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Exploration of day-to-day route choice models by a virtual experiment

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Abstract

This paper examines existing day-to-day models based on a virtual day-to-day route choice experiment using the latest mobile Internet technologies. With the realized day-to-day path flows and path travel times in the experiment, we calibrate several well-designed path-based day-to-day models that take the Wardrop's user equilibrium as (part of) their stationary states. The nonlinear effects of path flows and path time differences on path switching are then investigated. Participants' path preferences, time-varying sensitivity, and learning behavior in the day-to-day process are also examined. The prediction power of various models with various settings (nonlinear effects, time-varying sensitivity, and learning) is compared. The assumption of "rational behavior adjustment process" in Yang and Zhang (2009) is further verified. Finally, evolutions of different Lyapunov functions used in the literature are plotted, and no obvious diversity is observed.

Keywords: day-to-day flow dynamics; virtual route choice experiment; regression analysis; model calibration; model comparison.

1. Introduction and literature review

It is believed that travelers' historical traffic experience, as well as their prediction of future traffic conditions, would influence their trip decisions from day to day. Prediction of the traffic conditions in a future time epoch (e.g., traffic volume at the morning peak on a working day) can help transportation agencies arrange appropriate management and control strategies ahead of time. Prediction is especially useful when the network structure changes (Guo and Liu, 2011; He and Liu, 2012). To model the variation of traffic flows from epoch to epoch (Cascetta, 1989; Watling and Cantarella, 2015), a substantial stream of research on day-to-day dynamics has been developed. In general, two types of trip decision, i.e., route choice and departure time choice, are considered in the day-to-day context. This paper

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focuses solely on route choice. Readers interested in day-to-day departure time choices can refer to the work by Hu and Mahmassani (1997), Mahmassani (1990), Mahmassani and Chang (1986), Mahmassani et al., (1986), and more recently Xiao and Lo (2016), just to name a few.

Starting from the pioneer work by Smith (1984) and Horowitz (1984), the day-to-day route choice models are established to study how aggregate traffic flow changes based on current/historical flows and travel costs. The day-to-day model is a deterministic-process model if it is formulated as ordinary differential equations or difference equations, and the steady states can be different kinds of user equilibrium (UE), including deterministic UE (DUE, i.e., Wardrop's UE), stochastic UE (Cantarella and Cascetta, 1995; Smith and Watling, 2016), and boundedly rational UE (Di et al., 2015; Guo and Liu, 2011; Mahmassani and Chang, 1987; Ye and Yang, 2017). On the other hand, the stochastic-process models formulate flow dynamics as stochastic processes, and the steady state is the equilibrium probability distribution (Cascetta, 1989; Cascetta and Cantarella, 1991; Davis and Nihan, 1993; Hazelton, 2002; Hazelton and Parry, 2016; Hazelton and Watling, 2004; Parry and Hazelton, 2013; Watling and Cantarella, 2015).

The interaction between day-to-day dynamic route flows and other components of the transportation system has been widely studied in an analytical way, including the traffic information system (Bifulco et al., 2016; Cantarella, 2013; Cho and Hwang, 2005; Friesz et al., 1994), fixed or responsive signal control strategies (Cantarella et al., 2012; Huang et al., 2016; Liu and Smith, 2015; Smith et al., 2015; Smith and Mounce, 2011; Xiao and Lo, 2015), congestion pricing (Friesz et al., 2004; Farokhi and Johansson, 2015; Guo, 2013; Guo et al., 2016; Han et al., 2017; Liu et al., 2017; Tan et al., 2015; Wang et al., 2015; Xu et al., 2016; Yang, 2007; Yang and Szeto, 2006; Yang et al., 2007; Ye et al., 2015), and tradable credit schemes (Ye and Yang, 2013). The day-to-day dynamics of other travel modes, such as rail (Wu et al., 2013) and transit (Bar-Yosef et al., 2013; Cantarella et al., 2015; Li and Yang, 2016), were also studied.

In addition to theoretical development, the day-to-day dynamics of route choices have also been studied through simulations and laboratory experiments. Most of these studies were concerned with how travelers' route choices are affected by factors such as information, experience, risk, uncertainty, personality factors, as well as various transportation system components mentioned above (Avineri and Prashker, 2005, 2006; Ben-Elia et al., 2008, 2013; Hu and Mahmassani, 1997; Lotan, 1997; Lu et al., 2011; Mahmassani and Herman, 1990; Mahmassani and Stephan, 1988; Rapoport et al., 2014; Srinivasan and Mahmassani, 2003; Yang et al., 1993). The laboratory experiments were also used to test static UE theories such as the Braess Paradox and Downs-Thomson Paradox (Dechenaux et al., 2014; Morgan et al., 2009; Rapoport et al., 2009).

Our paper focuses on another interesting question that has not yet received sufficient attention in the research community: Are the various route-choice-based day-to-day models proposed so far good enough to reflect the real-life situation, and, if yes, what are the relative performances of these models? Regarding this question, some early and recent empirical studies have been conducted, such as Avineri and Prashker (2005), He and Liu (2012), Mahmassani and Jou (2000), Meneguzzer and Olivieri (2013) and Rapoport et al., (2014). To answer our question, we conducted a virtual route choice experiment and collected the participants' day-to-day route choice data via smart phone apps. Using the experimental data, we study a specific group of DUE-based day-to-day route choice models in the literature, which all have good stability and convergence properties but have not yet been empirically studied. The following aspects of these models are studied. First, these path-based day-to-day models are calibrated. Second, the nonlinear effects of path flows and path time differences on route switching are investigated. Third, the participants' preferences for different paths, variation of their sensitivity over time, and their learning behavior are examined. Fourth, the assumption of "rational behavior adjustment process" is verified. Fifth, the predictive power of various day-to-day models is compared. Finally, various forms of Lyapunov functions used for stability analysis in the literature are examined.

The rest of this paper is organized as follows. Section 2 introduces the settings and processes of the virtual route choice experiment. Section 3 provides the findings from the quantitative analyses of the data. Section 4 draws the conclusions and discusses possible future directions.

2. Introduction of the virtual route choice experiment

To mimic travelers' real-life decision-making processes from day to day, the traditional laboratory or virtual experiments usually involved a relatively small number of participants and/or required the participants to repeatedly make decisions within short periods of time. In order to better mimic the real world, we managed to involve a larger number of participants and allow longer periods for decision making with the help of the social networking app *WeChat*. The network in Figure 1 was used, where "O" and "D" are the origin and destination, respectively, and the link travel times were calculated as $t_a\left(v_a\right)=t_a^0\left[1+0.15\left(v_a/Y_a\right)^4\right]$, where v_a, t_a^0 and Y_a are respectively the flow, free flow time and capacity of link a; the values of t_a^0 and Y_a are given in Table 1. In our experiment, 268 participants took part for 26 rounds, where each round corresponded to a true calendar day. Most of the participants were students of Southwest Jiaotong University in China. On the first day, the route map and the free flow times on the three paths were provided to the participants at 8:00 a.m. The participants were asked to submit their route choice before 9:00 p.m. on the same day. When all the route choices were submitted, the path travel times were calculated based on the predetermined travel time functions. Notably, the travel time functions were unknown to the participants. The participants choosing the shortest path(s) were the winners for that day and immediately rewarded monetarily. The reward given to each winner was random, but the total amount was equal to the number of winners multiplied by one Chinese Yuan per winner. On the second day and afterwards, both the route map and the path travel times of the previous day were provided (in minutes, rounded to one decimal place) to the participants at 8:00 a.m. They then made and submitted decisions before 9:00 p.m. of the same day; the travel times were calculated at night and the winners were rewarded. This process continued until terminated by us.

