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Hedge Fund Index Engineering Methodologies: A Comparison and Demonstration

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We examine hedge fund (HF) index construction methodologies, by describing and analyzing case studies from two well-known database vendors and evaluating them using numerical examples on the same dataset. Despite the fact that they follow a similar due diligence process, there are great differences in the index engineering practices arising from different quantitative techniques, even for indices in the same HF category. However, those quantitative techniques provide similar results. The differences are rather due to the use of different HF universes and different inclusion criteria. This paper is the first to use actual numerical case studies to illustrate and compare how HF index engineering works. Having read it the reader will have a good understanding of how HF indices are formed.

Keywords: hedge funds, indices, indexes, classification, construction methodology

JEL codes: G12, G14

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1 Introduction

In this paper we shed light on hedge fund (HF) index construction methodologies and in particular on the classification processes. We are the first to compare different methodologies using the same dataset and demonstrate using real data how HF indices can end up with very different constituents. Our research objective is to close the gap in the literature concerning HF index construction methodologies, particularly when dealing with classification issues. To this end we examine two existing HF index engineering methodologies and compare them using a common database with practical examples, thereby providing a better understanding of how all HF indices are constructed. Through this investigation we answer the research question of why there are such large differences between HF indices from different vendors, even when they are supposed to represent the same strategy.

Many authors such as Harri and Brorsen (2004), Getmansky (2004), Ammann and Moerth (2005) and Bali, Brown and Caglayan (2011) simply used the classification scheme provided by the database vendors. Other authors such as Agarwal and Naik (2000), Jawadi and Khanniche (2012) and Meligkotsidou and Vrontos (2014) used HF indices provided by the database vendors, thus working on a strategy index level and obviating the need to consider whether those indices really were representative. Most papers use more than one database, due to the fact that inclusion on every database is a voluntary decision made by the fund managers. Authors who used more than one database such as Ackerman, McEnnally, and Ravenscraft (1999), Capocci and Hubner (2004), Joenvaara, Kosowski, and Tolonen (2012), and Patton and Ramadorai (2013) usually implemented a mapping between the strategies provided by the database vendors, whereas others such as Agarwal, Daniel and Naik (2004) and Kosowski, Naik and Teo (2007) made a broader classification of the database strategies provided by the database vendors. This classification consisted of mapping strategies into four groups; directional, relative value, security selection, and multi-process funds. Yet others such as Bares, Gibson, and Gyger (2003) used classification systems based on the investment process, the asset class, and the geographical period provided by the vendor. Lastly, Das (2003) examined a non-hierarchical clustering algorithm using the (disclosed) asset class, size, fees, leverage, and liquidity. However, this classification scheme does not focus on funds' strategy or style.

Overall it seems that the majority of authors have used the predefined classification schemes from the database vendors. Subsequently, some authors grouped HFs into broader styles or categories. However, none of those who did use the predefined classification schemes focused on the vendors' classification process itself. Our research motivation lies in the fact that although most authors use HF strategies and HF classifications that can have a significant impact on their research output, there is little or no examination in the extant literature of the issues arising from the various vendors' different HF classification processes. In other words, authors do not pay attention to the quantitative techniques used by these vendors when classifying HFs. This issue is important because HF classification can have a significant impact on the HF research results. In the literature there is a significant gap regarding HF index construction methods which is a fundamental element in HF studies. In order to fill this gap, we focus on two studies (Hedge Fund Research Inc, 2012 and Patel, Roffman, and Meziani, 2003) from two well-known database vendors, Hedge Fund Research Inc. and Standard and Poor's. These studies describe the processes and algorithms of their respective index construction methodology and form the basis of our study. They present their vendor's selection criteria, the classification method and the index construction process. Nevertheless, neither study provides any practical examples of their techniques, nor do they address that fact that other database vendors adopt different quantitative techniques that must end up giving different results. These issues are covered by this paper. We focus on HF classification processes using the same dataset because the index construction part (calculating NAV, GAV etc.) is similar between the vendors.

We report two new findings. First, both vendors use rigorous quantitative techniques, combined with qualitative processes through due diligence so as to ensure that they produce high quality representative indices. Nevertheless, these database vendors use different quantitative techniques, particularly when dealing with the classification process. Second, we show that these different quantitative techniques end up classifying HFs in a fairly similar way. The differences between indices' reported returns are instead mainly due to the different datasets used and the different inclusion criteria adopted by the database vendors. Hence, investors should worry more about different universes that database vendors use than the index construction process itself. Concerning our findings, it is surprising that little or no academic work has been done on the differences in similar HF strategy indices from different database vendors. Amenc and Martellini (2003) were the first who examined in a systematic way the differences between HF indices and their lack of success in accurate measuring. There are studies that dealt with the problems of measuring and interpreting indices (e.g. Brittain, 2001, Schneeweis, Kazemi and Martin, 2002), the attractiveness of investing in hedge fund indices (e.g. Brooks and Kat, 2002) or survivorship and selection biases in these indices (Fung and Hsieh, 2002) but none of these dealt with the differences between indices representing the same strategy with regard to the index construction process. Our study is complementary to the above studies as it examines HF indexing from a different perspective.

Our paper contributes to the literature by being the first to examine and explain the main principles and quantitative techniques used to build HF indices, through the use of real vendor cases explaining why there are differences in indices representing the same strategies. The usefulness of the insights developed in our paper allow investors to better understand the nature of the HF indices on offer rather than treating them as a "black box", helping them to feel more confident about choosing the right index benchmark for their investments (e.g. knowing their needs, understanding how each index is constructed and what exactly it shows, and assisting them in the 'right' decision). This is important as the selection of the right index benchmark is not a trivial process and may affect their investment decisions. Database vendors are helped to construct better indices, by understanding the methods of their rivals and combining new techniques in their index construction methodology, by collaborating with other vendors or even specializing in certain indices. Researchers can gain deeper knowledge of the HF indices that they use in their research. Finally, financial governance authorities could, through collaboration with database vendors, create a common HF pool to help investors, as the differences in indices are mainly due to the different HF universes used by the database vendors.

Following this introduction, we proceed to the HFR case and in the third section to the S&P case. Afterwards, we proceed to numerical calculations – demonstration of the two classification methods. We then compare and evaluate the two cases on a quantitative and qualitative basis. We conclude concerning index construction methods in the HF industry.

2 Hedge Fund Index Construction: HFR Case

In this section we present an analysis of the HF indexing methodology that is followed by Hedge Fund Research Inc. (2012). Later we present a case from S&P's Hedge Fund Indexing methodology (Patel, Roffman, and Meziani 2003), following the index engineering methodology step by step. As a result, the reader will see the differences in practical terms through these comparisons.

Hedge Fund Research, Inc. (HFR) contains more than 6800 funds and funds of funds worldwide. It constructs two main types of indices: the HFRX and HFRI indices (HFR, 2012). In February 2013 the firm also introduced the HFRU indices.

The HFRI Monthly indices are a range of benchmarks constructed so that they are able to represent the HF industry by equally weighted components of funds that are being reported by their managers to the HFR database. The HFRI index category ranges from the HFRI Fund Weighted Composite index that consists of 2,200 funds to particular sub-strategy classified indices. It is non-investable. The HFRX indices have various index-weighting methods (depending on each index), have different characteristics from HFRI and HFRU indices, and are investable. The newest HFRU (Euros) index category that is denominated in euros is equally-weighted, and is not investable. The HFRU composite index consists of over 600 funds.

