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Absolute versus convective helical magnetorotational instabilities in Taylor-Couette flows

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Abstract. We study magnetic Taylor-Couette flow in a system having nondimensional radii $r_i = 1$ and $r_o = 2$, and periodic in the axial direction with wavelengths $h \geq 100$. The rotation ratio of the inner and outer cylinders is adjusted to be slightly in the Rayleigh-stable regime, where magnetic fields are required to destabilize the flow, in this case triggering the axisymmetric helical magnetorotational instability (HMRI). Two choices of imposed magnetic field are considered, both having the same azimuthal component $B_\phi = r^{-1}$, but differing axial components. The first choice has $B_z = 0.1$, and yields the familiar HMRI, consisting of unidirectionally traveling waves. The second choice has $B_z \approx 0.1 \sin(2\pi z/h)$, and yields HMRI waves that travel in opposite directions depending on the sign of B_z . The first configuration corresponds to a convective instability, the second to an absolute instability. The two variants behave very similarly regarding both linear onset as well as nonlinear equilibration.

Keywords: Instabilities, Pattern formation, Magnetohydrodynamics, Taylor-Couette flows

1. Introduction

An important distinction in hydrodynamic stability theory is between absolute and convective instabilities (Brevdo and Bridges 1996, Chomaz 2005). This distinction arises almost inevitably if the underlying system is such that disturbances naturally travel in one direction only. An absolute instability is one where perturbations are seen to grow even if one focuses attention on a fixed point in space, while the disturbance travels past that point. In contrast, a convective instability is one where perturbations grow only as seen in a reference frame traveling with them. In a reference frame fixed in space, one sees only a temporary perturbation as the disturbance travels past a given point, but a return to stability thereafter. Absolute instability is thus a more stringent criterion than convective instability. The difference between the two can be crucial, especially in bounded versus unbounded systems: in bounded systems any global eigenmodes are necessarily absolute instabilities, whereas in unbounded systems instabilities can arise from infinity, and the entire concept of a global eigenmode is less clearly defined (Fox and Proctor 1998, Tobias *et al* 1998, Sandstede and Scheel 2000).

Taylor-Couette flow, the flow between differentially rotating concentric cylinders, is one of the oldest problems in classical fluid dynamics, and continues to attract considerable attention to this day, for both Newtonian (Grossmann *et al* 2016) and non-Newtonian (Fardin *et al* 2014) fluids. The basic setup by itself does not exhibit this required property that the system should have a natural direction in which disturbances travel. However, if an axial through-flow is added, that does establish a preferred direction, and absolute and convective instabilities have been widely studied in this modified problem (e.g. Tagg *et al* 1990, Babcock *et al* 1991, Langenberg *et al* 2004, Avila *et al* 2006, Martinand *et al* 2009, Miranda-Barea *et al* 2015). In this work we will also consider a Taylor-Couette geometry, but the preferred direction will be established by imposing magnetic fields, which can break the up/down symmetry of pure, non-magnetic Taylor-Couette flow (Knobloch 1996).

The earliest work on magnetohydrodynamic Taylor-Couette flow was by Chandrasekhar (1953), who showed that imposing an axial field suppresses the onset of Taylor vortices in the standard configuration where only the inner cylinder is rotating. The next development was by Velikhov (1959), who showed that an axial field could also have a destabilizing effect if both cylinders are rotating, and in such a way that the system is hydrodynamically stable according to the familiar Rayleigh (1917) criterion. This instability, subsequently known as the magnetorotational instability (MRI), is believed to play a key role in astrophysical accretion disks (Balbus 2003).

The MRI in a purely axial field is now also known as the ‘standard magnetorotational instability’ (SMRI), to distinguish it from the ‘helical magnetorotational instability’ (HMRI), in which a combination of axial and azimuthal fields is imposed. One advantage of the HMRI over the SMRI is that the critical Reynolds number is reduced by several orders of magnitude (Hollerbach and Rüdiger 2005), with the result that the HMRI has been obtained experimentally (Rüdiger *et al* 2006, Stefani *et al* 2006, 2009),

whereas the standard configuration has yielded other interesting results (Nornberg *et al* 2010, Roach *et al* 2012) but not yet the SMRI itself.