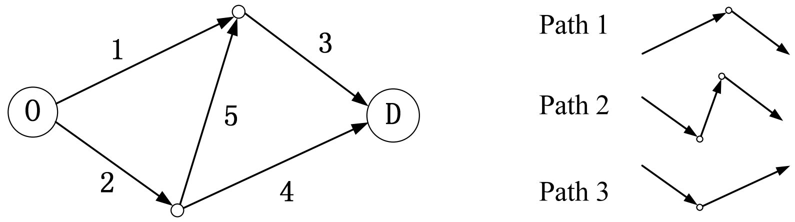
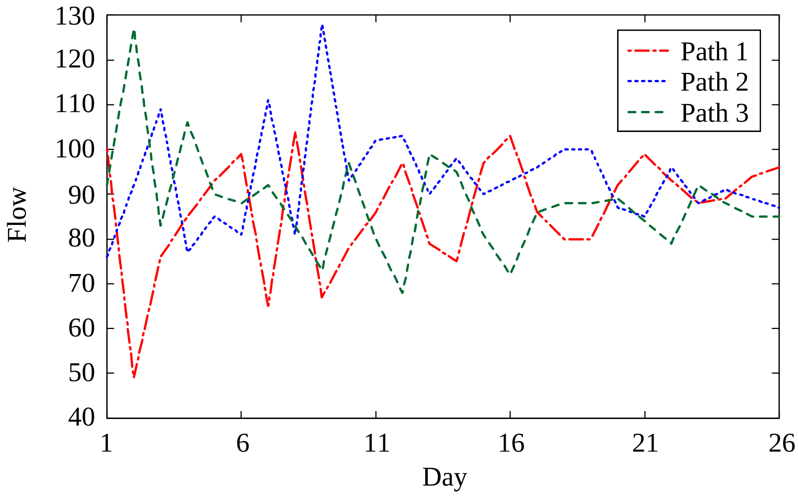


Figure 1. Network structure and paths.

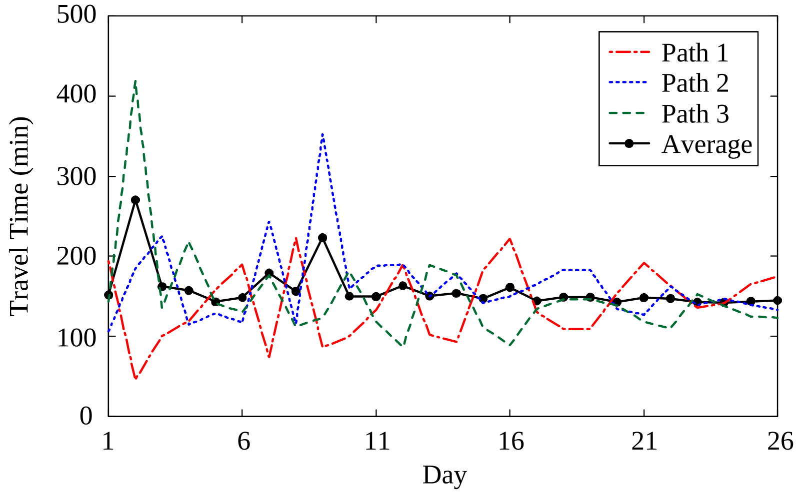
Table 1. Link characteristics

Link no. (a)	1	2	3	4	5
Free flow time (t_a^0) (min)	25	10	5	20	15
Capacity (Y_a)	40	80	80	40	40

To win the reward, the participants would have an incentive to choose the shortest path, which fits the participants' behavior into the assumption of the DUE. Therefore, our analyses in this paper will only focus on those day-to-day models whose equilibrium states are DUE. With the parameters given in Table 1, we can calculate a unique equilibrium path flow pattern of [89, 89, 89], with an identical path travel time of 142 min. After plotting the observed day-to-day path flows and path travel times in Figure 2, we found that as the experiment proceeded, the fluctuations in path flows and path travel times became smaller and smaller; on the 26th round/day, the network state was close to the equilibrium, so we terminated the experiment. Furthermore, the average path travel time fluctuated even less and was very close to the equilibrium path travel time, even in the early stage of the experiment.



(a)



(b)

Figure 2. Day-to-day evolution of (a) path flows and (b) path travel times.

3. Data analyses

This section will be devoted to exploring the existing day-to-day models by calibrating the parameters of these models based on the collected data. For subsequent analyses, denote by $d = 268$ the fixed origin-destination demand, $A = \{1,2,3,4,5\}$ the link set, and $R = \{1,2,3\}$ the path set. Let $f_r^{(n)}$ and $c_r^{(n)}$ be the flow and actual travel time on path $r \in R$ on day n, respectively, where $n = 1,\ 2,\ \ldots,\ 26$. Define $f^{(n)} = \left(f_r^{(n)}, r \in R\right)^{\mathrm{T}}$ and $c^{(n)} = \left(c_r^{(n)}, r \in R\right)^{\mathrm{T}}$ as the column vectors of path flows and path travel times, respectively, where “T” denotes the transpose operation.

The focus of this section will be on the first-order day-to-day models, in which the path flows on day $n+1$ are uniquely determined by the flows and travel times on day n. Denoting $g_{rs}^{(n+1)}$

as the flow swapping rate from path r to path s on day $n+1$, the general first-order day-to-day flow dynamics can be expressed as

$$g_{rs}^{(n+1)} = \alpha\phi_{rs}^{(n+1)}\left(f^{(n)}, c^{(n)}\right),\ \alpha > 0, \tag{1}$$

where $\phi_{rs}^{(n+1)}\left(f^{(n)}, c^{(n)}\right)$ is a function specifying how $f^{(n)}$ and $c^{(n)}$ determine the flow changing rate from path r to path s, and normally it satisfies $\phi_{rs}^{(n+1)} = -\phi_{sr}^{(n+1)}$. The first-order day-to-day models investigated in this paper include the proportional-switch adjustment process (PSAP) in Smith (1984),

$$\phi_{rs}^{(n+1)} = f_r^{(n)}\left[c_r^{(n)} - c_s^{(n)}\right]_+ - f_s^{(n)}\left[c_s^{(n)} - c_r^{(n)}\right]_+, \tag{2}$$

where $\left[x\right]_+ = \max\left(x, 0\right)$; the first-in-first-out (FIFO) dynamics in Jin (2007),

$$\phi_{rs}^{(n+1)} = f_r^{(n)} f_s^{(n)}\left(c_r^{(n)} - c_s^{(n)}\right); \tag{3}$$

the recent one in Xiao, Yang and Ye (2016), hereafter called XYY dynamics,

$$\phi_{rs}^{(n+1)} = c_r^{(n)} - c_s^{(n)}; \tag{4}$$

the evolutionary traffic flow dynamics (ETFD) in Yang (2005),

$$\phi_{rs}^{(n+1)} = f_r^{(n)}\left[\overline{c}^{(n)} - c_s^{(n)}\right]_+ - f_s^{(n)}\left[\overline{c}^{(n)} - c_r^{(n)}\right]_+, \tag{5}$$

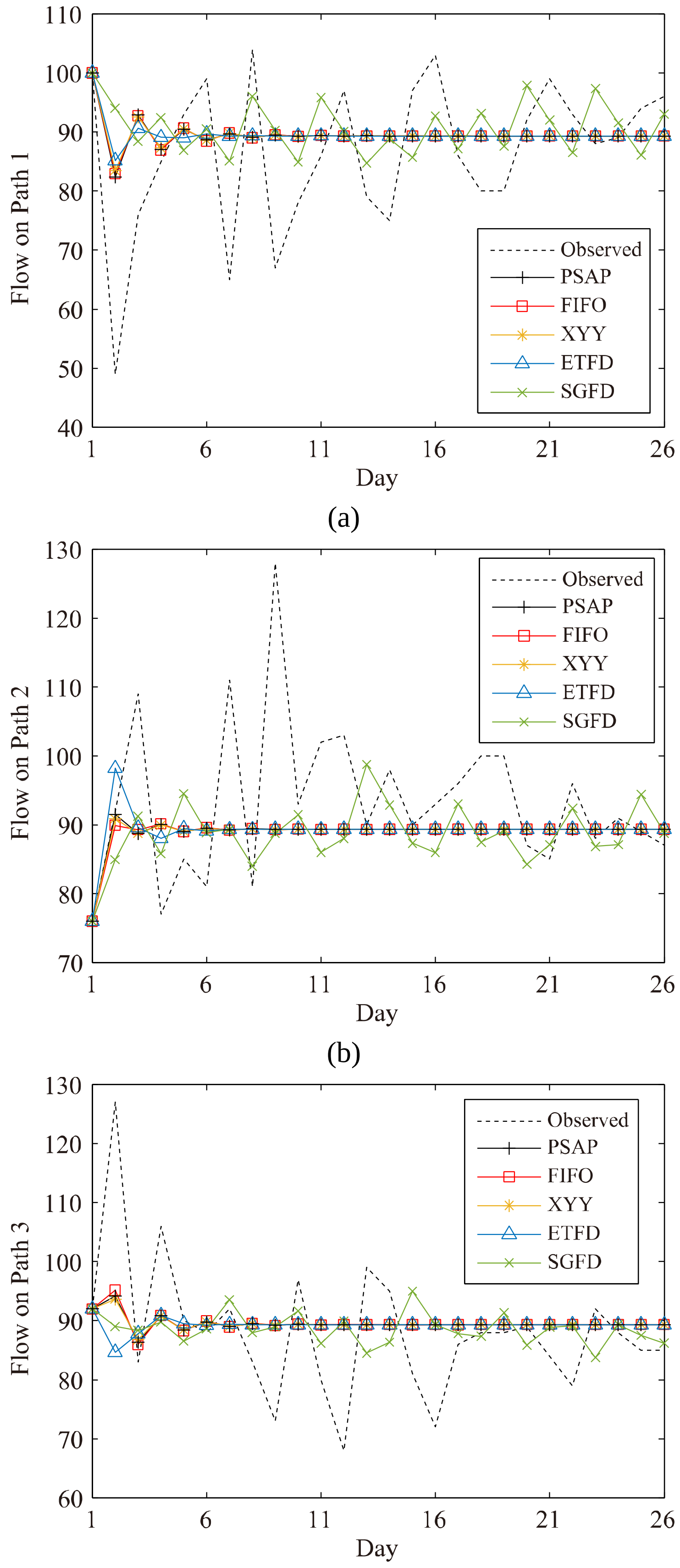
where $\overline{c}^{(n)} = \sum_{r\in R} f_r^{(n)} c_r^{(n)} \big/ d$ is the average path travel time on day n; and the simplex gravity flow dynamics (SGFD) in Smith (1983),