The HFRX Global Hedge Fund Index is composed of four main strategy indices that consist of other sub-indices representing various sub-strategies. The HFRX methodology (that is similar to HFRU) includes highly quantitative classification, cluster analysis, correlation analysis, cutting-edge optimization, and Monte Carlo simulations. This approach uses both quantitative and qualitative analysis in order to first, define whether the HF is being managed with transparency and second, check whether the manager complies with the requirements of the due diligence process that is followed by the Hedge Fund Research Inc. Using appropriate aggregated and weighted techniques this HFRX methodology produces the highest statistical probability that the return series would be adequately representative of the HF industry. The general processes are:

(i) Cluster Analysis: HFR screens approximately 7000 funds in its database. Funds with at least \$50M assets under management are included. Also, they must have at least two years' track record, consent to trade on a transparent basis and be open for new investment.

(ii) Correlation Analysis: used for grouping funds by appropriate strategies and to eliminate outliers.

(iii) Monte Carlo Simulation: also used for grouping funds by the most relevant strategies and to eliminate outliers.

(iv) Due Diligence Analysis: Selected funds from the initial screening must be transparent and pass the rigorous qualitative screening.

(v) Strategy Weighting: funds are weighted appropriately to maximize correlation with their group.

We cover each of these steps in more detail below.

The first step is the construction through initial database screening of pure clusters that are represented by specific strategies. Each cluster is for funds using a certain strategy and will be the base for the creation of HF sub-indices. So, there is an initial screening of the HFR database of open funds that (at least claim to) belong to a particular strategy class and comply with the criteria mentioned in the first section.

HFR chooses one representative HF in each strategy for each manager. It is common for successful and well-known fund managers to manage two or more separate funds that belong to the same strategy. Therefore, if there is such a situation and the most representative fund cannot determined then: (a) the fund having the longest track record will be regarded as representative (b) if the funds have the same time track record then the one with the larger assets under management is used.

The representative Hedge Fund Strategy Universe (also called the Strategy Universe-HFS) is obtained from the Global Hedge Fund Universe (HFU) that is contained in the HFR database. The funds that constitute the pure HFS are then filtered and passed only if they satisfy specific criteria such as having a minimum value of assets under management, net of fees reporting, a minimum reporting frequency, fund transparency, being open to new investments etc. If even one of these criteria is not met then the relevant formula used is equal to zero and the HF does not pass on to the next stage. This process is robust, objective and all criteria must be met by each fund for inclusion. Besides this, in the due diligence process (transparency screening) HFR examine other qualitative factors through fund manager interviews, examination of financial statements and organizational structure and other important elements. This qualitative process is complementary to the quantitative process in the database screening.

At the initial database screening the self-reported strategies and sub-strategies are used. However, there are biases in self-reported data that must be eliminated. Therefore, in order to verify style purity, cluster analysis is implemented at the sub-strategy level. If a fund belongs to outliers then it is excluded or reclassified.

The cluster analysis is implemented at a sub-strategy level using 24 consecutive monthly returns at the end of a prior quarter. HFR use the Euclidian distance in the space of monthly returns as the distance or distinction measure of HFs. They also use Ward's (1963) linkage rule. This rule minimizes the variance within clusters and maximizes it between the clusters at every move of the process. Using Euclidian distances or Ward's (1963) linkage rule is a type of cluster analysis instead of ANOVA that we present in section 4.

We mentioned above that funds that belong to outliers may be excluded or reclassified. For that reason HFR uses the trim parameter within the cluster analysis that eliminates some funds, for example the six percent that are least close to the rest of the group. The remaining funds constitute the strategy pure cluster, in other words, the pure strategy index as the remaining funds after the initial classification (through distance rules and Ward's linkage rule) minus the outliers.

The next process is to perform an additional screening called representation analysis. This denotes how dissimilar the returns of each fund are to the respective strategy's returns (sub-strategy or region). The analysis is based on monthly returns for the last twenty four months so as all funds have a complete available dataset. Those funds that have passed successfully cluster analysis and representation analysis are called the final strategy pure cluster. In each cluster all funds have equal weight.

HFR applies multiple representation analysis in order to calculate and rank (in ascending order) the Divergence Score (DS) for each fund. The Divergence Score measures the dissimilarity of a fund in relation to the cluster. Each fund is ranked by its return DS score according to specific measures mentioned below:

The smaller the DS of a fund, the smaller its difference compared to the underlying cluster, hence the higher its ranking. The general formula for the Divergence Score is:

$$DS_i = Information\ Ratio\ Score_i + Beta\ Score_i + Volatility\ Score_i \quad (1)$$

Where $IRS_i = \left(InfoRatio_{Cluster/Strategy} - InfoRatio_{i/Strategy}\right) + \left(InfoRatio_{Cluster/Substrategy} - InfoRatio_{i/Substrategy}\right) + \left(InfoRatio_{Cluster/Region} - InfoRatio_{i/Region}\right)$ (2)

The Information Ratio of fund i versus benchmark B is expressed by:

$$InfoRatio_{i/B} = \frac{\overline{(R_i - R_B)}}{\sigma(R_i - R_B)} \quad (3)$$

$\overline{(R_i - R_B)}$ is the average monthly difference in returns between the fund and the benchmark for twenty four month period. The $\sigma(R_i - R_B)$ is the standard deviation of the difference in returns. The benchmarks that are used are:

Strategy = Hedge fund strategy benchmark specific to fund's strategy (i.e. Event Driven). Sub-strategy = Hedge fund sub-strategy benchmark specific to fund's strategy (e.g. Merger Arbitrage). Region = Regional equity benchmark particular to fund's investment focus (i.e. Europe).

BetaScore$_i$ in equation 1 is defined as

$$BetaScore_i = \left|\beta_{Cluster/Strategy} - \beta_{i/Strategy}\right| + \left|\beta_{Cluster/Sustrategy} - \beta_{i/Substrategy}\right| + \left|\beta_{Cluster/Region} - \beta_{i/Region}\right| \quad (4)$$

The beta of fund i versus benchmark B is expressed as:

$$|1 - \beta_{i/B}| = |1 - \rho_{i/B} * \frac{\sigma_i}{\sigma_B}| \quad (5)$$

σ_i and σ_B are the standard deviation of fund i and the benchmark B respectively. $\rho_{i/B}$ is the correlation of fund i with the benchmark B and it is expressed as:

$$\rho_{i/B} = \frac{cov(R_i,B)}{\sigma_i \sigma_B} \quad (6)$$

Where R_i denotes the returns of the fund and B the returns of the benchmark.

The Volatility Score VS_i of the fund i in equation 1 is expressed as:

$$VS_i = \frac{|\sigma_i - \sigma_{Cluster}|}{\sigma_{Cluster}} \quad (7)$$

Where $\sigma_{Cluster}$ is the standard deviation of returns of the cluster during the evaluation period.

A high beta (correlation) and high volatility scores indicate that a fund is more directional / tactical in its classification. So, higher ranking funds are categorized as a market directional class whereas lower ranking funds are classified as being in the absolute return class. The middle group between them is not taken into consideration.

