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FULL LENGTH ARTICLE

Design for limit stresses of orange fruits (*Citrus sinensis*) under axial and radial compression as related to transportation and storage design

Christopher Chukwutoo Ihueze [a], Chika Edith Mgbemena [a,b,*]

[a] *Department of Industrial and Production Engineering, Nnamdi Azikiwe University, Awka, Nigeria*
[b] *School of Aerospace, Transport and Manufacturing, Cranfield University, Cranfield MK43 0AL, Bedfordshire, United Kingdom*

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von Mises stress

Abstract This article employed the Hertz contact stress theory and the finite element method to evaluate the maximum contact pressure and the limit stresses of orange fruit under transportation and storage. The elastic properties of orange fruits subjected to axial and axial contact were measured such that elastic limit force, elastic modulus, Poisson's ratio and bioyield stress were obtained as 18 N, 0.691 MPa, 0.367, 0.009 MPa for axial compression and for radial loading were 15.69 N, 0.645 MPa, 0.123, 0.010 MPa. The Hertz maximum contact pressure was estimated for axial and radial contacts as 0.036 MPa. The estimated limiting yield stress estimated as von Mises stresses for the induced surface stresses of the orange topologies varied from 0.005 MPa–0.03 MPa. Based on the distortion energy theory (DET) the yield strength of orange fruit is recommended as 0.03 MPa while based on the maximum shear stress theory (MSST) is 0.01 MPa for the design of orange transportation and storage system.

1. Introduction

Orange fruits and orange juices have several beneficial health and nutritive properties (Economos and Clay, 1999). They are rich in Vitamin C (or ascorbic acid) and folic acid, as well as a good source of fiber. They are fat free, sodium free and cholesterol free. In addition they contain potassium, calcium, foliate, thiamin, niacin, vitamin B6, phosphorus, magnesium and copper. They may help to reduce the risk of heart diseases and some types of cancer. They are also helpful to reduce the risk of pregnant women to have children with birth diseases; hence the need for a comprehensive design and characterization of the fruit for safe transportation.

* Corresponding author at: Department of Industrial and Production Engineering, Nnamdi Azikiwe University, Awka, Nigeria. Tel.: +447766676134, +234 8037582990.
E-mail addresses: ce.mgbemena@unizik.edu.ng, c.mgbemena@cranfield.ac.uk (C.E. Mgbemena).

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Nomenclature

σ_E	engineering stress	E_1	elastic modulus of plate
σ_T	true stress	E_2	elastic modulus of sample
A_i	instantaneous area	σ_x	Hertzian principal stress for x-axis
F	load	σ_y	Hertzian principal stress for y-axis
ε_T	true strain	σ_z	Hertzian principal stress for z-axis
L_0	original length	$P_{\max}$	maximum pressure
δ	displacement	z	distance from contact surface
A_r	area ratio	v	Poisson ratio
A_0	original area	G	modulus of rigidity
Ψ	sphericity	σ_1	maximum stress at principal plane 1
A_p	area of particle	σ_2	maximum stress at principal plane 2
V_p	volume of particle	σ_y	yield stress equivalent to von Mises stress

Topuz et al. (2005) studied the physical and nutritional properties of four varieties of orange. They presented results of measurements based on dimensions, volume, mean geometrical diameter, surface area, fruit density, pile density, porosity, packaging coefficient, and friction coefficient.

Khannali et al. (2007) in a study found eleven models for the prediction of orange mass based upon dimensions, volume and surface areas. Sharifi et al. (2007) examined some orange parameters, such as coefficient of sphericity, mean geometrical diameter, apparent specific mass, an orange pile specific mass, rind ratio, and packing coefficient. Pallottino et al. (2011) subjected orange fruit of Tarocco variety to conventional parallel plate compression tests while assessing precisely the contact area of the fruit under squeezing at different deformation levels via two different visual methods, and successfully converted the typical force–deformation curves into true stress–strain relationships, as an attempt to assess the real mechanical properties of Tarocco orange fruit and to develop more efficient on-line non-destructive sorting rules.

Conventionally, fruit firmness evaluation is carried out manually via the so called Magness-Taylor test (MT), using a hand-held penetrometer, also known as fruit pressure tester, which gives a direct measure of the peak force at rupture (Shmulevich et al., 2003). In citrus fruits the relationship between puncture force and firmness is concealed by the differences in the tissue types directly under the puncture probe. Moreover, such tests are generally inadequate for fruit sorting and should be replaced with another one capable of assessing the mechanical properties of citrus fruits in a more objective and reproducible way (Menesatti et al., 2008). Thus this study targets a holistic bridge of the gap of previous methods by utilizing a locally manufactured compression test rig of Ihueze et al. (2010) in determining some relevant physico-mechanical properties of orange fruit under axial and radial loading.

When a biological material with cylindrical or spherical shape is loaded, there exists both longitudinal and transverse deformation, that when one direction is tension the other will be in compression. This is to say that there may be a biaxial or triaxial stress state. However in practice; it is difficult to device experiments to cover every possible combination of critical stresses because each test is expensive and a large number is required. Since finite element method can solve a multidimensional problems; this study utilized experimentally derived mechanical properties of orange fruit to apply finite element method through MATLAB Partial Differential Equation Toolbox (MATLAB PDE Toolbox) in determining the limiting stresses for designing orange storage and transportation.