$$\phi_{rs}^{(n+1)} = \frac{f_r^{(n)}\left[\overline{c}^{(n)} - c_s^{(n)}\right]_+ - f_s^{(n)}\left[\overline{c}^{(n)} - c_r^{(n)}\right]_+}{\sum_{s\in R}\left[\overline{c}^{(n)} - c_s^{(n)}\right]_+}. \tag{6}$$

The network tatonnement process (NTP) (Friesz et al., 1994) and the projected dynamical system (PDS) (Zhang and Nagurney, 1996; Nagurney and Zhang, 1997) are not investigated here for two reasons: First, they will degenerate to XYY dynamics under mild conditions (Xiao et al., 2016); second, their parameters cannot be estimated by the regression method used in this paper. The parameter α in Eq. (1) can be calibrated by both simulation and regression, under different forms of $\phi_{rs}^{(n+1)}$ in Eqs. (2)-(6).

3.1. Simulation-based calibration

The simulation-based calibration is to find the values of the parameters under which the simulated evolution process can best fit the observed one (in the sense of minimizing the sum of squared error between the simulated and observed flow swapping rates). As shown in Figure 3, none of these five models can produce a fluctuation pattern close to the one observed, and the simulated trajectories of PSAP, FIFO and XYY almost overlap.



(a) (b) (c)

Figure 3. Best-fit trajectories based on simulations: (a) Path 1; (b) Path 2; (c) Path 3.

3.2. Regression-based calibration

The simulation-based calibration shows the difficulty of reproducing the day-to-day flow pattern by one particular model using only one parameter. Therefore, we turn to a relaxed problem: Given the path flows and costs of a particular day, how accurately can we predict the flows on the next day? For this problem, regression-based calibration can be used. The regression analyses are conducted with the help of the built-in function *regstats* in MATLAB R2016a. To clarify, $\hat{\alpha}$ denotes the calibrated value of parameter α; the *p*-value associated with $\hat{\alpha}$ is obtained by a two-sided test and indicates that the null hypothesis ($\alpha = 0$) is rejected at a significance level higher than this *p*-value. The heteroscedasticity is tested by the White test. The first-order autocorrelation is tested by the Ljung-Box Q test (or Q test for short), and the higher-order autocorrelations are not tested. The *p*-value of the White test (Q test) indicates that the null hypothesis of homoscedasticity (autocorrelation-free) is rejected at a significance level higher than this *p*-value. In this study, we consider a significance level of 5% when testing both heteroscedasticity and autocorrelation, so they might need to be dealt with if their associated *p*-values are smaller than 0.05.

3.2.1. The five original day-to-day models

For the convenience of model comparison, we define the day set $N = \{2,3,\cdots,25\}$ throughout all of Section 3.2. We begin the analyses with the five day-to-day models given in Eqs. (2)-(6). The regression is based on the following formulation:

$$g_{rs}^{(n+1)} = \alpha\phi_{rs}^{(n+1)} + \varepsilon_{rs}^{(n+1)},\ n \in N,\ (r,s) = (1,2),(1,3),(2,3), \tag{7}$$

where $\varepsilon_{rs}^{(n+1)}$ is the random error, and $\phi_{rs}^{(n+1)}$ is calculated by substituting the observed $f^{(n)}$ and user-informed $c^{(n)}$ into Eqs. (2)-(6). Notably, considering $g_{rs}^{(n+1)} = -g_{sr}^{(n+1)}$ and $\phi_{rs}^{(n+1)} = -\phi_{sr}^{(n+1)}$, only three path pairs are considered in the regression, and the intercept is excluded. The plots of $g_{rs}^{(n+1)}$ against $\phi_{rs}^{(n+1)}$ (Figure 4) show an origin-centric pattern and an positive correlation between them.

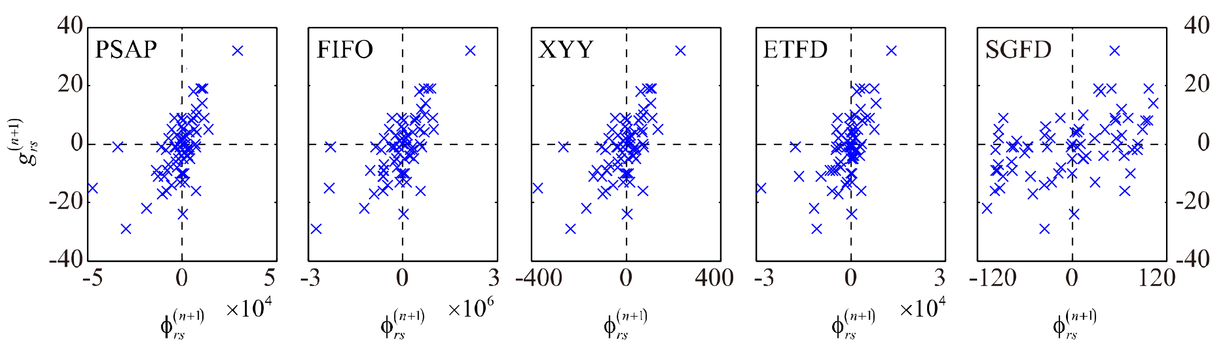


Figure 4. Plots of $g_{rs}^{(n+1)}$ against $\phi_{rs}^{(n+1)}$.

The results of the ordinary least square (OLS) regression based on Eq. (7) are listed in Table 2. The heteroscedasticity is detected in PSAP, XYY and ETFD. The autocorrelation is detected in SGFD. We will not try to correct for the autocorrelation of SGFD hereafter until we discuss the learning behavior in Section 3.2.5.

Table 2. Calibration results of the original models (OLS)

		PSAP	FIFO	XYY	ETFD	SGFD
α	$\hat{\alpha}$	6.35E-4	9.53E-6	7.51E-2	1.04E-3	7.83E-2
	p-value	2.16E-9	1.66E-10	7.51E-10	5.91E-8	6.32E-5
White test (*p*-value)		0.007*	0.304	0.027*	0.036*	0.636
Q test (*p*-value)		0.328	0.588	0.370	0.092	0.019**

* Homoscedasticity rejected at the significance level of 5%
** Autocorrelation-free rejected at the significance level of 5%

