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Investigation of Nonlinear Chirp Coding for Improved Second Harmonic Pulse Compression

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Abstract

In this paper, a nonlinear frequency-modulated (NLFM) chirp coding was investigated to improve the pulse compression of the second harmonic chirp signal by reducing the range sidelobes level. The problem of spectral overlapping between fundamental and second harmonic component (SHC) was also investigated. Therefore, two methods were proposed, method-I show the scenario of non-overlap condition and method-II with pulse inversion technique was used for overlap harmonic condition. In both methods, the performance of the NLFM chirp was compared with the reference linear frequency-modulated (LFM) chirp signals. Experiments were performed using a 2.25

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MHz transducer mounted coaxially at a distance of 5 cm with a 1 mm hydrophone in a water tank and the peak negative pressure of 300 kPa was set at the receiver. Both simulations and experimental results show that the peak sidelobe level (PSL) of the compressed SHC of NFLM chirp was improved by at least 13 dB in method-I and 5 dB in method-II when compared with the PSL of LFM chirps. Similarly, the integrated sidelobe level (ISL) of the compressed SHC of NFLM chirp was improved by at least 8 dB when compared with the ISL of LFM chirps. In both methods, the axial mainlobe width of the compressed NFLM chirp was comparable to the LFM signals. The signal-to-noise ratio of the SHC of NFLM was improved by up to 0.8 dB, when compared with the SHC of LFM signal having the same energy level. Results were also presented which show the robustness of NFLM chirp under a frequency dependent attenuation of 0.5 dB/[cm×MHz] up to penetration depth of 5 cm and a Doppler shift of up to 12 kHz.

Keywords: Ultrasound, Nonlinear chirp, Pulse compression, Pulse inversion, Harmonic imaging

1 Introduction

2 In recent years, ultrasound harmonic imaging has become prevalent in
3 commercial medical ultrasound imaging systems. Ultrasound harmonic imag-
4 ing relies on the second or higher order harmonic components. In tissue
5 harmonic imaging, these nonlinear harmonics are produced by finite ampli-
6 tude distortion of ultrasound waves propagating through biological tissue
7 (Duck, 2010). Whereas in ultrasound contrast imaging, these harmonics are
8 produced by the nonlinear scattering from contrast microbubbles (de Jong
9 et al., 2002; Maresca et al., 2014). Ultrasound images based on the nonlinear
10 second harmonic component (SHC) provide improved spatial resolution with
11 reduced reverberation artifacts when compared to conventional (fundamen-
12 tal) B-mode imaging (Jensen, 2007; Wells, 2006).

13 Coded excitation techniques were originally introduced in radar commu-
14 nication and now are widely used in medical ultrasound imaging systems
15 to provide improved signal-to-noise ratio (SNR) (Cook and Bernfeld, 1967;
16 Chiao and Hao, 2005). Coded excitation with long duration linear frequency
17 modulated (LFM) chirp signals offer the potential to improve the SNR of
18 the SHC. This can be done without increasing the peak excitation pressure
19 or mechanical index (MI) and without reducing the system frame-rate (Cob-
20 bold, 2007). However, on the receiving side of the system, harmonic matched
21 filters are typically used to extract and compress the SHC and to recover sig-
22 nal axial resolution (Kim et al., 2001; Arif et al., 2010a). The SNR of a
23 chirp signal depends on the time-bandwidth product (TBP) and can also
24 be improved by extending the signal bandwidth. However, in ultrasound
25 harmonic imaging application, the signal bandwidth extension is restricted

26 by the finite bandwidth of the ultrasound transducer to accommodate both
27 the fundamental and the SHC, and the spectral overlapping between the
28 fundamental and the SHC ([Averkiou, 2000](#)).

29 The power spectrum of an unweighted LFM chirp is approximately rect-
30 angular in shape and yields a sinc-like function after pulse compression. The
31 compressed chirp signal contains a peak sidelobe level (PSL) at ~ -13 dB.
32 This higher value of PSL will mask out the mainlobe width (MLW) from the
33 weak scatterer and will potentially degrade the image contrast by appearing
34 as false echoes. Therefore, the higher values of the PSL are unacceptable in
35 modern medical ultrasound imaging systems operating at a dynamic range
36 of more than 60 dB ([Johnston and Fairhead, 1986](#); [Misaridis and Jensen, 2005b](#)).
37

38 In order to reduce the higher PSL of the compressed chirp signal, a strong
39 weighting function is applied either on the transmitting signal or on the re-
40 ceived matched filter; the latter case is termed a mismatched filter. Window-
41 ing on the excitation signal causes a reduction in the transmitting energy and
42 hence penetration depth, whilst windowing on the matched filter results in
43 reduced gain in the SNR and axial resolution. Therefore, a tradeoff between
44 the MLW and PSL is exist in the pulse compression process of the LFM
45 signal ([Adams, 1991](#); [Milleit, 1970](#)).

46 Nonlinear frequency modulated (NLFM) chirp signals provide an alter-
47 native means to modify the rectangular power spectrum of the LFM chirp
48 into a desirable shape. The NLFM chirp can be designed to optimise the
49 signal transmitting energy and the shape of the power spectrum so that it
50 matches spectrally with the transfer function of the transducer. This re-

51 sults in more energy transmitted through the transducer which potentially
52 improve the SNR and penetration depth. Also, reduced PSL will be get af-
53 ter pulse compression without using any additional windowing function on
54 the matched filter ([Harput et al., 2013](#); [Arif et al., 2010b](#); [Gran and Jensen, 2007](#);
55 [Pollakowski and Ermert, 1994](#)).

