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# 1 Model and Design of a Four-Stage 2 Thermoacoustic Electricity Generator with Two 3 Push-Pull Linear Alternators

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6 **Abstract.** Recent work of the authors on a two-stage thermoacoustic  
7 electricity generator with a “push-pull” linear alternator has led to further  
8 investigations of multi-stage thermoacoustic engines. The investigation  
9 started with proposing a new feedback loop to reduce the length and the  
10 volume of the two-stage engine. The use of acoustic inertance-compliance  
11 reduced the engine total length from 16.1 m to 7.5 m and maintained the  
12 performance. A four-stage traveling wave thermoacoustic engine is  
13 considered working as an electricity generator with two push-pull linear  
14 alternators. The proposed engine considered the thermoacoustic core  
15 geometries of the two-stage engine. The engine consists of four identical  
16 quarter-wave stages connected in series to form one wave length  
17 thermoacoustic engine. The engine compact model is 5.2 m in length.  
18 Using pressurized helium at 28 bar as a working gas, the simulation  
19 showed that engine generates 261 W of electricity using heat source at  
20 exhaust gases temperature of the IC engines. The simulation has been done  
21 using DeltaEC package. The research shows that the four-stage engine  
22 does not require a compliance in the feedback loop as the core itself acts as  
23 a compliance. The results presented in this paper demonstrate that four-  
24 stage thermoacoustic engine has a good potential for waste heat recovery  
25 and inexpensive electricity generation.

## 26 1 Introduction

27 Day by day, the demand for energy rises all over the †world. In recent years a lot of  
28 environmental impacts have been discovered and proven as being caused by power  
29 generation technologies. This creates a thoughtful approach towards clean and  
30 environmentally friendly technologies. Thermoacoustic power generation technology could  
31 be considered as one such technology. Sound waves in fluids are normally regarded as  
32 coupled oscillations of pressure and velocity; however these are also associated with  
33 temperature oscillations. These temperature changes are too small to be noticed in the

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34 typical sound propagation processes in air at atmospheric pressure. However, in highly  
35 pressurised gases (e.g. at pressures of the order of 30-60 bar) and at high acoustic intensity  
36 the temperature changes become significant. The temperature effects can be utilised for  
37 energy conversion processes when the acoustic wave propagates next to the solid body.  
38 Using a sound source, a temperature gradient can be built up in the solid. Imposing a  
39 temperature gradient on the solid may lead to the generation of acoustic power. These  
40 processes form the back-bone of thermoacoustic technologies [1]. Thus, thermoacoustics is  
41 the interaction between thermodynamics and acoustics in the fluid medium inside a special  
42 solid configuration within acoustic resonance conditions.

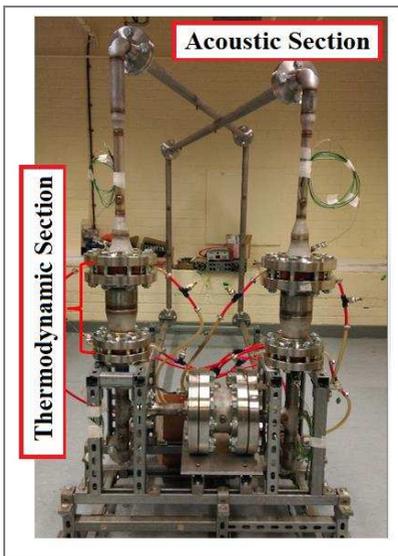
43 Internal combustion engines are heat engines that combust fuel with an oxidizer inside  
44 the engine. The thermal energy released by burning the fuel will leave the engine as thermal  
45 energy rejected by the cooling water, exhaust gases, miscellaneous losses or as brake  
46 power. Taymaz [2] reported results for the heat balance of a standard four stroke diesel  
47 engine with a capacity of 6.0 litres. The heat rejected to the exhaust system ranged from  
48 24% to 29% of the total energy released from the fuel depending on the engine load. The  
49 availability of this large amount of waste energy gives the potential to harvest waste heat  
50 and convert it to useful work. Johnson [3] outlined the technologies to generate cooling  
51 power from waste heat from exhaust gases. Thermoacoustic technology has been listed as  
52 one of the four vital technologies. Later on, Jadhao and Thombare [4] highlighted  
53 thermoacoustic technology as a direct electricity generator in their review of internal  
54 combustion engine exhaust gas heat recovery. The advantages of using thermoacoustic  
55 technology electricity generator as the system is elegant, reliable, low cost, environmentally  
56 safe and has no moving parts (in thermodynamic section). The disadvantages were low  
57 efficiency and power density.