As dramatic as the reduction in the critical Reynolds number is – see also Kirillov and Stefani (2010) for further insights into the scalings that cause this – for our purposes here the most important aspect of the HMRI is the fact that it breaks the up/down symmetry of the system. Imposing either purely axial or purely azimuthal fields preserves this symmetry, but a combination of both breaks it (Knobloch 1996). The HMRI is thus inevitably a unidirectionally traveling wave, and hence the above distinctions between absolute and convective instabilities become relevant, especially since the theory was done in a cylinder without ends, whereas the experiment necessarily had ends.

The purpose of this work therefore is to consider the HMRI in two different fields. First, the original combination of axial and azimuthal fields, which amounts to a convective instability in an unbounded cylinder. Second, a field which is similar locally, but has an axial component that reverses periodically on a sufficiently long axial lengthscale. This causes waves to travel in different directions in different portions of the cylinder, and hence yields an absolute instability, by creating effective boundaries even in an otherwise unbounded cylinder. We numerically compute both linear onset and nonlinear saturation results, and show that in both cases the absolute and convective instability versions are still qualitatively similar.

2. Equations

Consider a Taylor-Couette geometry with nondimensional radii $r_i = 1$ and $r_o = 2$. Periodicity is imposed in z , with a wavelength $h \geq 100$, sufficiently long to give the relevant structures space to develop. Both cylinders rotate, at rates Ω_i and Ω_o . The values relevant to the HMRI have $\Omega_o/\Omega_i > 0.25$, just slightly into the Rayleigh-stable regime where magnetic fields are necessary to destabilize the basic state and trigger the onset of Taylor vortices.

The nondimensional Navier-Stokes and magnetic induction equations are

$$\frac{\partial \mathbf{U}}{\partial t} = -\nabla p + Re^{-1} \nabla^2 \mathbf{U} - \mathbf{U} \cdot \nabla \mathbf{U} + Ha^2 Re^{-1} Rm^{-1} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = Rm^{-1} \nabla^2 \mathbf{B} + \nabla \times (\mathbf{U} \times \mathbf{B}), \quad (2)$$

along with $\nabla \cdot \mathbf{U} = \nabla \cdot \mathbf{B} = 0$. Length is scaled by r_i , time by Ω_i^{-1} , \mathbf{U} by $\Omega_i r_i$, and \mathbf{B} by the magnitude B_0 of the imposed field.

The three nondimensional parameters are the two Reynolds numbers

$$Re = \frac{\Omega_i r_i^2}{\nu}, \quad Rm = \frac{\Omega_i r_i^2}{\eta}, \quad (3)$$

which measure the ratios of the viscous and magnetic diffusive timescales to the rotational timescale, and the Hartmann number

$$Ha = \frac{B_0 r_i}{\sqrt{\mu \rho \nu \eta}} \quad (4)$$

which measures the magnitude of the imposed field \mathbf{B}_0 . The quantities ν , η , ρ and μ are the fluid's viscosity, magnetic diffusivity, density and permeability, respectively.

The Stefani *et al* (2006, 2009) HMRI experiments have $Re = O(10^3)$ and $Rm = O(10^{-3})$. Note how the ratio of the two, $Rm/Re = \nu/\eta$, is a material property of the fluid, which is very small for all liquid metals. The smallness of Rm can be used to effectively eliminate it. The magnetic field is first expanded as

$$\mathbf{B} = \mathbf{B}_0 + Rm\mathbf{b}, \quad (5)$$

where \mathbf{B}_0 is the imposed field (required to be curl-free but otherwise still unspecified), and $Rm\mathbf{b}$ is the induced field. Inserting Eq. (5) into (1) and (2) and neglecting $O(Rm)$ terms then yields the so-called inductionless limit

$$\frac{\partial \mathbf{U}}{\partial t} = -\nabla p + Re^{-1} \nabla^2 \mathbf{U} - \mathbf{U} \cdot \nabla \mathbf{U} + Ha^2 Re^{-1} (\nabla \times \mathbf{b}) \times \mathbf{B}_0, \quad (6)$$

$$\nabla^2 \mathbf{b} = -\nabla \times (\mathbf{U} \times \mathbf{B}_0), \quad (7)$$

which no longer involves Rm . That is, Rm enters only in the meaning ascribed to \mathbf{b} , but not in the equations to be solved.