56 The effects of shaping the transmitting spectrum using the NLFM chirp
57 for improved spectral matching with the transmitter were first studied by PS.
58 Brandon ([Brandon, 1973](#)). He had designed the nonlinear pulse compression
59 system for radar to get the high resolution with reduced loss in the SNR.
60 The NLFM chirp was designed using the least squares optimization method
61 for synthetic transmit aperture B-mode imaging ([Gran and Jensen, 2007](#)).
62 The NLFM signal with the quadratic instantaneous frequency function was
63 designed and implemented for tissue harmonic imaging ([Song et al., 2011](#)).

64 In this paper, NLFM chirp coding was investigated as an excitation
65 scheme in the area of ultrasound harmonic imaging. The aim was to re-
66 duce the PSL after pulse compression and to improve the SNR of the second
67 harmonic chirp component.

68 The remaining sections of the paper are divided as follows: Section-II
69 describes the basic theory and design methods of NLFM and reference LFM
70 chirp signals, Section-III describes about the proposed methods, Section-IV
71 describes the simulation and experimental procedures with the post process-
72 ing of nonlinear received signals, the results of harmonic pulse compression
73 and Doppler sensitivity evaluation of designed chirp signals are examined in
74 Section-V, and finally the performance of second harmonic pulse compression
75 achieved using the Nonlinear chirp coding is discussed in Section-VI.

76 **Theory and Signal Design**

77 *Nonlinear Frequency Modulated (NLFM) Signals*

78 In exponential form, the time domain chirp signal $x(t)$ can be expressed
79 as [Misaridis and Jensen \(2005a\)](#),

$$x(t) = p(t) e^{j2\pi\phi(t)} \quad (1)$$

80 where $p(t)$ and $\phi(t)$ are the amplitude and phase modulation functions
81 of the chirp signal, respectively.

82 The spectrum of the chirp signal given in (1) is expressed as,

$$X(\omega) = \int_{-\infty}^{\infty} p(t) e^{j\{-\omega t + \phi(t)\}} dt \quad (2)$$

83 The integrand $[-\omega t + \phi(t)]$ in (2) is an oscillating function which is varying
84 at the rate of $\frac{d}{dt}[-\omega t + \phi(t)]$. The major contribution to the chirp spectrum
85 occurs when the rate of change of oscillating function is minimal and is also
86 referred to as the stationary phase point, this can be expressed as

$$\frac{d}{dt}[-\omega t + \phi(t)] = 0 \quad (3)$$

87 In (3), ω and t are two independent variables. Therefore, the particular
88 value of t that can satisfy the condition given in (3) can be found by assum-
89 ing the value of ω . In the case of an LFM signal with a quadratic phase
90 modulation function, the integral in (2) can be solved analytically. However,
91 in the case of nonlinear phase modulation function, the second order Taylor
92 expansion of phase $\phi(t)$ around t_k which is the solution of the (3) at ω_k is
93 used to reduce the integral in (2) to Fresnel integral. Hence after algebraic

94 manipulation, the power spectrum of the chirp signal at ω_k is given by (Cook
95 and Bernfeld, 1967),(Collins and Atkins, 1999), (Arif, 2010)

$$|X(\omega_k)|^2 \approx 2\pi \frac{p^2(t_k)}{|a(t_k)|} \quad (4)$$

96 where $|X(\omega_k)|^2$ is the power spectrum of the chirp signal, $p(t_k)$ is the
97 amplitude modulation function, and $a(t_k)$ is the chirp-rate function of the
98 signal.

99 In the case of an LFM signal, the term chirp-rate function $a(t_k)$ in (4)
100 is constant and the shape of the power spectrum is controlled by modify-
101 ing the amplitude modulation function $p(t_k)$ of the signal using a time or
102 frequency domain windowing function. However, in the case of the NLFM
103 signal, the shape of the power spectrum is controlled by either modifying the
104 chirp-rate function and keeping the amplitude modulation function constant
105 (rectangular envelope) or by modifying both the chirp-rate and amplitude
106 modulation functions of the signal; the latter case is known as a hybrid de-
107 sign approach. The main advantage of the hybrid design approach is that
108 the designed NLFM signal will be less sensitive to a Doppler shift caused by
109 the moving blood or tissue. However, this advantage comes at the expense of
110 reduced SNR of the transmitting signal due to amplitude tapering (Johnston
111 and Fairhead, 1986).

112 In a chirp coded excitation system, a matched filter is used on the receiv-
113 ing side to perform pulse compression of the long duration signal to recover
114 axial resolution. The matched filter will cancel out the phase information of
115 the chirp signal and therefore phase has no contribution in the axial resolu-
116 tion. The output of the matched filter is the autocorrelation function (ACF)

117 of the chirp signal ([Girod et al., 2001](#)).

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t + \tau)x^*(t)dt \quad (5)$$

118 Also the ACF of the chirp signal is the inverse Fourier transform of the
119 power spectrum and is expressed as

$$\begin{aligned} R_{xx}(\tau) &= \int_{-\infty}^{\infty} \{|X(f)| e^{j\phi(f)}\} \{|X(f)| e^{-j\phi(f)}\} e^{j2\pi f\tau} df \\ &= \int_{-\infty}^{\infty} |X(f)|^2 e^{j2\pi f\tau} df \end{aligned} \quad (6)$$

120 Therefore, the mainlobe width and sidelobe level of the compressed chirp
121 signal will depend on the shape of the power spectrum. Hence choosing a
122 suitable window function as a shape of the power spectrum would yield a
123 low sidelobe level after pulse compression. In this paper, NLFM signals were
124 designed using the hybrid approach and a tapered cosine window (also known
125 as a Tukey window) was selected as a shape of the desired power spectrum
126 and is expressed as ([Harris, 1978](#)),

$$|X(\omega)|^2 = \begin{cases} 1, & 0 \leq |\omega| \leq \beta \frac{B}{2} \\ \frac{1}{2} \left[1 + \cos \left(\pi \frac{\omega - \beta \frac{B}{2}}{2(1-\beta)\frac{B}{2}} \right) \right], & \beta \frac{B}{2} \leq |\omega| \leq \frac{B}{2} \end{cases} \quad (7)$$

127 where B is the bandwidth of $X(\omega)$, and β is the envelope tapering ratio:
128 $\{0 \leq \beta \leq 1\}$.

