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Interaction of p modes with a collection of thin magnetic tubes

R. Jain,[1★] A. Gascoyne[1] and B. W. Hindman[2]

[1]*Department of Applied Mathematics, University of Sheffield, Sheffield S3 7RH*
[2]*JILA, University of Colorado at Boulder, Boulder, CO 80309-0440, USA*

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ABSTRACT

We investigate the net effect of a multitude of thin magnetic tubes on the energy of ambient acoustic p modes. A p mode, when incident on a thin magnetic flux tube, excites magneto-hydrodynamic (MHD) tube waves. These tube waves propagate vertically along the flux tube carrying away energy from the p-mode cavity resulting in the absorption of incident p-mode energy. We calculate the absorption arising from the excitation of *sausage* MHD waves within a collection of many non-interacting magnetic flux tubes with differing plasma properties. We find that the shape and magnitude of the absorption, when compared with the observationally measured absorption, favours a model with a maximum-flux boundary condition applied at the photosphere and a narrow distribution of plasma β in an ensemble with mean β value between 0.5 and 1.

Key words: MHD – Scattering – Sun: helioseismology – Sun: oscillations.

1 INTRODUCTION

The solar f and p modes of oscillations are stochastically excited in the solar interior and the measured relationship between their frequencies and wavelengths may be used to infer information about the structure and dynamics of the solar interior. It is well established that these oscillations are influenced by the properties of magnetic structures such as sunspots and plages. The studies by Braun, Duvall & LaBonte (1987, 1988), Bogdan & Braun (1995) and Braun & Birch (2008) suggest that both sunspots and plages are prolific absorbers of p-mode power. Thus, the role of subsurface field structures in modifying the properties of f and p modes has been the focus of many theoretical investigations seeking to understand the physical mechanism responsible for this absorption (e.g. Spruit 1991; Bogdan & Cally 1995; Bogdan et al. 1996; Crouch & Cally 2005; Jain et al. 2009). In particular, Jain et al. (2009, hereafter referred to as JHBB) modelled a plage as a collection of thin, untwisted, axisymmetric, vertical magnetic flux tubes and attributed the absorption observed in plage to the excitation of magnetohydrodynamic (MHD) tube waves by the pummeling of the flux tubes by the p-mode motions. They considered all the flux tubes comprising the plage to be identical, with the same magnetic flux ϕ and plasma β (the ratio of gas to magnetic pressure). They found that, depending on the boundary condition applied at the photosphere, absorption coefficients exceeding 50 per cent could be achieved. However, in reality, the flux tubes within a plage are not identical, and thus, in this paper, we consider a heterogeneous collection of thin flux tubes with varying plasma β. We then investigate the net effect of such a collection on the absorption of p modes.

2 THE EQUILIBRIUM

The thin magnetic flux tubes within the collection are all straight, vertically aligned and axisymmetric. They are embedded within a neutrally stable, polytropically stratified atmosphere. We thus model the field-free medium as a plane-parallel atmosphere with constant gravity, $\boldsymbol{g} = -g\hat{z}$. The pressure, density and sound speed vary with depth z as power laws with a polytropic index a. We truncate the polytrope at $z = -z_0$ which represents the model photosphere (see Bogdan et al. 1996 and Hindman & Jain 2008 for the details). Above the truncation depth $z > -z_0$ we assume the existence of a hot vacuum ($\rho_{\mathrm{ext}} \to 0$ with $T_{\mathrm{ext}} \to \infty$). For each flux tube in the collection, we assume that the tube is sufficiently thin that the tube is in thermal equilibrium with its field-free environment. A consequence of this assumption is that the plasma β is constant with depth within the tube and the radial variation of the magnetic field across the tube may be ignored (see Bogdan et al. 1996). However, while the value of β is constant within a given tube, β is allowed to vary from tube to tube. In particular, we consider the probability that any given tube has a given value of β is given by the distribution function,

$$P(\beta) = \mathcal{C}\beta\, \mathrm{e}^{-\frac{(\beta-\beta_0)^2}{2\sigma^2}}, \qquad \beta > 0 \tag{1}$$

where β_0 determines the peak of the distribution and σ provides the distribution's width. Further, the normalization constant $\mathcal{C}$ is given by

$$\mathcal{C}^{-1} = \int_0^\infty \beta\, \mathrm{e}^{-\frac{(\beta-\beta_0)^2}{2\sigma^2}}\, \mathrm{d}\beta. \tag{2}$$

