**Reasoning Under a Presupposition and the Export Problem: The Case of Applied Mathematics**

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ABSTRACT

 ‘Abstract expressionist’ accounts of applied mathematics seek to avoid the apparent Platonistic commitments of our scientific theories by holding that we ought only to believe their mathematics-free *nominalistic content*. The notion of ‘nominalistic content’ is, however, notoriously slippery. Yablo’s account of non-catastrophic presupposition failure offers a way of pinning down this notion. However, I argue, its reliance on possible worlds machinery begs key questions against Platonism. I propose instead that abstract expressionists follow Geoffrey Hellman’s lead in taking the assertoric content of empirical science to be irreducibly modal, using the ‘noninterference’ of mathematical objects as justification for detaching nominalistic consequences.

Keywords: algebraic approaches to mathematics; Hellman; Hilbert; nominalistic content.

‘If –thenism’ in the philosophy of mathematics has an impressive heritage, despite often being dismissed when discussed under that label. As Stephen Yablo points out in his extremely thought-provoking paper, Bertrand Russell’s *Principles of Mathematics* opens on an ‘if-thenist’ theme, defining “Pure Mathematics” as “the class of all propositions of the form “p implies q,” where *p* and *q* are propositions containing one or more variables, the same in the two propositions, and neither *p* nor *q* contains any constants except logical constants.” (Russell, 1903, p. 3) While we may (as Russell soon did) come to question whether all mathematical propositions really take this form, the sentiment behind Russell’s definition, according to which mathematics is in the business in working out what follows from what, remains in a number of approaches to pure mathematics, which (following terminology of Geoffrey Hellman’s (2003)) we can call *algebraic*.

The idea of an ‘algebraic’ approach to a mathematical theory arises from a dispute between Gottlob Frege and David Hilbert at the turn of the twentieth century, concerning the nature of mathematical axioms. The debate is pursued in a series of letters between Frege and Hilbert, where both present startlingly different views of the nature of mathematical axioms (Frege 1980). For Frege, mathematical axioms are assertions of truths about some independently given subject matter. We grasp axioms as true because we grasp the thoughts they express (and through this we know them to be consistent). For Hilbert, by contrast, axioms function as implicit definitions of their primitive terms. He writes,

In my opinion a concept can be fixed logically only by its relations to other concepts. These relations, formulated in certain statements, I call axioms, thus arriving at the view that axioms (perhaps together with propositions assigning names to concepts) are the definitions of the concepts. (Hilbert to Frege, 22/9/1900)

Consider the Peano axioms, for example, with primitive terms 0, number, successor. In Hilbert’s view the axioms tell us what would have to be true of any system consisting of objects falling under the predicate ‘number’, with a distinguished element ‘0’, and a distinguished mapping ‘s’, to count as a natural number system. There is no single interpretation required of these – or of any – mathematical axioms. Rather, we can read them as potentially applying to many different interpretations – or perhaps even to none. Assurance of the logical possibility, or ‘coherence’[[1]](#footnote-1) of such axiom systems thus becomes essential if they are not to fall into triviality, and we can establish consistency by finding an interpretation in which the axioms are true. But logical possibility is what really matters for our axioms, rather than literal truth of any particular interpretation.

Algebraic views such as that expressed by Hilbert see pure mathematical practice as involving inquiry into the consequences of coherent axiom systems. Some such views build this picture into an account of the *hidden logical form* of mathematical claims, and therefore into their content (as does the early Russell, when he reads mathematical propositions as, properly construed, elliptic for true logical implications ‘p implies q’). Others take the logical form of our mathematical claims at face value, but hold that *what matters* in mathematics is not the face-value truth of our axioms or theorems as claims about mathematical objects, but just whether our theorems follow logically from their axioms. Geoffrey Hellman’s modal structuralism presents a view of the first kind: a mathematical theorem T within an axiomatic theory with structure-characterizing axioms Axx is taken as really asserting, effectively, ‘it’s logically possible for there to be a system S of objects, predicates, and relations satisfying the axioms Axx, and, necessarily, in any such system S, T is true in S’. On the other hand, fictionalist accounts of mathematics such as Hartry Field’s (1989) (which I endorse) accept the face value reading of our mathematical axioms and theorems that is presented by their surface logical form, but argue that, although our axioms so understood may be, for all we can know, literally false, this does not matter for successful mathematical practice, since the value of mathematical theorizing comes not in its interpretation as a body of truths, but what it can tell us about what *would* be the case *were* there any objects, predicates, and relations satisfying our mathematical axioms (i.e., about what follows logically from the axioms).