The representation analysis is the second process in the two-tier screening process and assures the pure cluster group. Accuracy is assured by means of the divergence scores. The total number of funds that constitutes a pure strategy cluster may exceed 500. Because of the rapidly-changing nature of the HF industry it is virtually impossible to maintain such a large number of funds, all providing daily transparent reporting. For that reason HFR use Monte Carlo simulations in order to construct an index with fewer funds without significantly losing representativeness. The number of funds is different from strategy to strategy and may depend upon the number of funds in each cluster, the desired accuracy level, strategy diversity and volatility. This optimization model randomly selects different sized fund samples from a certain strategy cluster and then compares the correlation between each fund sample with the whole cluster. The optimization process not only determines the number of constituents that maximize the representation of the cluster but also their optimal weights. Monte Carlo simulation is therefore employed to examine the number of funds needed to constitute a strategy index that is representative of the strategy cluster. The next step is to find the optimal weights to maximize the representation of the cluster using the Generalized Reduced Gradient quasi-Newtonian Optimization Method. The optimum number of funds depends on the weights (that should lie between certain limits) and the Divergence Score for each fund, as described above.

The underlying HFR indices compute NAV (Net Asset Value) using the actual performance of the managed account by a single fund manager (hedge fund) that reports to the HFR database. The NAV is computed from the following formula:

$$\text{Net Asset Value}_{\text{(per share)}} = \frac{Market\ Value\ of\ Assets - Liabilities - Management\ Costs}{Shares\ Outstanding} \quad (8)$$

The basic HFR NAV index is 1000 and represents the value of the first day of trading. HFR's NAV index change is calculated from the percentage change from t to t+1, and this change depends on the weighted change of all fund-specific NAVs.

We now describe briefly how the global HFRX index, the single strategy index, and the weighted strategy index are structured. The index is organized as a tree structure. The HFRX Global Hedge Fund Index is constituted from other single strategy indices such as the Equity Hedge Fund Index, the Event Driven Hedge Fund Index, the Macro Hedge Fund Index and the Relative Value Arbitrage Hedge Fund Index. These represent the four basic categories according to HFR. The weights of each strategy are given by its assets under management in the fund universe as contained in the HFR database. We then move one level lower, to the HFRX Single (broader) Strategy Indices. Each index is represented by one of the above four categories. Each single (broader) strategy index is composed by the eligible sub-strategy indices that underlie that strategy.

3 Hedge Fund Index Construction: S&P Case

The S&P Hedge Fund Index is composed of three HF styles. Those are Arbitrage, Event-Driven, and Directional/Tactical. Each style is composed of various strategies in a tree structure similar to the HFR. The index construction equally weights the styles and strategies, and uses a rigorous quantitative and qualitative approach so as to select the appropriate funds. The whole index engineering process considers three complementary procedures.

The first procedure has to do with the number of funds that is required in order to build a representative and investable index. S&P apply stratified sampling and bootstrap simulation techniques and have concluded that a fund sample consisting of approximately thirty or forty funds corresponds sufficiently to the risk/return characteristics of a wider portfolio of funds.

The second procedure settles on a specific universe (pool) of appropriate candidates in order to be included in the index. This process begins by examining the strategy consistency of each fund through screening the fund sample for self-reporting bias and inconsistencies. The screening process may take into consideration style classification that uses two common quantitative approaches: Fundamental Style Analysis and Return-based Analysis. The process is essential so as to produce a pool that is cohesively characterized in terms of styles and strategies. Then this pool is additionally screened according to length of track record, investment capacity and assets under management in order to confirm that it is investable.

The third procedure is the due diligence process. S&P uses the due diligence process to qualitatively analyze the candidates for the index HFs This process verifies the management and investment policy, operational capabilities and management experience. Consequently, having gone through this process the remaining funds are investable and have passed the due diligence evaluation.

The fourth procedure is to apply an equal weight of styles and strategies, providing investors with broad diversification across major HF strategies. The index provider ensures that there is a completely clear and public annual announcement regarding potential construction methodology changes and index rebalances to equal weights.

According to S&P a portfolio of 30 or 40 randomly selected funds has a stabile distribution of risk and return characteristics. However, the range of these characteristics is wide. If there are two portfolios of funds (each containing twenty randomly selected funds) there may be a large difference in risk and return characteristics due to different risk exposures. To eliminate the effect of wide distribution of returns and risks in a HF portfolio, S&P used the stratified sampling technique in order to build HF portfolios with balanced risk exposures to tighten the return-risk characteristics.

The first step in the stratifying sample application is to identify the risk dimensions by using two approaches: first it examines the systematic market exposures of a particular investment doctrine and second, it statistically examines the returns history of particular investments. Under the first approach, one could allocate investments to style classifications. This is simple but may be inconsistent because hedge funds' style classifications are made by fund managers. As a result, there might be some biases or inconsistencies. Concerning the second approach, it is stricter but it suffers from the typical problems when dealing with historical returns analysis as well as translating the analysis into a transparent investment process.

Hence these two approaches constitute pools of single-strategy funds. As mentioned before they categorize HFs into three general styles. Those are Arbitrage, Event-Driven, and Directional/Tactical. Every style is composed of three strategies. Consequently, there are in total nine strategies that describe almost completely the investment styles and asset classes.

The second step in stratified sampling is to investigate the cohesiveness of each of the nine samples. Due to the fact that there is no consistency in style reporting, funds from different strategy groups are mixed so that the cross-section of return dispersion is high within these strategies. Also, because there is a wide spectrum of returns there is a need for a relatively large sample of funds in order to have an appropriate level of sampling precision. To enhance strategy cohesiveness there are four quantitative screens:

(i) For each fund of S&P's database they compare two correlation distributions[4] regarding returns. The first is correlation distributions with funds in the same industry and the second is correlation distributions with funds in all other strategies. Then using the Kolmogorov-Smirnov Test, they test whether the two distributions are different.

(ii) The next quantitative screen, after having tested that the two distributions are different, is to examine whether the median of the correlation distribution of funds within the same strategy is greater than the median of the correlation distribution of funds in all other strategies.

(iii) The next quantitative screen is to compare the degree of (return) correlation of each individual fund within the same group with other HF indices of similar strategy they want to examine.

(iv) The last quantitative screen is to compare the standard deviation of each fund to its peer group.

In order to cross-validate the statistical consistency of the nine strategy groups (quantitative screen (ii) above) S&P use ANOVA (analysis of variance). Its principle is to examine whether the standardized distance within groups is less than the standardised distance between groups. ANOVA (S&P) and correlation analysis/distance (HFR) produce similar results, i.e. they cluster funds in a similar way (see section 4).

To summarize, the construction of the index is begun by calculating the aggregate score of a fund followed by the four quantitative screenings above. The first two calculate whether the correlations of fund returns with other funds in the industry are different from correlations with funds in other strategies. The third statistic measures the correlation of the fund return with the proper HF sub-index. The fourth statistic compares the risk of a fund to the risk of other equivalent funds as they can be observed by the historical volatility.

To find out the number of funds that are needed in order to represent a strategy for the general index construction, a simulation is needed. For each strategy S&P run 600

[4] "Correlation distribution": if we have a group of HFs then we have a number of pair correlations, i.e. the correlation of each hedge fund's returns with the returns of each of the other HFs. Each HF has its own distribution of pair correlations, with a mean, standard deviation etc.

simulations, that is, 100 each for samples ranging from one to six funds. They use the simulated bootstrap model for which there is repeated random resampling from the original sample, using each bootstrapped sample to compute a statistic. The resulting empirical distribution of that statistic (in this case return dispersion as the number of funds increases within a certain group-strategy) is then examined and interpreted as an approximation to the true sampling distribution. S&P found that three to five funds per strategy (each of the three broader styles consists of three strategies) is a sufficient number to express the return distribution between funds.