2. Theoretical review of basic relations

The following relations were reviewed to aid evaluation of engineering and true stress parameters (Shigley and Mischke, 2001):

$$\sigma_E = F/A_0 \tag{1}$$

$$\varepsilon_E = \delta/L_0 \tag{2}$$

$$\sigma_T = F_i/A_i \tag{3}$$

$$\varepsilon_T = \ln(n+1) \tag{4}$$

$$A_r = \frac{A_i - A_0}{A_0} \tag{5}$$

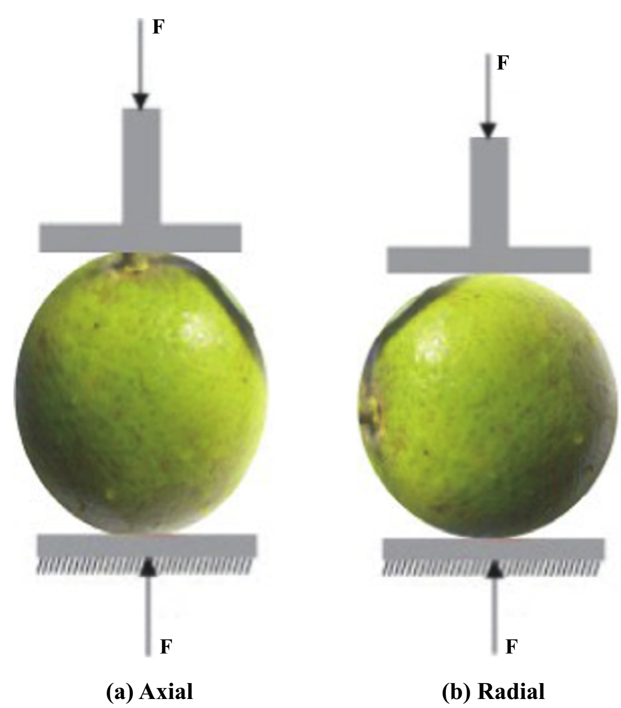

Figure 1 Depiction of loading schemes: (a) axial and (b) radial.

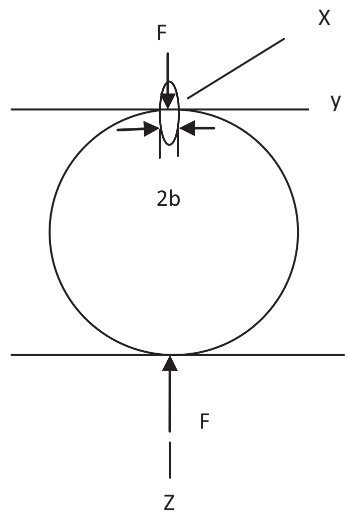

Figure 2 Depiction of elliptical distribution of stress across the contact zone of width 2b.

http://en.wikipedia.org/wiki/Sphericity reported the equation for the evaluation of the sphericity of a particle as:

$$\Psi = \frac{\pi^{\frac{1}{3}}(6V_p)^{\frac{2}{3}}}{A_p} \tag{6}$$

Shigley and Mischke (2001) also reported the following relations for computation of hertzian stresses for cylindrical shaped surfaces in contact.

$$b = \sqrt{\frac{2F}{\pi l}\frac{(1-v_1^2)/E_1 + (1-v_2^2)/E_2}{1/d_1 + 1/d_2}} \tag{7}$$

$$P_{\max} = \frac{2F}{\pi bl} \tag{8}$$

The maximum stresses occur on the z axis and these are principal stresses. Their values are:

$$\sigma_x = -2vP_{\max}\left(\sqrt{1+\frac{z^2}{b^2}}\right) - \frac{z}{b} \tag{9}$$

$$\sigma_y = -P_{\max}\left[\left(2 - \frac{1}{1+\frac{z^2}{b^2}}\right)\sqrt{1+\frac{z^2}{b^2}} - 2\frac{z}{b}\right] \tag{10}$$

Table 2 Average tensile material properties for sorted samples.

Properties	Axial loading	Radial loading
Modulus of elasticity (MPa)	0.691	0.645
Proportionality limit (MPa)	0.008	0.005
Elastic limit (MPa)	0.008	0.005
Bioyield point (MPa)	0.009	0.010
Poisson ratio	0.367	0.123
Ultimate strength (MPa)	0.029	0.020
Modulus of rigidity (MPa)	0.252	0.289
Load (elastic limit load) (N)	18.000	15.570

Table 3 Basic parameters for hertzian stresses.

Parameters	Value
Axial load, F (elastic limit load) (N)	18
Radial load, F (elastic limit load) (N)	4.159
Radius of contact, b (mm)	15.57
Maximum pressure (MPa)	0.036
Modulus of rigidity (MPa)	0.289

Table 4 Hertzian stress computations for sample (radial loading).

Z	$\sigma_x/P_{\max}$	$\sigma_y/P_{\max}$	$\sigma_z/P_{\max}$	$\tau_{xz}/P_{\max}$
0	−0.276	−1.000	−1.000	0.362
0.5	−0.207	−0.142	−0.894	0.344
1	−0.114	−0.879	−0.707	0.297
1.5	−0.084	−0.047	−0.555	0.235
2	−0.065	−0.025	−0.447	0.191
2.5	−0.053	−0.014	−0.367	0.157
3	−0.045	−0.008	−0.316	0.136

$$\sigma_z = \frac{-P_{\max}}{\sqrt{1+z^2/b^2}} \tag{11}$$

The classical relation for computation of shear modulus and Poisson ratio is expressed as

$$G = \frac{E}{2(1+v)} \tag{12}$$

Table 1 Analysis of the longitudinal section of sample (axial loading).