The first way to tackle heteroscedasticity is to modify the model forms. Comparing the forms of ETFD and SGFD in Eqs. (5) and (6), SGFD shares the same term as ETFD but includes extra functions of travel times in the denominator. Enlightened by this, we simply modify PSAP, FIFO, XYY and ETFD by dividing the average travel time, leading to

$$\text{PSAP (new): } \phi_{rs}^{(n+1)} = \frac{f_r^{(n)}\left[c_r^{(n)} - c_s^{(n)}\right]_+ - f_s^{(n)}\left[c_s^{(n)} - c_r^{(n)}\right]_+}{\bar{c}^{(n)}}, \tag{8}$$

$$\text{FIFO (new): } \phi_{rs}^{(n+1)} = \frac{f_r^{(n)} f_s^{(n)}\left(c_r^{(n)} - c_s^{(n)}\right)}{\bar{c}^{(n)}}, \tag{9}$$

$$\text{XYY (new): } \phi_{rs}^{(n+1)} = \frac{c_r^{(n)} - c_s^{(n)}}{\bar{c}^{(n)}}, \tag{10}$$

$$\text{ETFD (new): } \phi_{rs}^{(n+1)} = \frac{f_r^{(n)}\left[\bar{c}^{(n)} - c_s^{(n)}\right]_+ - f_s^{(n)}\left[\bar{c}^{(n)} - c_r^{(n)}\right]_+}{\bar{c}^{(n)}}. \tag{11}$$

The calibration (Table 3) of these four new models gives statistically significant $\hat{\alpha}$ (at the significance level of 1%), and the heteroscedasticity of PSAP, XYY and ETFD are corrected. Interestingly, the new ETFD model in Eq. (11) and the SGFD model in Eq. (6) have the same numerator but different denominators, while the former is autocorrelation-free but the latter is not. It is worth pointing out that model modification should not alter the properties of the original day-to-day model in terms of steady states and stability. Here, with our modifications, the steady states are unchanged, but the stability requires revisiting.

Table 3. Calibration results of the modified models (OLS)

		PSAP (new)	FIFO (new)	XYY (new)	ETFD (new)
α	$\hat{\alpha}$	0.132	1.84E-3	14.5	0.221
	p-value	7.38E-10	3.22E-10	9.98E-10	1.69E-08
White test (*p*-value)		0.062	0.162	0.186	0.410

Q test (p-value)	0.508	0.655	0.456	0.181

Bearing in mind the restrictions of doing model modifications, we adopt the weighted least square (WLS) that does not need to change the model forms. In particular, the following WLS is used:

$$\frac{g_{rs}^{(n+1)}}{\sqrt{\left|\phi_{rs}^{(n+1)}\right|}} = \alpha \frac{\phi_{rs}^{(n+1)}}{\sqrt{\left|\phi_{rs}^{(n+1)}\right|}} + \frac{\varepsilon_{rs}^{(n+1)}}{\sqrt{\left|\phi_{rs}^{(n+1)}\right|}},\ n \in N,\ (r,s) = (1,2),(1,3),(2,3). \tag{12}$$

Notably, the samples with $\phi_{rs}^{(n+1)} = 0$ will be ruled out in the WLS regression. The results are given in Table 4. Compared with the OLS results in Table 3, the heteroscedasticity of PSAP, XYY and ETFD are corrected, although the autocorrelation of SGFD holds up. However, the significance level of $\hat{\alpha}$ is only 5% for SGFD and even higher for the other four models; for the latter four models, the 95% confidence interval (CI) shows that there is a tiny chance for α to be zero or even negative. Based on these observations, we will adhere to OLS unless heteroscedasticity appears, and in this case, WLS will be applied instead.

Table 4. Calibration results of the original models (WLS)

Model	α			White test (p-value)	Q test (p-value)
	$\hat{\alpha}$	p-value	95% CI		
PSAP	6.75E-4	0.079	[-0.074, 1.3]×1E-3	0.376	0.587
FIFO	8.97E-6	0.062	[-0.044, 1.7]×1E-5	0.310	0.583
XYY	7.12E-2	0.054	[-0.012, 1.3]×1E-1	0.250	0.503
ETFD	1.26E-3	0.061	[-0.054, 2.4]×1E-3	0.356	0.197
SGFD	8.80E-2	0.038	[0.048, 1.6]×1E-1	0.137	0.003*

* Autocorrelation-free rejected at the significance level of 5%

3.2.2. Nonlinear effects of flows and cost differences on route switching

Mounce and Carey (2011) suggested incorporating nonlinear effects in the original PSAP formulations. Following this idea, we define the following bivariate function $h(x,p)$,

$$h(x,p) = \begin{cases} x^p & x > 0 \\ 0 & x = 0 \\ -(-x)^p & x < 0 \end{cases}, \tag{13}$$

and extend Eqs. (2)-(5) as follows,

$$\text{PSAP: } \phi_{rs}^{(n+1)}(p,q) = h\left(f_r^{(n)}, p\right) h\left(\left[c_r^{(n)} - c_s^{(n)}\right]_+, q\right) - h\left(f_s^{(n)}, p\right) h\left(\left[c_s^{(n)} - c_r^{(n)}\right]_+, q\right), \tag{14}$$

$$\text{FIFO: } \phi_{rs}^{(n+1)}(p,q) = h\left(f_r^{(n)}, p\right) h\left(f_s^{(n)}, p\right) h\left(c_r^{(n)} - c_s^{(n)}, q\right), \tag{15}$$

$$\text{XYY: } \phi_{rs}^{(n+1)}(p,q) = h\left(c_r^{(n)} - c_s^{(n)}, q\right), \tag{16}$$

$$\text{ETFD: } \phi_{rs}^{(n+1)}(p,q) = h\left(f_r^{(n)},p\right)h\left(\left[\bar{c}^{(n)} - c_s^{(n)}\right]_+,q\right) - h\left(f_s^{(n)},p\right)h\left(\left[\bar{c}^{(n)} - c_r^{(n)}\right]_+,q\right), \quad (17)$$

where the parameters p and q capture the degrees of nonlinearity. Finding the best values of p and q in each model can be treated as the following nonlinear regression problem:

$$\min_{p,q,\alpha} \sum_{n\in N} \sum_{(r,s)} \left[g_{rs}^{(n+1)} - \alpha\phi_{rs}^{(n+1)}(p,q) \right]^2 .$$

The results (Table 5) prefer an increasing and concave relationship between cost differences and swapping rates for all four models, while the relationships between path flows and swapping rates are quite different: Although they all suggest convex relationships, PSAP suggests a decreasing one, while FIFO and ETFD suggest an increasing one. The day-to-day models in Eqs. (14)-(17) with these optimal p and q values are then calibrated, and the results are shown in Table 6.

Table 5. Optimal parameter values for capturing nonlinear effects

	PSAP	FIFO	XYY	ETFD
p	-0.69	1.20	-	3.12
q	0.99	0.92	0.89	0.31

Table 6. Calibration based on optimal p and q values (OLS)

		PSAP	FIFO	XYY	ETFD
α	$\hat{\alpha}$	2.07	2.35E-6	0.13	1.20E-6
	p-value	5.73E-10	1.42E-10	6.00E-10	1.21E-9
White test (*p*-value)		0.084	0.475	0.073	0.057
Q test (*p*-value)		0.369	0.629	0.406	0.483

3.2.3. Path preferences

Being curious about whether the participants treated paths differently when making route choices, we write

$$g_{rs}^{(n+1)} = \gamma_{rs} + \alpha_{rs}\phi_{rs}^{(n+1)} + \varepsilon_{rs}^{(n+1)},\ n \in N,\ (r,s) = (1,2),(1,3),(2,3), \quad (18)$$

where γ_{rs} represents participants' preference between paths r and s, and α_{rs} is the path-specific sensitivity. The following WLS is used if heteroscedasticity is detected (at the significance level of 5%) in OLS:

$$\frac{g_{rs}^{(n+1)}}{\sqrt{\left|\phi_{rs}^{(n+1)}\right|}} = \gamma_{rs}\frac{1}{\sqrt{\left|\phi_{rs}^{(n+1)}\right|}} + \alpha_{rs}\frac{\phi_{rs}^{(n+1)}}{\sqrt{\left|\phi_{rs}^{(n+1)}\right|}} + \frac{\varepsilon_{rs}^{(n+1)}}{\sqrt{\left|\phi_{rs}^{(n+1)}\right|}},\ n \in N,\ (r,s) = (1,2),(1,3),(2,3). \quad (19)$$

The results are given in Table 7. The $\hat{\gamma}_{rs}$ values show that Path 2 is the most preferred path; however, this is not evident according to the *p*-value and the 95% CI. Again, autocorrelation is detected (at the significance level of 5%) in SGFD between the path pair (2,3).

