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Leveraging D2D to Maximize the Spectral Efficiency of Massive MIMO Systems

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***Abstract*—In this paper, we investigate offloading of UEs in D2D mode for a massive MIMO system, where the base station (BS) is equipped with a large, but finite number of antennas and the total number of UEs is kept fixed. We derive closed-form expressions for the bounds of the overall capacity of the system. Our results reveal that there exists an optimal user offload fraction, which maximizes the overall capacity. This fraction is strongly coupled with the network parameters such as the number of antennas at the BS, D2D link distance and the transmit SNR at both the UE and the BS. Given a set of network parameters, careful tuning of the offload fraction can provide up to $5\times$ capacity gains.**

I. Introduction

The last decade has witnessed a tremendous growth in the demand for wireless data services. According to a recent report by Cisco, the global IP traffic is projected to further increase over three-fold in the next five years with mobile and wireless devices accounting for nearly 70% of this traffic [1]. As a consequence, the current research on 5G networks is focusing on the transformation of existing cellular infrastructure to cater for a bulk of simultaneously active devices requesting high data rates. The three ways to achieve this goal are i) increasing the resource pool, ii) network densification, and iii) improving spectral utilization [2]. In this paper, we focus on the ways to improve spectral utilization in cellular networks. We explore the coexistence of the two key emerging techniques used in this domain called Massive MIMO and device-to-device (D2D) communication.

In case of massive MIMO, a large antenna array is deployed at the base station (BS). The data streams are spatially multiplexed and multiple user equipments (UEs) are served simultaneously at the same time/frequency resource [3]. The distinct feature of massive MIMO is that the number of antennas is much larger than the UEs and this allows for significant improvements in link reliability and data rates due to increased spatial directivity. The additional degrees of freedom alleviate the need for sophisticated signal processing techniques and simple linear processing achieves near-optimal performance [4]. Furthermore, low-cost individual antennas can be deployed as the power radiated by an individual antenna can be reduced without compromising the performance.

Device-to-device (D2D) communication is a promising technique to further enhance the spectral efficiency (SE) (measured in bps/Hz/cell) of cellular networks. It enables direct communication between UEs in close proximity without the intervention of the BS [5]. The short range of D2D communication improves coverage and hence the data rate. It also reduces the burden of access on the BS and the core network. In case of network-assisted D2D communication, the BS handles the device discovery and resource management of D2D UEs. The two main design problems governing network-assisted D2D communication are resource allocation and mode selection [6].

Even though D2D been studied extensively in the context of cellular networks with BSs equipped with a single antenna, the analysis of D2D with massive MIMO is still in its infancy. In [7] and [8], the authors analyze an isolated cell with a single cellular UE and D2D pair and investigate how the excess antennas at BS can eliminate the interference at the D2D receiver. The sum capacity of an isolated cell with a fixed number of cellular UEs and a random number of D2D pairs has been studied in [9] for the case of cellular uplink (UL). Expressions for signal-to-interference-and-noise ratio (SINR) are derived for both the cellular and D2D cases for fixed spatial locations of UEs and the randomness is accounted for in simulations. The corresponding downlink (DL) analysis is conducted in [10] and the density of D2D pairs maximizing the sum capacity is explored.

The research on massive MIMO with D2D thus far does not consider dynamic mode selection for the UEs. It is only in [11] that the authors consider mode switching for a UE (between cellular and D2D) in cellular UL for a simple network setting with a single D2D pair. The optimality region for D2D mode satisfying the link SE requirements is defined around the D2D transmitter. The obtained results, however, cannot be directly translated to DL and scaled for multiple D2D pairs case as the location of interfering UEs is assumed to be fixed. The interference from the active D2D pairs is highly dependent on their distance from the UE under consideration and will significantly impact the findings. Also, the link SE metric does not cater for the rate experienced by all UEs.

Motivated by this, we study the offloading problem for a single cell scenario in DL, where a fixed number of UEs N is distributed uniformly around the BS. We focus on D2D in DL time slot as it is more suited for massive MIMO scenario. This is because the BS can make use of the excess degrees of freedom to interference at the D2D receivers, whereas this is not possible in the UL with single antenna UEs [7], [8], [12], [13]. While D2D communication between UEs in close proximity can provide high data rates, the transmit power of BS is much higher than a UE and it is not clear under what circumstances offloading is a better choice. There must exist a trade off between the offloaded UEs and the overall SE. The incentive of this work is to answer the following question: *Given a certain number of UEs inside a cell, what is the optimal offload fraction which maximizes the sum capacity in a massive MIMO system?*. Our main contribution is to explore this trade off and derive closed-form expressions for the approximation of the unconditional overall capacity.

The rest of the paper is organized as follows. Section II, provides the system model and preliminary analysis to compute the received SINR at the UE. Section III is the main technical section of the paper, which presents the derivation of the SE of a UE in both cellular and D2D modes. Section IV validates our analysis with numerical results. Section V concludes the paper.