58 The first looped tube thermoacoustic engine was presented by Yazaki et al. [5]. The  
59 configuration of travelling wave engine was a one wavelength loop which contained the  
60 thermoacoustic core at a specific location. The experimental results showed that the  
61 travelling wave engine uses the temperature gradient at the regenerator to perform as an  
62 acoustic amplifier. Kitadani et al. [6] investigated the electricity generation from a looped  
63 tube engine. A loudspeaker was connected within the loop to convert the acoustic power to  
64 electricity and was placed at the end of a branch optimised to be a quarter wavelength. The  
65 dependence of sound amplification on the phase difference between the acoustic pressure  
66 and velocity was highlighted here, and it was clarified that there are not many controlling  
67 parameters to adjust the phase difference. The engine generated 1.1 W of electricity out of  
68 330 W input heat. Kang et al. [7] constructed a two-stage looped tube engine having two  
69 loudspeakers in different configurations; within the loop line and in a branch. This engine  
70 used pressurized helium (18 bar) as working gas. The idea was to put a thermoacoustic core  
71 in each of the two high impedance zones and a loudspeaker in each of the two low  
72 impedance zones, to avoid acoustic losses. The loudspeaker connected at the branch helps  
73 to tune and set the acoustic phasing difference (velocity and pressure). A ball valve was  
74 introduced as an acoustic load to correct the acoustic field. At 171 Hz working frequency,  
75 the maximum generated electric power was 204 Watts at 3.41% thermal-to-electric  
76 efficiency and a maximum efficiency of 3.43% was obtained at 183 W electric power.

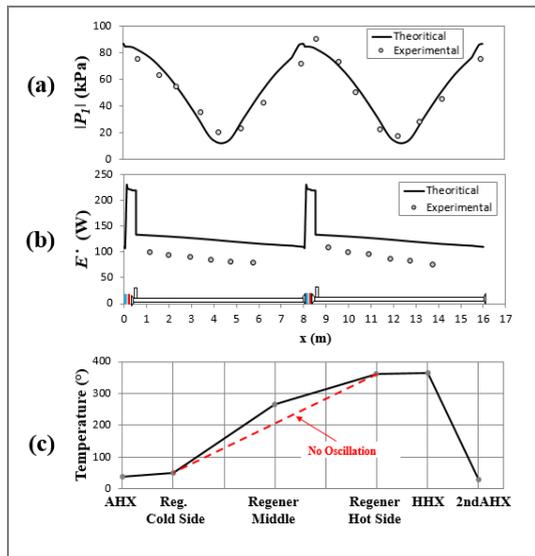
77 Four-stage thermoacoustic engines were pioneered by de Blok [8,9]. Four novel engines  
78 were built with four identical self-matching stages. Basically, they have low acoustic loss  
79 because of lower acoustic dissipation in the resonance and feedback loop. The identical  
80 four stages were presented as feasible from the construction point of view because of  
81 having identical components per stage. The largest engine in this group is named  
82 ThermoAcoustic Power (TAP) generated 1.64 kW of electricity using available waste heat  
83 of 20 kW, at working frequency of 40 Hz. Senga and Hasegawa [10] built a four-stage  
84 engine similar to the de Blok [9] configuration, using air at atmospheric pressure as

