



Deposited via The University of Sheffield.

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/id/eprint/120266/>

Version: Accepted Version

---

**Article:**

Parker, R.J. and Goodwin, S.P. (2009) The role of cluster evolution in disrupting planetary systems and discs: the Kozai mechanism. *Monthly Notices of the Royal Astronomical Society*, 397 (2). pp. 1041-1045. ISSN: 0035-8711

<https://doi.org/10.1111/j.1365-2966.2009.15037.x>

---

**Reuse**

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

**Takedown**

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing [eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk) including the URL of the record and the reason for the withdrawal request.

# The role of cluster evolution in disrupting planetary systems and disks: the Kozai mechanism

Richard J. Parker\* and Simon P. Goodwin

*Department of Physics and Astronomy, University of Sheffield, Sheffield, S3 7RH, UK*

## ABSTRACT

We examine the effects of dynamical evolution in clusters on planetary systems or protoplanetary disks orbiting the components of binary stars. In particular, we look for evidence that the companions of host stars of planetary systems or disks could have their inclination angles raised from zero to between the threshold angles ( $39.23^\circ$  and  $140.77^\circ$ ) that can induce the Kozai mechanism. We find that up to 20 per cent of binary systems have their inclination angles increased to within the threshold range. Given that half of all extrasolar planets could be in binary systems, we suggest that up to 10 per cent of extrasolar planets could be affected by this mechanism.

**Key words:** stars: formation – open clusters and associations – planetary systems – methods:  $N$ -body simulations

## 1 INTRODUCTION

The majority of star formation is believed to occur in clustered environments (Lada & Lada 2003, and references therein). Furthermore, around 60 per cent of solar-type stars are observed in binary systems in the field (Duquennoy & Mayor 1991). It is believed that a higher percentage of these stars (and probably *all* of them) form not as singles, but in binary or higher order systems (e.g. Goodwin & Kroupa 2005; Goodwin et al. 2007), although see Lada (2006) for an alternative interpretation.

Several studies (e.g. Kroupa 1995a,b; Parker et al. 2009a) have demonstrated that intense dynamical evolution occurs for stellar systems in typical (i.e. Orion-like) clusters. If most stars form in such clusters, then this dynamical processing may also have a significant effect on any planet formation that occurs around binary stars.

Over 300 extrasolar planets have been discovered to date<sup>1</sup>. Of the nearby extrasolar planets (i.e. within 200 pc, see Butler et al. 2006)<sup>2</sup>, around 40 are known to orbit a component of a binary system (Desidera & Barbieri 2007). Indeed, Bonavita & Desidera (2007) suggest that after incompleteness has been taken into account, the numbers of planets orbiting binary and single stars may be equal.

Given the significant fraction of extrasolar planets

orbiting the components of binary systems, the dynamical history of such stellar systems becomes important for understanding the properties of the planets therein. The Kozai mechanism (Kozai 1962) has been shown by several authors, (e.g. Innanen et al. 1997 – see also Holman, Touma & Tremaine 1997; Takeda & Rasio 2005; Malmberg, Davies & Chambers 2007a) to disrupt the orbits of planets (see e.g. Malmberg et al. 2007a, their fig. 1) that form in binary systems.

In this paper we address the possibility of whether the dynamical evolution of a cluster may provide a way of inducing the Kozai mechanism in a significant fraction of binary stars. We describe the Kozai mechanism in Section 2, we outline our method in Section 3, we present our results in Section 4 and we conclude in Section 5.

## 2 THE KOZAI MECHANISM

The Kozai mechanism (Kozai 1962) was used to quantify how the orbits of inclined asteroids were influenced by Jupiter. It assumes that the mass of the asteroid is negligible compared to that of Jupiter and the Sun – the same assumption can be made for a planet orbiting one of the components in a binary system (Innanen et al. 1997).

If the inclination angle of the orbit exceeds  $39.23^\circ$ , then the Kozai mechanism states that there is a cyclical exchange of angular momentum to the asteroid, causing the eccentricity of the asteroid to vary periodically. The same effect is predicted for planetary systems (and protoplanetary disks) in binary systems.

It should be noted that these Kozai cycles can be switched off if the inclination angle of the orbit exceeds

\* E-mail: r.parker@sheffield.ac.uk

<sup>1</sup> The number of discoveries is increasing on an almost daily basis; for the most up-to-date catalogue maintained by J. Schneider see <http://exoplanet.eu/>.