The appropriate number of funds chosen per strategy is based on a quantitative evaluation of the simulation results as well as the number of sub-strategies within each strategy. S&P found that portfolios of 30 to 40 funds based on quantitative techniques sufficiently narrowed the range of risk/return characteristics, but that more funds did not narrow it significantly further. Also, stratified sampling facilitates further narrowing of the spectrum of standard deviations, returns, and correlations with well-established asset classes.

So far we have described the quantitative screening of HFs as well as the quantitative method used in order to have the appropriate number (a target) of funds that will constitute the S&P HFI (first and second procedures). The initial candidate pool consists of funds that have the highest quantitative scores within each strategy. The third process is Due Diligence. S&P's due diligence process is described below. It consists of three main components and includes interviews with fund managers regarding each fund's pure style, trading strategy and practices, infrastructure and operations.

(i) An initial screening of selected funds takes place with sufficiently long track records to provide a preliminary indication of their performance, taking into consideration the assets under management of these funds in order to verify their appeal to investors and the sustainability of their strategy.

(ii) A preliminary examination of the track record, strategy, operating setup, and personnel is performed. This is designed to identify the quality of management, risk and operational management, strategy implementation and capacity limits.

(iii) The Due Diligence Process is a continuous process that is able to detect any changes to how the fund is being operated and managed.

There are interviews of and questionnaires for fund managers and other key staff with periodic visits. The content that is investigated is: general questions regarding the funds, management team backgrounds, investment strategy detailed questions, risk profiles and polices, portfolio construction, systems and infrastructures, service providers, performance analysis, and intensity of strategy cohesiveness. It should be clear that S&P follows a transparent and rigorous methodology in their due diligence process.

At the beginning of this case we referred to the fact that the S&P HFI equally weights styles and the strategies. Contrary to the capitalization-weighted indexes, equally-weighted indexes avoid favouring large funds or strategies that attract noticeable capital flows. Generally, fund-weighted or equally-weighted indices, unlike asset-weighted indices, present a broader view concerning the HF universe. Any biases in favour of larger funds are eliminated because there are no changes in weights. This is particularly important for strategies that contain a relatively small number of funds.

After considering via the quantitative and qualitative process the appropriate funds as well their (equal) weights, the final process is the calculation of the index value. It is calculated through the common NAVs (Net Asset Values) formula of the underlying HFs.

$$\text{Net Asset Value}_{\text{(per share)}} = \frac{Market\ Value\ of\ Assets - Liabilities - Management\ Costs}{Shares\ Outstanding} \qquad (9)$$

$$\text{Gross Asset Value}_{\text{(per share)}} = \frac{Total\ Value\ of\ Assets\ (excl.liabilities\ and\ mgt.costs)}{Shares\ Outstanding} \qquad (10)$$

Thus, the composite index is computed as:

$$\text{NAV Index: } \sum_{i=1}^{F} Number\ of\ shares\ of\ fund_i \times \frac{NAV_i}{Divisor} \qquad (11)$$

$$\text{GAV Index: } \sum_{i=1}^{F} Number\ of\ shares\ of\ fund_i \times \frac{GAV_i}{Divisor} \qquad (12)$$

Where F=Number of funds in the index, Number of shares of fund$_i$ = number of shares allocated to the fund at the last rebalancing to initiate index participation at the appropriate weight, NAV$_i$ = net asset value of the fund (equally weighted according to S&P), GAV$_i$ = gross asset value of the fund and Divisor = initial translation factor to start index at 1000. The S&P HFI tracks a hypothetical portfolio of its components with no capital inflows or outflows, which holds the divisor constant.

As the Diligence Process is an on-going procedure some funds may be added or removed to/from the S&P HFI if they do (or do not) meet certain criteria. A fund can be excluded from the index if it violates qualitative due diligence standards, does not conform to the reporting process, there is a significant strategy shift, there are legal and regulatory issues, major management changes, or concerns for excessive growth or redemptions. Additions can take place not only to replace other HFs. If there is a fund that complies with all the previously mentioned criteria and rules and it will generate a more representative group for a given strategy then it may be added in alignment with the committee perspective.

For an index to make sense there must be a base. So Standard & Poor's constructed an index as of 30 September 2002 that is called the S&P HFI Pro Forma Index. This index is equally-weighted and is rebalanced annually. It uses monthly performance data for the time period January 1998 to September 2002. The S&P HFI index uses this Pro Forma Index as a reference. It is similar to that used by HFR.

4 HFR and S&P Classifications - Demonstration

In this section we illustrate the calculations used in implementing the HFR and S&P index engineering approaches. We use data from two database vendors: EurekaHedge and BarclayHedge containing live and dead funds providing a long coverage (monthly data, 01/1990 to 03/2014). We follow a strict database cleaning and merging approach.[5] We map strategies between the different databases and we end up with: CTA (CT), Event Driven (ED), Global Macro (GM), Long Only (LO), Long Short (LS), Market Neutral (MN), Multi Strategy (MS), Relative Value (RV), Sector (SE),

[5] The algorithms and processes we followed for database cleaning and merging are available on request.

Short Bias (SB) and Others (OT) (includes funds that do not belong to the previous strategies). Each portfolio of a specific strategy is represented by its average time series returns. We classify these strategies in broad categories (groups).

Using numerical examples we demonstrate in practical terms the way that indices are constructed. As noted, index construction concerning the NAV or GAV calculation is the same. Nevertheless, the clustering and classification process is different between database vendors. We simulate the two different index engineering methodologies on the same dataset and then we compare the results to examine whether there are differences between them. We find that those quantitative techniques provide similar results in the index construction process. Differences in the indices between the vendors are mainly because they have different HF universes and different inclusion criteria in their due diligence process. The quantitative parts of their processes, although different, nevertheless provide similar results. In our examples the steps followed are:

For HFR:

1. We used part of the HFR methodology in the index engineering process that calculates the distances between those HF strategies.

2. We implemented the Divergence Score for these HF strategies.

For S&P:

1. We used part of the S&P methodology, measuring the correlations with strategies in the same category (group) and then measuring the correlations with strategies in other categories.

2. We compared the (return) correlation of each individual strategy with the index in the same group.

3. We compared the standard deviation of a strategy to its peer group.

4. We used ANOVA to examine whether the standardized distance within groups is less than the standardized distance between groups.

4.1 Distances Between Strategies (HFR)

We compare the distances between the eleven fund strategies. In table 1, LO, SE and LS are relatively close compared to GM, ED, and SE and even more so for SB, CT, and GM. More specifically, the average distance between LO, SE and LS is 23.850 units; for GM, ED and SE it is 40.413; and for SB, CT and GM it is 90.510 units. Hence, the SE strategy should be allocated with LO and LS and not with GM and ED. Similarly, GM is better allocated with ED and SE rather than SB and CT. Another potential group is ED, RV and MS with average distance 19.917, which is one of the lowest among the hedge fund strategies.

Table (1) about here

Figure 1 shows the distances between fund strategies graphically. We expect that fund strategies that have small distances would be allocated to the same category. The SB strategy follows an opposite direction toward the market index with negative exposures. Hence, we would expect SB to have a large distance compared to the other strategies. Figure 1 confirms this.