85 working medium. The main difference is that it has one load and hence the cross section  
 86 area of the regenerators increased with the acoustic power flow direction after the load. The  
 87 acoustic power generated did not reach 1 W on this rig. Zhang and Chang [11] numerically  
 88 studied the onset temperature, mean pressure, working gas, hydraulic radius and the number  
 89 of stages of a four-stage engine similar to the de Blok [9] configuration. The results were  
 90 used to develop another numerical study, of replacing one of the engine stages with a  
 91 refrigerator stage by Zhang [12]. The simulation results showed that it can reach a relative  
 92 Carnot coefficient of performance of 28.5% at a refrigeration temperature of 5°C.

93 This paper starts with a description of the design and construction of two stage  
 94 thermoacoustic engine which generate electricity by running a push-pull linear alternator.  
 95 Followed by the first performance measurements results of the engine. A model having a  
 96 developed feedback loop is proposed to reduce the length of the engine. A four-stage  
 97 engine running two push-pull linear alternator will be given at the end of this paper.



98 **Fig 1.** Photograph of the experimental apparatus.



**Fig 2.** Simulation and experimental results (a) pressure amplitude, (b) acoustic power flow along the engine (c) thermoacoustic core temperature distribution.

102 **2 Experimental Results**

103 The apparatus is a one wavelength, 16.1 m long, looped tube engine filled with 28 bar  
 104 helium. It consists of two identical stages each having a power extraction point and the  
 105 linear alternator connecting these two points, as shown in Figure 1. Each stage consists of  
 106 thermodynamic section where heat is transferred to or from the working gas and acoustic  
 107 section which comprises of pipes transmitting the acoustic power from one place to another.  
 108 The details of the rig could be found in [13,14].

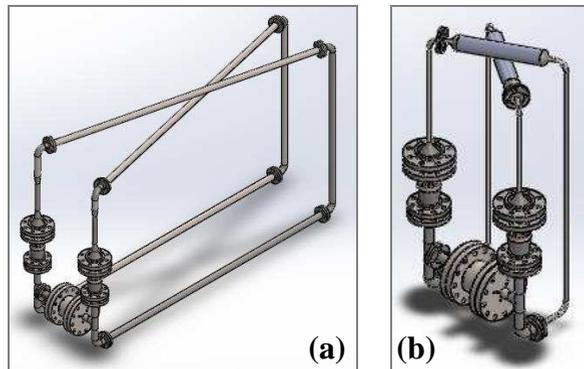
109 The engine runs at a frequency of 54.68 Hz and the optimum load resistance is 30 Ω.  
 110 The maximum electricity generated was 48.6 W at 2.7% thermal-to-electric efficiency. The  
 111 regenerator temperature difference was 310 K. The pressure amplitude distribution has  
 112 been measured in sixteen locations along the engine loop. Additionally, the simultaneous  
 113 measurement of pressure signals and phase difference between each pair of adjacent  
 114 transducers allows calculating the acoustic power at the midpoint between adjacent  
 115 transducers. As shown in Figure 2a, the distribution of the pressure amplitude along the

116 engine shows a good match between the experimental the theoretical results. In addition,  
117 Figure 2b compares the experimental acoustic power to the simulation results. Generally,  
118 the trend of two results is in agreement, while there is a 20-30% discrepancy in absolute  
119 power levels. Figure 2c, shows that all temperature distribution along the thermoacoustic  
120 core. The non-linear temperature distribution graph confirms that there is Gedeon  
121 streaming, compared to the linear distribution at no oscillation run.