★E-mail: R.Jain@sheffield.ac.uk

3 THE GOVERNING EQUATION

The incident p mode in the non-magnetic medium can be described by a single partial differential equation for the displacement potential Φ:

$$\frac{\partial^2 \Phi}{\partial t^2} = c^2 \nabla^2 \Phi - g \frac{\partial \Phi}{\partial z}, \tag{3}$$

where c^2 is the square of the adiabatic sound speed. Writing

$$\mu = \frac{(a+1)}{2}, \quad \nu^2 = \frac{a\omega^2 z_0}{g}, \quad \kappa_n = \frac{\nu^2}{2k_n z_0},$$

equation (3) can be transformed into an ordinary differential equation that supports plane wave solutions of the form

$$\Phi(\boldsymbol{x}, t) = \mathcal{A}\, \mathrm{e}^{-i\omega t} \mathrm{e}^{i k_n x} Q_n(z), \tag{4}$$

where

$$Q_n(z) = (-2k_n z)^{-(\mu+1/2)} W_{\kappa_n,\mu}(-2k_n z).$$

Here ω is the temporal frequency and $k_n(\omega)$ the wavenumber eigenvalue; $\mathcal{A}$ is the complex wave amplitude and $Q_n(z)$ is the vertical eigenfunction proportional to Whittaker's W function (Abramowitz & Stegun 1964). Assuming that the Lagrangian pressure perturbation vanishes at the truncation depth, we calculate the eigenvalues and eigenfunctions for the truncated polytrope.

In the framework of MHD, thin flux tubes support both longitudinal (sausage) waves and transverse (kink) waves but in this paper we only study the absorption of p modes due to the excitation of the sausage waves [see Hindman & Jain (2008) for details of the excitation of kink waves; see also Spruit (1984) for a derivation of governing equations for tube waves]. When a thin flux tube is pummeled by a p-mode wave, the vertical fluid displacement of sausage waves within the tube can be described by

$$\left[\frac{\partial^2}{\partial t^2} + \frac{2gz}{H}\frac{\partial^2}{\partial z^2} + \frac{g(1+a)}{H}\frac{\partial}{\partial z}\right]\xi_\parallel = \frac{(1+a)(\beta+1)}{H}\frac{\partial^3 \Phi}{\partial z \partial t^2}, \tag{5}$$

where

$$H = 2a + \beta(1+a).$$

Note that magnetic flux tubes, in general also allow torsional Alfvén waves but we consider a neutrally stable atmosphere for which acoustic-gravity waves are irrotational and cannot couple to torsional waves. Furthermore, we do not consider the excitation of tube waves by incident f modes because the absorption measurements reported by Braun & Birch (2008) that we are trying to model lack measurements for the f mode. The observationally measured absorption by Braun & Birch (2008) are for p modes with radial orders 1–4 and show a 'bell-shaped' frequency dependence; absorption increases with frequency with a peak of roughly 20 per cent between 3 and 4 mHz followed by a decrease beyond 4 mHz. Azimuthal averaging in these observations results in the measured absorption coefficients *mostly* being sensitive to axisymmetric p modes (Aaron Birch, private communication). Hence we are justified in only considering the excitation of sausage waves.