D (mm)	L (mm)	δ	F (N)	ε_E	σ_E	ε_T	σ_T	A_i	A_r
63.39	60.50	0	0	0	0	0	0	2876.0	−0.000014
63.46	59.90	0.6	19	0.01	0.007	0.01	0.007	2819.3	−0.020
63.90	58.40	2.1	26	0.035	0.009	0.034	0.01	2679.8	−0.068
64.15	56.50	4	26	0.066	0.009	0.064	0.01	2508.3	−0.128
65.90	55.00	5.5	35	0.091	0.012	0.087	0.015	2376.9	−0.174
66.36	53.40	7.1	35	0.117	0.012	0.111	0.016	2240.6	−0.221
66.44	51.40	9.1	44	0.15	0.015	0.14	0.021	2075.9	−0.278
67.33	49.50	11	44	0.182	0.015	0.167	0.023	1925.3	−0.331
67.44	47.60	12.9	55	0.213	0.019	0.193	0.031	1780.3	−0.381
68.46	45.90	14.6	63	0.241	0.022	0.216	0.038	1655.4	−0.424
69.00	44.40	16.1	71	0.266	0.025	0.236	0.046	1548.9	−0.461
69.20	42.40	18.1	83	0.299	0.029	0.262	0.059	1412.6	−0.509

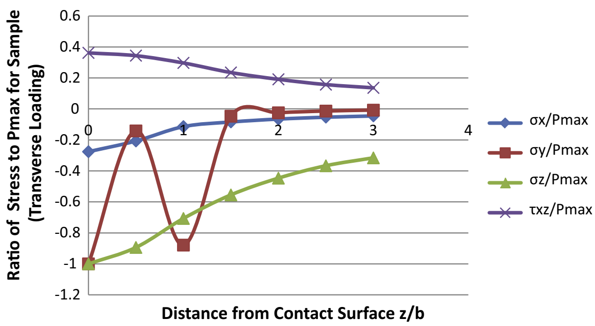

Figure 3 Ratio of stress to $P_{\max}$ as a function of distance from contact surface.

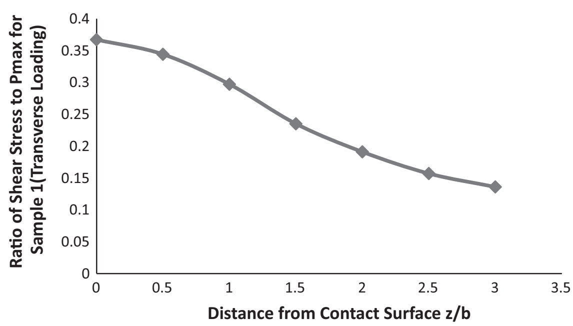

Figure 4 Ratio of shear stress to $P_{\max}$ as a function of distance from contact surface for sample.

For a plate and cylinder surface contact Eq. (7) reduces to

$$b = \sqrt{\frac{2F}{\pi l}\frac{(1-v_1^2)/E_1 + (1-v_2^2)/E_2}{1/d_2}} \tag{13}$$

One major concern of a design engineer is to place limit to use of material or system. It is therefore necessary to determine the limit stresses to be applied to the orange fruits in transportation and storage. Classical relations exist for the prediction of limit stresses of ductile and brittle materials. Experience may suggest that the outer layer of orange fruit may be ductile in this study concentrate on the model for the prediction of ductile failure.

The von Mises Criterion, also known as the maximum distortion energy criterion, octahedral shear stress theory, or Maxwell–Huber–Hencky–von Mises theory is often used to estimate the yield of ductile materials (Shigley and Mischke, 2001; Ihueze et al., 2014). The von Mises criterion states that failure occurs when the energy of distortion reaches the same energy for yield/failure in uniaxial tension. Mathematically, this is expressed as

$$\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right] \le \sigma_y^2 \tag{14}$$

In the cases of plane stress, $\sigma_3^2 = 0$. The von Mises criterion reduces to,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \le \sigma_y^2 \tag{15}$$

3. Materials and methods

For the purpose of this study, Valencia oranges, a variety of the common oranges were used. A compression test rig was used to measure the tensile data for evaluation of the biomechanical properties of orange fruit under axial and radial compression and the appropriate relations of Eqs. (1)–(13) used for unknown parameters under investigation. The loading schemes for axial and radial loading conditions are as shown in Figs. 1 and 2 respectively. MATLAB PDE Toolbox was used to apply finite element method for the evaluation of limiting stresses of orange fruit under transportation and storage.

4. Results and discussion

4.1. Experimental methods

Orange fruit samples were loaded in axial and radial compression as depicted in Fig. 1 from where mechanical properties