### 122 3 Feedback Loop Optimization

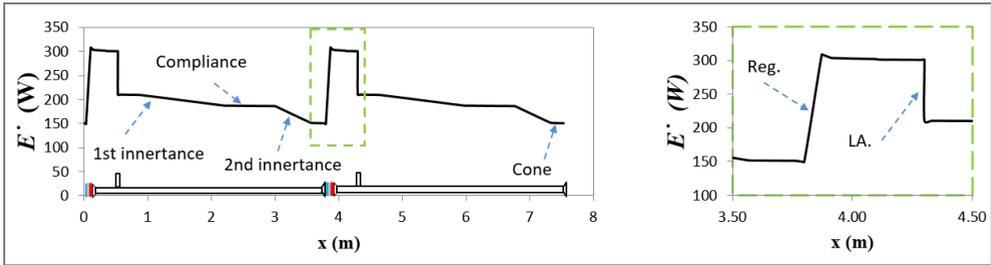
123 The main purpose of the feedback loop is to deliver acoustic power from the end of a  
124 stage to the beginning of the other at a convenient acoustic phasing. The built two-stage  
125 thermoacoustic engine is 16.1 m long, each stage is 8.05 m long. The feedback loop length  
126 of each stage is approximately 7.5 m long. The feedback loop consists of approximately  
127 7.05 m long, 1.5 inch (40.9 mm) diameter pipe, 275 mm long, 1 inch (26.6 mm) diameter  
128 pipe, and reducers. This feedback loop could be made shorter by changing the cross-section  
129 of the pipes.

130 Firstly, the feedback loop has been modelled separately investigating a shorter  
131 configuration delivering sound at the same phasing. Secondly, the feedback loop of the  
132 engine model was changed from the long straight loop to the new short multi-cross-section  
133 loop. The length of the previous feedback loop was needed to shift the acoustic phasing of  
134 the pressure and volume flow velocity by  $180^\circ$  between stages. This could be achieved by  
135 using multi-cross-section feedback loop. The wide cross-section pipe shifts the volumetric  
136 flow velocity phase by acting as acoustic compliance, while narrow pipe shifts the pressure  
137 phase by acting as acoustic inertance. The combination of compliance-inertance shifts the  
138 acoustic phasing in much shorter length. The new proposed feedback loop consists of  
139 inertance-compliance-inertance. The first section is a 300 mm long,  $\frac{1}{2}$  inch diameter pipe  
140 which is part of the previous configuration, followed by a cone leading to 1313 mm long  
141 pipe of  $\frac{3}{4}$  inch (20.9 mm) diameter. A non-standard 50 mm long reducer is used to connect  
142 the  $\frac{3}{4}$  inch pipe to a 3 inch pipe. A 420 mm long and 3 inch (77.9 mm) diameter pipe acts  
143 as an acoustic compliance which shifts the volumetric flow velocity phase with  
144 approximately  $40^\circ$ . A non-standard 50 mm long reducer is used to connect the 3 inch  
145 compliance to a  $\frac{3}{8}$  inch inertance. The inertance is a 584 mm long and  $\frac{3}{8}$  inch diameter pipe  
146 which shifts the pressure phase of approximately  $62^\circ$ . The last part is 189 mm reducer  
147 (combination of standard reducers) connecting the  $\frac{3}{8}$  inch pipe to the 4 inch core. The  
148 lengths of the two inertances and compliance has been optimized carefully aiming to  
149 achieve the acoustic conditions at a shorter length possible. Figure 3 compares the engine  
150 configuration for both feedback loops.



151  
152 **Fig 3.** Thermoacoustic engine (a) with previous feedback loop, (b) with proposed feedback loop.

153 The engine model is 7.5 m long and runs at 56.8 Hz. At a 375 K regenerator  
 154 temperature difference, the engine generates 130 W of electricity. Figure 4 shows the  
 155 simulation results of the engine. Figure 4 shows the calculated acoustic power  
 156 distribution along the engine. Clearly the acoustic power is generated in the regenerators and  
 157 mainly extracted/dissipated at the linear alternator junctions. The figure shows that the inrtance  
 158 dissipates acoustic power much more than the compliance. Also, the smaller diameter  
 159 inrtance ( $\frac{3}{8}$  inch pipe) dissipates acoustic power more than the bigger diameter ( $\frac{3}{4}$  inch)  
 160 pipe.