Figure (1) about here

The above process is implemented by HFR for every fund (in pairs) with 24 months' returns so as to discover the distances between them. Some funds have a small distance between them, hence they should form a group or an index.

4.2 Correlations (S&P)

We measure the correlations of strategies in the same group (category) and then measure the correlations of strategies in other groups.

Table 2 presents all correlations between strategies. The correlations between strategies that are high, indicate a similar group; correlations between strategies that are low, indicate strategies belonging to different groups. For example (we use the same strategies with our previous demonstration regarding distances) LO, SE, LS have an average correlation among them equal to 0.917; the strategies GM, ED, SE have 0.563 and the strategies SB, CT, GM have -0.030. Similar to the distance example, the SE strategy should be allocated with LO and LS and not with GM and ED. Similarly, GM is better allocated with ED, SE rather than SB, CT. Another potential group is ED, RV, MS with average pair correlation of 0.770.

Table (2) about here

Figure 2 shows the correlations graphically. We expect fund strategies that are highly positively correlated to belong to the same group. Strategies that are either uncorrelated or negatively correlated (e.g. CT and ED) we expect not to belong to the same group.

Figure (2) about here

So far both processes (Euclidian Distance and correlation techniques) have produced similar results.

Using table 2 we calculate the correlation distribution for strategies that belong to the same group. Thus, the correlation distribution (its standard deviation)[6] for LO, SE, LS is 0.032; for GM, ED, SE it is 0.196; and for SB, CT, GM it is 0.296. Based on the correlation distribution, it is preferable that SE should belong to the same group as LO, LS compared to the candidate group GM, ED. Similarly, GM should preferably belong to the ED, SE group compared to the CT, SB group. Similarly within the group of ED, RV, MS the distribution correlation is equal to 0.056, which is relatively low.

The correlation distribution between all strategies (both within and between groups) is: standard deviation 0.511 with mean 0.410, median 0.479, and mean to standard deviation ratio 0.803. The correlation distribution for the groups (e.g. LO, SE, LS or

[6] We compute the standard deviation of the pair correlations (correlation of each fund or strategy with each of the other funds or strategies) within the group.

ER, RV, MS) as we expected, is narrower than all strategies together, having a larger mean-to-standard deviation ratio.

Based on table 2, we computed the correlation of each strategy with its group. LO has correlation with its group (SE, LS) of 0.914; SE with its group (LO, LS) of 0.923; LS with its group (LO, SE) of 0.966.

ED's correlation with its group (RV, MS) is 0.852; RV's with its group (ED, MS) is 0.829; MS's with its group (ED, RV) is 0.781.

As expected, strategies are highly correlated with the group that they belong to. The same process is followed by S&P at the fund level for all individual funds with the indices that they belong to, for verification purposes.

4.3 Standard Deviation

We compare the standard deviations of strategies that belong to the same group.

LO, SE, LS have an average standard deviation equal to 3.120 and the distribution of their standard deviation[7] is equal to 0.405; GM, ED, SE have an average standard deviation equal to 2.372 and the distribution of their standard deviation is equal to 0.773, i.e. higher than the previous group; SB, CT, GM have an average standard deviation equal to 3.543 and the distribution of their standard deviation is equal to 0.594, which is also higher than the first group. Strategies that belong to the same group (LO, SE, LS) have a narrower standard deviation distribution compared to the other two groups in our example (GM, ED, SE and SB, CT, GM).

We now compare the standard deviation of a strategy to its peer group: LO has standard deviation of 3.437 compared to 2.919 for its peer group (SE, LS). SE has standard deviation equal to 3.259 compared to 2.997 for its peer group (LO, LS). LS has standard deviation of 2.663 compared to 3.245 for its peer group (LO, SE).

[7] "Distribution of their standard deviation": If we have a group of funds within a strategy then each member of this group has its own standard deviation of returns. Hence, we have many standard deviations in this group (one value for each fund). Thus, we can plot the overall distribution (of all fund-specific standard deviation values) represented by a mean, standard deviation etc. for this group. The lower the standard deviation of the overall distribution for the group (of funds or strategies) the better it is, because this group is more homogenous.

Similarly, ED has standard deviation equal to 1.840 compared to 1.373 for its peer group (RV, MS). RV has standard deviation equal to 1.238 compared to 1.669 for its peer group (ED, MS). MS has standard deviation equal to 1.713 compared to 1.475 for its peer group (ED, RV). Not surprisingly, the standard deviation is similar between each strategy and its related group.

4.4 Analysis of Variance (S&P)

In order to validate the statistical consistency of the strategy groups, S&P uses ANOVA by examining whether the standardized distance within groups is less than the standardized distance between groups. In other words, they check whether the mean vectors are the same and, if not, which mean components differ significantly.

The analysis of variance is based upon a decomposition of the observations:

X_{li}		$\bar{X}$		$\bar{X}_l - \bar{X}$		$X_{li} - \bar{X}_l$
Observation (SSobs)	=	**overall sample** mean (SSmean)	+	**estimated treatment effect** (SStr -between samples-)	+	**Residual** (SSres –within samples-)

This decomposition into sums of squares allocates variability in the combined samples into mean, treatment, and residual (error) components.

Table 3 presents pair ANOVAs between funds (or strategies) in our sample.

Table (3) about here

The average pairs ANOVA of the group LO, SE, LS is 1.913. Between non-groups such as GM, ED, SE and SB, CT, GM it is much higher, equal to 4.514 and 102.783 respectively.

Figure 3 presents a visual representation of the ANOVAs. For example, the SB and SE strategies have one of the highest ANOVAs between them compared to other pair ANOVAs. On the other hand MN and ED have one of the lowest ANOVAs between them compared to other pair ANOVAs.

Figure (3) about here

We show two example within-group ANOVA calculations in tables 4 and 5. First we compute the ANOVA within group LO, SE, LS (table 4). The F-value is less than the critical value F_{crit} hence we do not reject the null hypothesis that the variables are the same. There is relatively large variance within each strategy but all these strategies behave in the same way.

Table (4) about here

In table 5 we compute the ANOVA between three candidate groups: (LO, SE, LS), (OT, GM, RV) and SB (it has the highest distances and opposite correlations with almost all the other strategies). The F-value is larger than the F_{crit} which means that we accept the alternative hypothesis: the variances are not the same between these three groups.

Table (5) about here

We have shown with the use of ANOVA that the standardized distance within the group (LO, SE, LS) is low (3.83), whereas the standardized distance between groups (LO, SE, LS), (OT, GM, RV) and SB, is considerably larger at 177.14.

4.5 Divergence Score (HFR)

The divergence score (DS) measures the dissimilarity of a fund in relation to the group (cluster). It is used by HFR in their representation analysis as a second quantitative screening. The smaller the score, the better it is (less difference compared to the cluster). The score for each HF is defined as:

$$\text{Divergence Score (DS)} = \text{Information Ratio Score (IR)} + \text{Beta Score (BS)} + \text{Volatility Score (VS)}$$

In our example we compute the DS of LS against the group (SE, LO); then we compute the DS of RV against the same group (SE, LO). To do this we first compute IR, BS and VS for each.

<u>Information Ratio:</u>

The Information Ratio is given by:

$$InfoRatio_{i/B} = \frac{R_i - R_B}{\sigma(R_i - R_B)}$$

$\overline{(R_i - R_B)}$ is the average monthly difference in returns between the fund and the benchmark, usually for at least 24-month period.