II. System Model

We consider a TDD DL transmission scenario where the BS is equipped with M antennas and $N < M$ single antenna UEs are distributed uniformly in an annular region of inner radius R_{min} and outer radius R_{max} centered at the BS as shown in Fig. 1. K out of N UEs are served directly by the BS, while the remaining $N-K$ UEs are offloaded to D2D mode. Each of the $N-K$ D2D receiving UEs is associated to a unique D2D transmitter UE located randomly at the perimeter of a disk of radius r_{d2d} centered at the UEs. These transmitters can be thought of as UEs which are not receiving data in the current time slot and can establish D2D connections with their neighboring UEs to share previously downloaded files [14].

Without loss of generality, the set of all N UE locations can be written as $\mathcal{U} = \{\mathbf{x}_1, .., \mathbf{x}_K, \mathbf{x}_{K+1}, .., \mathbf{x}_N\}$. Assuming that the BS is located at the origin, the distance between the kth UE and the BS $r_{k0} = \|\mathbf{x}_k\|$ is distributed as

$$f_{r_{k0}}(x) = \frac{2x}{R_{max}^2 - R_{min}^2}, R_{min} \leq x \leq R_{max}. \quad (1)$$

r_{d2d} We adopt a simple power-law path loss model where the signal power attenuates according to $r^{-\alpha_m}, m = \{c, d\}$, where r is the distance separation and α_m denotes the path loss exponent in mode m. The BS-UE and UE-links suffer from small scale Rayleigh fading. This implies that the channel gain is independent and identically distributed (i.i.d) complex Gaussian variable with zero mean and unit variance. We further assume that the D2D pairs share the same resources as the cellular UEs and hence, both the BS-UE and UE-UE links interfere with each other. The BS is considered to have full channel state information (CSI) of the UEs and it employs zero-forcing beamforming (ZFBF) precoding. As a result, there is no signal leakage within the cellular UEs. The BS transmits a total power p_b, which is equally distributed for cellular UEs and the D2D UEs transmit a fixed power p_d, where $p_d < p_b$. The preliminary analysis for the SINR at the UEs in cellular and D2D modes is presented as follows.

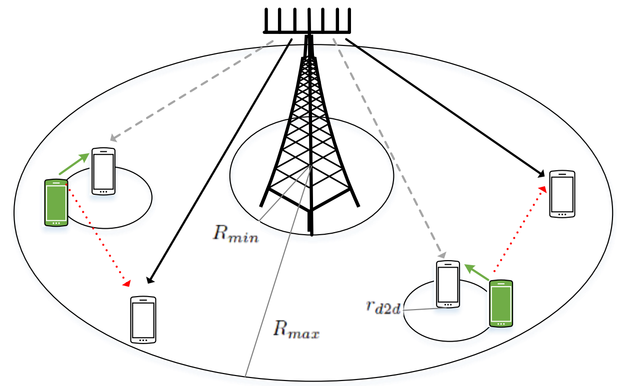


Figure 1: System Model.

A. Cellular Mode

The signal received at the kth cellular UE under ZFBF can be written as

$$y_k = \sqrt{\frac{p_b r_{k0}^{-\alpha_c}}{K}} \left(\mathbf{h}_{k0}^{BS-UE}\right)^H \mathbf{w}_{k0}^{BS} s_{k0}^{BS-UE} + \sqrt{p_d} \sum_{l=1}^{N-K} \sqrt{r_{kl}^{-\alpha_d}} h_{kl}^{UE-UE} s_l^{UE} + v_k^{BS}, \quad (2)$$

where, $\mathbf{h}_{k0}^{BS-UE} \in \mathbb{C}^{M\times 1}$ is a vector of M channel gains, v_k^{BS} is the zero mean additive white Gaussian noise (AWGN) with variance σ_{BS}^2, the complex scalar signal is such that $\mathbb{E}\left[\left\|s_{k0}^{BS-UE}\right\|^2\right] = 1$ and $\mathbf{w}_k \in \mathbb{C}^{M\times 1}$ is the precoding vector. To satisfy the maximum BS power constraint, $\mathbf{w}_{k0}^{BS} = \mathbf{g}_{k0}^{BS}/\|\mathbf{g}_{k0}^{BS}\|^2$ is normalized such that $\|\mathbf{w}_k\|^2 = 1$. The un-normalized precoding vector $\mathbf{g}_{k0}^{BS}$ for ZFBF is given as

$$\mathbf{G}^{BS} = \mathbf{H}^{BS-UE}\left(\left(\mathbf{H}^{BS-UE}\right)^H \mathbf{H}^{BS-UE}\right)^{-1}, \quad (3)$$

where $\mathbf{H}^{BS-UE} = \{\mathbf{h}_{10}^{BS-UE}, .., \mathbf{h}_{K0}^{BS-UE}\}$ and $\mathbf{G}^{BS-UE} = \{\mathbf{g}_{10}^{BS-UE}, .., \mathbf{g}_{K0}^{BS-UE}\}$. The second term in (2) denotes the interference signal from all the $(N-K)$ active D2D transmitters to the kth cellular UE, where $r_{kl} = \|\mathbf{x}_k - \mathbf{x}_l\|$ is the distance between the kth UE and the lth D2D transmitter and s_l^{UE} is the information symbol transmitted by the lth D2D transmitter. The average SE for the kth UE in cellular mode can be written as $SE_k^{BS-UE} = \mathbb{E}\left[\log_2\left(1 + SINR_k^{BS-UE}\right)\right]$, where

$$SINR_k^{BS-UE} = \frac{\gamma_b r_{k0}^{-\alpha_c}\left\|\left(\mathbf{h}_{k0}^{BS-UE}\right)^H \mathbf{w}_{k0}^{BS}\right\|^2}{\gamma_d \sum_{l=1}^{N-K} r_{kl}^{-\alpha_d}\left\|h_{kl}^{UE-UE}\right\|^2 + 1}, \quad (4)$$

where $\gamma_b = p_b/\sigma_{BS}^2$ is the transmit signal-to-noise ratio (SNR).