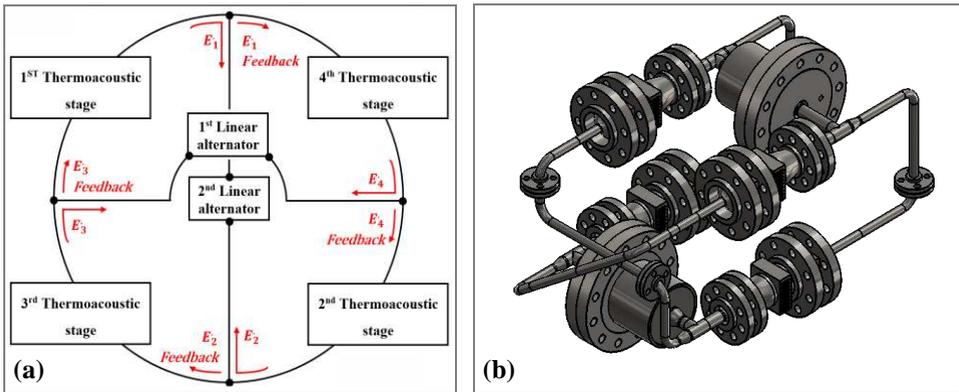


161  
 162 **Fig 4.** Simulation acoustic power flow along the engine.

### 163 4 Four-stage thermoacoustic engine

164 Basically, the development of the two-stage engine to a four-stage engine involves  
 165 changing the acoustic section only by adopting the use of acoustic inrtance-compliance.  
 166 All the parts of the thermodynamic section will be kept the same, so that the parts of the  
 167 current two-stage engine could be used in the next research.

168 The configuration consists of four identical stages each having a power extraction  
 169 points, and the linear alternators connecting these four points as shown in Figure 5a.  
 170 Clearly, there is a power extraction at each stage followed by a feedback loop which leads  
 171 to the next stage. Theoretically, the flow pressure amplitude and volumetric flow rate of  
 172 each stage is identical in all the stages. Figure 5b shows a proposed layout of the engine.



173  
 174 **Fig. 5** (a) Conceptual drawing of the proposed four-stage engine, (b) A proposed layout of the four-  
 175 stage engine.

176 The modelling was done using DeltaEC package created by the Los Alamos National  
 177 Laboratory [15]. The DeltaEC shooting method cannot run multi-identical stages. The  
 178 modelling has been done as a quarter of the engine which is one stage and the other three  
 179 stages were represented as a self-excited hypothetical flow. There are two self-excited  
 180 hypothetical flows in this engine, each has a specific flow characterisation based on the

181 understanding of the identical stages and push-pull connection. The first self-excited  
 182 hypothetical flow is the flow entering the first stage which represents the flow at the end of  
 183 the fourth stage. The boundary conditions of the this flow were set in the following manner:  
 184 the temperature, pressure amplitude, volumetric flow rate and total power were set to be  
 185 equal at the beginning and the end of the simulated stage (at  $X=0$  and  $X=\lambda/4$ ), as

$$186 \quad T_{x=0} = T_{x=\lambda/4} \quad (1)$$

$$187 \quad |P_{x=0}| = |P_{x=\lambda/4}| \quad (2)$$

$$188 \quad |U_{x=0}| = |U_{x=\lambda/4}| \quad (3)$$

$$189 \quad \dot{H}_{2,x=0} = \dot{H}_{2,x=\lambda/4} \quad (4)$$

190 where  $T$  is temperature,  $P$  is pressure amplitude,  $U$  is volumetric flow, and  $\dot{H}_2$  is the total  
 191 power. The phases of pressure and velocity were set to be shifted by  $180^\circ$  at the end of the  
 192 stage with reference to the beginning (at  $X=0$  and  $X=\lambda/4$ ), as

$$193 \quad Ph(P)_{x=0} = Ph(P)_{x=\lambda/4} + 180^\circ \quad (5)$$

$$194 \quad Ph(U)_{x=0} = Ph(U)_{x=\lambda/4} + 180^\circ \quad (6)$$

195 where  $Ph(P)$  is pressure phase and  $Ph(U)$  volumetric flow phase.