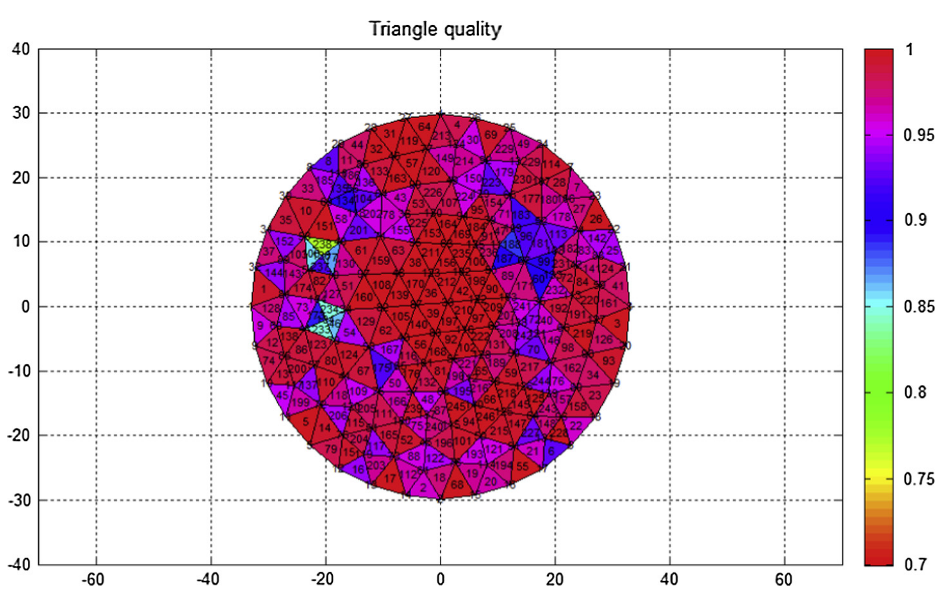

Figure 5 Finite element model for axially compressed orange fruit.

from subsequent analysis will be derived and results recorded as in Table 3 with a compression test rig of Ihueze et al. (2010).

Table 1 clearly shows that orange fruit has tensile strength of 0.029 MPa. By employing (15) and Eqs. (8)–(13), the load and other parameters required for the computation of the Hertzian stresses were obtained as presented in Table 2.

4.1.1. Evaluation of modulus of elasticity

The slopes of the linear portion of graphs of engineering stress–strain plots of Tables 1 and 2 give the elastic modulus of the sample as recorded in Table 2.

4.1.2. Evaluation of Poisson's ratio

The relationship for evaluation of Poisson's ratio is expressed as

$$\mu = \frac{\text{Transverse strain}}{\text{Axial strain}} = \frac{\left\{\frac{(A_i - A_0)}{A_0}\right\}}{\text{Axial strain}} \tag{16}$$

So that plotting the graph of transverse strain and axial strain of true stress–strain response or area ratio and axial strain of true stress–strain response and finding the slope of the linear section of their graphs gives the Poisson's ratio as expressed in Eq. (16), the Poisson's ratio is estimated and

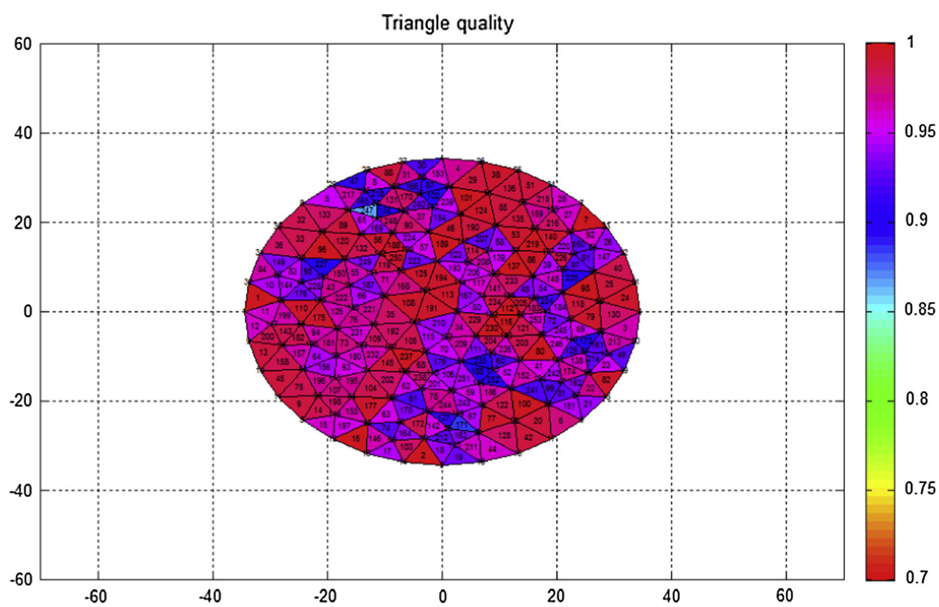

Figure 6 Finite element model for radially compressed orange fruit.

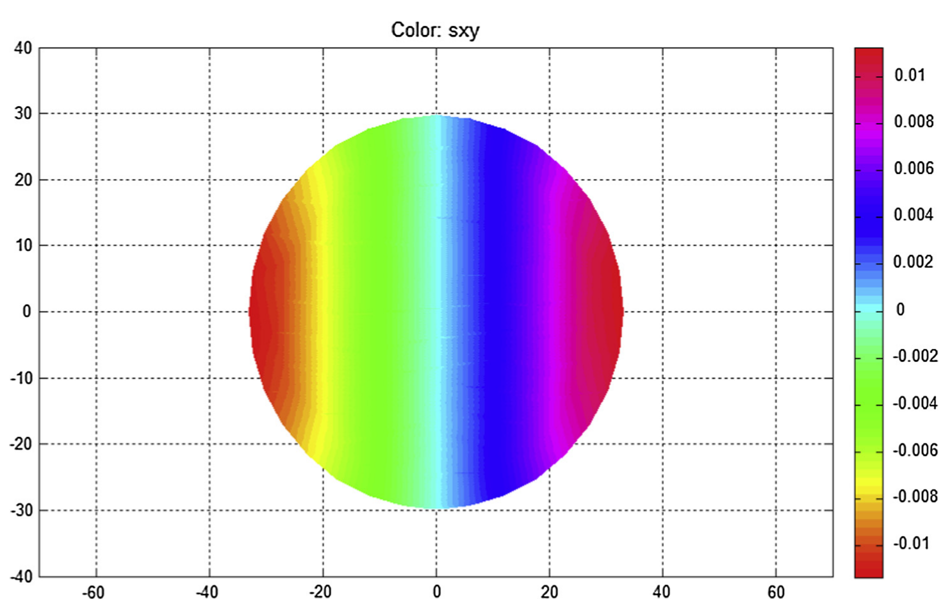

Figure 7 Matlab depiction of axial orange loading shear stresses.

presented as in Table 2. Other mechanical properties of orange fruits were also evaluated from the engineering stress–strain graphs of Table 1 and presented in Table 3.