$$SE_k^{BS-UE}|r_{kl}\left(\beta_{kl}\right) \approx \frac{\log_2\left(1+\beta_{kl}\right)}{R_{max}^2} + \frac{2\sqrt{\beta_{kl}}}{\ln\left(2\right)}\tan^{-1}\left(\sqrt{\frac{1}{\beta_{kl}}}\right) \quad (5)$$

$$SE_j^{UE-UE}|r_{jl}\left(\beta_{jl1},\beta_{jl2}\right) \approx \frac{1}{R_{max}^2}\log_2\left(1+\frac{\gamma_d r_{d2d}^{-4}}{\beta_{jl1}+\gamma_b/R_{max}^4}\right) + \frac{2}{\ln\left(2\right)}\left[\sqrt{\frac{\gamma_b/R_{max}^4}{\beta_{jl2}+\gamma_d r_{d2d}^{-4}}}\tan^{-1}\left(\sqrt{\frac{\beta_{jl2}+\gamma_d r_{d2d}^{-4}}{\gamma_b/R_{max}^4}}\right) - \sqrt{\frac{\gamma_b/R_{max}^4}{\beta_{jl2}}}\tan^{-1}\left(\sqrt{\frac{\beta_{jl2}}{\gamma_b/R_{max}^4}}\right)\right] \quad (6)$$

B. D2D Mode

The signal received at the jth UE x_j in D2D mode from its corresponding dth D2D transmitter can be written as

$$y_j = \sqrt{p_d r_{d2d}^{-\alpha_d}}\, h_{jd}^{UE-UE} s_d^{UE} + I_j^{UE-UE} + I_j^{BS-UE} + v_d^{UE},$$

where, v_d^{UE} is the zero mean additive white Gaussian noise (AWGN) with variance σ_{UE}^2, $I_j^{UE-UE} = \sqrt{p_d}\sum_{l\neq d}^{N-K}\sqrt{r_{jl}^{-\alpha_d}}h_{jl}^{UE-UE}s_l^{UE}$ is the interference signal received by the jth UE in D2D mode from other active D2D transmitters and $I_j^{BS-UE} = \sqrt{\frac{p_b r_{j0}^{-\alpha_c}}{K}}\sum_{k=1}^{K}\left(\mathbf{h}_{j0}^{BS-UE}\right)^H \mathbf{w}_k^{BS}\mathbf{s}_k^{BS-UE}$ is the interference from the BS. The average SE for the jth UE in D2D mode can then be written as $SE_j^{UE-UE} = \mathbb{E}\left[\log_2\left(1+SINR_j^{UE-UE}\right)\right]$, where

$$SINR_j^{UE-UE} = \frac{\gamma_d r_{d2d}^{-\alpha_d}\left\|h_{jd}^{UE-UE}\right\|^2}{\left\|I_j^{BS-UE}\right\|^2+\left\|I_j^{UE-UE}\right\|^2+1} \quad (7)$$

where $\gamma_d = p_d/\sigma_{UE}^2$ is the transmit signal-to-noise ratio (SNR).

III. Spectral Efficiency Analysis

This is the main technical section of the paper. The goal of this work is to evaluate the optimal fraction of UEs to be offloaded in D2D mode. We define our performance determining metric as follows.

Definition 1. Given a fixed number of UEs N, the maximum attainable overall capacity is given by the following optimization problem

$$C_{tot} = \mathbf{max}_K \sum_{k=1}^{K} SE_k^{BS-UE} + \sum_{j=K+1}^{N} SE_j^{UE-UE}$$

where $\mu^* = (N-K^*)/N$ is the optimal offload fraction.

In the following subsections, we present our analysis pertaining to the cellular and D2D SEs.

A. Cellular Mode

The following Lemma provides the SE of a UE in cellular mode conditioned on UE locations.