129 The tapered cosine window with a tapering ratio of 100% ($\beta = 1$) was
130 carefully chosen for two reasons. Firstly, it matches with the transfer function
131 of the ultrasound transducer used in the experiments and secondly it provides
132 reduced PSL after pulse compression of the SHC of the received NLFM signal.

133 The nonlinear instantaneous frequency function, $f_i(t)$, of the NLFM sig-
 134 nal is expressed as (Collins and Atkins, 1999),

$$f_i(t) = f_c + \frac{B}{2} \left[\frac{\alpha \tan\left(\frac{2\gamma t}{T}\right)}{\tan(\gamma)} + \frac{2(1-\alpha)t}{T} \right] \quad (8)$$

135 where the parameters α and γ are used to control the nonlinear curve of
 136 the instantaneous frequency function. B is the sweeping bandwidth, f_c is the
 137 centre frequency, and T is the chirp duration.

138 The chirp-rate function, $a(t)$, of the NLFM signal was found by taking
 139 the derivative of the instantaneous frequency function, $f_i(t)$, in (8).

$$a(t) = \frac{d}{dt}(f_i(t)) = \frac{B}{T} \left[\frac{\alpha\gamma(1 + \tan^2\left(\frac{2\gamma t}{T}\right))}{\tan(\gamma)} + (1-\alpha) \right] \quad (9)$$

140 The amplitude modulation function, $p(t)$, of the NLFM signal was ob-
 141 tained by rearranging (4) and substituting the values of power spectrum and
 142 chirp-rate function from (7) and (9).

$$p(t_k) \approx \sqrt{|X(\omega_k)|^2 |a(t_k)|} \quad (10)$$

143 The phase modulation function, $\phi(t)$, of the NLFM signal was obtained
 144 by taking the integral of the instantaneous frequency function, $f_i(t)$, in (8)
 145 with respect to time t .

$$\phi(t) = \int_0^t f_i(t) dt, \quad 0 \leq t \leq T \quad (11)$$

146 Finally, the NLFM signal was found by substituting the values of the
 147 amplitude modulation function $p(t)$, and the phase modulation function $\phi(t)$
 148 from (10) and (11) into (1).

149 *Linear Frequency Modulated (LFM) Signals*

150 The LFM signals were designed using equation (1), where the phase mod-
151 ulation function of the LFM signal is expressed as:

$$\phi(t) = \frac{B}{2T}t^2 + (f_c - \frac{B}{2})t + \varphi, \quad 0 \leq t \leq T \quad (12)$$

152 where B is the sweeping bandwidth, T is the chirp duration, f_c is the
153 centre frequency, and φ is the initial phase of the chirp signal and have
154 values of 0 and 0.5 for pulse inversion.

155 In this study, two LFM signals were used as a reference excitation. Each
156 signal had a duration (T) of 20 μ s, and a centre frequency (f_c) of 2.25 MHz.
157 The difference in each LFM signal was the transmitting energy which was set
158 by the application of tapered cosine window with envelope tapering ratios (β)
159 of 10% and 80%. Respectively after the application of a windowing function
160 the LFM signals will be termed as LFM-W10 and LFM-W80. The energy of
161 the LFM-W80 signal was set equal to the NLFM signal whereas the energy
162 of the LFM-W10 signal was 43% higher than the NLFM signal. The LFM-
163 W10 and LFM-W80 signals were designed to show the effect of different
164 windowing on the transmitting energy, power spectrum and harmonic pulse
165 compression.

166 *Excitation Signals*

167 The design parameters of the NLFM and reference LFM excitation signals
168 are shown in Table 1. The instantaneous frequency functions, $f_i(t)$, of the
169 NLFM, LFM-W80, and LFM-W10 signals are shown in Fig. 1(Top). The
170 instantaneous frequency function of the NLFM signal is nonlinear and is

171 similar in appearance to a reverse ‘S’ shape. However, the instantaneous
172 frequency functions of the LFM signals are linear. The NLFM and LFM
173 signals have equal sweeping bandwidth (B) and time duration (T).

174 The chirp-rate function, $a(t)$, of the NLFM signal shows higher values at
175 the edges and lower values in the middle part of the signal and appears as a
176 ‘U’ shape. However, the chirp-rate functions of the LFM signals are constant
177 throughout the signal’s duration as shown in Fig. 1(Bottom). Note that both
178 the instantaneous frequency and chirp-rate functions of LFM signals have
179 equal magnitude because of the same sweeping bandwidth and duration.

180 The amplitude envelopes, $p(t)$, of the NLFM, LFM-W80, and LFM-W10
181 excitation signals are shown in Fig. 2(Top). The amplitude envelope of
182 the LFM-W80 chirp was designed so that it can transmit acoustical energy
183 equal to the NLFM signal. The amplitude envelope of the LFM-W10 chirp
184 is nearly flat throughout the signal duration. This is due to the fact that less
185 amplitude tapering ($\beta = 0.1$) was applied. Therefore, the LFM-W10 signal
186 can transmit 43% more acoustical energy than the NLFM and LFM-W80
187 signals.