4.2. Hertz contact stress theory and computations for Hertzian stresses

The Hertzian stresses also regarded as principal stresses are computed with Eqs. (8), (9), (10), (11) and (13) and presented in Table 3 and 4 for orange samples and expressed in Figs. 3 as a function of distance, z from the contact surface. It must be noted that Hertzian stresses were evaluated at certain distances below the contact surface as shown in Fig. 3. In Fig. 3, shearing stress was a maximum at a point of contact with maximum value of 0.01 MPa, where the maximum contact pressure is 0.036 MPa. Unlike the Hertz contact stress theory the finite element method evaluated the induced surface stresses within the topology of the orange which are found to be tensile forces and are greater than the compressive normal principal stresses evaluated using Hertz contact stress theory. The contact stresses were found to decrease as the distance from the contact surface decreases as depicted in Figs. 3 and 4.

4.3. Finite element method and evaluation of induced stresses

MATLAB PDE Toolbox solves finite elements function of a field function. The finite element solution provides values of stresses at various points on the orange shape as provided

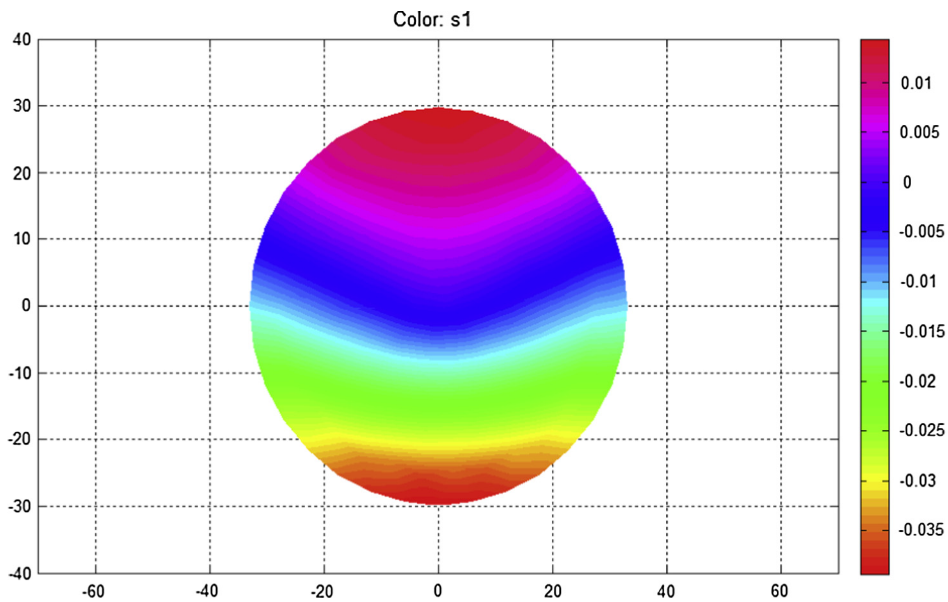

Figure 8 Matlab depiction of axial orange loading first principal stresses.

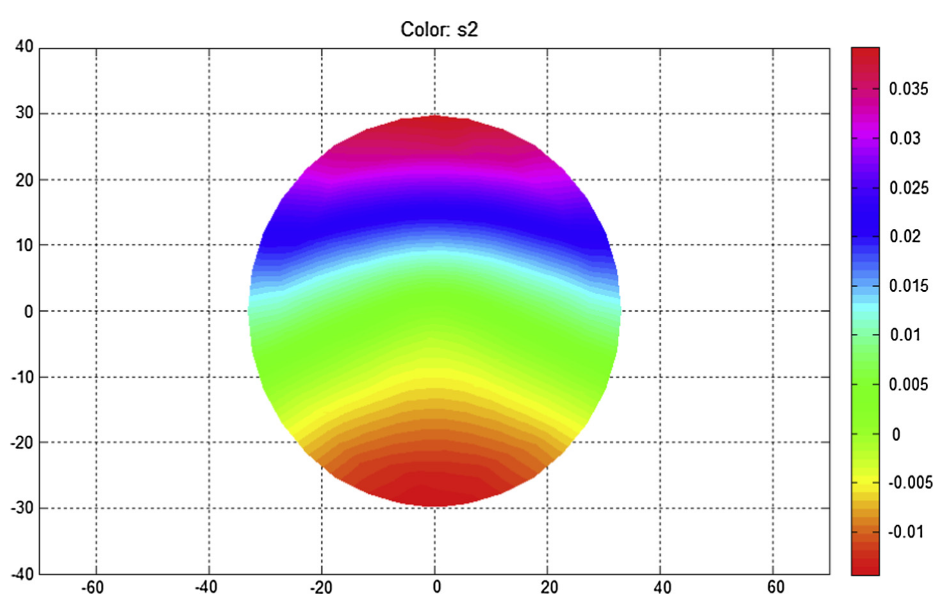

Figure 9 Matlab depiction of axial orange loading second principal stresses.