Lemma 1. *Conditioned on the location of the UEs, the average SE of a UE in cellular mode can be approximated as*

$$SE_k^{BS-UE}|r_{j0},r_{jl} \approx \log_2\left(1+\frac{\gamma_b\left(M-K\right)r_{k0}^{-\alpha_c}}{K\left(1+\gamma_d\sum_l^{N-K}r_{nl}^{-\alpha_d}\right)}\right). \quad (8)$$

Proof: Since $\log\left(1+x^{-1}\right)$ is convex in x, we employ Jensen's inequality to obtain

$$SE_k^{BS-UE}|r_{j0},r_{jl} \approx \log_2\left(1+\mathbb{E}\left[\left(SINR_k^{BS-UE}\right)^{-1}\right]^{-1}\right). \quad (9)$$

The desired power in (4) is a chi-squared random variable such that $2\left\|\left(\mathbf{h}_{k0}^{BS-UE}\right)^H\mathbf{w}_{k0}^{BS}\right\|^2 \sim \chi^2_{2(M-K+1)}$. This is because the isotropic M dimensional vector is projected onto $M-K+1$ dimensional beamforming space [12]. The average channel power is then calculated as $\mathbb{E}\left[\left(\left\|\left(\mathbf{h}_{k0}^{BS-UE}\right)^H\mathbf{w}_{k0}^{BS}\right\|^2\right)^{-1}\right] = (M-K)^{-1}$. The interference power from each D2D UE is a unit mean exponential random variable. $\left\|h_{kl}^{UE-UE}\right\|^2 \sim \exp(1)$. Exploiting the independence of these random variables and plugging in the expected power values in (9), we obtain (8). ■

We de-condition $SE_k^{BS-UE}|r_{j0},r_{jl}$ in (8) with respect to distances in the following Lemma and Proposition.

Lemma 2. *The average SE of an arbitrary UE in cellular mode conditioned on the location of interfering D2D UEs can be approximated in closed-form for $\alpha_c=\alpha_d=4$ as (5) where $\beta_{kl}\left(\psi\right)=\frac{\gamma_b(M-K)}{K(1+\gamma_d\psi)R_{max}^4}$ and $\psi=\sum_{l=1}^{N-K}r_{kl}^{-4}$.*

Proof: The proof follows by averaging (8) over r_{k0} which is distributed according to (1). ■

Proposition 1. *The bounds on the unconditional average SE of a UE in cellular mode $SE_{k,LB}^{BS-UE} \leq SE_k^{BS-UE} \leq SE_{k,UB}^{BS-UE}$ can be written in closed-form as*

$$SE_{k,UB}^{BS-UE} = SE_k^{BS-UE}|r_{kl}\left(\beta_c^{UB}\right), \quad (10)$$

where $\beta_c^{UB}=\beta_{kl}\left(\psi_c^{UB}\right)$ with $\psi_c^{UB}=(N-K)\,\mathbb{E}\left[r_{kl}\right]^{-4}$ and $\mathbb{E}\left[r_{kl}\right]=\frac{128}{4\pi}R_{max}$ for the upper bound and

$$SE_{k,LB}^{BS-UE} = SE_k^{BS-UE}|r_{kl}\left(\beta_c^{LB}\right), \quad (11)$$

Parameter	Value
BS antennas M, Total users N	200, 100
BS coverage radius (Outer R_{max}, Inner R_{min})	200 m, 2m
UE deployment scheme	Uniform random
D2D range r_{d2d}	12 m
Path loss exponents: α_c, α_d	4, 4
Ratio of cellular and D2D SNR $(\gamma_b/\gamma_d)_{\text{dB}}$	30 dB

Table I: List of simulation parameters.

where $\beta_c^{LB} = \beta_{kl}\left(\psi_c^{LB}\right)$, *with* $\psi_c^{LB} = (N-K)\,\mathbb{E}\left[r_{kl}^{-4}\right]$ *and*

$$\mathbb{E}\left[r_{kl}^{-4}\right] \approx \rho_g^{-1}\Bigg[-\frac{3\sqrt{(4R_{max}^2-1)}}{4R_{max}^2}, + \left(1+\frac{1}{2R_{max}^2}\right)\cos^{-1}\left(\frac{1}{2R_{max}}\right)\Bigg]$$

$\rho_g = \sqrt{(4R_{max}^2-1)}\left(\frac{2R_{max}^2+1}{8R_{max}^2}\right)$ *for the lower bound.*

Proof: Since the terms in (5) are of the form $\log\left(1+\left(A+B\,r_{kl}^{-4}\right)^{-1}\right)$ and $\left(A+B\,r_{kl}^{-4}\right)^{-1/2}\tan^{-1}\left(\left(A+B\,r_{kl}^{-4}\right)^{1/2}\right)$. The functions are both concave in r_{kl} and convex in r_{kl}^{-4} for $A, B > 0$. We make use of Jensen's inequality to shift the expectation operator inside these functions. The D2D UEs are i.i.d distributed and their respective transmitters are uniformly located at a fixed distance r_{d2d}. For tractability, we assume that $R_{min} = 0$. This does not impact the result as $R_{max} \gg R_{min}$. The effective distance between the kth UE and lth transmitting D2D UE is then distributed according to [15]

$$f_{r_{kl}}(x) = \frac{2x}{\pi R_{max}^2}\Bigg(2\cos^{-1}\left(\frac{x}{2R_{max}}\right) - \frac{x}{R_{max}}\sqrt{1-\left(\frac{x}{2R_{max}}\right)^2}\Bigg),\ 0 \le x \le 2R_{max},$$