For the imposed field \mathbf{B}_0 , we consider the two choices

$$\mathbf{B}_c = r^{-1} \hat{\mathbf{e}}_\phi + \delta \hat{\mathbf{e}}_z, \quad (8)$$

$$\mathbf{B}_a = r^{-1} \hat{\mathbf{e}}_\phi + \delta [\sin(\kappa z) I_0(\kappa r) \hat{\mathbf{e}}_z - \cos(\kappa z) I_1(\kappa r) \hat{\mathbf{e}}_r], \quad (9)$$

where $\kappa = 2\pi/h$, and I_0 and I_1 are the modified Bessel functions. \mathbf{B}_c is essentially the same as in Hollerbach and Rüdiger (2005), except that it is more convenient here to fix the azimuthal component and have δ as the magnitude of the axial component, rather than vice versa. Because \mathbf{B}_c has no variation in z , the waves necessarily travel in the same direction throughout the entire cylinder, and hence can be said to arise 'from infinity'. The linear onsets computed by Hollerbach and Rüdiger (2005) are thus a convective instability.

In contrast, the field \mathbf{B}_a has the same azimuthal component as \mathbf{B}_c , but non-azimuthal components that vary periodically in z . Using the asymptotic properties $I_0(\kappa r) \approx 1$ and $I_1(\kappa r) \ll 1$ for $\kappa r \ll 1$, we obtain further that

$$\mathbf{B}_a \approx r^{-1} \hat{\mathbf{e}}_\phi + \delta \sin(\kappa z) \hat{\mathbf{e}}_z. \quad (10)$$

The specific functions I_0 and I_1 are required only in order to satisfy $\nabla \cdot \mathbf{B}_a = 0$ and $\nabla \times \mathbf{B}_a = \mathbf{0}$ for the exact expression (9). We see therefore that locally \mathbf{B}_a is much the

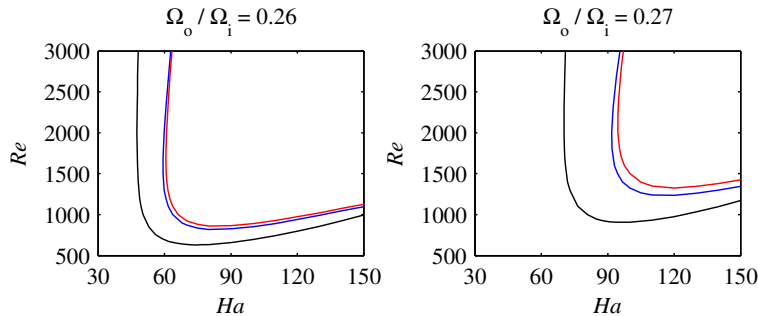


Figure 1. Stability boundaries in the (Ha, Re) plane, for $\Omega_o/\Omega_i = 0.26$ and 0.27 as indicated. In each panel the black curves are the convective instability obtained for \mathbf{B}_c . The blue and red curves are the absolute instability obtained for \mathbf{B}_a , with $h = 100$ and 200 , respectively. (The convective instability curves are independent of h , being a one-dimensional eigenvalue problem involving only r .)

same as the z -independent \mathbf{B}_c , except that its axial component reverses sign over the global extent of the cylinders.

The presence of this $\sin(\kappa z)$ factor in the axial component of \mathbf{B}_a has two consequences. First, near integer multiples of $h/2$, where $\sin(\kappa z) \approx 0$, the HMRI cannot exist at all, since it relies crucially on both axial and azimuthal components of the imposed field being non-zero. Second, in between these nodes, where the axial component has opposite signs, the waves will necessarily also travel in opposite directions, because it is the relative sign of the axial and azimuthal components that determines the up/down symmetry breaking (Knobloch 1996). We see therefore that even though the cylinders are infinite in extent, any waves that develop cannot arise from infinity, but inevitably have to emerge at a particular location, travel a certain distance, and then fade away again. That is, these waves correspond to an absolute instability, at least as far as global eigenmodes are concerned.