$\sigma(R_i - R_B)$ is the standard deviation of the difference in returns.

In our simple example we compute the information ratio of the strategies LS and RV in relation to the same candidate group.

LS case:

Absolute average monthly difference in returns between LS and the group: 0.681

Standard Deviation of the difference of returns of LS and the group: 0.962

Hence, Information Ratio for LS is: $\frac{0.681}{0.962} = 0.708$

RV case:

Absolute average monthly difference in returns between RV and the group: 1.912

Standard Deviation of the difference of returns of RV and the group: 2.404

Hence, Information Ratio for RV is: $\frac{1.912}{2.404} = 0.795$

The above process is implemented by S&P for different levels of benchmarks i.e. strategy, sub-strategy and region.

Beta Score:

The Beta Score is defined as: $|1- \beta_{i/B}| = |1 - \rho_{i/B} * \frac{\sigma_i}{\sigma_B}|$

σ_i *and* σ_B are the standard deviation of fund i and the benchmark B, respectively. $\rho_{i/B}$ is the correlation (beta) of fund i with the benchmark B and is defined as:

$$\rho_{i/B} = \frac{cov(R_i,B)}{\sigma_i \sigma_B}$$

LS case:

Standard Deviation of Benchmark (group): 3.245

Standard Deviation of LS: 2.663

Thus, $(\sigma_\iota/\sigma_B) = \frac{2.663}{3.245} = 0.821$

Correlation of LS with the Benchmark (group): 0.966

So, the Beta Score for LS is $|1\text{-}\ 0.966 \times 0.821| = 0.207$

RV case:

Standard Deviation of Benchmark (group): 3.245

Standard Deviation of RV: 1.238

Thus, $(\sigma_\iota/\sigma_B) = \frac{1.238}{3.245} = 0.381$

Correlation of RV with the Benchmark (group): 0.782

So, the Beta Score for RV is $|1\text{-}\ 0.782 \times 0.381| = 0.702$

<u>Volatility Score:</u>

The Volatility Score is defined as: $VS_i = \frac{|\sigma_i - \sigma_{Cluster}|}{\sigma_{Cluster}}$

Where $\sigma_{Cluster}$ is the standard deviation of the cluster (SE, LO group in our case) and σ_i is the standard deviation of the strategy (LS or RV).

$|\sigma_i - \sigma_{Cluster}|$ for LS is $|2.663 - 3.245| = 0.582$

$|\sigma_i - \sigma_{Cluster}|$ for RV is $|1.238 - 3.245| = 2.007$

Thus, volatility score for LS is $\frac{0.582}{3.245} = 0.179$

And volatility score for RV is $\frac{2.007}{3.245} = 0.619$

Finally, we compute the Divergence Score for LS and RV:

DS	=	IR	+	BS	+	VS		
Divergence Score for LS	=	0.708	+	0.207	+	0.179	=	1.094
Divergence Score for RV	=	0.795	+	0.702	+	0.619	=	2.115

The LS strategy has a Divergence Score of 1.094, which is barely more than half that of RV (2.115). As previously mentioned, the Divergence Score denotes how much the fund is different from the benchmark (SE, LO in our case). So LS is closer to the cluster (group) than RV. In unreported results, we also tested LS against all the other strategies with regard to the benchmark (SE, LO group), and found that the differences in DS scores were similarly high. Thus, LS is better allocated to the group (SE, LO) than to any other strategy group.

Lastly we tested LO vs RV against the group (LS, SE) which gave Divergence Scores of 0.980 and 2.061 respectively; also SE vs RV against the group (LS, LO) which gave 0.803 and 2.063 respectively. Hence, we can conclude that, of all strategies, only LO should be allocated to the group LS, SE.

To summarize, we have used the HFR and S&P methodologies to give practical examples on how indices are constructed using classification. We found that the calculation of distances along with divergence scores (HFR) provides similar results to calculating correlations, standard deviation analysis and ANOVA (S&P). Strategies such as (LO, SE, LS) or (ED, RV, MS) are clustered in a similar way despite the different methods. This evidence suggests that differences of the indices between the vendors is mainly because they have different HF universes and different inclusion criteria in their due diligence process.

5 Comparison between HFR and S&P Cases

Both vendors use rigorous quantitative techniques, combined with qualitative processes through due diligence so as to produce high quality representative indices. Nevertheless, they use some different technical quantitative methods. HFR uses cluster analysis using Ward's (1963) linkage rule (that is very similar to the ANOVA methodology) and correlation - representation analysis through the Divergence Score. On the other hand, S&P uses a stratified sampling technique considering systematic market exposures and statistically examines the returns history of the funds. Then, in order to bring out the cohesiveness and the differences among HFs it uses the ANOVA methodology. Both firms use simulations to find the appropriate number of funds within the index and perform due diligence analysis. HFR follows somewhat more rigorous quantitative rules concerning the initial screening process. This is because they use specific formulae and eliminate any subjectivity that a stratified method may realize. Concerning the second screening and HFs allocation to specific indices (strategy groups), both vendors use robust and clear techniques with several sub-processes to ensure that the construction methodology is appropriate. Ultimately, both database vendors use quantitative techniques that produce very similar results, in other words, they cluster funds in a similar way. Furthermore, both vendors use rigorous qualitative due diligence processes with interviews, visits etc. The qualitative due diligence process is a very important element as there are some qualitative criteria not captured by quantitative processes.

Both vendors use simulation techniques to construct a relatively small number of funds that are representative in a HF index. However, there is one great difference concerning the weights that each fund has in the index. For S&P it is equally weighted whereas HFR use a more advanced method using an optimization process. In favour of asset-weighted indices, investors tend to allocate their money to larger companies and rebalance their portfolios' constructions according to the performance results of individual assets. Conversely, asset-weighted indices may sometimes be distorted due to large funds' performance. However, in the traditional markets there is a tendency towards capitalization weights that correspond better to investors' preferences (they invest in larger companies).

Regarding the index structure and calculation, both vendors use a 'tree' framework and the general principles of the NAV calculations are the same using a base index equal to 100 or 1000, hence enabling them to compute index changes in a meaningful way. Concerning the Net Asset Value (NAV) the formula is similar with the same principles and compounding rules.

Nevertheless, there is almost a decade of age difference between these two construction methodologies (2003 for S&P and 2012 for HFR). However, our purpose is not to favor one or the other. It is rather to demonstrate and present to the reader detailed index engineering construction processes in a practical way.

6 Conclusion

This paper is the first to present and analyze in an integrated and practical way HF index engineering processes and particularly classification. We have demonstrated the methods followed by two database vendors as examples that use rigorous quantitative techniques, and also qualitative processes through the due diligence process, in order to ensure that they produce high quality representative indices. The fourth section presents numerical examples emulating their quantitative processes using real data.

Our findings are that, even though database vendors use different methods or quantitative approaches, they are able to cluster funds in a somewhat similar way. This implies that the differences between the index vendors are primarily due to different datasets and different selection criteria. It is almost inevitable that indices in the same category have great differences. This is because the vendors use different datasets, have different selection criteria and use different quantitative techniques. This was demonstrated by Amenc and Martellini (2003). Other studies dealt with measurement and interpretation issues of HF indices (Brittain, 2001; Schneeweis et at. 2002), investment attractiveness of HF indices (Brooks and Kat, 2002) or with survivorship and selection biases of indices (Fung and Hsieh, 2002). Our study complements the previous studies and assists investors to understand and select better benchmarks for their investments; it helps database vendors to construct, collaborate or specialize in certain indices; it helps government authorities to collaborate with database vendors to form a common HF pool with indices; and it assists researchers to gain a better knowledge of HF fund indices as our study is complementary to other studies regarding indices.