4 ABSORPTION COEFFICIENT FOR A SINGLE MAGNETIC FLUX TUBE

The observationally measured absorption coefficient is traditionally defined as the ratio of the difference between the ingoing and outgoing power to the ingoing power at the same frequency ω, radial order n and azimuthal order m. In the current work we assume that part of the ingoing p-mode energy goes into the excitation of the magnetic tube waves and there is a deficit in the outgoing p-mode energy as a result of this. We thus, theoretically compute the absorption coefficient, α, associated with this damping mechanism. The absorption coefficient due to the excitation of sausage tube waves is given by (see JHBB for the details of the derivation)

$$\begin{aligned} \alpha_n &= \mathcal{F}_n(\beta, \omega)\phi\ , \\ \mathcal{F}_n(\beta, \omega) &= \frac{\pi\beta}{2(2+\gamma\beta)(\beta+1)} \frac{(2k_n z_0)^{a+1}}{\nu^2 \mathcal{H}_n} \frac{1}{B_0 z_0^2} \\ &\quad \times (|\Omega + \mathcal{I}^*|^2 + |\mathcal{I}|^2 - |\Omega|^2 + \mathcal{S}), \\ B_0^2 &= \frac{8\pi}{\beta+1} P_0, \end{aligned} \tag{6}$$

where $\mathcal{H}_n$ is the energy flux for an outgoing p mode; B_0 is the tube's photospheric magnetic field strength, ϕ is the flux tube's magnetic flux, and P_0 is the photospheric pressure in the field-free atmosphere. Also, $\mathcal{I}$ is the interaction integral between the incident p mode and the sausage tube wave ($\mathcal{I}^*$ is the complex conjugate of $\mathcal{I}$) and Ω is a boundary condition parameter applied at the photosphere (see Hindman & Jain 2008 for details). The $\mathcal{S}$ term is due to the p-mode driver at the surface and is essentially the product of the p-mode eigenfunction and the real part of the sausage tube wave displacement.

5 ABSORPTION COEFFICIENT FOR A MODELLED PLAGE

Ignoring the excitation of acoustic jacket waves (Bogdan & Cally 1995) and the effects of multiple scattering, we model a plage as a collection of many non-interacting flux tubes and estimate the total absorption coefficient for the plage as follows. First, we calculate the absorption due to each magnetic flux tube using the previous expression. Next, the absorption coefficient $\alpha_n(\boldsymbol{r}, \omega)$ measured at position $\boldsymbol{r}$ by local helioseismic techniques such as ridge-filtered holography (Braun & Birch 2008) are related to the tubes through a spatial weighting function or kernel $K_n(\boldsymbol{r}, \omega)$,

$$\alpha_n(\boldsymbol{r}, \omega) = \sum_i \mathcal{F}_n(\beta_i, \omega)\, \phi_i\ K_n(\boldsymbol{r}_i - \boldsymbol{r}, \omega), \tag{7}$$

where each flux tube in the plage is labelled by an index i, and $\boldsymbol{r}_i$, ϕ_i and β_i are the position, magnetic flux and plasma β parameter, respectively, for tube i.

If all the flux tubes were identical, the quantity $\mathcal{F}_n(\beta_i, \omega)$ could be taken outside the summation sign and the remaining sum would be simply the kernel-weighted magnetic flux, $\Theta_n(\boldsymbol{r}, \omega)$, which is a measurable quantity. This is exactly how the absorption coefficient for a simulated plage was estimated in JHBB where all flux tubes had the same β and ϕ. Here, however, we will be considering a particular distribution (see equations 1 and 2) of flux tubes. Thus, we compute an ensemble average of the absorption coefficient,

$$\langle \alpha_n(\boldsymbol{r}, \omega)\rangle = \left\langle \sum_i \mathcal{F}_n(\beta_i, \omega)\phi_i K_n(\boldsymbol{r}_i - \boldsymbol{r}, \omega)\right\rangle. \tag{8}$$