Table 7. Calibration results on the path preference

Model	(r,s)	γ_{rs}			α_{rs}		Is OLS?***	White test (p-value)	Q test (p-value)
		$\hat{\gamma}_{rs}$	p-value	95% CI	$\hat{\alpha}_{rs}$	p-value			
PSAP	(1,2)	0.06	0.927	[-1.18, 1.29]	7.02E-4	0.0477	No	0.459	0.609
	(1,3)	0.13	0.934	[-3.08, 3.34]	5.10E-4	0.0009		0.793	0.891
	(2,3)	-2.69	0.160	[-6.52, 1.14]	9.77E-4	0.0000		0.860	0.597
FIFO	(1,2)	0.09	0.887	[-1.21, 1.39]	9.46E-6	0.0369	No	0.397	0.595
	(1,3)	-0.12	0.935	[-3.10, 2.86]	9.31E-6	0.0002		0.704	0.818
	(2,3)	-2.32	0.233	[-6.23, 1.59]	1.15E-5	0.0001		0.858	0.455
XYY	(1,2)	0.03	0.956	[-1.17, 1.24]	7.37E-2	0.0360	No	0.353	0.583
	(1,3)	0.12	0.936	[-2.94, 3.18]	6.25E-2	0.0003		0.751	0.962
	(2,3)	-2.97	0.132	[-6.90, 0.95]	1.16E-1	0.0001		0.927	0.550
ETFD	(1,2)	1.12	0.554	[-2.75, 4.99]	8.86E-4	0.0080		0.367	0.386
	(1,3)	0.63	0.691	[-2.61, 3.87]	8.58E-4	0.0008		0.838	0.862
	(2,3)	-3.36	0.090	[-7.28, 0.56]	2.24E-3	0.0000		0.771	0.757
SGFD	(1,2)	0.60	0.749	[-3.23, 4.44]	7.94E-2	0.0136		0.136	0.084
	(1,3)	-0.36	0.819	[-3.56, 2.84]	8.50E-2	0.0010		0.212	0.886
	(2,3)	-2.65	0.347	[-8.37, 3.06]	8.83E-2	0.1235*		0.870	0.031**

* Null hypothesis $\alpha_{rs} = 0$ not rejected at the significance level of 5%

** Autocorrelation-free rejected at the significance level of 5%

*** Blank cells indicate "Yes"

3.2.4. Time-varying parameters

Horowitz (1984) assumed that travelers' sensitivities to the path time differences can change during the evolution process. Under this circumstance, the parameters calibrated from historical data may not work well for predicting future traffic conditions. To examine this effect, we set up a time window of 15 days and calibrate the day-to-day processes in Eqs. (2)-(6) with observations in this time window, to see how $\hat{\alpha}$ changes as the time window rolls forward. The time window is $M = \{m-14, m-13, \cdots, m\}$, and m rolls from 16 to 25. Again, WLS

$$\frac{g_{rs}^{(n+1)}}{\bar{c}^{(n)}} = \alpha_{rs}\frac{\phi_{rs}^{(n+1)}}{\bar{c}^{(n)}} + \frac{\varepsilon_{rs}^{(n+1)}}{\bar{c}^{(n)}},\ n \in M,\ (r,s) = (1,2),(1,3),(2,3), \tag{20}$$

will be used if OLS rejects homoscedasticity at the significance level of 5%. Here, the denominator is different from $\sqrt{\left|\phi_{rs}^{(n+1)}\right|}$ used in earlier subsections in order to obtain a statistically significant and heteroscedasticity-free result. For all regressions, $\alpha = 0$ is rejected at the significance level of 5%, and homoscedasticity is not rejected at the significance level of 5%. Unfortunately, autocorrelation-free is rejected at the significance level of 5% in 4 out of 10 regressions for SGFD. The evolution of $\hat{\alpha}$ is demonstrated in Figure 5. As we can see, $\hat{\alpha}$ in the SGFD model has an obvious decreasing trend, which is less obvious in the other four models. However, we stress that the trend in SGFD is problematic due to the existence of autocorrelation.

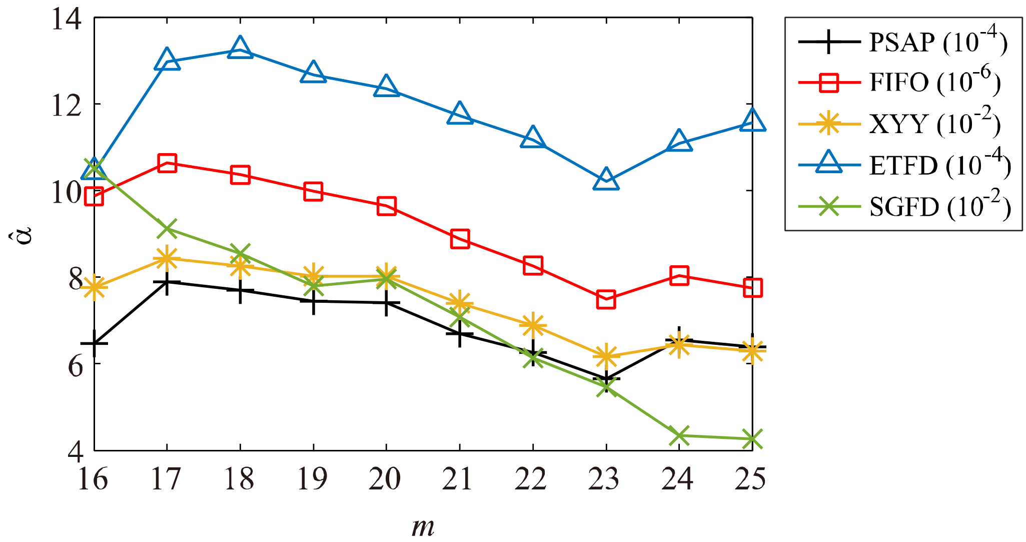


Figure 5. Evolution of $\hat{\alpha}$ with a rolling time window.

To confirm this trend, we assume a linear relationship between parameter α and day n as

$$\alpha = \alpha(n) = \theta n + \mu . \quad (21)$$

Substituting it into Eq. (1) leads to

$$g_{rs}^{(n+1)} = \theta \times (n+1)\phi_{rs}^{(n+1)} + \mu \times \phi_{rs}^{(n+1)} . \quad (22)$$

The following WLS is used if heteroscedasticity is detected (at the significance level of 5%) in OLS:

$$\frac{g_{rs}^{(n+1)}}{\overline{c}^{(n)}} = \theta \times \frac{(n+1)\phi_{rs}^{(n+1)}}{\overline{c}^{(n)}} + \mu \times \frac{\phi_{rs}^{(n+1)}}{\overline{c}^{(n)}} + \frac{\varepsilon_{rs}^{(n+1)}}{\overline{c}^{(n)}} . \quad (23)$$

The calibration results of Eq. (22) with respect to the five models in Eqs. (2)-(6) are given in Table 8. The $\hat{\theta}$ values indicate a decreasing trend of α; however, it is not significant at the 5% level in all models except SFGD; however, not surprisingly, SGFD is not autocorrelation-free. An interesting finding is that by comparing the formulations of ETFD and SGFD, the inclusion of the denominator $\sum_{s\in R}\left[\overline{c}^{(n)} - c_s^{(n)}\right]_+$ changes α from time-invariant to time-varying. The explanation might be that as the experiment proceeded, the system evolved closer to the equilibrium, and thus the value of $\sum_{s\in R}\left[\overline{c}^{(n)} - c_s^{(n)}\right]_+$ gradually decreased. Such a decreasing trend would counteract the decreasing trend of α, so the ETFD model shows no time-dependency. It is unclear why ETFD is autocorrelation-free while SGFD is not.

Table 8. Calibration results on time-varying parameters

Model	OLS/WLS	θ			μ		White test	Q test
		$\hat{\theta}$	p-value	95% CI	$\hat{\mu}$	p-value	(p-value)	(p-value)
PSAP	WLS	-1.13E-5	0.595	[-5.37, 3.10] ×1E-5	8.04E-4	8.30E-4	0.416	0.477
FIFO	WLS	-3.39E-7	0.222	[-8.89, 2.10] ×1E-7	1.32E-5	8.65E-5	0.517	0.581
XYY	WLS	-2.36E-3	0.291	[-6.78, 2.07] ×1E-3	0.1019	1.82E-4	0.518	0.451
ETFD	WLS	-2.69E-8	0.999	[-7.74, 7.74] ×1E-5	0.0012	7.72E-3	0.260	0.220
SGFD	OLS	-8.00E-3	0.004*	[-1.34, -0.26] ×1E-2	0.1932	2.50E-5	0.437	0.022**

[*] Null hypothesis $\theta = 0$ rejected at the significance level of 1%
[**] Autocorrelation-free rejected at the significance level of 5%

To confirm our conjecture above on the cause of the difference between ETFD and SGFD, we redo the regression of Eq. (22) with $\phi_{rs}^{(n+1)}$ from Eqs. (8)-(11). The results in Table 8 suggest a time-varying α for the new FIFO model. What we can learn from this subsection is that the assumption of time-varying parameters is actually associated with the model.