Further research is needed towards the reproduction of our study using multiple datasets and focusing at the fund level. Examining various classification techniques of HFs, a researcher could get more robust results on the efficiency and the similarities of the quantitative methods used by the underlying vendors. This extension could include the use of further quantitative techniques beyond those used by the database vendors. Little statistical work has been done to determine the best methods for different end users. Another extension would be the evaluation and identification of the best possible construction methods or practices in the HF index composition process. This could include either evaluating specific quantitative techniques according to predefined criteria, or evaluating currently available indices against other benchmarks such as an index of indices or fund of funds index.

References

Ackermann, C., McEnally, R., and Ravenscraft, D. (1999). The Performance of hedge funds: Risk, return, and incentives, The Journal of Finance, 54(2), pp.833-874.

Agarwal, V., and Naik N. (2000). On taking the alternative route: risks, rewards, and performance persistence of hedge funds, Journal of Alternative Investments, 2(4), pp.6-23.

Agarwal, V., Daniel, N. and Naik, N. (2004). Flows, performance, and managerial incentives in hedge funds, Working Paper, [Online]. Available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=424369&download=yes [Accessed 18 July 2015].

Amenc, N., and Martellini, L. (2003). The alpha and omega of hedge fund performance measurement, [Online]. Available at http://www.edhec-risk.com/events/EDHEC_Events/Event.2004-12-09.1402/attachments/alpha%20and%20omega.pdf [Accessed 01 October 2015].

Ammann, M. and Moerth, P. (2005). Impact of fund size on hedge fund performance, Journal of Asset Management, 6(3), pp.219-238.

Bali, T., Brown, S., and Caglayan, M. (2011). Do hedge funds' exposures to risk factors predict their future returns?, Journal of Financial Economics, 101(1), pp.36-68.

Bares, P.-A., Gibson, R. and Gyger, S. (2003). Performance in the hedge funds industry: An analysis of short-and long-term persistence, The Journal of Alternative Investments, 6(3), pp.25-41.

Brittain, W.H. (2001). Hedge funds and the institutional investor, Journal of International Financial Management and Accounting, 12(2), pp.225-234.

Brooks, C. and Kat, H. (2002). The statistical properties of hedge fund index returns and their implications for investors, The Journal of Alternatives Investments, 5(2), pp.26-44.

Capocci, D. and Hubner, G. (2004). Analysis of hedge fund performance, Journal of Empirical Finance, 11(1), pp.55-89.

Das, N. (2003). Development of an analytical framework for hedge fund investment, Working Paper, [Online]. Available at http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.198.2161&rep=rep1&type=pdf [Accessed 18 July 2015].

Fung, W. and Hsieh, D. (2002). Hedge-fund benchmarks: Information context and biases, Financial Analysts Journal, 58(1), pp.22-34.

Getmansky, M. (2004). The life cycle of hedge funds: Fund flows, size and performance, Working Paper, [Online]. Available at http://web.mit.edu/mgetman/www/Files/JobMarket_Mila_Getmansky.pdf [Accesses 14 July 2015].

Harri, A. and Brorsen, B. (2004). Performance persistence and the source of returns for hedge funds, Applied Financial Economics, 14(2), pp.131-141.

Hedge Fund Research (2012). HFRX hedge funds indices methodology, [Online]. Available at https://www.hedgefundresearch.com/pdf/HFRX_formulaic_methodology.pdf [Accessed 18 January 2013].

Jawadi, F. and Khanniche, S. (2012). Modeling hedge fund exposure to risk factors, Economic Modeling, 29(4), pp.1003-1018.

Joenvaara, J., Kosowski, R., and Tolonen, P. (2012). New ‘Stylized facts’ about hedge funds and database selection bias, Working Paper, [Online]. Available at http://www.efmaefm.org/0EFMAMEETINGS/EFMA%20ANNUAL%20MEETINGS/2013-Reading/papers/EFMA2013_0410_fullpaper.pdf [Accessed 27 July 2015].

Kosowski, R., Naik, N. and Teo, M. (2007). Do hedge funds deliver alpha? A Bayesian and bootstrap analysis, Journal of Financial Economics, 84(1), pp.229-264.

Meligkotsidou, L. and Vrontos, I. (2014). Detecting structural breaks in multivariate financial time series: evidence from hedge fund investment strategies, Journal of Statistical Computation and Simulation, 84(5), pp.1115-1135.

Patel, S., Roffman, P. and Meziani, J. (2003). Standard & Poor’s hedge fund index: Structure, methodology, definitions, and practices, The Journal of Alternative Investments, 6(2), pp.59-82.

Patton, A. and Ramadorai, T. (2013). On the high-frequency dynamics of hedge fund risk exposures, The Journal of Finance, 68(2), pp.597-635.

Schneeweis, T., Kazemi, H., and Martin, G. (2002). Understanding hedge fund performance: research issues revisited – Part I, The Journal of Alternative Investments, 5(3), pp.6-22.

Ward, J.H. (1963). Hierarchical grouping to optimize an objective function, Journal of the American Statistical Association, 58(301), pp.236-244.

Table 1. Distances Between Strategies

	CT	ED	GM	LO	LS	MN	MS	RV	SE	SB	OT
CT	0.000										
ED	66.331	0.000									
GM	62.341	36.067	0.000								
LO	85.650	34.622	53.126	0.000							
LS	73.417	26.850	39.608	23.404	0.000						
MN	58.205	32.370	34.030	58.099	43.992	0.000					
MS	58.298	20.954	35.383	43.075	30.461	29.395	0.000				
RV	62.189	18.242	33.928	43.696	33.267	25.085	20.564	0.000			
SE	81.106	37.083	48.088	28.353	19.804	54.335	38.168	43.399	0.000		
SB	102.137	111.854	107.050	140.945	129.134	90.934	108.589	103.473	137.920	0.000	
OT	59.395	27.408	33.086	49.168	35.058	25.547	25.390	24.204	103.576	103.576	0.000

Table 2. Correlations Between Strategies

	CT	ED	GM	LO	LS	MN	MS	RV	SE	SB	OT
CT	1.000										
ED	-0.005	1.000									
GM	0.174	0.400	1.000								
LO	-0.076	0.875	0.444	1.000							
LS	0.011	0.816	0.536	0.930	1.000						
MN	0.197	0.219	0.285	0.193	0.308	1.000					
MS	0.247	0.765	0.391	0.709	0.748	0.336	1.000				
RV	-0.001	0.831	0.329	0.798	0.736	0.099	0.723	1.000			
SE	-0.018	0.776	0.514	0.879	0.943	0.280	0.765	0.715	1.000		
SB	0.112	-0.626	-0.366	-0.806	-0.811	-0.057	-0.545	-0.606	-0.790	1.000	
OT	0.144	0.531	0.393	0.572	0.651	0.276	0.548	0.421	0.640	-0.506	1.000