Assuming that the locations $\boldsymbol{r}_i$, magnetic fluxes ϕ_i and plasma parameters β_i of the tubes are not correlated, we have

$$\langle \alpha_n(\boldsymbol{r}, \omega)\rangle = \overline{\mathcal{F}}_n(\omega)\Theta_n(\boldsymbol{r}, \omega), \tag{9}$$

with

$$\overline{\mathcal{F}}_n(\omega) \equiv \mathcal{C} \int_0^\infty \mathcal{F}_n(\beta, \omega)\beta \mathrm{e}^{-\frac{(\beta-\beta_0)^2}{2\sigma^2}}\ \mathrm{d}\beta\ , \tag{10}$$

$$\Theta_n(\mathbf{r},\omega) \equiv \left\langle \sum_i \phi_i K_n(\mathbf{r}_i - \mathbf{r},\omega) \right\rangle$$
$$\approx \int \mathrm{d}\mathbf{r}' \; |B(\mathbf{r}')| \; K_n(\mathbf{r}' - \mathbf{r},\omega) \; . \tag{11}$$

We show the results for stress-free and maximum-flux boundary conditions (see Hindman & Jain 2008) satisfied by the sausage waves at the photosphere. The stress-free boundary condition requires the total stress to vanish at the photosphere, and thus, the energy flux through the surface is zero. The maximum-flux boundary condition, on the other hand, is obtained by maximizing the energy flux that can pass upwards through the model photosphere. Hence, the maximum-flux boundary condition is also a minimal reflection condition where we have an upper limit on the amount of sausage tube wave energy that can be transmitted into the regions above the surface.

6 RESULTS AND DISCUSSION

We compute equation (9) for different distributions of β values (differing β_0 and σ). The magnetic flux Θ_n and the kernel functions K_n are determined from observation and are identical to those used by JHBB.

In Fig. 1, we plot the absorption coefficient for a modelled plage as a function of frequency. We have shown the results for maximum-flux (top panel) and stress-free (bottom panels) boundary conditions. The symbols in this figure are for a modelled plage that is composed of a host of magnetic flux tubes whose β values have been drawn from a distribution defined by equations (1) and (2). The three horizontal panels correspond to different β_0 values in the distribution, while the different symbols for the curves denote two different distribution widths σ (0.2 and 2). The solid and dashed curves correspond to absorption by a plage comprised of a multitude of identical flux tubes, all with the same value of β equal to the mean of the distribution i.e. $\beta = \bar{\beta}$ (see Table 1). As expected, when the distribution of β values is narrow, the absorption coefficient for the distribution of tubes is well matched by a collection of identical tubes, all with $\beta = \bar{\beta}$. When the distribution is broad,

Table 1. Mean beta, $\bar{\beta}$ for the modelled distribution function.

β_0	0.5	1.0	1.5
$\bar{\beta}$ for $\sigma = 0.2$	0.58	1	1.5
$\bar{\beta}$ for $\sigma = 2$	2.73	2.98	3.25

substantial differences arise between the distribution of tubes and the collection of identical tubes for stress-free case. This is because the absorption for a single tube is nearly a linear function of β for the maximum-flux boundary condition while for the stress-free boundary condition, the absorption is a rapidly decreasing function of β. This can be seen in Fig. 2 where we have plotted the absorption coefficient, α, for a single tube as a function of plasma β. The different line-style curves are for various radial order p modes. The near-linear dependence of α on β is clear for the maximum-flux boundary condition case.