<sup>2</sup> See <http://exoplanets.org/planets.shtml> for the census by Butler et al. (2006) of the extrasolar planets within 200 pc of the Earth.

140.77°. We define a range of inclination angles of systems susceptible to the Kozai mechanism as

$$39.23^\circ < i_{\text{Koz}} < 140.77^\circ. \quad (1)$$

Malmberg et al. (2007a) used the Kozai mechanism as a method to disrupt planetary systems and showed that it induces a highly eccentric orbit for the outer planet, which then crosses the orbits of the inner planets. In some cases this process leads to the ejection of one or more of the planets. Malmberg et al. (2007a) use intermediate ( $a \sim 100$  AU) separation binary systems to invoke this mechanism.

In our simulations, we investigate whether the inclination angle between the components of binary systems can be increased from zero (at the birth of the cluster) to an angle in the range  $i_{\text{Koz}}$  (the *Kozai angle*). We assume that the binaries at the cluster birth act as gyroscopes and hence any systems with an angle greater than the Kozai angle have undergone significant dynamical processing and could be subjected to intense perturbations.

### 3 METHOD

#### 3.1 Cluster set-up

We closely follow the method described by Parker et al. (2009a) to set up the clusters and binary systems in our simulations. The clusters are designed to mimic a ‘typical’ star cluster, similar to Orion with a mass  $\sim 10^3 M_\odot$ .

For each set of initial conditions, we create a suite of 20 clusters, identical apart from the random number seed used to initialise the simulations.

We set our clusters up as initially virialised Plummer spheres (Plummer 1911) using the prescription given in Aarseth, Hénon & Wielen (1974). The Plummer sphere provides the positions and velocities of the centre of mass of stellar systems. We adopt three different half-mass radii for our clusters; 0.1, 0.2 and 0.4 pc. Parker et al. (2009a) argue that the initial half-mass radius of Orion was in the range 0.1 – 0.2 pc, but we include simulations with a half-mass radius of 0.4 pc for comparison.

#### 3.2 Binary properties

We create all our clusters with an initial binary fraction,  $f_{\text{bin}} = 1$  (i.e. all stars form in binary systems; there are no singles or triples, etc.), where

$$f_{\text{bin}} = \frac{B}{S + B}, \quad (2)$$

and  $S$  and  $B$  are the numbers of single and binary systems, respectively.

The mass of the primary star is chosen randomly from a Kroupa (2002) IMF of the form

$$N(M) \propto \begin{cases} M^{-1.3} & m_0 < M/M_\odot < m_1, \\ M^{-2.3} & m_1 < M/M_\odot < m_2, \end{cases} \quad (3)$$

where  $m_0 = 0.1 M_\odot$ ,  $m_1 = 0.5 M_\odot$ , and  $m_2 = 50 M_\odot$ . For simplicity we do not include brown dwarfs (BDs) in our simulations.

Secondary masses are drawn from a flat mass ratio distribution with the constraint that if the companion mass

is  $< 0.1 M_\odot$  it is reselected, thereby removing the possibility of choosing secondaries with BD-like masses. This constraint maintains the underlying binary fraction, but biases the masses of low-mass systems towards unity (see Kouwenhoven et al. 2009a).

The generating function for orbital periods are the log-normal distributions observed by Duquennoy & Mayor (1991) and Fischer & Marcy (1992) of the form

$$f(\log P) = C \exp \left\{ \frac{-(\log P - \overline{\log P})^2}{2\sigma_{\log P}^2} \right\}, \quad (4)$$

where  $\overline{\log P} = 4.8$ ,  $\sigma_{\log P} = 2.3$  and  $P$  is in days.

Eccentricities of binary stars are drawn from a thermal eccentricity distribution (Kroupa 1995a, 2008) of the form

$$f_e(e) = 2e. \quad (5)$$

Binaries with small periods but large eccentricities would expect to undergo the tidal circularisation shown in the sample of G-dwarfs in Duquennoy & Mayor (1991). We account for this by reselecting the eccentricity if it exceeds the following period-dependent value  $e_{\text{tid}}$ :

$$e_{\text{tid}} = \frac{1}{2} (0.95 + \tanh(0.6P - 1.7)). \quad (6)$$

This ensures that the eccentricity–period distribution matches the observations of Duquennoy & Mayor (1991), as we expect that tidal circularisation occurs before cluster evolution takes place (Parker et al. 2009b). Finally, the periods are converted to semi-major axes.