188 The power spectra of the NLFM, LFM-W80, and LFM-W10 signals are
189 shown in Fig. 2(Bottom). The transfer function (TF) of the 2.25 MHz
190 transducer used in the experiment is also shown in the figure. The TF of the
191 transducer was accounted in the design process of the NLFM signal. The
192 NLFM and reference LFM signals have same sweeping bandwidth. However,
193 their -6 dB bandwidths were different due to the different amplitude tapering
194 used in the design process. The power spectrum of the LFM-W10 signal
195 contains Fresnel-ripples in the pass band due to less amplitude tapering ($\beta =$

196 0.1) and their -6 dB bandwidth was 40% higher than the NLFM signal.
197 Similarly, the power spectrum of the LFM-W80 signal is smooth and contains
198 no ripples in the pass band. However, their -6 dB bandwidth was 11% higher
199 than the NLFM signal.

200 **Proposed Methods**

201 In this paper, two methods were proposed to show the scenarios of non-
202 overlap and overlap harmonic conditions. The proposed system flow charts
203 are shown in Fig. 3.

204 *Method I: For Non-Overlap Harmonic Condition*

205 Method I require a single transmission of a chirp signal with a fractional
206 bandwidth (FBW) of 20%. In the power spectrum of the received signal,
207 the nonlinear spectral harmonics do not overlap with each other due to the
208 use of narrow bandwidth chirp excitation. Therefore, the application of a
209 harmonic matched filter would yield the extraction and compression of the
210 second harmonic chirp component. Note that the bandpass filter response
211 is inherent within the harmonic matched filter specification as shown in Fig.
212 3(Top).

213 *Method II: For Overlap Harmonic Condition*

214 In ultrasound harmonic imaging, the bandwidth of the SHC is twice the
215 bandwidth of the fundamental frequency component [Chiao and Hao \(2005\)](#);
216 [Misaridis and Jensen \(2005b\)](#). Therefore, increasing the bandwidth of an ex-
217 citation signal will cause spectral overlapping between the fundamental and
218 the SHC. The direct application of a second harmonic matched filter under

219 an overlap harmonic condition will result in the production of higher range
 220 sidelobes after pulse compression. Therefore, in this method before pulse
 221 compression, the pulse inversion (PI) technique was proposed to extract the
 222 SHC under an overlap harmonic condition. PI requires two transmissions
 223 of 45% fractional bandwidth chirp signals with a phase difference of 180°.

224 The summation of the corresponding received echoes will cancel out the fun-
 225 damental component and will extract the SHC under an overlap harmonic
 226 condition (Simpson et al., 1999; Ma et al., 2005). After the extraction of
 227 the SHC, the application of a harmonic matched filter would yield the pulse
 228 compression of the second harmonic chirp component as shown in Fig. 3(Bot-
 229 tom).

230 **Simulation and Experimental Investigation**

231 *Simulations*

232 The nonlinear propagation of ultrasound waves in an attenuating medium
 233 was simulated using the following nonlinear model (Wojcik et al., 1998).

$$\rho \frac{\partial^2 u}{\partial t^2} = -\nabla p, \quad p = -\rho c^2 \left(\nabla \cdot u + \frac{1}{2} \frac{B}{A} (\nabla \cdot u)^2 \right) \quad (13)$$

234 where u is the particle displacement vector, c is the speed of sound, p
 235 is the acoustic pressure, ρ is the material density, and B/A is the nonlinear
 236 coefficient.

237 The model was simulated in MATLAB (The MathWorks Inc., Natick,
 238 MA, USA) using the time-domain based pseudo-spectral method. A detailed
 239 description of the simulation method can be found in (Anderson, 2000). The

240 simulation model was noise free and does not account for any transducer
241 geometry, therefore only plane waves were applied in simulations.

242 The performance of the pulse compression system is also affected by the
243 spectral mismatched of the receiving filter. This is caused by the frequency
244 dependent attenuation of tissue, which also limit its implementation in real
245 time imaging systems (Ramalli et al., 2015). In the case of ultrasonic atten-
246 uation, the power spectrum of the backscattered signal is frequency shifted
247 and distorted as a function of signal bandwidth and propagation depth. The
248 higher frequencies in the bandwidth are more attenuated than the lower fre-
249 quencies. This results in a reduction of the effective TBP of the signal and
250 hence the SNR (Rao, 1994). Therefore, the effect of frequency dependent at-
251 tenuation as a function of bandwidth was also investigated in all simulations.
252 The frequency dependent attenuation of $0.5 \text{ dB}/[\text{cm} \times \text{MHz}]$ was considered
253 up to penetration depth of 5 cm. The simulation parameters are shown in
254 Table 2, (Hallaj et al., 2001).

255 *Experimental Setup and Procedure*

256 In order to validate the proposed methods, experiments were performed
257 in a tank containing deionised, degassed filtered water at the temperature
258 of $21 \pm 1^\circ\text{C}$. The schematic diagram of the experimental setup is shown in
259 Fig. 4. The NLFM and reference LFM excitation signals were designed in
260 Matlab (The MathWorks Inc., Natick, MA, USA) and then loaded into a
261 programmable arbitrary waveform generator (AWG) (33250A Agilent Tech-
262 nologies Inc., 80 MHz, Santa Clara, CA, USA). The generated signals from
263 AWG were then amplified using an RF power amplifier (A150 Electronics &
264 Innovation Ltd., 55 dB, Rochester, NY, USA). The amplified signals were