Note that for a wide distribution of tubes ($\sigma = 2.0$), the absorption coefficient for maximum-flux boundary condition is greater than 1 for the p1 and p2 modes.This is clearly not desirable and is a consequence of *the weak absorption approximation* used in our formalism. We assume that each individual flux tube absorbs energy from the incident p mode independently of other flux tubes around it. However, in reality when there are many tubes present and their net absorption is significant, the ingoing p-mode energy sampled by an individual tube is far less than that of the incident p mode. So, the assumption of weak absorption considered here overestimates the absorption. Also, the absorption coefficient is sensitive to the upper boundary condition. The stress-free boundary condition (complete reflection) produces weak absorption for high-β tubes and maximum-flux boundary condition (minimum reflection) yields very high absorption for high-β tubes. The two boundary conditions considered here represent lower and upper bounds on the wave reflection at the surface but in *reality* the actual reflection at the solar surface lies between these two extreme cases.

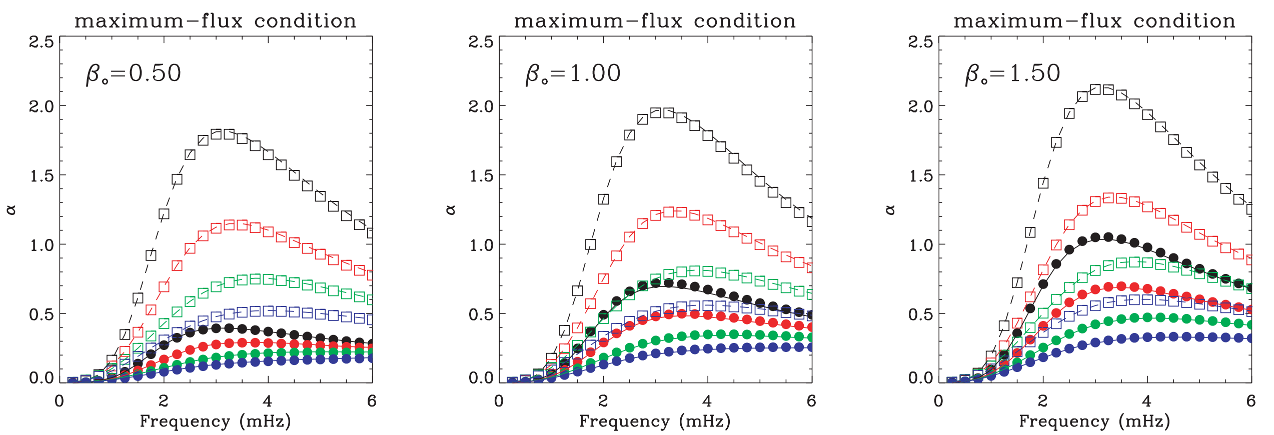

Figure 1. Absorption coefficient as a function of frequency for an ensemble of many thin, magnetic flux tubes with plasma β varying between 0 and ∞. The top and bottom panels are for maximum flux and stress-free boundary conditions, respectively, at the photosphere. We chose the distribution function given in equation (1), characterized by the parameters β_0 and σ. The filled circles and squares are for $\sigma = 0.2$ and 2.0, respectively. The solid and dashed lines represent the corresponding absorption coefficients for a collection of identical tubes in which the β value for each tube was selected to be the same as the mean value of the distributions defined by equations (1) and (2). Each mode order is denoted by a different colour (online): black (p1), red (p2), green (p3), blue(p4). Note that absorption decreases with increasing radial order.