By combining the primary and secondary masses of the binaries with their semi-major axes and eccentricities, the relative velocity and radial components of the stars in each system are determined. These are then placed at the centre of mass and centre of velocity for each system in the Plummer sphere.

Simulations are run using the `kira` integrator in `Starlab` (e.g. Portegies Zwart et al. 1999, 2001, and references therein) and evolved for 10 Myr.

#### 3.3 Finding susceptible systems

For each cluster we identify the binary systems<sup>3</sup> that have been preserved since the birth of the cluster (i.e. those that have not been broken up by dynamical processing). We also identify the binary systems formed through dynamical interactions in the clusters as these systems are likely to be susceptible to the Kozai mechanism (Malmberg et al. 2007b).

Simulations show that disk fragmentation is probably a major mode of binary formation (e.g. Goodwin, Whitworth & Ward-Thompson 2004; Goodwin et al. 2007) and hence we expect that the orbits of the companion and the planets/disk will be roughly coplanar (observations of circumstellar disks in binary systems have shown that they are usually inclined by less than 10 – 20° to the component stars, e.g. Jensen et al.

<sup>3</sup> We use the nearest-neighbour algorithm described by Parker et al. (2009a) (and independently verified by Kouwenhoven et al. 2009b) to determine whether a star is in a bound binary system.

2004; Monin et al. 2006). However, this is not an assumption that affects our result as we examine the shift in the binary inclination relative to whatever initial inclination the planets/disk have.

We measure the change in the inclination of the binary orbit to any planetary system or protoplanetary disk by examining the change in the orbital angular momentum vector from formation. We assume that the inclination of the planets/disk will not be strongly effected by any interaction that changes the inclination of the binary and hence any change in the inclination of the binary orbit will be a change relative to the planets/disk.

For the preserved systems we use any change in the angular momentum vector of the binary to ascertain whether the system has an inclination angle in the range  $i_{\text{Koz}}$  after each Myr (assuming a birth inclination angle of zero). For the systems formed during the cluster's lifetime, we calculate the change in angular momentum with respect to the parental binary or binaries<sup>4</sup> and use the largest angle in our calculations<sup>5</sup>.

As we are interested in the effect of cluster evolution on planetary systems (or protoplanetary disks), we define a periastron distance that allows for the formation of such a system without interference from the secondary component of the binary system. We take the standard definition of periastron distance  $r_{\text{peri}}$ :

$$r_{\text{peri}} = a(1 - e), \quad (7)$$

where  $a$  and  $e$  are the semi-major axis and the eccentricity of the binary system, respectively.

We determine  $r_{\text{peri}}$  for each system, and assume that a stable planetary system (or disk) could have developed for any binary with  $r_{\text{peri}} > 100$  AU. Of the 43 planets orbiting a binary component in the sample of Desidera & Barbieri (2007), only 5 are in binary systems with separations less than 100 AU. If a binary exceeds this periastron threshold we determine its inclination angle from the change in the orbital angular momentum vector.

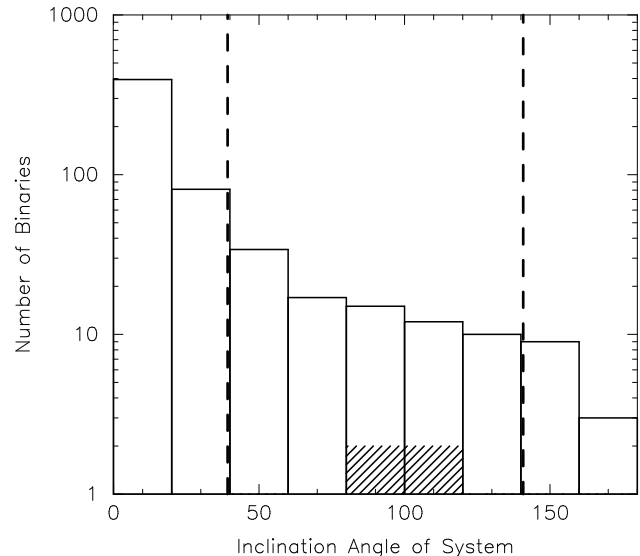
## 4 RESULTS

### 4.1 The fraction of systems susceptible to the Kozai mechanism

We show the distribution of inclination angles for systems with  $r_{\text{peri}} > 100$  AU after 1 Myr in Fig. 1. Whilst only 1 per cent of *all* systems have an inclination angle within the threshold range ( $39.23^\circ$  to  $140.77^\circ$ ), many of the binaries are tight ( $< 30$  AU) and hence not susceptible to dynamical processing (Parker et al. 2009a).