265 then applied to a single element immersion transducer (V323-SM, Olympus
266 NDT Inc., Waltham, MA, USA) with a pulse repetition frequency of 100 Hz.
267 The transducer has an active element diameter of 6.35 mm, central frequency
268 of 2.25 MHz with a -6 dB FBW of 56%, and a focal length is in between 8.9
269 mm to 11.4 mm. The transducer was mounted coaxially with a hydrophone
270 at a distance of 50 ± 0.1 mm. All measurements were performed in the far
271 field of the transducer. The distance and alignment between the transducer
272 and hydrophone were controlled using a custom built computer numerical
273 control (CNC) scan system. A needle-type Polyvinylidene Fluoride (PVDF)
274 hydrophone with an active element diameter of 1.0 mm (calibrated from 1 to
275 20 MHz, Precision Acoustics Ltd., Dorchester, UK) was used to receive the
276 nonlinear signals. The pressure level of each waveform was calibrated and
277 a mechanical index (MI) of 0.2 (peak negative pressure of 300 kPa at 2.25
278 MHz) was received by the hydrophone. The average noise measurement for
279 the overall system was performed before starting the experiments. This was
280 done with the same experimental setup except that the AWG output being
281 switched off. The signals received from the hydrophone were captured 64
282 times and averaged using a digital oscilloscope (44Xi LeCroy Corporation,
283 400 MHz, Chestnut Ridge, NY, USA) at a sampling frequency of 100 MS/s
284 with 8-bit resolution. All the acquired data were stored in a computer system
285 and processed off-line using MATLAB (The MathWorks Inc., Natick, MA,
286 USA). All the received signals were corrected using an inverse filter designed
287 according to the frequency response of the hydrophone where the calibration
288 data were supplied by the manufacturer (Precision Acoustics Ltd., Dorch-
289 ester, UK).

290 *Post Processing of Nonlinear Received Signals*

291 *NLFM Received Signals*

292 In the case of the NLFM signal, the shaping of the power spectrum was
293 done on the transmitting side. Therefore on the receiving side of the system,
294 the harmonic matched filter was used to perform pulse compression of the
295 SHC of the received signal. The impulse response of the harmonic matched
296 filter was the time-reversed conjugate of the NLFM chirp with twice the
297 centre frequency and bandwidth parameters so that it matches with the
298 SHC of the received signal. Also the same windowing function was used in
299 the design of the harmonic matched filter as used in the excitation signal
300 ([Borsboom et al., 2005](#)).

301 *LFM Received Signals*

302 Since a strong amplitude tapering ($\beta = 0.8$) was used in the design process
303 of the LFM-W80 signal, therefore a harmonic matched filter was used to
304 perform pulse compression of the SHC of the received signal. However, in the
305 case of the LFM-W10 signal, less amplitude tapering ($\beta = 0.1$) was applied to
306 the excitation signal. Hence, instead of a harmonic matched filter, a harmonic
307 mismatched filter was used on the receiving side to perform second harmonic
308 pulse compression. The harmonic mismatched filter was designed by applying
309 a Chebyshev window of 100 dB attenuation to the harmonic matched filter.
310 This results in the further reduction of PSL after pulse compression.

311 *Performance Evaluation of Second Harmonic Pulse Compression*

312 The performance of second harmonic pulse compression of NLFM, LFM-
313 W80 and LFM-W10 received signals was quantitatively assessed by using the

314 three quality parameters, namely: the mainlobe width (MLW), peak sidelobe
315 level (PSL), and integrated sidelobe level (ISL). In the compressed second
316 harmonic chirp signal, the axial resolution was evaluated by measuring the
317 MLW at -20 dB in microseconds. The PSL and ISL provide a measure of self
318 induced noise and contrast resolution. The PSL was evaluated by taking the
319 ratio of the highest sidelobes peak to mainlobe peak in decibels. The ISL
320 was computed by taking the ratio of total sidelobes power to the mainlobe
321 power in decibels ([Arshadi et al., 2007](#)), ([Misaridis et al., 2000](#)).

322 **Results and Discussion**

323 *Simulation Results: Method I*

324 The power spectra of the simulated nonlinear received signals with a FBW
325 of 20% are shown in Fig. 5. The SHC of the received NLFM, LFM-W80 and
326 LFM-W10 signals were respectively -23.5 dB, -23.8 dB and -23.3 dB below
327 the fundamental frequency component. The SHC of NLFM was improved by
328 0.3 dB when compared with the SHC of the LFM-W80 signal of same energy
329 level. The SHC of NLFM was 0.2 dB below with the SHC of the LFM-W10
330 signal.

331 The envelopes of the simulated compressed second harmonic chirp sig-
332 nals are shown in Fig. 6. The pulse compression results of second harmonic
333 chirp signals are shown in Table 3. The results showed that at least 15 dB
334 improvement in the PSL of the compressed second harmonic signal of the
335 NLFM when compared with the PSL of LFM-W80 and LFM-W10 signals.
336 The ISL of the compressed NLFM signal was improved by 9 dB when com-
337 pared with the ISL of the compressed LFM-W80 signal. Similarly the ISL of

338 the compressed NLFM signal was improved by 13.9 dB when compared with
339 the ISL of the compressed LFM-W10 signal. The MLW of the compressed
340 second harmonic signal of NLFM was comparable to the MLW of LFM-W80
341 and LFM-W10 signals.

342 *Simulation Results: Method II*

343 The power spectra of the simulated nonlinear received NLFM, LFM-W80
344 and LFM-W10 signals with a FBW of 45% are shown in Fig. 7. The SHC
345 of the NLFM was -24.1 dB, whereas the SHC of the LFM-W80 and LFM-10
346 received signals were respectively -25.0 dB and -23.5 dB below the funda-
347 mental frequency component. The SHC of NLFM was improved by 0.9 dB
348 when compared with the SHC of the LFM-W80 signal of same energy level.
349 The SHC of NLFM was 0.6 dB below with the SHC of the LFM-W10 signal.
350 The figure shows the spectral overlapping of the second harmonic with the
351 fundamental and third harmonic components. The PI technique was applied
352 to these nonlinear received signals in order to extract the SHC. After PI pro-
353 cessing the SHC of NLFM, LFM-W80 and LFM-W10 signals were improved
354 by 12 dB.