It is not only nominalists who adopt elements of the algebraic approach to mathematical theories. Indeed, in Leng (2009) I argue that an algebraic approach to mathematical theorizing is present also in so-called ‘full blooded Platonism’ and platonist *ante rem* structuralism. Nevertheless, from a nominalist perspective the prospect for a trade of ontology for modality that is offered by an algebraic understanding of mathematical practice is particularly appealing. No longer is the existence of a domain of abstracta to serve as referents for mathematical terms required to account for the objective correctness of the theorems of pure mathematics. Rather, we deal just in coherence and logical consequence, requiring only that our mathematical assumptions are logically possible and that our theorems correctly characterize the logical consequences of those assumptions. So long as nominalists can resist reductionist attempts to interpret claims about logical possibility and logical consequence in model theoretic terms as claims about what is true in all set theoretic models of the axioms, then commitment to mathematical objects is replaced by commitment to primitive modal facts that, at least on the face of it, seem less problematic. (See Field (1984, 1991) and Leng (2007) for defences of this trade of ontology for modality.)

So much for pure mathematics. The real difficulty with any such ‘if-thenist’ or algebraic approach to mathematics comes when we turn to the ‘mixed’ mathematical/empirical claims that appear when we apply mathematics. Here, if we try to take Hellman’s account of mathematical claims as claims about what *would* be the case *were* there any objects satisfying our mathematical axioms, we are faced with the dual problem of (a) explaining what assumptions the apparently unconditional mixed mathematical/empirical claims of empirical science are really conditionalised on, and (b) explaining why modal truths about what *would* be the case *were* certain hypothetical assumptions true are in any way relevant to the matters that interest concerning what actually *is* the case in the actual world. On the other hand, if we take the fictionalist construal of the axioms of pure mathematics as, for all we can know, false, but of mathematical theorems as nevertheless, fictionally ‘correct’, since true-in-the-story set out by the axioms (i.e., following logically from consistent axioms), the question again arises as to how to interpret the ‘correctness-if-not-truth’ of mathematical claims when they appear in empirical scientific theories, and in particular, of how this correctness-if-not-truth can be of any empirical use to us in explaining and predicting empirical phenomena. Christopher Pincock calls this the “export” challenge for mathematical fictionalism, the challenge being “to provide rules that will indicate, for a given context, which claims can be extracted from the fiction and taken literally as claims about the actual world.” (Pincock (2012), p.252) An ‘if-thenist’ construal of our mathematically-stated empirical theories will have to first explain in what sense the apparently unconditional claims of our empirical scientific theories are really conditionalized on undischarged assumptions, and secondly to explain, given that they are dependent on such assumptions, how it is that we can use these claims to draw absolute, rather than merely conditional, conclusions about the physical world.