<sup>4</sup> We define a parental binary as a system that was a binary initially and contains one of the stars in a newly formed binary. The Duquennoy & Mayor (1991) period generating function leads to very wide (unphysical) binaries in the clusters and so not all the primordial binaries created in the simulations are detected by our algorithm (see Parker et al. 2009a, for a more detailed discussion). The result of this is that some of the newly formed binaries may only have one parental binary.

<sup>5</sup> In practice, in the very few binaries created through these intense dynamical interactions, both angles tend to be in the range  $i_{\text{Koz}}$  so the angle chosen is irrelevant.



**Figure 1.** The distribution of inclination angles of systems with  $r_{\text{peri}} > 100$  AU after 1 Myr for clusters with an initial half-mass radius of 0.1 pc. Binary systems from 20 different realisations of the same cluster have been binned together in this histogram. The open histogram represents primordial binaries, preserved since the birth of the cluster; the hashed histogram represents binary systems formed during the dynamical evolution of the cluster. The dashed lines indicate the threshold angles ( $39.23^\circ$  and  $140.77^\circ$ ), between which the Kozai mechanism may act upon any planets in orbit around the binary stars.

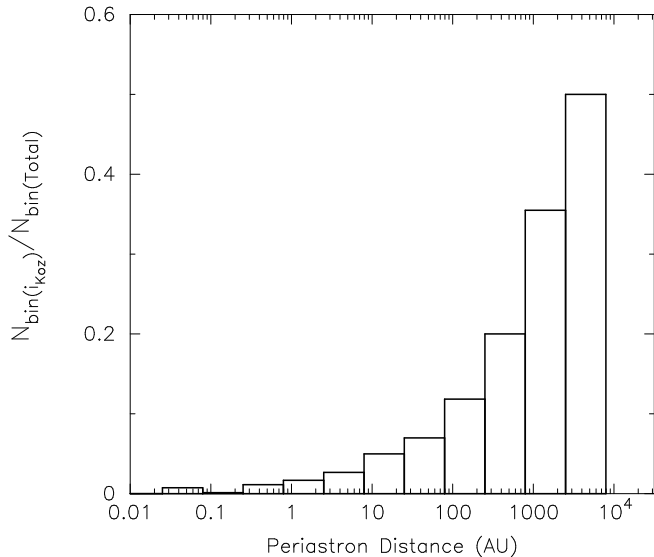
This is readily demonstrated in Fig. 2, where we plot the fraction of binaries with inclination angles in the threshold range  $i_{\text{Koz}}$  as a function of  $r_{\text{peri}}$ . Binaries with  $r_{\text{peri}} > 100$  AU are far more likely to have inclination angles in the range  $i_{\text{Koz}}$  than the tighter systems. Additionally, a binary with a periastron distance  $r_{\text{peri}} < 100$  AU is unlikely to form a stable planetary system, as that system will be perturbed by the other component of the binary system.

The contribution from binaries that are formed during the cluster's evolution is minimal. As can be seen in Fig. 1, only four newly formed systems with  $r_{\text{peri}} > 100$  AU lie within the threshold range  $i_{\text{Koz}}$ , compared to the many tens of primordial binaries.