196 The second self-excited hypothetical flow is applied to the other side of the linear  
 197 alternator. The boundary conditions of the second self-excited hypothetical flow were set  
 198 based on the physics of push-pull operation. The pressure amplitude, volumetric flow and  
 199 velocity phase were set to be equal on both sides of the alternator piston. Only the pressure  
 200 phase was set to be out of phase (phase difference of  $180^\circ$ ). The three boundaries were set  
 201 as targets at locations A and B which are before and after the linear alternator piston, as

$$202 \quad |P_A| = |P_B| \quad (7)$$

$$203 \quad Ph(P)_A = Ph(P)_B + 180^\circ \quad (8)$$

$$204 \quad \dot{H}_{2,A} = \dot{H}_{2,B} \quad (9)$$

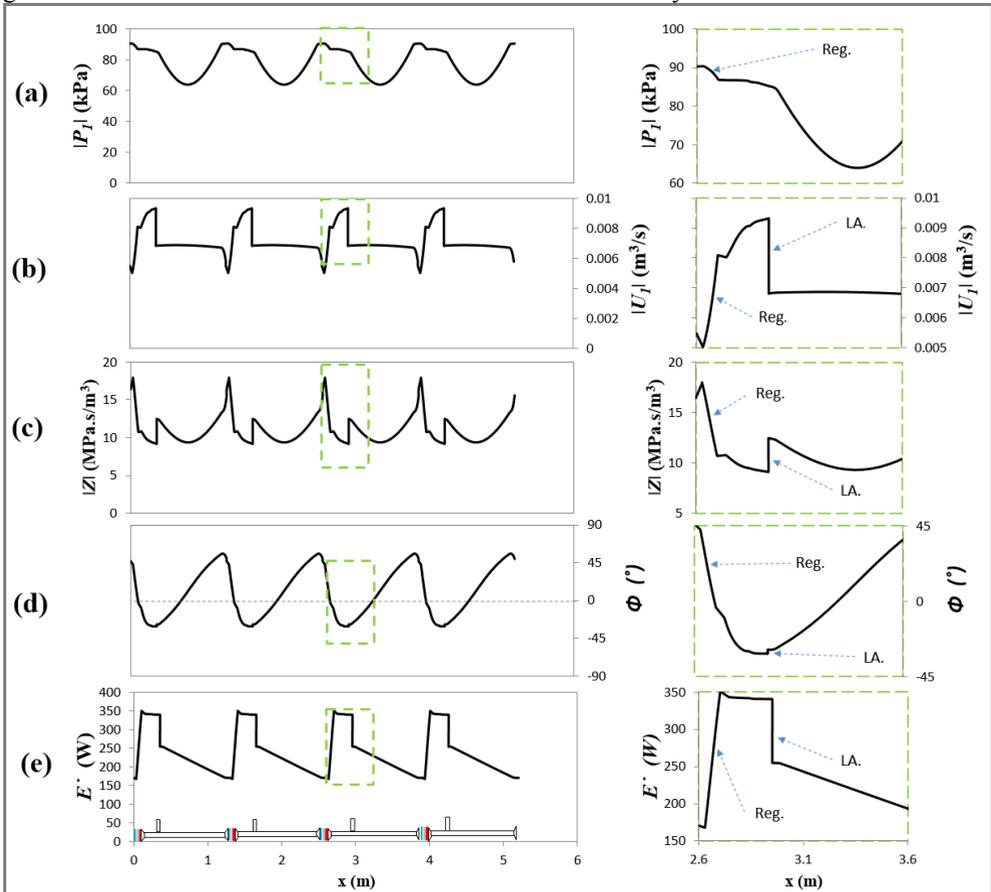
205 As the required acoustic impedance and thermodynamic section dimensions  
 206 optimization have already been done in the previous design [13,14], the layout and  
 207 feedback loop is optimised in this part. The optimum feedback loop tube diameter was 10  
 208 mm. As explained before in Section 3, the narrow pipe acts as an acoustic inertance and  
 209 shifts the pressure phase by approximately  $75^\circ$ . There is no need for an acoustic compliance  
 210 as the volumetric flow velocity phase already shifts at the thermoacoustic core by  
 211 approximately  $65^\circ$ . A feedback loop of 840 mm was found to be sufficient to match the  
 212 stages. The simulation optimization introduced the dimensions of the physical parts. Figure  
 213 5b shows a proposed layout of the four-stage engine.