Table 3. Analysis of Variance

	CT	ED	GM	LO	LS	MN	MS	RV	SE	SB	OT
CT	0.000										
ED	8.907	0.000									
GM	9.124	0.001	0.000								
LO	4.971	0.570	0.626	0.000							
LS	3.677	1.138	1.217	0.097	0.000						
MN	62.813	24.413	24.058	32.442	36.096	0.000					
MS	2.162	2.293	2.403	0.577	0.200	41.670	0.000				
RV	19.200	1.953	1.853	4.632	6.073	12.558	8.477	0.000			
SE	0.161	6.675	6.863	3.345	2.300	56.621	1.144	15.848	0.000		
SB	186.275	113.716	112.948	130.384	137.610	32.750	148.303	85.867	175.494	0.000	
OT	1.267	3.455	3.590	1.218	0.627	46.236	0.119	10.602	0.526	156.812	0.000

Table 4. ANOVA Within Group (LO, SE, LS)

Groups	Sum	Average	Variance	
LO	290.77	0.999	11.813	
LS	298.3	1.025	7.093	
SE	334.89	1.151	10.620	
Source of Variation	SS	MS	F	F crit
Between Groups	3.828	1.914	0.194	3.006
Within Groups	8562.512	9.842		

Table 5. ANOVA Between Groups (LO, SE, LS), (OT, GM, RV) and SB

Groups	Sum	Average	Variance	
SB	15.3	0.053	27.004	
Group LO, SE, LS (Average)	307.987	1.058	9.178	
Group OT, GM, RV (Average)	275.98	0.948	1.401	
Source of Variation	SS	MS	F	F crit
Between Groups	177.141	88.570	7.070	3.006
Within Groups	10898.988	12.528		

Figure 1. Distances Between Strategies

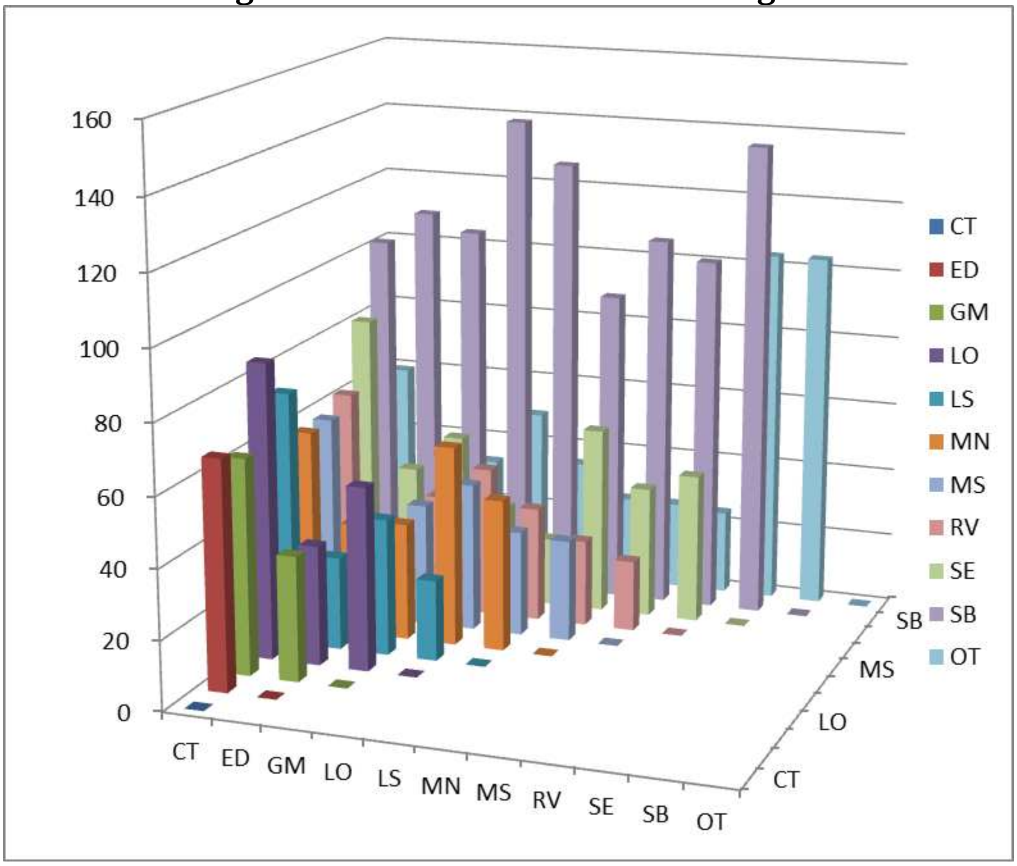

Figure 2. Correlations Between Strategies

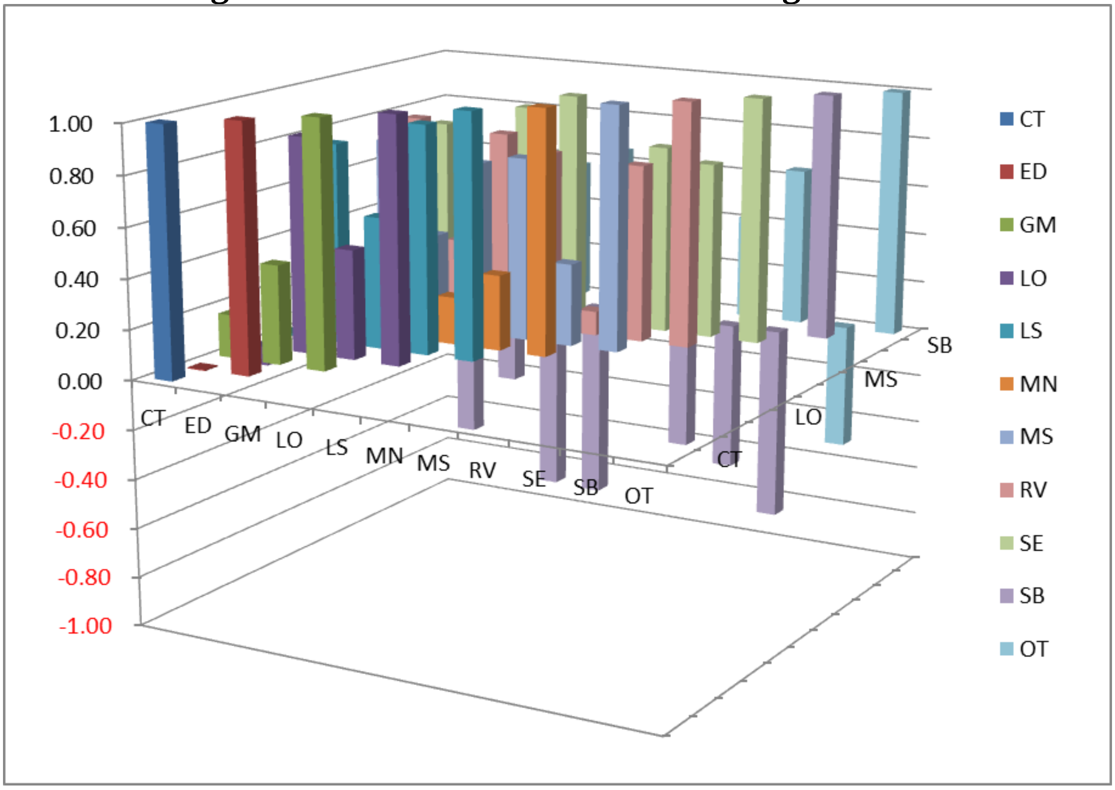

Figure 3. ANOVA Chart