In my (2010), I follow a suggestion of Yablo’s (first indicated in his (1998)) in this regard, which is to view empirical science too as a kind of fiction whose value arises out of its ability to represent some real empirical content. Yablo draws on work of Kendall Walton’s (1993) on fiction as a kind of make-believe, and in particular on the use that can be made of utterances made within the context of a make-believe to say things about reality (Walton calls this ‘prop oriented make-believe’). In Walton’s picture, a game of make-believe generates *prescriptions to imagine*, via its *props* and *principles of generation*. A sentence S is *fictional* in a given game, if there is a prescription to imagine that S is true (given the props and principles of generation that generate these prescriptions). If we utter a sentence S within the context of the game we should not be read as asserting that S is true, but only as playing the game. Nevertheless, when we utter such a sentence in the context of playing the game, we are in part indicating by our utterance a genuine belief, that being our belief that the props and principles of generation are such as to make that utterance appropriate (i.e., fictional)*.* And this can tell us something about how we take things to be with the props themselves (that they are the way they would have to be to make our utterance appropriate against the backdrop of the fiction’s generative assumptions).

Adapting this picture to mathematics, the approach I took in my (2010) was to take *physical reality*, and in particular, physical objects, to play the role of props, and to take the axioms of ZFU (ZFC set theory with urelements) to provide the principles of generation, of the ‘fiction’ of mathematically stated empirical science. Taking physical objects as the urelements in ZFU allows them to be collected into sets and therefore paves the way for the application of mathematics to physical reality. When we say ‘the number of planets is eight’, on this Yablo/Walton inspired view we should be understood as making a move within the fiction that is generated by the axioms of ZFU with whatever physical objects there actually are as urelements. What makes this utterance appropriate – fictional – is a combination of how things actually are with the planets, taken together with the mathematical axioms. In particular,the fact that there are eight planets (something that can be expressed in entirely nominalistic terms), taken together with the axioms of ZFU and appropriate definitions, makes it *fictional* that the number of planets is eight (i.e., that the set whose members are all and only the planets has cardinality 8). So given that we take it that the principles of generation that allow physical objects to be considered as collected into sets do not change any of the purely physical facts about these physical objects, the *fictionality* of ‘the number of planets is eight’ implies the *truth* of the nominalistic claim ‘there are eight planets’.[[2]](#footnote-2)

This idea that *platonistic* assumptions can be used to express some purely *nominalistic content* is key to answering Pincock’s ‘export challenge’. If we think of the hypothesis that physical objects can be collected into sets satisfying the ZFU axioms as being ‘undischarged’ in scientific theorizing, then the export challenge is to say how we can move from conditional claims made against the backdrop of this undischarged assumption to absolute claims about the physical world. The challenge is met by saying that it is the *nominalistic content* of claims made against the backdrop of the undischarged hypothesis that physical objects can be collected into sets satisfying the axioms of ZFU that is available for export, since the addition of the hypothesis that physical objects can be collected into sets leaves everything as it is as concerns the physical arrangements of physical things (an assumption that Hartry Field stresses lies behind the traditional Platonist view of mathematical theories as *necessarily* true, and thus placing no constraints on contingent facts about physical objects).

This idea of our theories having a *mathematics-free* content, which exhausts what we are committed to believe when we make use of those theories, is central to a number of recent (and indeed not so recent) accounts of the use of mathematics in science. Yablo cites Terence Horgan’s work as an early proposal along these lines, with Horgan’s notion of a sentence’s being ‘concretely adequate’ playing the role of nominalistic adequacy in his account. But similar notions arise elsewhere in what David Liggins (2014) has nicely referred to as ‘abstract expressionist’ accounts of the use of abstract mathematics in natural science to express claims about the concrete world. Examples include

1. Mark Balaguer’s (1998) account of the nominalistic content of a mixed sentence (A) concerning a physical system S as being as being the claim “that S holds up *its end* of the “(A) bargain”, that is, S does *its part* in making (A) true.” (Balaguer 1998, p. 133)
2. Joseph Melia’s (2000) ‘weaseling’ interpretation of scientists as putting forward ‘mixed’ mathematical/empirical claims as if true but then ‘taking back’ their platonistic content, leaving behind only the restrictions they place on the arrangement of concrete things (which looks like a prime example of Yablo’s ‘ϕ ~ ψ’ construction).
3. Gideon Rosen’s (2001) notion of the nominalistic adequacy of a mixed sentence S, where “S is nominalistically adequate iff the concrete core of the actual world is an exact intrinsic duplicate of the concrete core of some world at which S is true—that is, just in case things are in all concrete respects as if S were true.” (Rosen 2001, p. 75),

as well as my own borrowing of Yablo’s Waltonian notion of fictionality in claiming that an utterance of a mixed mathematical/empirical sentence S can be informative in indirectly expressing the claim that S is fictional.