214 The modelling was done using DeltaEC package created by the Los Alamos National  
 215 Laboratory [15]. Figure 6 shows the simulation results. The graphs on the right are  
 216 magnified areas marked by green dashed lines of the graphs on the left. Figure 6a shows the  
 217 calculated pressure amplitude distribution along the engine loop. There are peaks at each  
 218 regenerator of the four stages. There is a major pressure drop at the regenerator caused by  
 219 the flow resistance. Figure 6b shows the distribution of volumetric velocity along the  
 220 thermoacoustic engine. The engine has been designed to have the lowest volumetric flow

221 rate at the regenerator to minimize the viscous dissipation. There is a volumetric flow rate  
 222 drop at the linear alternator branch caused by the power extraction at the linear alternator.

223 Figure 6c is the acoustic impedance profile along the engine. It can be seen that the  
 224 acoustic impedance is nearly maximum at the regenerators which is one of the design  
 225 strategies. The acoustic impedance drops within the regenerator length, which is caused by  
 226 the pressure drop and velocity amplification. Figure 6d shows the phase difference between  
 227 the velocity and pressure oscillation along the engine. This graph illustrates that the phase  
 228 difference is zero within the regenerator limited which is preferred, and couldn't be  
 229 maintained in the previous two-stage design. Figure 5e shows the acoustic power  
 230 distribution along the engine. Clearly, the narrow feedback loop dissipates a big portion of  
 231 the generated acoustic power comparing to the previous design.

232 The simulation considers a heat source at a temperature of internal combustion engine  
 233 exhaust gases, which is able to maintain a regenerator temperature difference of 375 K.  
 234 The simulation results showed that the engine ran at 56.6 Hz and the total engine length  
 235 was 5.2 m. Each alternator generated 130.5 W of electricity, and hence, the engine  
 236 generated 261 W at the theoretical thermal to electric efficiency of 16.2%.



237  
 238 **Fig. 6** Simulation results (a) pressure amplitude, (b) volumetric velocity, (c) acoustic impedance, (d)  
 239 phase difference angle and (e) acoustic power flow along the engine.

## 240 **5 Conclusion**

241 The experiments showed that the thermoacoustic technology can be used to convert heat at  
242 internal combustion engine exhaust gases to useful electricity. A two-stage configuration  
243 can run a linear alternator in push-pull mode. The simulation of the feedback loop showed  
244 that the feedback loop length could be reduced from approximately 7.5 m to 3.25 m by  
245 using a combination of acoustic inertance and compliance. The simulation showed that the  
246 engine could be developed from two-stages with one linear alternator to four-stages with  
247 two linear alternators. The results illustrated that it ran at 56.6 Hz and the total engine  
248 length was 5.2 m. Each alternator generates 130.5 W of electricity, and hence, the engine  
249 generates 261 W at the theoretical thermal to electric efficiency of 16.2%.

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