for radial and axial loading of orange samples. The principal stresses and von Mises stresses were also evaluated in order to predict failure as presented in Figs. 7–14. The MATLAB PDE Toolbox was applied with the basic parameters found in Table 4 and on the approximation of orange shape as sphere following Eq. (6) and evaluation of surface area and volume of sphere as found in en.wikipedia.org/wiki/Sphere, where:

$$V_p = \frac{4}{3}\pi r^3 \tag{17}$$

$$A_p = 4\pi r^2 \tag{18}$$

By using half of transverse dimension as r = 33 mm for orange fruit as shown in Table 5 in Eqs. (17) and (18)

$$V_p = 150223.488\ \text{mm}^3, \quad A_p = 13690.908\ \text{mm}^2$$

So that by Eq. (6), Ψ = 1.04. This is the sphericity of the orange which is a measure of its roundness and is usually unity (1) as found in http://en.wikipedia.org/wiki/Sphere.

4.3.1. Analysis with MATLAB PDE Toolbox

The evaluation of the sphericity of orange as 1.04 with Eq. (6) suggests that the orange shape is spherical or an ellipsoid. So that in the application of MATLAB PDE Toolbox elliptical shape was assumed and variables of Table 5 used for the application of proportionality limit loads of 18 N and 15.569 N for axial (longitudinal) and radial (transverse) loading of orange fruit respectively and results presented as in Figs. 7. The finite element models of axially and radially compressed orange fruits are shown in Figs. 5 and 6.

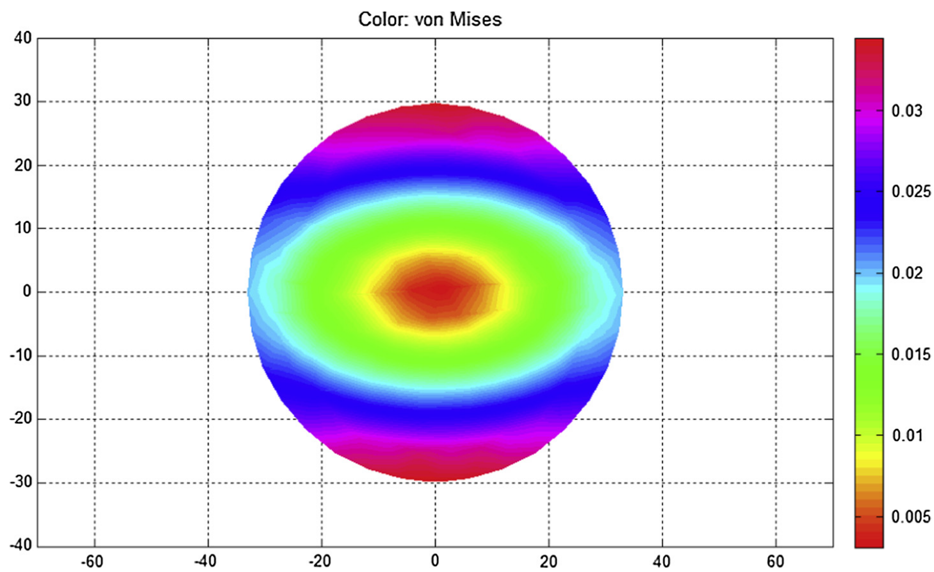

Figure 10 Matlab depiction of axial orange loading von Mises stresses.

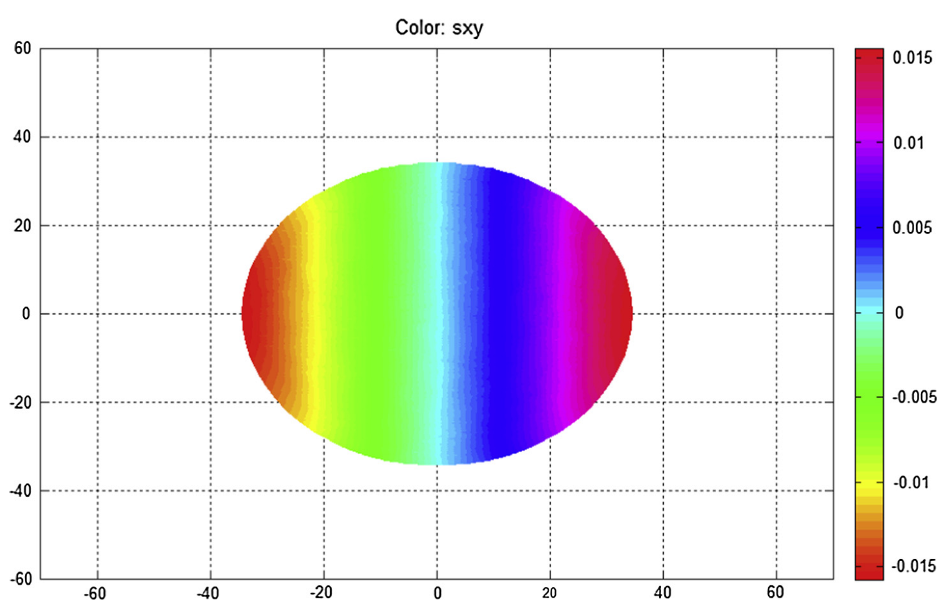

Figure 11 Matlab depiction of radial orange loading shearing stresses.