where $\mathbb{E}\left[r_{kl}\right] = \frac{128}{4\pi}R_{max}$. It is slightly tricky to obtain $\mathbb{E}\left[r_{kl}^{-4}\right]$. Since $f_{r_{kl}}(0) = 2/R_{max}^2 > 0$, it implies that the expectation $\mathbb{E}\left[r_{kl}^{-4}\right]$ is unbounded even when the cell size is large. To tackle this issue and avoid singularity, we introduce a minimum separation distance of 1m. Therefore, we have

$$\mathbb{E}\left[r_{kl}^{-4}|r_{kl}\ge 1\right] = \int_{x=1}^{2R_{max}} x f_{r_{kl}}(x|r_{kl}\ge 1)\,dx$$

where $f_{r_{kl}}(x|r_{kl} \ge 1) = f_{r_{kl}}(x)/\mu_g, 1 \le x \le 2R_{max}$ and $\mu_g = \mathbb{P}\left[r_{kl}\ge 1\right]$. This completes the proof. ■

B. D2D Mode

The SE of a UE in D2D mode conditioned on UE locations is given by the following Lemma.

Lemma 3. *Conditioned on the location of the UEs, the average SE of a UE in D2D mode can be approximated as*

$$SE_j^{UE-UE}|r_{j0}, r_{jl} \approx \log_2\left(1+\frac{\gamma_d r_{d2d}^{-\alpha_d}}{1+\gamma_d\sum_{l\neq d}^{N-K} r_{jl}^{-\alpha_d}+\gamma_b r_{j0}^{-\alpha_c}}\right). \tag{12}$$

Proof: We follow a different approach compared to the proof of Lemma 1. Since the desired power is exponentially distributed $\left\|h_{jd}^{UE-UE}\right\|^2 \sim \exp(1)$, the expected value of its inverse does not exist. We therefore exploit the concavity of $\log(1+x)$ to obtain $\log_2\left(1+\mathbb{E}\left[SINR_j^{UE-UE}\right]\right)$. We can write $\mathbb{E}\left[SINR_j^{UE-UE}\right] = \int_0^\infty \mathrm{e}^{-s_z}\mathbb{E}\left[\exp\left(-s_z I_j{}^{BS-UE}\right)\right]\mathbb{E}\left[\exp\left(-s_z I_j{}^{UE-UE}\right)\right]dz$, where $s_z = {}^{z\,r_{d2d}^{-\alpha_d}}/\gamma_b$. Since $\left(\mathbf{h}_{j0}^{BS-UE}\right)^H$ is independent of $\mathbf{w}_k^{BS}$, $\left\|\left(\mathbf{h}_{j0}^{BS-UE}\right)^H\mathbf{w}_k^{BS}\right\|^2 \sim \exp(1)$. $\left\|I_j{}^{BS-UE}\right\|^2$ is the superposition of K independent data streams, which implies $\sum_{k=1}^K 2\left\|\left(\mathbf{h}_{j0}^{BS-UE}\right)^H\mathbf{w}_k^{BS}\right\|^2 \sim \chi_{2K}^2$. For the D2D interference power $\left\|I_j{}^{UE-UE}\right\|^2$, we have $\left\|h_{jl}^{UE-UE}\right\|^2 \sim \exp(1)$. To ensure tractability, we invoke Jensen's inequality once again to draw the expectation inside the exponential to obtain (12). ■

Similar to the analysis for cellular mode, we derive the expressions for unconditional SE of a UE in D2D mode as follows.

Lemma 4. *The average SE of an arbitrary UE in D2D mode conditioned on the location of interfering D2D UEs can be approximated in closed-form for* $\alpha_c = \alpha_d = 4$ *as (6) where* $\beta_{jl1}(\psi) = \beta_{jl2}(\psi) = 1+\gamma_d\psi$ *and* $\psi = \sum_{l\neq d}^{N-K} r_{jl}^{-4}$.

Proof: The proof follows by averaging (12) over r_{k0}. ■

Proposition 2. *The bounds on the unconditional average SE of a UE in D2D mode* $SE_{j,LB}^{UE-UE} \le SE_j^{UE-UE} \le SE_{UB}^{UE-UE}$ *can be written in closed-form as*

$$SE_{j,UB}^{UE-UE} = SE_j^{UE-UE}|r_{jl}\left(\beta_d^{UB}, \beta_d^{LB}\right), \tag{13}$$

where $\beta_d^{UB} = \beta_{jl1}\left(\psi_d^{UB}\right)$ *with* $\psi_d^{UB} = (N-K-1)\,\mathbb{E}\left[r_{kl}\right]^{-4}$ *and* $\beta_d^{LB} = \beta_{jl1}\left(\psi_d^{LB}\right)$ *with* $\psi_d^{LB} = (N-K-1)\,\mathbb{E}\left[r_{kl}^{-4}\right]$ *for the upper bound and*

$$SE_{j,LB}^{UE-UE} = SE_j^{UE-UE}|r_{kl}\left(\beta_d^{LB}, \beta_d^{UB}\right), \tag{14}$$

for the lower bound.

