 model, ISPH_MPAPD is applied to study five benchmark cases of wave-structure
539 interaction, including the dam break flow and solitary wave impact on a vertical wall,
540 solitary wave impact on a slope, solitary wave overtopping on an impermeable
541 seawall and regular wave slamming on the subface of an open-piled structure.

542 According to the comparison between numerical results computed by ISPH_MPAPD,
543 ISPH_MP and ISPH_APD and experimental data, the performance of ISPH_MPAPD
544 is found to be most satisfactory in view of its accuracy, stability and efficiency in
545 dealing with the instabilities caused by the particle clustering and fracturing. Future
546 work is needed to improve the method for more challenging applications in the wave
547 interactions with a movable structure.

548 However, as documented in the benchmark study of Nair and Tomar (2015) and
549 Pahar and Dhar (2016), any particle shifting technique can violate the conservation of
550 volume. The sensitivity test on the particle volume for solitary wave case in Fig. 18

551 disclosed that the relative volume errors are 1.45% for ISPH_MP, 1.24% for
552 ISPH_APD and 0.71% for ISPH_MPAPD, respectively, but this small deviation of
553 the volume could significantly improve the stability of numerical results by
554 effectively regularizing the particle distributions. So the benefit of shifting scheme
555 well outweighs the drawback caused by the particle volume errors. On the other hand,
556 as for ISPH_MP, it may violate the momentum conservation but only to some extent.
557 In the context of particle methods, it would be impossible to satisfy both the
558 momentum conservation and the Taylor-series consistency at the same time.
559 ISPH_MP tends to provide approximate pressure gradient, i.e. not perfectly
560 momentum conservative, but being closer to the Taylor-series consistency. Recently,
561 it has been found that the Taylor-series consistency appears to be more important than
562 the exact local conservation of the momentum (Khayyer et al., 2017b).

563 Besides, we should also be aware that the present SPH accuracy is influenced by
564 various factors. Turbulence is one of the issues whose influence is case-dependent. In
565 present study the main objective is to evaluate the combined correction scheme. Also,
566 in the numerical simulations the effect of sub-particle-scale turbulence on the
567 macroscopic hydrodynamics, such as water surface deformation and impact pressure,
568 seems to be trivial due to the use of sufficiently small particle size. However, if the
569 coarser particles are used in larger practical domains, the SPS turbulence modelling
570 must be considered due to the significant increase of turbulence levels.

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