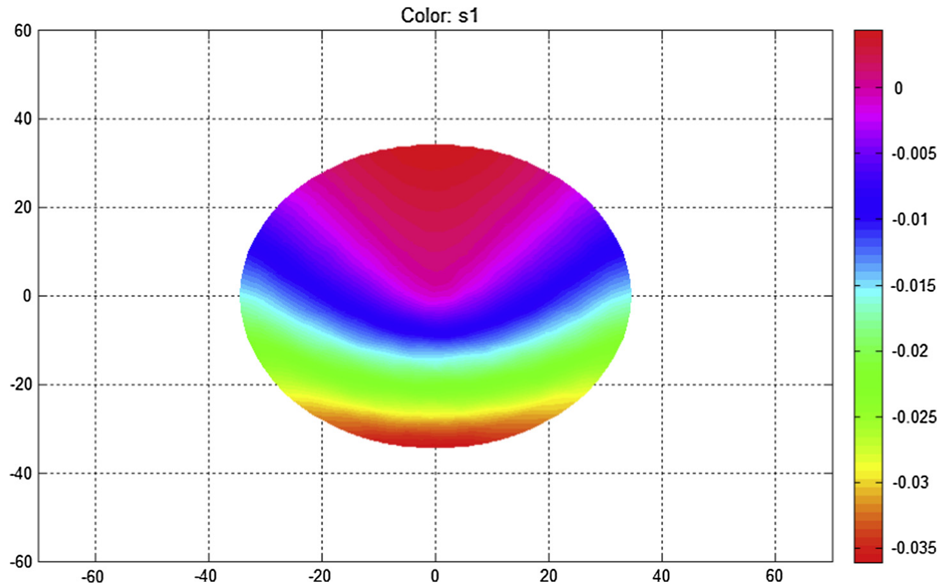

Figure 12 Matlab depiction of radial orange loading first principal stresses.

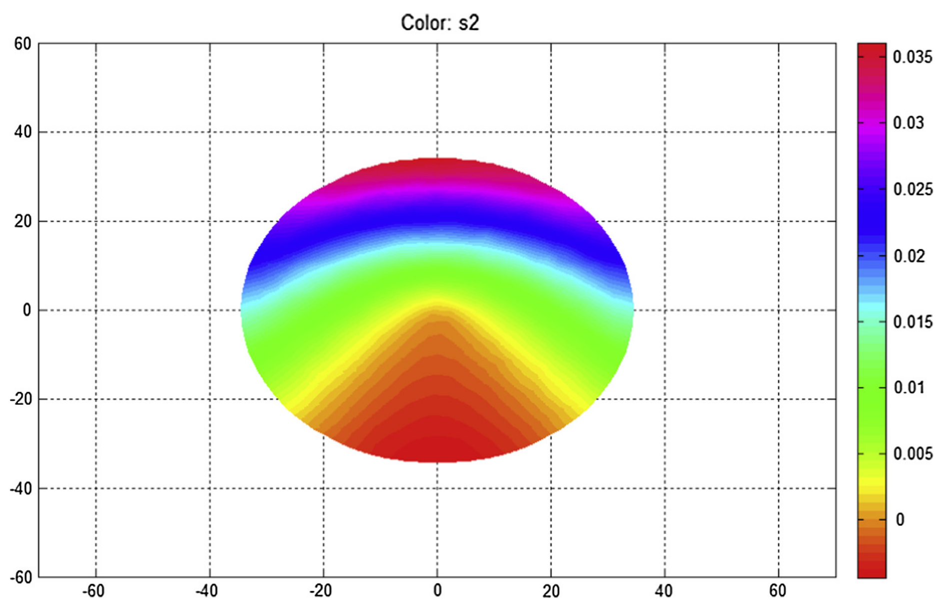

Figure 13 Matlab depiction of radial orange loading second principal stresses.

The result of loading of oranges in axial and radial compression is shown in Figs. 7–14. Fig. 7 shows that the limiting shearing stress of axially compressed orange fruit is in the range −0.01 MPa–0.01 MPa, while Figs. 8 and 9 express the first and second principal stresses of axially compressed orange fruit as −0.035 MPa–0.01 MPa and −0.01 MPa–0.035 MPa respectively. The von Mises stresses for axially compressed orange fruit are also given in Fig. 10 to be within the range 0.005–0.03 MPa. This also sets a limit for the maximum contact pressure evaluated with Hertz contact stress theory as 0.036 MPa.

Fig. 11 shows that the limiting shearing stress of radially compressed orange fruit is in the range −0.015 MPa–0.01 MPa, while Figs. 12 and 13 express the first and second principal stresses of radially compressed orange fruit as −0.035 MPa–0.00 MPa and 0.00–0.025 MPa respectively. The von Mises stresses for radially compressed orange fruit are also given in Fig. 14 to be within the range 0.005–0.03 MPa. This also sets a limit for the maximum contact pressure evaluated with Hertz contact stress theory as 0.036 MPa.