Proof: The proof is similar to that of Prop. 1 with the exception that there is a negative sign inside the second term of (6). It can be easily shown that for $f(A) = \left(A+B\,r_{jl}^{-4}\right)^{-1/2}\tan^{-1}\left(\left(A+B\,r_{jl}^{-4}\right)^{1/2}\right)$, we have $f(A_1) \le f(A_2)$ for $A_1 \ge A_2$ or $A_1 - A_2 = \gamma_d r_{d2d}^{-4} > 0$. Therefore, if we re-write $SE_j^{UE-UE}|r_{jl} = T_1 + T_2$, then T_2 exhibits the opposite behavior of T_1. It is concave in r_{jl}^{-4} and

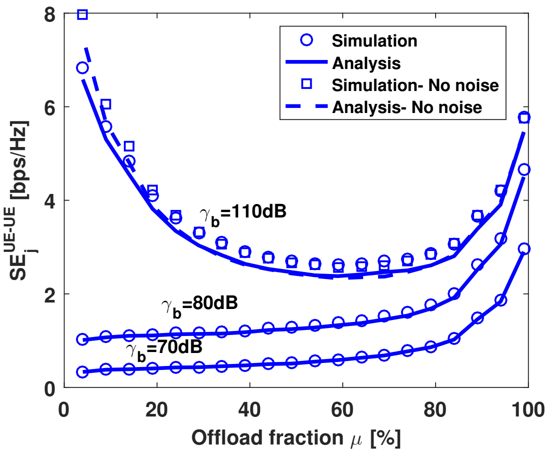


Figure 2: Effect of the offload fraction on the SE of an arbitrary UE in cellular mode.

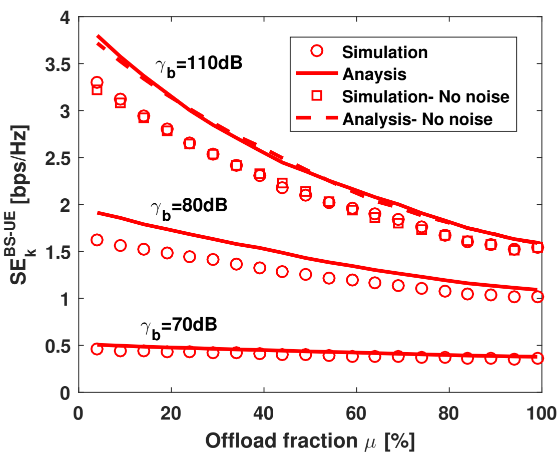


Figure 3: Effect of the offload fraction on SE of an arbitrary UE in D2D mode.

convex in r_{jl}. The coefficient $(N-K-1)$ in ψ_1 and ψ_2 denotes the number of interfering D2D pairs, excluding the one on which the performance is being measured. ■

IV. Results and Discussion

We now present the numerical results to study how the offloading mechanism is linked with the overall capacity. As a first step, we validate our analysis in (8) and (12) with the help of Monte Carlo simulations in Figs. 2 and 3 respectively. The simulation parameters are listed in table I unless stated otherwise. The simulations are repeated for 10^4 network realizations for each offload fraction. In each realization, the SE is measured at an arbitrary UE operating in cellular or D2D mode. The SE obtained from (8) and (12) and the simulations is averaged over all the realizations and hence, the effect of link distances is also averaged out. We plot the average SE per UE against the UE percentage offload fraction $\mu = (N-K)/N$. We see that the analysis for both SE_k^{BS-UE} and SE_j^{UE-UE} is in good agreement with the simulations for various transmit SNR values γ_b and γ_d[1]. We also plot the SE for the case when there is no noise, i.e. $v_k^{BS}, v_d^{UE} = 0$ or alternatively $\gamma_d, \gamma_b \to \infty$. In that case, the analysis in (8) and (12) reduces to

$$\lim_{\gamma_b,\gamma_d\to\infty} SE_k^{BS-UE}|r_{j0},r_{jl} \approx \log_2\left(1+\frac{\gamma_b/\gamma_d\,(M-K)\,r_{k0}^{-\alpha_c}}{K\sum_l^{N-K} r_{nl}^{-\alpha_d}}\right)$$