Table 9. Calibration results on time-varying parameters in new models (OLS only)

Model	θ			μ		White test (*p*-value)	Q test (*p*-value)
	$\hat{\theta}$	*p*-value	95% CI	$\hat{\mu}$	*p*-value		
PSAP (new)	-4.61E-3	0.173	[-1.13, 0.21] ×1E-2	0.18	7.88E-6	0.029[**]	0.413
FIFO (new)	-9.44E-5	0.032[*]	[-1.80, -0.08] ×1E-4	2.78E-3	3.65E-7	0.076	0.509
XYY (new)	-0.68	0.057	[-1.38, 0.02]	21.39	1.63E-6	0.097	0.378
ETFD (new)	-4.97E-3	0.429	[-1.74, 0.75] ×1E-2	0.27	2.02E-4	0.078	0.155

[*] Null hypothesis $\theta = 0$ rejected at the significance level of 5%
[**] Homoscedasticity rejected at the significance level of 5%

3.2.5. User learning in the day-to-day process

Previous research also tried to explicitly model how travelers predict future travel costs based on their experience, and usually the exponential smoothing rule is used (Bie and Lo, 2010; Cascetta and Cantarella, 1993; Cantarella and Cascetta, 1995; Horowitz, 1984; Watling, 1999; Xiao et al., 2016; Ye and Yang, 2013). In Xiao et al. (2016), the XYY model in Eq. (4) is modified by replacing the experienced travel time $c_r^{(n)}$ with the perceived/predicted travel time $C_r^{(n+1)}$ on path $r \in R$ on day $n+1$, i.e.,

$$g_{rs}^{(n+1)} = \alpha\left(C_r^{(n+1)} - C_s^{(n+1)}\right),\ \alpha > 0, \tag{24}$$

and $C_r^{(n+1)}$ is updated through the following exponential smoothing rule,

$$C_r^{(n+1)} = \beta c_r^{(n)} + (1-\beta) C_r^{(n)},\ 0 < \beta \le 1. \tag{25}$$

Substituting Eq. (25) into Eq. (24) yields

$$g_{rs}^{(n+1)} = (1-\beta) g_{rs}^{(n)} + \alpha\beta\left(c_r^{(n)} - c_s^{(n)}\right),\ \alpha > 0,\ 0 < \beta \le 1. \tag{26}$$

Unrigorously, the general day-to-day model in Eq. (1) can be extended in a similar way, which leads to a general day-to-day model with learning:

$$g_{rs}^{(n+1)} = (1-\beta) g_{rs}^{(n)} + \alpha\beta\phi_{rs}^{(n+1)},\ \alpha > 0,\ 0 < \beta \le 1, \tag{27}$$

where $\phi_{rs}^{(n)}$ can take those forms in Eqs. (2)-(6). The calibration results of this learning model are given in Table 10, where the WLS is based on

$$\frac{g_{rs}^{(n+1)}}{\sqrt{\phi_{rs}^{(n+1)}}} = (1-\beta)\frac{g_{rs}^{(n)}}{\sqrt{\phi_{rs}^{(n+1)}}} + \alpha\beta\frac{\phi_{rs}^{(n+1)}}{\sqrt{\phi_{rs}^{(n+1)}}} + \frac{\varepsilon_{rs}^{(n+1)}}{\sqrt{\phi_{rs}^{(n+1)}}}. \tag{28}$$

Again, we have some interesting observations.

(1) The assumption of learning is not supported in PSAP, FIFO, XYY or ETFD. If we only

look at the value of $\widehat{1-\beta}$, all four methods suggest $1-\beta<0$, or equivalently $\beta>1$. In particular, for the XYY model, according to Eq. (25), $\beta>1$ obviously violates the widely used assumption of $0<\beta\le 1$ in the literature. However, is this completely impossible? By rewriting Eq. (25) into the following form,

$$C_r^{(n+1)} = C_r^{(n)} + \beta\left(c_r^{(n)} - C_r^{(n)}\right), \tag{29}$$

the learning process can now be interpreted in this manner: travelers will correct their previous perception/prediction by adding or subtracting a proportion β of the difference between actual and perceived/predicted travel times. As a result, the perceived/predicted time would increase if $c_r^{(n)} > C_r^{(n)}$ and decrease otherwise. From a practical point of view, both $0<\beta\le 1$ and $\beta>1$ could happen in reality: With $0<\beta\le 1$, travelers are relatively "conservative" when learning the travel times, and they are relatively "aggressive" if $\beta>1$.

(2) The SGFD model also suggests $1-\beta=-0.317$, which is statistically significant at the significance level of 1%. The 95% CI also shows that $1-\beta$ is quite likely to be negative, which reveals a learning-like pattern. However, different from the XYY model, such "learning" is not on travel cost but on $\phi_{rs}^{(n+1)}$.

(3) The SGFD is finally autocorrelation-free. Recalling the autocorrelation of SGFD in Table 2, such a learning formulation in Eq. (27) is actually a standard form of correcting for autocorrelation. Moreover, since the other four models in Table 2 are autocorrelation-free, it is not surprising that they show no learning behavior here. The different results for ETFD and SGFD again tell us that the learning behavior is also model-dependent.

Table 10. Calibration results for the user learning assumption

Model	OLS/WLS	$1-\beta$			$\alpha\beta$		White test (p-value)	Q test (p-value)
		$\widehat{1-\beta}$	p-value	95% CI	$\widehat{\alpha\beta}$	p-value		
PSAP	WLS	-0.086	0.379	[-0.28, 0.11]	5.47E-4	0.131	0.609	0.681
FIFO	OLS	-0.107	0.300	[-0.31, 0.10]	8.55E-6	9.39E-7	0.309	0.736
XYY	OLS	-0.109	0.308	[-0.32, 0.10]	6.67E-2	4.35E-6	0.122	0.498
ETFD	OLS	-0.185	0.090	[-0.40, 0.03]	8.35E-4	1.35E-4	0.195	0.262
SGFD	OLS	-0.317	0.002*	[-0.52, -0.12]	5.42E-2	5.65E-3	0.806	0.420

* Null hypothesis $1-\beta=0$ rejected at the significance level of 1%

We can now go back to handle the autocorrelation reported earlier in SGFD (Table 7 and Table 8). The path-specific SGFD with learning is constructed as follows,

$$g_{rs}^{(n+1)} = \gamma_{rs} + \left(1-\beta_{rs}\right) g_{rs}^{(n)} + \alpha_{rs}\beta_{rs}\phi_{rs}^{(n+1)},\ n\in N,\ \left(r,s\right)=\left(1,2\right),\left(1,3\right),\left(2,3\right), \tag{30}$$

and the OLS results are given in Table 11. The autocorrelation between the path pair (1,3) is corrected. Path preferences are not found according to the statistics on γ_{rs}. The path switching between path pairs (1,2) and (1,3) shows no obvious learning behavior but is positively correlated to $\phi_{rs}^{(n+1)}$, although for the path pair (1,2), there is a small possibility that $\alpha_{12}\beta_{12}$ could be zero or negative. For the path pair (2,3), the learning-like pattern is possible, with $1-\beta_{23}<0$; however, its flow swapping now seems not quite determined by $\phi_{12}^{(n+1)}$.