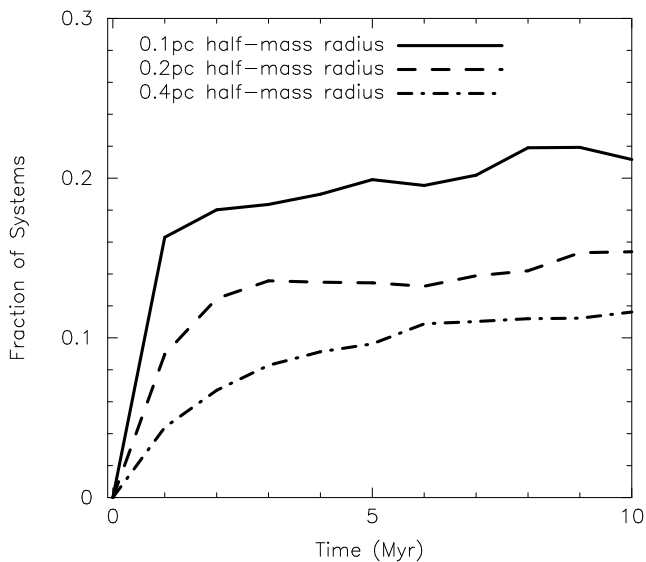
Of the systems that have  $r_{\text{peri}} > 100$  AU, we determine the fraction of systems that have an inclination angle between  $39.23^\circ$  and  $140.77^\circ$ . We show the evolution of this fraction during the cluster's lifetime in Fig. 3. We show the fraction of systems that could be subjected to the Kozai mechanism for three different initial half-mass radii; 0.1 pc (the solid line), 0.2 pc (the dashed line) and 0.4 pc (the dashed-dot line).

For the most dense clusters ( $r_{1/2} = 0.1$  pc), the fraction of systems that could potentially undergo the Kozai mechanism is roughly constant, at  $\sim 20$  per cent. For the less dense clusters, the fraction is less;  $\sim 13$  per cent for  $r_{1/2} = 0.2$  pc, and  $\sim 10$  per cent for  $r_{1/2} = 0.4$  pc. This decrease in affected systems with increasing half-mass radius is simply due to there being fewer interactions in the less-dense clusters.

The fraction of systems that could be subjected to the Kozai mechanism remains roughly constant after 1 Myr for

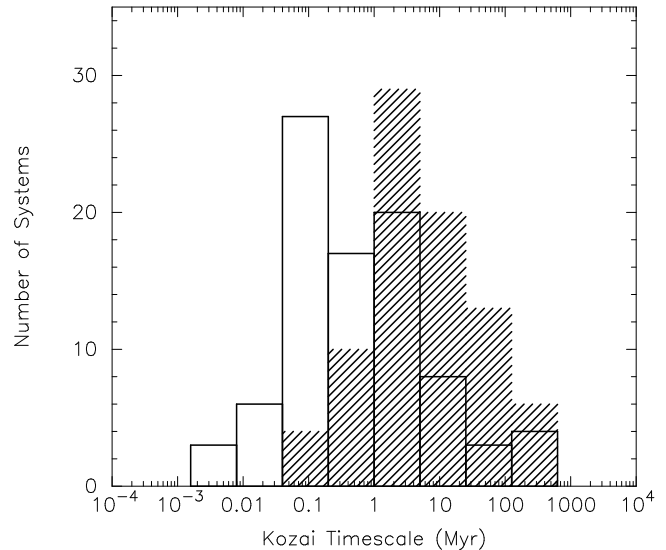


**Figure 2.** The fraction of binaries in the range  $i_{\text{Koz}}$  as a function of periastron distance  $r_{\text{peri}}$  after 1 Myr of cluster evolution (initial half-mass radius,  $r_{1/2} = 0.1$  pc).



**Figure 3.** The fraction of systems with  $r_{\text{peri}} > 100$  AU and an inclination angle within the Kozai threshold range  $i_{\text{Koz}}$  ( $39.23^\circ - 140.77^\circ$ ) as a function of time. We show the results for three different initial cluster half-mass radii; 0.1 pc, 0.2 pc and 0.4 pc. For the most dense cluster, the fraction of systems that could be subjected to the Kozai mechanism remains at  $\sim 20$  per cent during the first 10 Myr of cluster evolution.

the most dense clusters ( $r_{1/2} = 0.1$  & 0.2 pc). This is entirely due to the crossing time of the cluster being only  $\sim 0.2$  Myr (Parker et al. 2009a) and therefore the cluster has relaxed and the majority of binaries have already reached dynamical equilibrium.



**Figure 4.** The distribution of Kozai timescales,  $\tau_{\text{Koz}}$ , in Myr, for two hypothetical planets. The open histogram is the distribution for a planet with Neptune’s mass ( $5 \times 10^{-5} M_\odot$ ) and period (165 years), and the hashed histogram is the distribution for a planet with Jupiter’s mass ( $9 \times 10^{-4} M_\odot$ ) and period (12 years).