Attempts such as these to make sense of the idea of there being a ‘mathematics-free’ content that is expressed by mathematically-stated empirical theories have not gone unchallenged. Jeffrey Ketland (2011) presents a model-theoretic characterization of the notion of nominalistic adequacy (modelled on van Fraassen’s account of empirical adequacy), but notes that such an account seems unavailable to the nominalist who wishes to eschew unconditionalized quantification over set theoretic models. Eschewing models means, according to Ketland (2011, p. 213), the claim “‘the concreta behave as if sentence token t is true’ cannot be true unless ‘behave as if …’ is a *primitive* notion in instrumentalese”. Stathis Psillos (2010, p. 951) puts in question the abstract/concrete divide noting that it would be hard to say very much at all about the physical world without using some abstract vocabulary, and commitment at the very least to non-mathematical abstract objects (such as “the equator, the center of mass of the solar system, directions, shapes, and semantic type”), and suggesting that as a result “the very idea of an abstract entities–free n[ominalistic]-content of theories is hollow”, since “very little interesting” can be said about the physical world in purely concrete terms. And Jody Azzouni (2011) challenges the defender of nominalistic content to show that their account meets what he calls, “the *one-one demand* on nominalistic contents”, according to which at the very least,

scientific statements with distinguishable assertoric and deductive roles have distinctive nominalistic contents. (Azzouni, 2011, p. 40)

In relation to this, Azzouni notes that, where we might hope nominalists to provide some way of identifying the nominalistic content of a given mixed mathematical/empirical statement, the most that is usually provided is a few candidate examples (such as the ‘number of planets is eight’ example given above). “[S]uch examples”, Azzouni complains,

don’t even offer a recipe for understanding why all indispensably utilized statements that quantify or refer to undesirable entities have target nominalistic contents that they can be taken to stand for, let alone different ones for proxy statements that differ in their assertional and deductive roles. (Azzouni, 2011, p. 40)

In light of such concerns, Yablo’s proposal to put the machinery of an account of ‘non-catastrophic presupposition failure’ when reasoning against the backdrop of a presupposition to the service of the notion of the mathematics-free nominalistic content of mixed mathematical/empirical claims is particularly welcome. Can Yablo’s account do any better than the alternatives approaches outlined above in making adequate sense of this notion? And does Yablo’s picture suffice to provide the recipe Azzouni requests?

On the face of it, it looks like Yablo’s proposal is indeed designed to provide such a recipe. He asks, how we are to solve “C ~ A = R” for R, where in our case C is a mixed mathematical/empirical utterance, A is the mathematical assumption on which it is conditionalised (e.g., the assumption that, on top of any actual physical objects acting as urelements, there are also sets satisfying the axioms of ZFU), and R is the content that is left behind from an utterance of C once we have ‘taken back’ its platonistic implications. To find R, we start with a ‘home’ world in which both C and A are true, and ask effectively *why* C is true in that world *given that it is an A-world*. Thus, the reason for ‘the # of dragons = 0’ being true in a world where objects can be collected into sets and therefore numbered isn’t *that there are numbers*, as this is already assumed. Rather, the reason is *that there are no dragons* (rather than one or two or…). This reason, if you like, encapsulates the minimal additional information that must be added to the assumptions A to ensure that C is true – it is what Yablo calls a “targeted truthmaker” for A ⊃ C, that being “a truthmaker consistent with A…and making optimal use of A” (16). Moving to an ‘away’ world where A is not true, C~A will be true in such a world if the same reason that made C true-given-A in the home world is still present in the away world (i.e., just in case it A ⊃ C has a targeted truthmaker in that world).