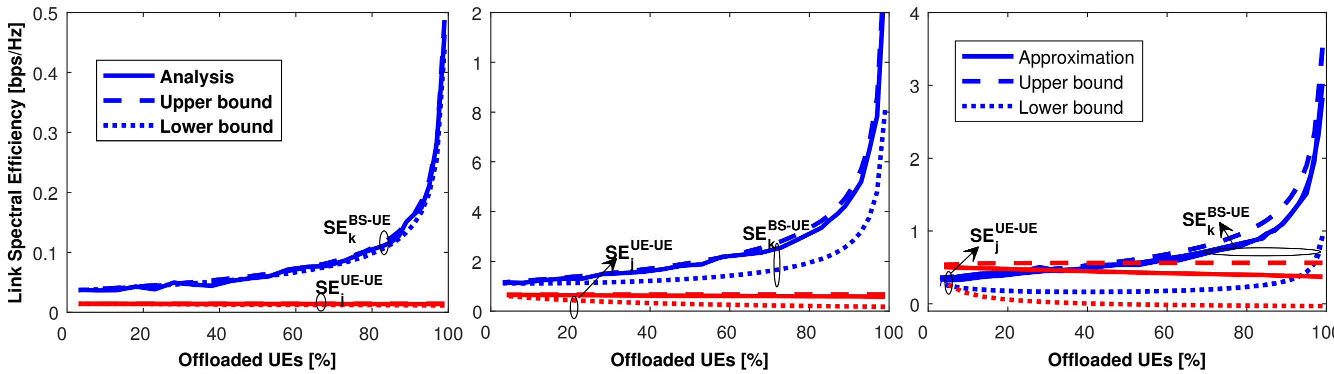
and

$$\lim_{\gamma_b,\gamma_d\to\infty} SE_j^{UE-UE}|r_{j0},r_{jl} \approx \log_2\Bigg(1 + \frac{r_{d2d}^{-\alpha_d}}{\sum_{l\neq d}^{N-K} r_{jl}^{-\alpha_d} + \gamma_b/\gamma_d\, r_{j0}^{-\alpha_c}}\Bigg).$$

We observe that for low transmit SNR values γ_b and γ_d, SE_k^{BS-UE} increases monotonically, while there is a drop in SE_j^{UE-UE} with the increase in μ. We refer to this as the low-SNR (LS) regime. The rise in SE_k^{BS-UE} with the increase in μ is because as more UEs are offloaded to D2D mode, the number of the cellular UEs inside the cell K decreases and the power allocated to each cellular UE by the BS increases. The fall in SE_j^{UE-UE}, on the other hand, is due to the increasing interference from D2D UEs. And this gap becomes more pronounced for higher values of γ_d . A different behavior is observed for SE_k^{BS-UE} in the high SNR (HS) regime which closely resembles the case when $\gamma_d, \gamma_b \to \infty$, i.e. the system is interference-limited. The SE_k^{BS-UE} is initially quite high when no UEs are offloaded. At offload percentage of 1%, the BS power is being distributed over $N-1$ UEs. Because of negligible D2D interference, a smaller allocated power is still sufficient to counter the $BS-UE$ link path loss in HS scheme. As more UEs are offloaded, the allocated power for cellular UE increases, but SE_k^{BS-UE} decreases steadily. This is because the increase in the BS power per UE is unable to cope with the increase in the D2D interference. After a certain fraction of UEs has been offloaded ($\mu \sim 50\%$), SE_k^{BS-UE} begins to rise again. This rise is now dominated by the increase in the allocated power per cellular UE. The value of SE_k^{BS-UE} at $\mu = 100\%$ is lower compared to that at $\mu = 1\%$ because of the adverse effects of the aggregate D2D interference power. In the rest of this paper, we will focus on the LS regime as HS regime is more suited for multi-cell environment, where inter-cell interference also plays a critical role. An interesting observation from Figs. 2 and 3 is that while SE_k^{BS-UE} monotonically increases in the LS regime and SE_j^{UE-UE} monotonically increases, there must exist an optimal offload fraction $\mu = \mu^*$ which maximizes C_{tot}.

After validation of our analysis, we study the accuracy of the bounds derived in Prop. 1 and 2. Fig. 4 shows that the

[1]The variation in transmit SNRs $\gamma_b = p_b/\sigma_{BS}^2$ and $\gamma_d = p_d/\sigma_{UE}^2$ is governed by several parameters including the BS and UE transmit powers p_b and p_d, the noise spectral density, BS and UE noise figures, carrier frequency, available transmission bandwidth, reference path loss, etc. In this paper, we implicitly treat the effect of these parameters by varying γ_b and γ_d directly to assess the performance of our setup. To ensure a fair comparison, a fixed, positive ratio γ_b/γ_d is maintained.



(a) $\gamma_b = 50$dB (b) $\gamma_b = 60$dB (c) $\gamma_b = 70$dB

Figure 4: Bounds on SE_k^{BS-UE} and SE_j^{UE-UE} from Prop. 1 and 2.

bounds closely match SE_k^{BS-UE} and SE_j^{UE-UE} from (8) and (12) respectively. The bounds are fairly tight especially for low values of γ_b and γ_d. For high values of γ_b and γ_d, the bounds on SE_j^{UE-UE} begin to deviate significantly while the bounds on SE_k^{BS-UE} still remain tight. The upper bound is tighter compared to the lower bound for both SE_j^{UE-UE} and SE_k^{BS-UE}. For the rest of the discussion, we use the upper bounds $SE_{j,UB}^{UE-UE}$ and $SE_{k,UB}^{BS-UE}$ to analyze the overall capacity C_{tot}.

We study the behavior of C_{tot} with respect to μ in Figs. 5-7. We also explore the impact of key design parameters on the optimal offload fraction $\mu = \mu^*$and the corresponding C_{tot}. These parameters include, the number of antennas M at the BS, D2D link distance r_{d2d} and the transmit SNRs γ_b and γ_d. From (5), we see that the SE of cellular UE SE_k^{BS-UE} increases with the increase in M, while the SE of D2D UE SE_j^{UE-UE} in (6) does not depend on M. As M increases, more and more UEs can be offlaoded to D2D mode as seen from Fig. 5. When $M = N = 100$, it is better to offload 75% UEs in D2D mode while only 6% UEs should to be offloaded when $M = 400$. Another important observation is that the selection of μ is crucial for smaller M. We can see that when $M = N = 100$, $C_{tot} = 2$bps/Hz for $\mu = 3\%$, whereas $C_{tot} = 10$bps/Hz for $\mu = 75\%$ giving 5 times better performance.