355 The envelopes of the simulated compressed second harmonic chirp signals
356 after processed with the PI are shown in Fig. 8. The results indicated that the
357 PSL of the compressed second harmonic signal of the NLFM was improved by
358 27.6 dB and 15 dB when compared with the PSL of LFM-W80 and LFM-W10
359 signals, respectively. Similarly the ISL of the compressed second harmonic
360 signal of the NLFM was improved by 13.6 dB and 12.8 dB when compared
361 with the ISL of LFM-W80 and LFM-W10 signals, respectively. The MLW of
362 the NLFM compressed second harmonic signal was higher, but comparable

363 to the LFM-W80 and LFM-10 signals.

364 *Experimental Results: Method I*

365 The power spectra of the measured nonlinear received signals at a depth
366 of 5 cm are shown in Fig. 9. The NLFM and LFM spectrum plots were com-
367 pared by normalising each spectrum with its own maximum value so that the
368 fundamental component of each signal was aligned to the 0 dB. The figure
369 shows the existence of the fundamental and nonlinear second and third har-
370 monic components. These nonlinear harmonic components were produced
371 due to the nonlinear propagation of ultrasound waves through water at the
372 higher acoustic pressure exerted by the transducer. The spectra of the re-
373 ceived signals show that the nonlinear harmonic components do not overlap
374 with each other due to the use of 20% FBW excitation. The third harmonic
375 component exists in the received signal because a broadband hydrophone
376 was used as a receiver. In a clinical setting for an ultrasound second har-
377 monic imaging, the third harmonic component will be filtered out or further
378 suppressed by a limited bandwidth of the receiving transducer.

379 In the frequency domain, the SHC of the received NLFM, LFM-W80
380 and LFM-W10 signals were respectively -18.8 dB, -19.6 dB and -18.4 dB
381 below the fundamental frequency component. The SNR was estimated in
382 the frequency domain by measuring the peak height of the second harmonic
383 relative to the mean noise value. The SNR of the SHC of NLFM was improved
384 by 0.8 dB when compared with the SHC of the LFM-W80 signal. The 0.8 dB
385 improvement in the SNR was small because both signals contained the same
386 energy level, sweeping bandwidth, and duration. The SNR of the SHC of
387 NLFM was comparable with the SHC of the LFM-W10 signal even though

388 the NLFM signal contained 57% of the energy of the LFM-W10 signal.

389 The envelopes of the compressed second harmonic chirp signals are shown
390 in Fig. 10. The pulse compression results of the SHC are shown in Table
391 4. The PSL of the compressed second harmonic signal of the NLFM was
392 improved by 13 dB when compared with the PSL of the LFM-W80 signal.
393 Similarly the PSL of the compressed second harmonic signal of the NLFM
394 was improved by 15.6 dB when compared with the PSL of the LFM-W10
395 signal. The compressed second harmonic signal of the LFM-W10 contains
396 higher PSL due to the existence of Fresnel ripples in the power spectrum.
397 Another reason for the higher PSL is the slight overlapping of the SHC with
398 the fundamental and third harmonics. This occurs as the LFM-W10 signal
399 contains 40% higher -6 dB bandwidth than the NLFM signal due to less
400 amplitude tapering. The ISL of the compressed second harmonic signal of
401 the NLFM was improved by 8.4 dB and 18.6 dB when compared with the ISL
402 of LFM-W80 and LFM-W10 signals, respectively. The improvement in PSL
403 and ISL will potentially improve the image quality and increase the contrast
404 resolution.

405 By comparing the MLW of the compressed second harmonic chirp signals
406 it was found that the compressed NLFM chirp had a slightly higher value of
407 MLW. This result was expected due to the fact that the NLFM chirp had 11%
408 and 40% lower -6 dB bandwidth than the LFM-W80 and LFM-W10 signals,
409 respectively. The MLW of the compressed chirp signal is inversely propor-
410 tional to the bandwidth of the signal. Therefore, in a pulse-echo mode for
411 ultrasound harmonic imaging, the MLW of the compressed second harmonic
412 NLFM and LFM chirps will be further increased due to the bandwidth re-

413 duction of the SHC caused by the frequency dependent attenuation in tissue
414 and limited bandwidth of the receiving transducer.

415 *Experimental Results: Method II*

416 The power spectra of the measured nonlinear received signals with a FBW
417 of 45% are shown in Fig. 11. In the frequency domain, the SHC of the
418 received NLFM, LFM-W80 and LFM-W10 signals were respectively -18.8 dB,
419 -19.5 dB and -17.8 dB below the fundamental frequency component. The
420 SNR of the SHC of NLFM was improved by 0.7 dB when compared with the
421 SHC of the LFM-W80 signal of same energy level. However, the SNR of the
422 SHC of NLFM was 1 dB below with the SHC of the LFM-W10 signal due to
423 less energy.

424 In method II, all the received NLFM, LFM-W80 and LFM-W10 signals
425 were processed using PI. After the PI process, the SNR of the SHC of NLFM,
426 LFM-W80 and LFM-W10 signals was improved by 6 dB and the fundamental
427 frequency components were suppressed by 37 dB. The PI process applied to
428 the nonlinear received NLFM signals is shown in Fig. 12 whereas the PI
429 process for the LFM signals is shown in Fig. 13. After the application of PI,
430 the extracted second harmonic chirps of NLFM, LFM-W80 and LFM-W10
431 signals were processed using associated harmonic matched and mismatched
432 filters to perform second harmonic pulse compression.