The possible worlds formulation brings out the closeness of Yablo’s picture to that of Gideon Rosen. Rosen’s account appears to differ from Yablo’s in taking the entire *concrete core* of a C world to be a truthmaker for C ~ A. On the other hand, Yablo’s appeal to the *reason* for C’s truth-given-A makes it look like we might be able to find a more fine-grained truthmaker than a world’s entire concrete core. However, we need to be careful here, given the presumed indispensability of mathematics in expressing nominalistic content. Yablo conveniently chooses an example where the nominalistic content of the sentence in question (“*The # of dragons = 0*”) is nominalistically expressible (as “*There are no dragons*”) . But, by hypothesis, in the cases that really matter to us there will be no such proxy-sentence that neatly expresses the nominalistic content we intend. In the absence of such sentences, we need to find a way to express the content directly. Rosen’s account does this by (a) thinking of C ~ A as saying, to borrow from Ketland’s rather disparaging characterisation, ‘the concreta behave as if C is true’, and then (b) using possible worlds semantics to make good on the ‘the concreta behave as if’ idiom by picking out precisely those worlds which are concretely identical to worlds in which C is true. Yablo’s alternative takes it that “A reason for *C* to be true-given-*A* is a truthmaker *X* for *A ⊃* Cthat is consistent with *A* and makes the fullest possible use of *A*”. This *may*, depending on what truthmakers look like, pick out something less than a full concrete core of a world, but in the absence of further detail, Rosen’s use of concrete cores may seem appealing by comparison.

I will not go into this issue further, as I think that both Yablo’s and Rosen’s use of possible worlds to clarify the notion of nominalistic content face a major problem. This, in fact, is noted already in Horgan’s (1984) discussion (p. 537), where he considers the modal status of the hypothesis that there are sets. Horgan considers the question of the nominalist’s assumptions about the modal status of this hypothesis (is it, for them, necessarily false or merely contingently so), noting that whichever way the nominalist jumps makes some difficulties. But issues are also thorny if we consider how the nominalist’s platonist opponent will react to the attempt to use mathematics-free possible worlds in a characterization of nominalistic content. For many traditional platonists, mathematical objects exist of necessity, so there will be no ‘away’ worlds where the mathematical implications of C fail to hold. For such platonists, if we take the content of ‘the concreta behave as if C’, or ‘C, but (perhaps), for the existence of some abstracta’, to be picked out by some range of possible worlds, these will all be A worlds, so such a picture will not distinguish between the content of ‘C ~ A’ and the content of ‘C’ (both will be true in all the same worlds). On the other hand, a number of platonists (including, prominently, Mark Colyvan (2001)) involved in the recent debate over the indispensability argument do take the existence of mathematical objects to be a contingent matter, but take it to be contingent *on* whether mathematics is required to describe the world. For such platonists, a world with no mathematical objects will be concretely very different from our own (Newtonian worlds might fit the bill, for example, assuming that Field is right about the dispensability of mathematics in Newtonian physics). But, they argue, a world where the correct scientific laws can only be described mathematically *just is* a world in which there are mathematical objects. So any world that is *concretely* as if the laws of our current scientific theories are true will also be *abstractly* as if they are, too. And if so, mathematics-free ‘away’ worlds that are nevertheless concretely the same as our own are again out of the picture. The machinery of possible worlds may be very useful in considering alternative hypotheses about the concrete (as in many of Yablo’s non-mathematical examples of non-catastrophic presupposition failure), but when the alternatives that are being considered are worlds where there do or do not exist abstract objects, the worry is that all the relevant questions against the platonist will be begged.