Eqs. (6) and (7), together with no-slip boundary conditions for \mathbf{U} , and insulating for \mathbf{b} , were numerically solved using an axisymmetric, pseudo-spectral code (Hollerbach 2008). \mathbf{U} and \mathbf{b} are expanded as

$$\mathbf{U} = \nabla \times (\psi \hat{\mathbf{e}}_\phi) + v \hat{\mathbf{e}}_\phi, \quad \mathbf{b} = \nabla \times (a \hat{\mathbf{e}}_\phi) + b \hat{\mathbf{e}}_\phi, \quad (11)$$

then ψ , v , a and b are further expanded in 5400 Fourier modes in z and 50 Chebyshev polynomials in r . The time-stepping of (6) is second-order Runge-Kutta, modified to treat the diffusive terms implicitly. At each time-step of (6), (7) is inverted directly for \mathbf{b} .

3. Results

Fig. 1 shows how the linear onset results depend on Ha and Re , for $\delta = 0.1$, and $\Omega_o/\Omega_i = 0.26$ and 0.27 . For parameter values greater than these stability boundaries

any infinitesimal perturbations will grow, eventually becoming the global eigenmodes shown in later figures. Note how absolute instability is indeed a more stringent condition than convective instability, requiring larger Hartmann and Reynolds numbers. Doubling the length of the cylinders, from $h = 100$ to 200 , has almost no effect on the absolute instability boundary; the contrast is between convective and absolute, not the precise cylinder length in which the absolute instability is allowed to develop.

Comparing the two rotation ratios, we see that increasing Ω_o/Ω_i from 0.26 to 0.27 causes both instabilities to shift to larger Hartmann and Reynolds numbers, but the shift is greater for the absolute instability boundaries. This is in agreement with the WKB analyses of Priede and Gerbeth (2009) and Priede (2011), who found that the absolute instability extends less far into the Rayleigh-stable regime than does the convective instability.

Finally, note that while the convective and absolute instability curves in fig. 1 look similar, computationally they are obtained very differently. The convective problem reduces to a one-dimensional eigenvalue formulation, just as before in Hollerbach and Rüdiger (2005), and requires minimal computational effort. In contrast, the absolute problem requires a two-dimensional time-stepping approach, and involved extensive calculations of many different (Ha, Re) combinations.

Having obtained the linear onset boundaries, we are next interested in the spatial structure of the eigenmodes in the fully nonlinear regime. We expect the Taylor vortices to develop as roughly round cells in the meridional circulation ψ . Since there is relatively little structure in r , we focus on ψ at mid-gap only. Simply noting whether ψ there is positive or negative is already sufficient to determine whether the corresponding vortices are clockwise or counter-clockwise. We can then conveniently capture the entire space-time dynamics of the solutions in so-called Hovmöller plots, showing $\psi(z, t)$ at this fixed $r = 1.5$.

Fig. 2 shows such plots, for \mathbf{B}_c with $h = 100$, as well as \mathbf{B}_a with $h = 100, 200$ and 400 . For the axially uniform field \mathbf{B}_c , we see exactly what we would expect: unidirectionally traveling waves that neither begin nor end at any particular locations, but simply travel throughout the entire periodic extent of the cylinder. In contrast, for the axially periodic field \mathbf{B}_a with $h = 100$, the waves now begin in specific locations, travel in different directions in the two halves where the axial component of \mathbf{B}_a is reversed in sign, and finally die out again in specific locations.

For \mathbf{B}_a with $h = 200$ we show only the lower half $z \in [0, 100]$; the upper half is exactly as one would expect based on the $h = 100$ panel. We see how the waves again begin and end in particular locations. For both the $h = 100$ and 200 results we also note how the waves are very regular and almost periodic when they first start to grow, but gradually become more irregular and broken-up as they travel. This effect is stronger for $h = 200$, where they have more room to travel and develop. Not surprisingly, it is stronger still for $h = 400$, where we show only the central portion $z \in [50, 150]$. At least in this region, the pattern is now qualitatively essentially identical to the original \mathbf{B}_c with $h = 100$, even though one is a convective instability with no beginning or ending