## 4.2 The Kozai timescale

A planet orbiting the component of a binary star will undergo Kozai cycles on the following timescale (Kiseleva, Eggleton & Mikkola 1998; Takeda, Kita & Rasio 2008; Verrier & Evans 2009):

$$\tau_{\text{Koz}} \simeq \frac{2}{3\pi} \frac{P_{\text{bin}}^2}{P_p} (1 - e_{\text{bin}}^2)^{3/2} \frac{m_1 + m_2 + m_p}{m_2}, \quad (8)$$

where  $P_{\text{bin}}$  is the period of the binary,  $P_p$  is the period of the planet,  $e_{\text{bin}}$  is the eccentricity of the binary,  $m_1$  and  $m_2$  are the masses of the primary and secondary components of the binary respectively, and  $m_p$  is the mass of the planet.

For the densest clusters ( $r_{1/2} = 0.1$  pc), we use Eqn. 8 to determine the Kozai timescale for two hypothetical planets orbiting a component of each binary. One planet has a period and mass similar to that of Neptune (165 years and  $5 \times 10^{-5} M_\odot$  respectively), the other has a period and mass similar to Jupiter (12 years and  $9 \times 10^{-4} M_\odot$  respectively).

Our results are shown in Fig. 4. We determine the Kozai timescale for the hypothetical planets for all binaries with a periastron distance  $r_{\text{peri}} > 100$  AU and an inclination angle  $> 39.23^\circ$ . We show the distribution of Kozai timescales for the Neptune analog in the open histogram, and the Kozai timescales for the Jupiter analog in the hatched histogram.

For the Neptune analog, there are a large number of systems in which the Kozai timescale is much less than 1 Myr. This means that such planets could be subjected to the Kozai mechanism very early on in the dynamical evolution of the cluster, and given this timescale it is more pertinent to ascribe these effects to a protoplanetary disk rather than a system of fully formed planets. It should also be noted that Kozai cycles could occur several times in 1 Myr for each planetary system.

The Kozai timescale is longer for the Jupiter analog, and we would expect on average only one Kozai cycle per

Myr for such a planet. However, if a system consists of more than one planet, it is the timescale for the outer planet in the system that is important (Takeda et al. 2008). Therefore a Jupiter-like planet could undergo more than one Kozai cycle per Myr if there were also planets orbiting the star with longer periods.

## 5 CONCLUSIONS

We use  $N$ -body simulations to dynamically evolve a typical Orion-like star cluster for 10 Myr. We examine the effect of dynamical evolution on binary systems that could host a stable planetary system or protoplanetary disk. To this end we determine the fraction of systems with a periastron distance  $r_{\text{peri}} > 100$  AU where the secondary component of the binary is shifted with respect to the primary by an angle within the Kozai threshold range ( $39.23^\circ < i_{\text{Koz}} < 140.77^\circ$ ).

For typical clusters with an initial half-mass radius corresponding to that of Orion originally (0.1 – 0.2 pc; see Parker et al. 2009a, for a discussion of the primordial half-mass radius of Orion), we find that 20 per cent of binary systems with  $r_{\text{peri}} > 100$  AU have inclination angles in the range  $i_{\text{Koz}}$  that could induce the Kozai mechanism (Kozai 1962). The Kozai mechanism has been shown to drastically alter the orbital properties of planets (Innanen et al. 1997; Malmberg et al. 2007a) and in particular the eccentricities of extrasolar planets (Malmberg & Davies 2009).

In these dense clusters, we place two hypothetical planets in systems that could be subjected to the Kozai mechanism in order to determine the Kozai timescale, i.e. the length of time it takes for a planet to undergo Kozai cycles. For Neptune-like planets, we find that the Kozai mechanism has the potential to occur more than once a Myr, with a longer timescale for Jupiter-like planets.

For less dense clusters, the fraction of planetary systems in  $> 100$  AU binaries that could be subjected to the Kozai mechanism is still between 10 and 20 per cent, i.e. a not insignificant fraction.

Around a quarter of the extrasolar planets discovered to date are orbiting a component of a binary system with a separation exceeding 100 AU (Desidera & Barbieri 2007). Due to incompleteness in these observations, it is suggested that half of all extrasolar planets may be in binary systems (Bonavita & Desidera 2007). We therefore propose that simple dynamical processing of binary stars in clusters could feasibly affect up to 10 per cent of *all* extrasolar planetary systems by inducing the Kozai mechanism.