Fig. 6 shows the effect of D2D link length r_{d2d} on C_{tot} and μ^*. The increase in r_{d2d} aggravates D2D link path loss and degrades SE_j^{UE-UE}, while SE_k^{BS-UE} is independent of r_{d2d}. We see that a high overall capacity C_{tot} can be achieved with smaller values of r_{d2d} and it is better to offload UEs in D2D mode if their respective D2D transmitter is located close by. We further notice that even a slight increase of a few meters in r_{d2d} significantly reduces gains in C_{tot} from offloading, thereby causing μ^* to drop. As a consequence, the BS has to carefully evaluate the offloading strategy based on the D2D link distances before scheduling UEs for transmission.

We also study the effect of γ_b and γ_d in Fig. 7. We observe from (5) and (6) that as γ_b increases while γ_b/γ_d is fixed, both

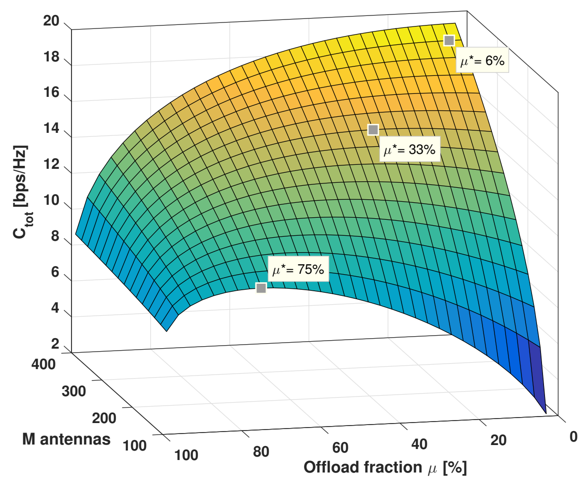


Figure 5: Effect of the number of antennas on C_{tot} and K^*: $\gamma_b = 60$dB.

SE_k^{BS-UE} and SE_j^{BS-UE} increase causing C_{tot} to increase. The increase in SE_j^{BS-UE}, however, is more that the increase in SE_k^{BS-UE} as evident from Figs. 3 and 4. This implies that with increasing SNR, more UEs should be offloaded to D2D mode to maximize C_{tot}. We see that for a 10dB rise in γ_b and γ_d, up to 30% more UEs can be offloaded to maximize C_{tot}.

V. Conclusion

In this paper we studied the performance gains achieved by network-assisted D2D communication in massive MIMO system, where a BS offloads a certain number of UEs in D2D mode to maximize the overall capacity. We derived closed-form expressions for spectral efficiency of an arbitrary UE in the cell in both D2D and cellular modes. Our results reveal that with careful selection of the offload fraction, given a set of network parameters, the overall capacity can be improved up to $5\times$.

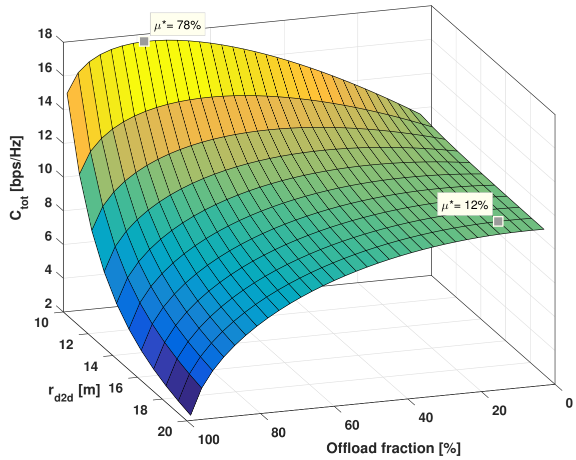


Figure 6: Effect of D2D link distance r_{d2d} on C_{tot} and K^*: $\gamma_b = 60$dB

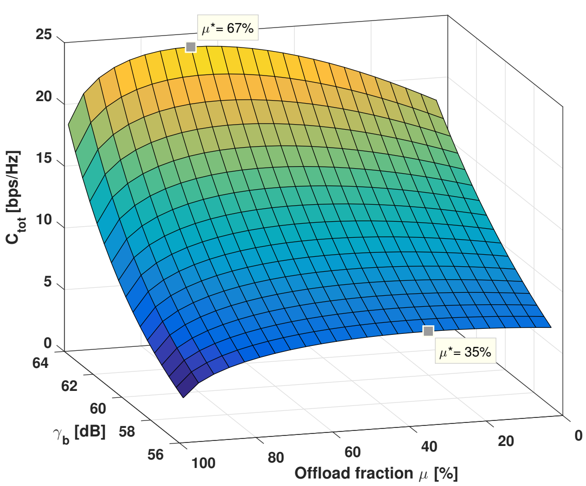


Figure 7: Effect of γ_d on C_{tot} and K^*.

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