Horgan’s response to this worry about the usefulness of possible worlds semantics in accounting for nominalistic content is instructive – he himself is led to consider possible worlds through the analysis of counterfactuals, given that he takes empirical scientific claims to have a counterfactual form, and, having opted against the possible worlds analysis of counterfactuals he goes for the metalinguistic alternative: “a counterfactual ‘A > C’ is true iff C is a logical consequence of the conjunction of A and certain cotenable supplementary sentences.” (538) There are some difficulties in general with the notion of cotenability, in working out what to hold fixed in the counterfactual circumstances. But in fact in the mathematical case these problems seem less stark because the ‘counterfactual’ world is meant to be just like the actual world but (perhaps) for the possible existence of mathematical objects. In fact, setting aside the ‘counterfactual’ understanding for a moment, what is important to Horgan’s account is that it puts the notion of *logical consequence* at the centre stage, as one might hope for an account that picks up on the ‘if-thenist’/’algebraic’ account of mathematics as being interested in what follows from what. Attempts to reduce this notion via the machinery of possible worlds (or indeed mathematical models) are best avoided by the nominalist. After all, already in the ‘algebraic’ approach to pure mathematics an appeal to primitive (unreduced) modality has been accepted.

Rather than Horgan’s development of the ‘if-thenist’ picture of science, I would like to turn to a proposal along the lines of Geoffrey Hellman’s (1989) modal structural alternative, which is in a similar spirit to Horgan’s though more fully worked out. As we have said, in the case of pure mathematical theories Hellman takes it that what is really expressed by a mathematical theorem T is that T is a logical consequence of logically possible axioms (this compares with the fictionalist who takes a face-value reading of T but says that the mathematical correctness of T (it’s ‘truth in the story’ of the coherent axiomatic theory to which it belongs). When he turns to applied mathematics, Hellman suggests that the relevant logical consequence claims concern what follows from the assumption that there are systems of mathematical objects satisfying our axioms that *do not interfere with* how things actually are non-mathematically. Hellman’s *non-interference proviso* (Hellman, 1989, p. 99) means that, effectively (leaving aside details of formulation), Hellman’s interpretation of the mixed mathematical/empirical claims of our scientific theories replaces the unconditional claim C with the claim that, ‘it is coherent to assume the existence of a system of objects satisfying the axioms of ZFU over and above whatever material objects there actually are, and it follows logically from the assumption that there is a system of objects satisfying the axioms for ZFU, together with the assumption that the material world is just as it actually is, that C’.[[3]](#footnote-3) In fact, put this way, this account of the content of the mixed claim C is effectively the same as the claim that C is fictional in the game generated by assuming the physical world is just as it is and that physical objects can be collected into sets satisfying the axioms of ZFU, using Walton’s notion of fictionality.

As with the case of pure mathematics, Hellman and the fictionalists disagree on whether to take a face-value reading of mixed mathematical/empirical claims or interpret them modally. However, this difference is minor: in both cases, it is the modal reading that accounts for the appropriateness of the mixed mathematical/empirical utterance. What I wish to suggest, then, is that if we are looking for a *non-mathematical content* that is expressed by the mixed mathematical/empirical claims of empirical science, we should take that content to be given by Hellman’s *modal* rendering of such claims. But if this *modal content* (rather than some claim solely about the physical world) is what’s expressed by a mixed utterance, this raises quite starkly Pincock’s ‘export challenge’. If empirical science is a body of modal truths about what follows from the assumption that whatever physical objects there are can be collected into sets satisfying the axioms of ZFU, then, given that we do not know whether this assumption is actually satisfied, how can empirical science be used to tell us anything about the actual world? The temptation (that abstract expressionists including myself have generally succumbed to) is to try to find some *further* non-modal surrogate for each such modal claim which indicates precisely what *that claim* says about the physical world, that being its presupposition-free content. But should we succumb to this temptation?