## ACKNOWLEDGEMENTS

We thank Melvyn Davies and Daniel Malmberg for useful discussions and comments on the original text. RJP acknowledges financial support from STFC. This work has made use of the Iceberg computing facility, part of the White Rose Grid computing facilities at the University of Sheffield.

## REFERENCES

Aarseth S. J., Hénon M., Wielen R., 1974, *A&A*, 37, 183  
 Bonavita M., Desidera S., 2007, *A&A*, 468, 721  
 Butler R. P., Wright J. T., Marcy G. W., Fischer D. A., Vogt S. S., Tinney C. G., Jones H. R. A., Carter B. D., Johnson J. A., McCarthy C., Penny A. J., 2006, *ApJ*, 646, 505  
 Desidera S., Barbieri M., 2007, *A&A*, 462, 345  
 Duquennoy A., Mayor M., 1991, *A&A*, 248, 485  
 Fischer D. A., Marcy G. W., 1992, *ApJ*, 396, 178  
 Goodwin S. P., Kroupa P., 2005, *A&A*, 439, 565  
 Goodwin S. P., Kroupa P., Goodman A., Burkert A., 2007, in Reipurth B., Jewitt D., Keil K., eds, *Protostars and Planets V The Fragmentation of Cores and the Initial Binary Population*. pp 133–147  
 Goodwin S. P., Whitworth A. P., Ward-Thompson D., 2004, *A&A*, 414, 633  
 Holman M., Touma J., Tremaine S., 1997, *Nature*, 386, 254  
 Innanen K. A., Zheng J. Q., Mikkola S., Valtonen M. J., 1997, *AJ*, 113, 1915  
 Jensen E. L. N., Mathieu R. D., Donar A. X., Dulligan A., 2004, *ApJ*, 600, 789  
 Kiseleva L. G., Eggleton P. P., Mikkola S., 1998, *MNRAS*, 300, 292  
 Kouwenhoven M. B. N., Brown A. G. A., Goodwin S. P., Portegies Zwart S. F., Kaper L., 2009a, *A&A*, 493, 979  
 Kouwenhoven M. B. N., Goodwin S. P., Parker R. J., Kroupa P., 2009b, *MNRAS*, in prep.  
 Kozai Y., 1962, *AJ*, 67, 591  
 Kroupa P., 1995a, *MNRAS*, 277, 1491  
 Kroupa P., 1995b, *MNRAS*, 277, 1507  
 Kroupa P., 2002, *Science*, 295, 82  
 Kroupa P., 2008, in Aarseth S. J., Tout C. A., Mardling R. A., eds, *Lecture Notes in Physics*, Berlin Springer Verlag Vol. 760 of *Lecture Notes in Physics*, Berlin Springer Verlag, *Initial Conditions for Star Clusters*. p. 181  
 Lada C. J., 2006, *ApJ*, 640, L63  
 Lada C. J., Lada E. A., 2003, *ARA&A*, 41, 57  
 Malmberg D., Davies M. B., 2009, *MNRAS*, 394, L26  
 Malmberg D., Davies M. B., Chambers J. E., 2007a, *MNRAS*, 377, L1  
 Malmberg D., de Angeli F., Davies M. B., Church R. P., Mackey D., Wilkinson M. I., 2007b, *MNRAS*, 378, 1207  
 Monin J.-L., Ménard F., Peretto N., 2006, *A&A*, 446, 201  
 Parker R. J., Goodwin S. P., Kroupa P., Kouwenhoven M. B. N., 2009a, *MNRAS*, in press (arXiv: 0905.2140)  
 Parker R. J., Attwood R. E., Goodwin S. P., Whitworth A. P., 2009b, *MNRAS*, submitted  
 Plummer H. C., 1911, *MNRAS*, 71, 460  
 Portegies Zwart S. F., McMillan S. L. W., Hut P., Makino J., 2001, *MNRAS*, 321, 199  
 Portegies Zwart S. F., Makino J., McMillan S. L. W., Hut P., 1999, *A&A*, 348, 117  
 Takeda G., Kita R., Rasio F. A., 2008, *ApJ*, 683, 1063  
 Takeda G., Rasio F. A., 2005, *ApJ*, 627, 1001  
 Verrier P. E., Evans N. W., 2009, *MNRAS*, 394, 1721