4.4. Failure prediction with von Mises stress criterion

The von Mises criteria require that at any point of material the maximum stress (principal stress) must be less than the von Mises stress at that point. It must be noted that the von Mises stress of the point of material corresponds to the yield stress of that point usually when failure mode is ductile or

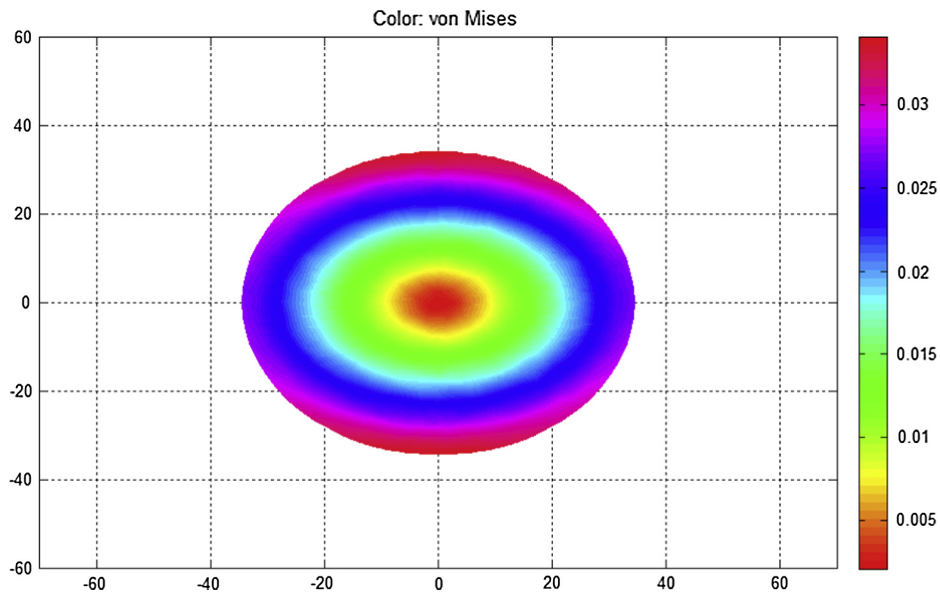

Figure 14 Matlab depiction of radial orange loading von Mises stresses.

Table 5 Basic parameters employed for the MATLAB PDE Toolbox modeling of orange fruit as elliptical shape.

Parameters	Axial loading	Radial loading
Diameter or transverse dimension (mm)	65.3	68.8
Longitudinal (mm)	59.6	68.8
Elastic modulus (MPa)	0.691	0.645
Proportionality limit load (N)	18.000	15.569
Sphericity	1.04	1.04

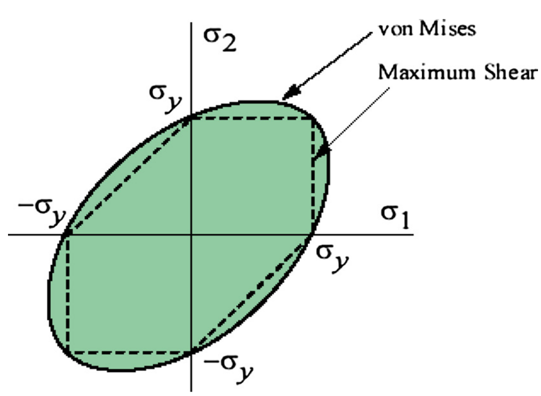

Figure 15 Depiction of principal stress ellipse for von Mises failure prediction: σ_1 = maximum stress at principal plane 1, σ_2 = maximum stress at principal plane 2, and σ_y = yield stress equivalent to von Mises stress.

fatigue. The concept of this failure prediction can be expressed as shown in Fig. 15. Fig. 15 clearly supports the predictions of the safe limits of this study as the von Mises stresses are less than the principal stresses which are the maximum stresses at various locations of the orange fruit. Since the shear stresses evaluated were less than the principal stresses and the von

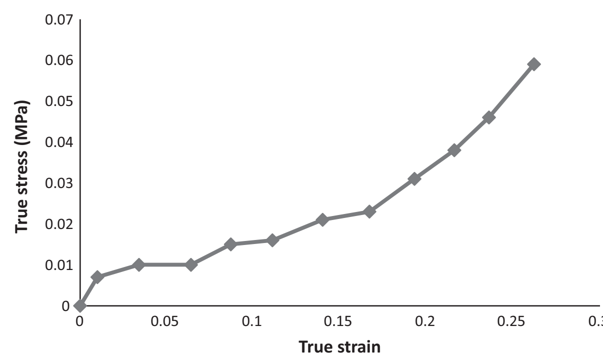

Figure 16 True stress as function of true strain longitudinal deformation during axial compression.

Mises stresses at some locations one may be constrained to specify the safe stress based on the maximum shear stress theory which is more conservative.

4.5. Validation and material characterization

Before contemplating on the appropriate failure theory to apply, the orange fruit material needs to be characterized. Shigley and Mischke (2001) reported that a material may be reported to be ductile if the true strain at fracture is more than 5% (0.05). By plotting the true stress–strain graphs of Table 1 as depicted in Fig. 16, the true strain at fracture was estimated at 0.262 for orange fruit. This value confirms that orange fruit is a ductile material hence the application of von Mises or deformation energy theory in failure prediction of this study is appropriate.

5. Conclusion

This article employed the Hertz contact stress theory and the finite element method to establish that:

1. The elastic properties of orange fruits subjected to axial and radial contact has elastic limit force, elastic modulus, Poisson's ratio and bio yield stress as 18 N, 0.691 MPa, 0.367, 0.009 MPa for axial compression and for radial loading are 15.69 N, 0.645 MPa, 0.123, 0.010 MPa.
2. The Hertz maximum contact pressure for orange fruit was estimated 0.036 MPa.
3. The limiting shearing stress is 0.01 MPa and the estimated yield stress is 0.03 MPa

Based on the distortion energy theory (DET), the limiting yield strength of orange fruit is recommended as 0.03 MPa while based on the maximum shear stress theory (MSST) is 0.01 MPa.

Conflict of interest

There is no conflict of interest.

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