I would like to suggest an alternative, modelled on Hartry Field’s (1980) account of the role played by mathematically-stated scientific theories. On Field’s account, if we know our mathematically-stated scientific theories to be conservative extensions of nominalistically stated alternatives that we reasonably believe to be true, we can reasonably believe any of the *nominalistic consequences* of these mathematically-stated theories, because those consequences will, by virtue of the conservativeness of mathematics, already follow from the nominalistic theories that we do believe. We do not have to think of the individual claims of our mathematically statedtheories as expressing any particular non-modal nominalistic content, but taken together our theories have some nominalistic consequences that we are entitled to believe. We are, of course, considering a situation where we assume that we do not have nominalistically stated alternatives to our ordinary scientific theories, so we cannot follow Field’s account directly. But if we take our theories to express true claims about what follows from the assumption that the nonmathematical world is exactly as it is, together with the assumption that nonmathematical objects can be collected together to form sets satisfying the axioms of ZFU, then one consequence of this is that any mathematics-free consequences of these assumptions should be true (as a consequence of the non-interference proviso). So we have a way of meeting the export challenge that does not proceed via finding individual ‘concrete contents’ for each of the mixed claims of empirical science. If we wish to find a mathematics-free content for such claims we should rest with Hellman’s modal construal. Such claims do not themselves have purely concrete contents, but given the non-interference proviso, we can infer from the truth of such a modal claim the truth of its consequent whenever that consequent is expressed in mathematics-free terms. So the claims of empirical science, suitably modalized, can be put to predictive use, even if we do not believe that there are any of the mathematical objects presupposed in their antecedents.

I have focussed in this discussion on the proposal to use Yablo’s account of the content of C ~ A to avoid commitment to abstracta in mathematics and in particular in our use of mathematics in empirical science. Yablo’s account is intended to account for more than just surplus abstract content of empirical claims. It is meant also to help to provide an account of noncatastrophic presupposition failure for utterances with empirical presuppositions, and corresponding ‘ϕ, but possibly for ψ’ locutions. I have suggested that, while the account may well be helpful in these concrete world cases, it may do little more than is already available in Rosen’s alternative approach to understanding the mathematics-free content of mixed mathematical/empirical utterances. Furthermore, given the reliance of both accounts on the machinery of possible worlds, and on the assumption that there can be worlds that are concretely identical but differing with respect to the presence or absence of mathematical objects, I have suggested that this approach begs some crucial questions against the Platonist. Instead, I have argued, we should accept the modality present in the ‘if-thenist’ construal of mathematical claims as a primitive logical consequence relation, and should take the content expressed by individual such claims as irreducibly modal. Rather than trying to identify purely concrete contents corresponding to each such modal claim, we should recognise that the value of such modal claims comes via the *non-interference proviso*, which ensures that, where such claims have nominalistic consequents, we will have reason to believe those consequences. Thus mathematically stated scientific theories can reasonably be put to predictive use.

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1. The word ‘coherence’ is Stewart Shapiro’s (1997), introduced to distinguish what we might think of as the *semantic* notion of consistency (logically possible truth) from deductive consistency (deriving no contradiction). For second-order theories, semantic and deductive consistency come apart. E.g., the conjunction of the second-order Peano axioms with the negation of the Gödel sentence G for that theory is deductively consistent (as G is not derivable), but this theory has no model. [↑](#footnote-ref-1)
2. My account differs from Yablo’s in offering the fictionalist account as a *revolutionary* proposal for making sense of the use of mathematics in science, as opposed to a *hermeneutic* account of what scientists themselves intend by their utterances when they express their empirical theories mathematically, but this difference is inessential in what follows. In both accounts, the notion of a mathematics-free nominalistic content, that either *is*, or *ought to be*, the believed content of our scientific theories is central. [↑](#footnote-ref-2)
3. Hellman prefers to minimize controversial axioms so suggests that the system Z+, which consists of the second-order Zermelo axioms with urelements, together with a further axiom stating that “There is just one limit ordinal” is sufficient, rather than making use of full ZFU [↑](#footnote-ref-3)