433 The envelopes of the compressed second harmonic chirps of NLFM, LFM-
434 W80 and LFM-W10 signals after processed with the PI are shown in Fig. 14
435 and the pulse compression results are shown in Table 4. The results indicated
436 that the PSL of the compressed second harmonic signal of the NLFM chirp
437 was improved by 17.6 dB and 5 dB when compared with the PSL of LFM-

438 W80 and LFM-W10 signals respectively. Compared to the results in method-
439 I, the PSL of the compressed second harmonic signal of the LFM-W10 was
440 improved by 7.2 dB due to the application of PI. Similarly the ISL of the
441 compressed second harmonic signal of the NLFM was improved by 13.5 dB
442 and 10.8 dB when compared with the ISL of the LFM-W80 and LFM-W10
443 signals, respectively.

444 The MLW of the compressed second harmonic chirps of NLFM, LFM-
445 W80 and LFM-W10 signals was reduced by more than 50% in method II
446 due to the application of the 45% FBW excitation signals which therefore
447 can improve the axial resolution. Similar to the method-I, the MLW of the
448 compressed second harmonic chirp of the NLFM was higher due to the lower
449 -6 dB bandwidth but comparable to the LFM-W80 and LFM-W10 signals.
450 In method II, the pulse compression performance of the harmonic matched
451 filter greatly relies on the PI. Therefore the MLW of the compressed second
452 harmonic chirp will be further increased if the PI fails to extract the complete
453 SHC under the tissue motion.

454 *Doppler Sensitivity Evaluation of Designed Chirp Signals*

455 The pulse compression performance of the harmonic matched filter will
456 reduce when its frequency response is not spectrally matched with the SHC
457 of the received signal. In the human body this occurs when the backscattered
458 echoes are received from moving blood (or tissue) causing a Doppler shift in
459 the signal spectrum. The amount of Doppler shift is directly proportional to
460 the velocity of the moving blood and is inversely proportional to the speed
461 of sound in the tissue. The Doppler shift can be expressed as [Jensen \(1996\)](#),

$$f_D = -\frac{2v}{c}f_c \quad (14)$$

462 where f_D is the Doppler shift in frequency, v is the velocity of the moving
 463 blood, c is the speed of sound in the tissue, and f_c is the centre frequency of
 464 the excitation signal.

465 In order to measure the Doppler tolerance of the NLFM and reference
 466 LFM signals, the Doppler shift frequency (f_D) was calculated and added to
 467 the centre frequency (f_c) of the harmonic matched filter. It was assumed
 468 that the velocity (v) of the moving blood is 2 m/s, the speed of sound (c) in
 469 the tissue is 1500 m/s, and the centre frequency (f_c) of the SHC is 4.5 MHz.

470 A Doppler shift frequency (f_D) of 12 kHz was computed using (14) and
 471 added to the centre frequency of the harmonic matched filter to mimic the
 472 Doppler shift caused by the moving blood. The measured nonlinear signals
 473 from the hydrophone with a FBW of 20% were processed using associated
 474 harmonic matched and mismatched filters with an added Doppler shift fre-
 475 quency of 12 kHz. The compressed second harmonic chirp signals are shown
 476 in Fig. 15. It was found that all three chirp waveforms were robust to a
 477 Doppler shift of up to 2 m/s with no significant differences were observed in
 478 the MLW, PSL and ISL of the compressed signals when compared with the
 479 experimental results of method I shown in Table 4.

480 Discussion and Conclusion

481 In this study, NLFM and LFM could use long signals because a single
 482 element probe was employed in the experiments. Moreover, the long dura-
 483 tion signal also increases the total excitation energy and thus improve the

484 SNR, which was required to generate the nonlinear harmonics in the water
485 experiments. However, the long duration signal has a limitation imposed
486 by the medical probes used in practical imaging systems (O'Donnell, 1992).
487 Therefore, the duration of the chirp signals should be optimized for medical
488 probes used in imaging applications.

489 In Method I, chirp signals with a fractional bandwidth (FBW) of 20%
490 were proposed with a single transmission. The low bandwidth was used in
491 order to avoid spectral overlapping between the required second harmonic
492 and fundamental components in the nonlinear received signals. Therefore,
493 a simple harmonic matched filter at the receiver was able to extract and
494 compress the second harmonic component. The bandwidth of the SHC is
495 twice the fundamental component, therefore the MLW of the SHC was im-
496 proved compared to the MLW of the fundamental component. Moreover, the
497 requirement of a single transmission of a chirp signal can avoid the motion
498 artifacts and reduction of the system frame-rate as required by the pulse in-
499 version process with the 45% FBW chirp signals in Method II. However, for
500 imaging applications the MLW of the SHC is still so large, due to the use of
501 20% FBW, which degrades the axial resolution of the image.

502 In Method II, figures 7 and 11 show less spectral overlapping for NLFM
503 and LFM-W80 chirps compared to the LFM-W10 signal due to the strong
504 amplitude tapering used in the design process. Although all applied signals
505 have same sweeping bandwidth, however, their -6 dB bandwidths were altered
506 due to the application of different amplitude tapering. The -6 dB bandwidth
507 of the NLFM chirp was respectively 40% and 11% lower than the LFM-W10
508 and LFM-W80 signals. In this method, the MLW of the compressed second

509 harmonic chirp signals was improved by 50% due to the use of 45% FBW
510 chirp signals which can improve the axial resolution. However, due to the
511 use of PI for the extraction of SHC, the system frame-rate was reduced by
512 half compared to the Method I.

513 The results of second harmonic pulse compression were also compared
514 with the previous study of nonlinear quadratic chirp for tissue harmonic
515 imaging (Song et al., 2010). It was observed that the PSL of the compressed
516 second harmonic signal of the proposed NLFM chirp was improved by at
517 least 5 dB in Method-I and at least 2 dB in Method II with the comparative
518 axial MLW.