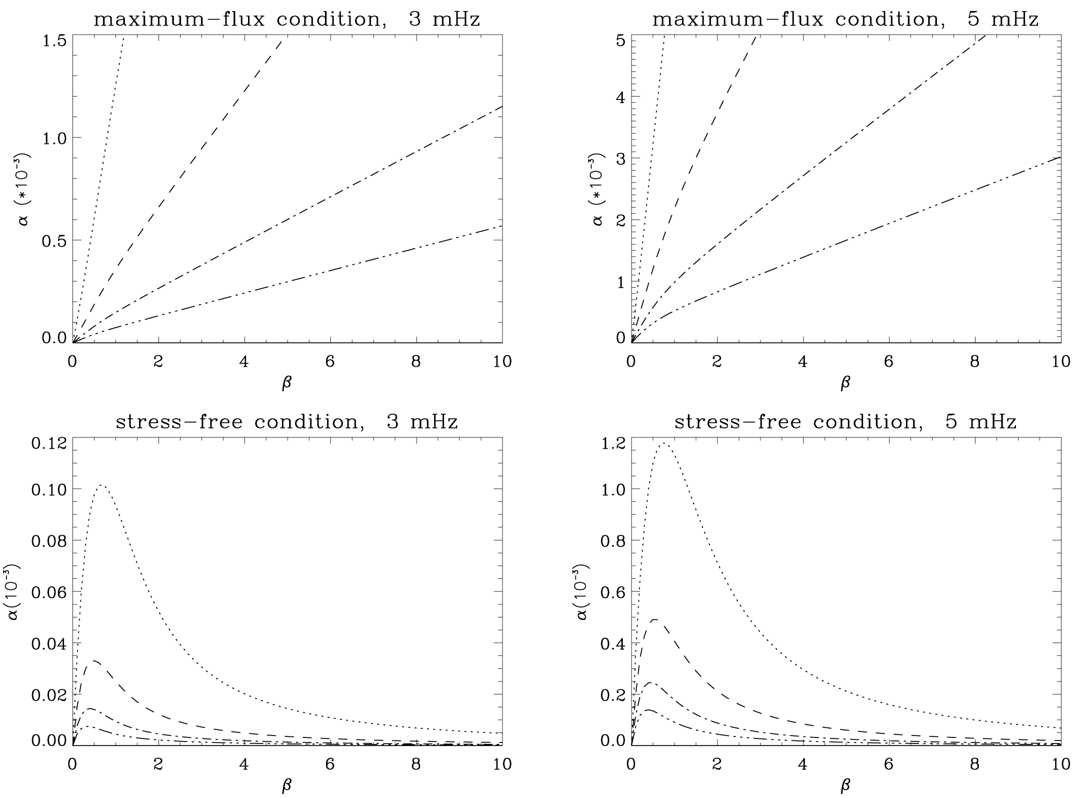

Figure 2. Absorption coefficient for a single tube as a function of plasma β for a given frequency. Each mode order is denoted by a different linestyle: dashed (p1), dot–dashed (p2), dash–dot–dotted (p3) etc. Note different scale on *y*-axis.

7 CONCLUSIONS

We compute absorption coefficients for a modelled plage assuming that the plage is comprised of a collection of non-interacting, vertical, axisymmetric, thin, magnetic-flux tubes. We have shown that reflection of sausage waves at the solar surface and the plasma properties of the magnetic flux tubes in an ensemble are some of the factors that strongly affect the energy loss of p modes in a magnetic structure. In particular, we have shown that as long as the plasma parameter β has a narrow range of variation with $\beta_0 < 1$, the macroscopic absorption coefficient of the collection effectively depends only on the mean value of β for different beta flux tubes. For a stress-free boundary, the width of the distribution plays an important but lesser role due to the non-linear dependence of α on β. In conclusion, the shape and magnitude of the absorption, when compared with the observationally measured absorption, favours a model with maximum-flux boundary condition and a narrow distribution of plasma β in an ensemble with mean β value between 0.5 and 1. Such conditions produce a collective absorption coefficient with significant amplitude (>0.1) that reaches a maximum between 3 and 4 mHz. Wide distributions ($\sigma > 1.0$) or distributions with $\beta_0 > 1$ produce absorption coefficients that are too large in magnitude. This finding is consistent with photospheric observations of the spatial distribution of magnetic field within plage. A range of β values between 0.5 and 1.0 corresponds to a range of photospheric field strengths between 1.2 and 1.4 kG. Direct observations of the field strength distribution within plages typically peak around 1.3 kG with a width of 0.3 kG (Rabin 1992; Martínez Pillet, Lites & Skumanich 1997; Lagg et al. 2010).

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