Table 11. Calibration of SGFD with user learning and path preference (OLS)

(r,s)	parameter	calibrated	*p*-value	95% CI	White test (*p*-value)	Q test (*p*-value)
	γ_{rs}	0.163	0.928	[-3.52, 3.84]		
(1,2)	$1-\beta_{rs}$	-0.296	0.083	[-0.631, 0.040]	0.176	0.326
	$\alpha_{rs}\beta_{rs}$	0.061	0.056	[-0.001, 0.123]		
	γ_{rs}	-0.356	0.824	[-3.63, 2.92]		
(1,3)	$1-\beta_{rs}$	-0.014	0.935	[-0.360, 0.332]	0.294	0.943
	$\alpha_{rs}\beta_{rs}$	0.084	0.003*	[0.031, 0.137]		
	γ_{rs}	-1.597	0.523	[-6.68, 3.48]		
(2,3)	$1-\beta_{rs}$	-0.550	0.012**	[-0.965, -0.135]	0.581	0.392
	$\alpha_{rs}\beta_{rs}$	0.012	0.838	[-0.104, 0.127]		

*Null hypothesis $\alpha_{rs}\beta_{rs}=0$ rejected at the significance level of 1%

**Null hypothesis $1-\beta_{rs}=0$ rejected at the significance level of 2%

The SGFD with learning, time-varying α and constant β is written as

$$g_{rs}^{(n+1)}=(1-\beta)\,g_{rs}^{(n)}+\theta\beta\times(n+1)\,\phi_{rs}^{(n+1)}+\mu\beta\phi_{rs}^{(n+1)}, \tag{31}$$

and the OLS result is given in Table 12. Not surprisingly, the autocorrelation is corrected, and the homoscedasticity persists. The learning-like behavior is possible, with $1-\beta<0$. It is also possible that α decreases with time since $\beta>1$ and $\theta\beta<0$.

Table 12. Calibration of SGFD with user learning and time-varying parameter (OLS)

parameter	calibrated	*p*-value	95% CI	White test (*p*-value)	Q test (*p*-value)
$1-\beta$	-0.26	0.0113	[-0.46, -0.06]		
$\theta\beta$	-6.38E-3	0.0204	[-11.7, -1.0]×1E-3	0.627	0.267
$\mu\beta$	0.15	0.0012	[0.06, 0.24]		

3.3. Model comparison based on regression results

To evaluate and compare the predictive power of all the day-to-day models that we have investigated so far, the root mean square error (RMSE), defined as $RMSE=\sqrt{\sum_{n\in N}\sum_{(r,s)}\left(\hat{g}_{rs}^{(n+1)}-g_{rs}^{(n+1)}\right)^2\Big/(3\times 24)}$, is a standard indicator, where $\hat{g}_{rs}^{(n+1)}$ is the predicted flow swapping rate based on the calibrated parameter value(s). Since RMSE is not intuitive in explaining the accuracy of prediction, we further define an indicator AE-*x* (*x*=10 or 20) as the proportions of those samples that satisfy $\left|\hat{f}_r^{(n+1)} - f_r^{(n+1)}\right| \le x$ in the total 72 samples, where $\hat{f}_r^{(n+1)}$ is the predicted path flow based on the calibrated parameter value(s).

The values of RMSE, AE-10 and AE-20 are calculated based on the calibrated parameter values in Section 3.2 and given in Table 13. Before discussing them, we must emphasize that our conclusion given below is very rough. The heteroscedasticity and autocorrelation are not corrected in some cases. Sometimes WLS replaces OLS for tackling heterogeneity; however, the WLS forms used in different places can be different, and WLS does not necessarily have the same sample set as that of OLS. OLS minimizes the RMSE, but WLS does not. All of these affect the numbers listed in Table 13. The comparisons are made below.

(1) Regarding RMSE, FIFO>PSAP>XYY>ETFD>SGFD (where ">" means "better than"), but the gaps between the first three models are small. Regarding AE-10 and AE-20, FIFO~PSAP~XYY>ETFD>SGFD (where "~" means "similar to").
(2) From (I) and (II), altering the function form of the day-to-day models does not necessarily improve the prediction power compared with the original form.
(3) From (I) and (III), WLS does not necessarily improve the prediction power compared with OLS.
(4) From (I) and (IV), compared with the original day-to-day models, the forms incorporating nonlinear effects unsurprisingly reduce the RMSE, and the AE-20 is also better off, but the AE-10 is worse off.
(5) Although considering the assumptions of time-varying parameter or learning behavior ought to lead to smaller RMSE values (but not necessarily smaller AE-10 and AE-20) than without considering them, such improvement is mild in our study, which might be because neither assumptions are detected, and the WLS also counteracts the improvement.

Table 13. Performance of different models in prediction

		PSAP	FIFO	XYY	ETFD	SGFD
(I) Original form (OLS) Eqs. (2)-(6)	RMSE	8.247	7.959	8.127	8.631	9.490
	AE-10	70.8%	72.2%	72.2%	68.1%	55.6%
	AE-20	93.1%	94.4%	94.4%	90.3%	83.3%
(II) New form (OLS) Eqs. (8)-(11)	RMSE	8.125	8.032	8.159	8.484	-
	AE-10	75.0%	66.7%	65.3%	69.4%	-
	AE-20	95.8%	91.7%	93.1%	91.7%	-
(III) Original form (WSL)	RMSE	8.248	8.013	8.172	8.662	9.493
	AE-10	72.2%	73.6%	70.8%	69.4%	55.6%
	AE-20	93.1%	93.1%	93.1%	88.9%	83.3%
(IV) Nonlinear effect (OLS)	RMSE	8.096	7.942	8.102	8.180	-
	AE-10	68.1%	68.1%	69.4%	65.3%	-
	AE-20	95.8%	94.4%	95.8%	93.1%	-
(V) Time-varying (original form)	RMSE	8.331	7.984	8.188	8.664	8.953
	AE-10	70.8%	73.6%	70.8%	69.4%	59.7%
	AE-20	93.1%	94.4%	94.4%	88.9%	87.5%

(VI) Time-varying (new form)	RMSE	8.017	7.770	7.948	8.446	-
	AE-10	75.0%	72.2%	75.0%	72.2%	-
	AE-20	94.4%	97.2%	97.2%	91.7%	-
(VII) User learning	RMSE	8.177	7.897	8.066	8.454	8.885
	AE-10	73.6%	70.8%	70.8%	69.4%	61.1%
	AE-20	93.1%	94.4%	95.8%	91.7%	86.1%

3.4. Rational behavior adjustment process

The evolution processes in Eqs. (2)-(6), as well as NTP and PDS, have commonality that the total travel cost of the network would decrease based on the previous day's path travel costs, i.e., $\left(f^{(n+1)}-f^{(n)}\right)\cdot c^{(n)}<0$, until DUE is reached. This property was noticed by Zhang et al. (2001) and Yang and Zhang (2009) and named the "rational behavior adjustment process" (RBAP) by the latter. Guo et al. (2013, 2015) pointed out that the same feature can be found in the link-based models such as those in He et al. (2010), Han and Du (2012) and Smith and Mounce (2011). The RBAP-like models with elastic demand were proposed in Sandholm (2002, 2005), Yang (2007) and Li et al. (2012).

To verify the RBAP property, we plot $\left(f^{(n+1)}-f^{(n)}\right)\cdot c^{(n)}$ against n in Figure 6: the maximum value is 510 and the minimum value is -14072. Among the 25 points, only 8 (marked as "×") are nonnegative. Therefore, the assumption of RBAP is well satisfied in our experiment. An additional observation is that the absolute value of $\left(f^{(n+1)}-f^{(n)}\right)\cdot c^{(n)}$ gradually shrinks to zero along the day-to-day process.

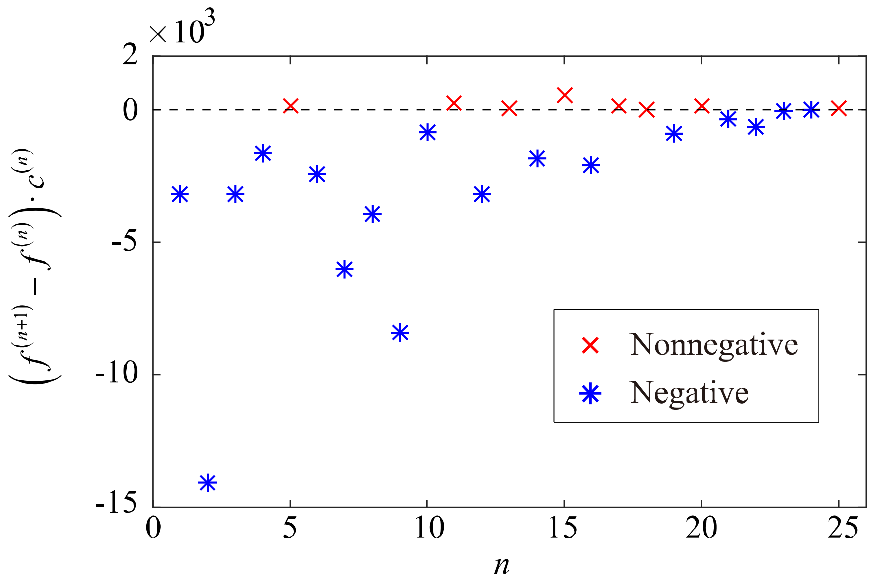


Figure 6. Verification of RBAP.