519 The nonlinear chirp was carefully designed with the customised window
520 so that the signal spectrum should match with the frequency response of the
521 transducer and maximum energy was transmitted through the transducer. At
522 the receiving side, this results in improvement in the SNR of the SHC and
523 reduction of the PSL after second harmonic pulse compression. Moreover, for
524 the NLFM chirp, the second harmonic pulse compression was done without
525 the need of an additional windowing on the harmonic matched filter, which
526 can potentially avoid the reduction of axial resolution and the gain in the
527 SNR.

528 The NLFM chirp and associated harmonic matched filter were also robust
529 to the Doppler shift caused by the moving blood of up to 2 m/s and frequency
530 dependent attenuation of 0.5 dB/[cm×MHz] up to penetration depth of 5
531 cm with no significant degradation of the MLW, PSL and ISL after pulse
532 compression.

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641 **Figure Captions**

642 **Figure 1:** Instantaneous frequency functions (Top), and the chirp-rate func-
643 tions of the excitation signals of 45% fractional bandwidth (FBW) (Bot-
644 tom).

645 **Figure 2:** Amplitude envelopes (Top), and the power spectra of the exci-
646 tation signals of 45% FBW along with the measured transfer function
647 (TF) of the transducer used in the experiments (Bottom).

648 **Figure 3:** Proposed system flow chart illustrating the signal processing chain
649 for the pulse compression of the second harmonic component (SHC)
650 using the harmonic matched filter. Method I (Top), and Method II
651 (Bottom).

652 **Figure 4:** Schematic diagram of the experimental setup.

653 **Figure 5:** Power spectra of the simulated nonlinear received signals with a
654 FBW of 20%.

655 **Figure 6:** The simulation results of method I show the envelopes of the
656 compressed second harmonic chirp signals with a FBW of 20%.

657 **Figure 7:** Power spectra of the simulated nonlinear received signals with a
658 FBW of 45%.

659 **Figure 8:** The simulation results of method II showing the envelopes of the
660 compressed second harmonic chirp signals of 45% FBW after processed
661 with the PI.

662 **Figure 9:** Power spectra of the measured nonlinear received signals with a
663 FBW of 20%.

664 **Figure 10:** The experimental results of method I show the envelopes of the
665 compressed second harmonic chirp signals with a FBW of 20%.

666 **Figure 11:** Power spectra of the measured nonlinear received signals with
667 a FBW of 45%.

668 **Figure 12:** Illustration of the PI process applied to the experimental data
669 of NLFM chirp with a FBW of 45%. Received Rx1 signal (Top-Left)
670 and associated power spectrum (Top-Right), Received Rx2 signal with
671 a phase difference of 180° (Middle-Left) and associated power spectrum
672 (Middle-Right), and summation of Rx1 and Rx2 NLFM signals in time
673 domain (Bottom-Left) and associated power spectrum showing the en-
674 hancement of the SHC and suppression of the fundamental and third
675 harmonic components (Bottom-Right).

676 **Figure 13:** Illustration of the PI process applied to the experimental data
677 of LFM-W80 (Left) and LFM-W10 (Right) chirps with a FBW of 45%.
678 (Top) power spectrum of the received Rx1 signal, (Middle) power spec-
679 trum of the received Rx2 signal with a phase difference of 180° , (Bot-
680 tom) power spectrum of the time domain summation of Rx1 and Rx2
681 LFM signals showing the enhancement of the SHC and suppression of
682 the fundamental and third harmonic components.

683 **Figure 14:** The experimental results of method II showing the envelopes
684 of the compressed second harmonic chirp signals of 45% FBW after

685 processed with the PI.

686 **Figure 15:** Envelopes of the compressed second harmonic chirp signals with
687 a FBW of 20% processed by associated harmonic matched and mis-
688 matched filters with and without an added Doppler shift frequency of
689 12 kHz.

690 **Tables**

691 **Table 1:** Design parameters of excitation signals.

Excitation Signals		NLFM	LFM
Parameters	Symbols	Values	
Sampling Frequency	f_s	40 MS/s	40 MS/s
Centre Frequency	f_c	2.25 MHz	2.25 MHz
Fractional Bandwidth	Δf	20%, 45%	20%, 45%
Time Duration	T	20 μs	20 μs
Gamma	γ	1.20	
Alpha	α	0.40	

692

693 **Table 2:** Simulation parameters and their values.

Parameters	Symbols	Values
Peak pressure	p	1 MPa
Axial distance	z	5 cm
Speed of sound	c	1500 m/s
Material density	ρ	1000 kg/m
Nonlinear coefficient	B/A	5.2
Frequency dependent attenuation	α_o	0.5 dB/[MHz·cm]

694

695 **Table 3:** Simulation results showing the performance evaluation parameters of second harmonic pulse compression.

Compressed Chirp Signals	Method I			Method II		
	MLW (μs)	PSL (dB)	ISL (dB)	MLW (μs)	PSL (dB)	ISL (dB)
NLFM	5.5	-42.6	-43.7	2.4	-48.8	-47.8
LFM-W80	4.6	-26.9	-34.7	2.0	-21.2	-34.2
LFM-W10	5.0	-26.5	-29.8	2.2	-33.8	-35.0

696

697 **Table 4:** Experimental results showing the performance evaluation parameters of second harmonic pulse compression.

Compressed Chirp Signals	Method I			Method II		
	MLW (μ s)	PSL (dB)	ISL (dB)	MLW (μ s)	PSL (dB)	ISL (dB)
NLFM	5.4	-41.2	-42.9	2.4	-38.2	-44.2
LFM-W80	4.5	-28.1	-34.5	2.0	-20.6	-30.7
LFM-W10	5.1	-25.6	-24.3	2.3	-33.2	-33.4

698