3.5. Perspective of Lyapunov function

Lyapunov's second theorem is widely used in the literature for proving the stability and convergence of a day-to-day model, with reliance on strictly decreasing Lyapunov functions. As pointed out by Xiao et al. (2016), the Lyapunov functions can represent the energies of the transportation networks; conversely, the energies of the transportation networks, once defined, can be used as the Lyapunov functions to investigate the stability of a day-to-day model. In this subsection, we examine the evolution of different Lyapunov functions that were previously used in the literature.

The most widely used Lyapunov function is the Beckmann's transformation, given as

$$V^{(n)} = \sum_{a \in A} \int_0^{v_a^{(n)}} t_a(\omega) \mathrm{d}\omega - \sum_{a \in A} \int_0^{v_a^*} t_a(\omega) \mathrm{d}\omega, \tag{32}$$

where v_a^* is the DUE flow on link $a \in A$, as in Guo et al. (2013), Han and Du (2012), Jin (2007), Peeta and Yang (2003) and Smith and Mounce (2011). Other forms of Lyapunov functions include the following one in Smith (1984),

$$V^{(n)} = \sum_{r \in R} \sum_{s \in R} f_r^{(n)} \left[c_r^{(n)} - c_s^{(n)} \right]_+^2, \tag{33}$$

and the Euclidean distance between the actual and DUE path flows (Nagurney and Zhang, 1997) or link flows (Guo et al., 2015), i.e.,

$$V^{(n)} = \sum_{a \in A} \left(v_a^{(n)} - v_a^* \right)^2 \tag{34}$$

or

$$V^{(n)} = \sum_{r \in R} \left(f_r^{(n)} - f_r^* \right)^2, \tag{35}$$

where f_r^* is the DUE flow on path $r \in R$. Notably, all four of these Lyapunov functions are originally used for continuous-time day-to-day models. When being used here for the discrete-time models, they may not be strictly decreasing with time anymore. To compare these Lyapunov functions, we plot $V^{(n)} / \max_n V^{(n)}$ against n in Figure 7. The four different forms in Eqs. (32)-(35) evolve quite similarly and all gradually approach zero with obvious fluctuations. An unexpected peak appears on the 9th day, consistent with the peak of flow on Path 2 on the same day (see Figure 2), which is mainly caused by the massive flow switch from Paths 1 and 3 to Path 2 without explicable reasons.

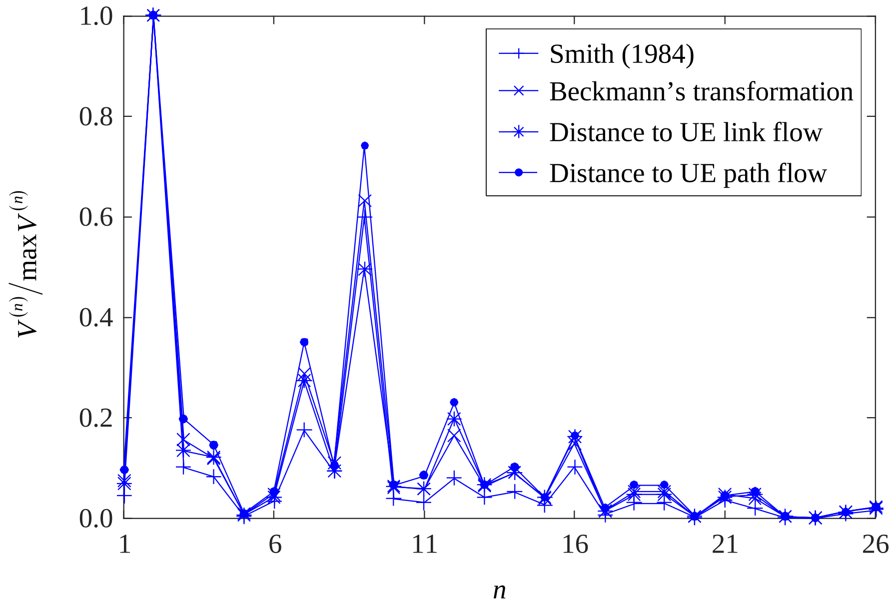


Figure 7. Evolution of the Lyapunov functions.

4. Conclusions and future research directions

In this paper, we adopted regression analysis to investigate the existing day-to-day models based on a virtual route choice experiment. We identified some issues (such as heterogeneity and autocorrelation) to which attention should be paid in such qualitative research; we also observed some interesting properties of the day-to-day model that can be considered in both empirical and theoretical research. The heterogeneity and autocorrelation found in the regression analyses could hint at missing features in a day-to-day model. Modifying the model form can help reduce the heterogeneity and autocorrelation; however, it may deprive a day-to-day model of the key mathematical properties such as steady states and stability. Alternatively, we adopted the WLS method that keeps the model forms intact. Various assumptions on participants' route choice behaviors are examined. The findings include:

(1) The influence of the path cost difference on the flow swapping rate is increasing and concave in PSAP, FIFO, XYY and ETFD; however, the influence of path flow varies.

(2) The path preference was not detected, which might be due to the experimental settings. The participants were encouraged to achieve the minimum travel time, so they had no reason to prefer one path to others.

(3) The parameter is time-varying in SGFD but not in PSAP, FIFO, XYY or EGFD, while an altered form of FIFO showed a time-varying parameter. This implies that whether the model parameters are time-varying or not is dependent also on the model form.

(4) The learning-like behavior was found only in SGFD, which is consistent with the autocorrelation found in the original SGFD form. Such an observation links the autocorrelation in an econometric analysis with the cognitive assumption in decision making. The failure to detect the learning behavior in most models may be attributed to the fact that we did not explicitly provide the historical travel times when the participants

were making choices, or that their cognitive behaviors during decision making were much more complex than the simple learning process assumed.

(5) The comparison between ETFD and SGFD illustrated how the modification of model forms can lead to distinct conclusions on the time-varying parameter and learning behavior.

(6) PSAP, FIFO and XYY showed similar predictive power with acceptable accuracy for the path flow prediction, while ETFD and SGFD performed only slightly less well.

(7) The assumption of the rational behavior adjustment process is well satisfied.

(8) The four Lyapunov functions used in the literature for stability analysis evolve similarly and all tend to approach zero.

Among the findings are some unsolved questions, such as the assumptions of participants' path preference and learning behavior, as well as whether and how the flows will affect the flow swapping, although intuitively they should be related to each other. Answering these questions requires richer data, either from the virtual experiments under more practical settings, or from real urban road networks.

Owing to the experimental settings, we are only able to examine the DUE-based day-to-day models. For future research, we are also interested in investigating those day-to-day models based on broader behavioral settings, such as stochastic UE and boundedly rational UE. In addition, it is good to explore, via virtual experiments, how the day-to-day models perform when modeling the scenarios with traffic disruptions or with the provision of traffic information. The demand can be well fixed in a laboratory experiment; however, this is never the case in the real world. Thus, another interesting extension is to consider an environment with varying-demand which could be either elastic with respect to cost or varying with departure time. Interestingly, there are less elastic-demand day-to-day models than fixed-demand models in the literature; therefore, empirical studies may help enrich the elastic-demand models in a bottom-up manner. Moreover, incorporating departure time choice (Mahmassani, 1990; Mahmassani and Stephan, 1988; Mahmassani et al., 1986; Xiao and Lo, 2016) would add another dimension to the route choices considered in a general network.

We are also enlightened by another idea of revealing individual-level characteristics by investigating aggregate-level models. It would be particularly interesting to develop a methodology for incorporating individual-level heterogeneity in a model which is based on aggregate-level measurements (such as flows), and detecting such heterogeneity from empirical data. Also, as some day-to-day models are built upon the assumption of travelers' perceiving behavior on travel times, and such perception is generally difficult to measure, it would be valuable to figure out a way to calibrate such models based on measurable variables such as flows and travel times. Finally yet importantly, quantitative analysis at the individual level, as the conventional way of studying route changing behavior, could provide valuable information for analysis at the macroscopic level. Unlike the above-mentioned idea of detecting individual-level heterogeneity in an aggregate-level model, the microscopic analysis could conversely provide a reference for macroscopic-level analysis.

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