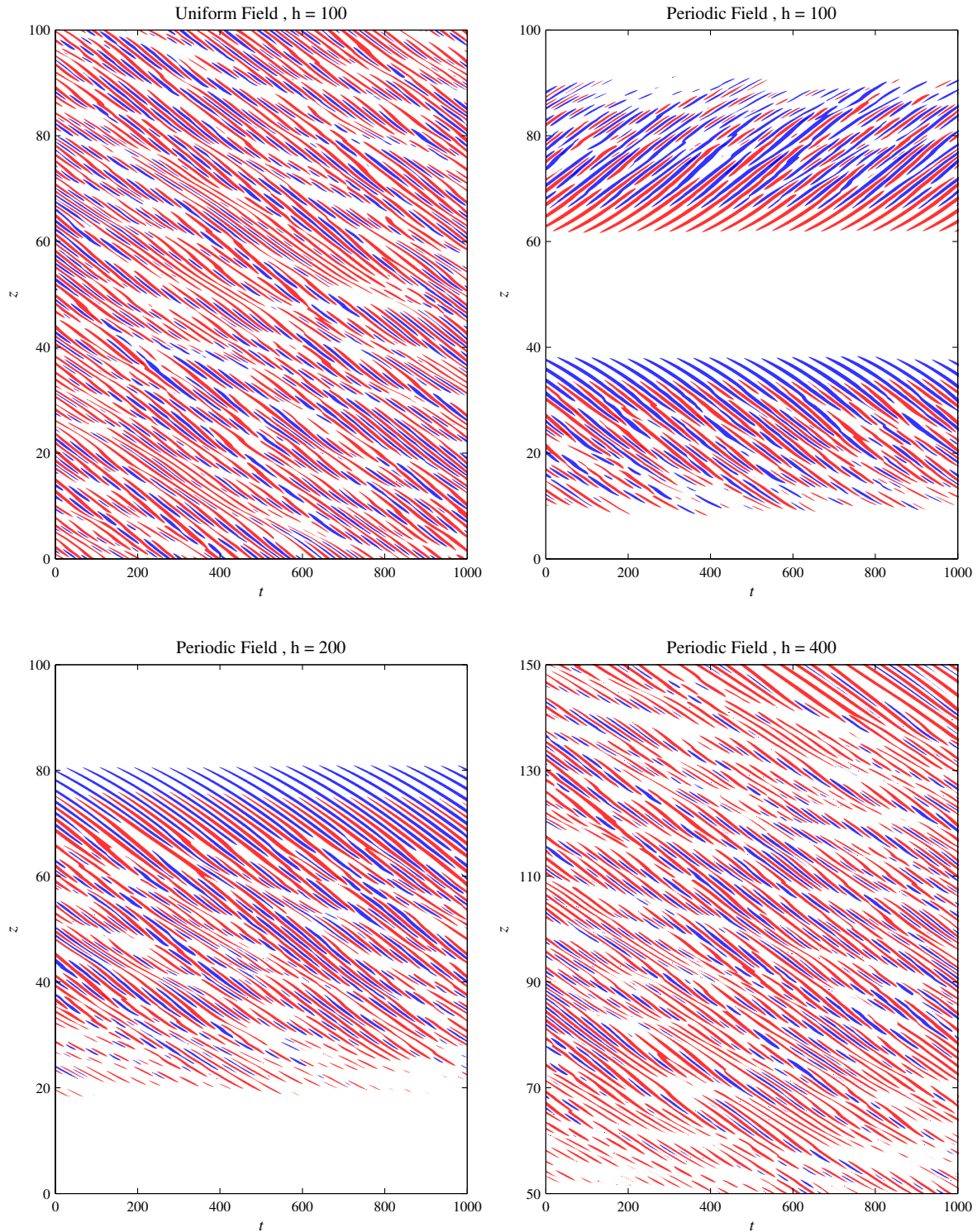


Figure 2. Hovmöller plots showing the space-time evolution of $\psi(z, t)$ at mid-gap, $r = 1.5$. Time is scaled on the rotational timescale Ω_i^{-1} . Blue and red denote opposite signs, hence oppositely rotating Taylor vortices. \mathbf{B}_c with $h = 100$, and \mathbf{B}_a with $h = 100, 200$ and 400 , as indicated. For the $h = 200$ and 400 results in the bottom row, only portions of the full cylinders are shown. All four results are for $\Omega_o/\Omega_i = 0.26$, $\delta = 0.1$, $Ha = 120$ and $Re = 2500$.

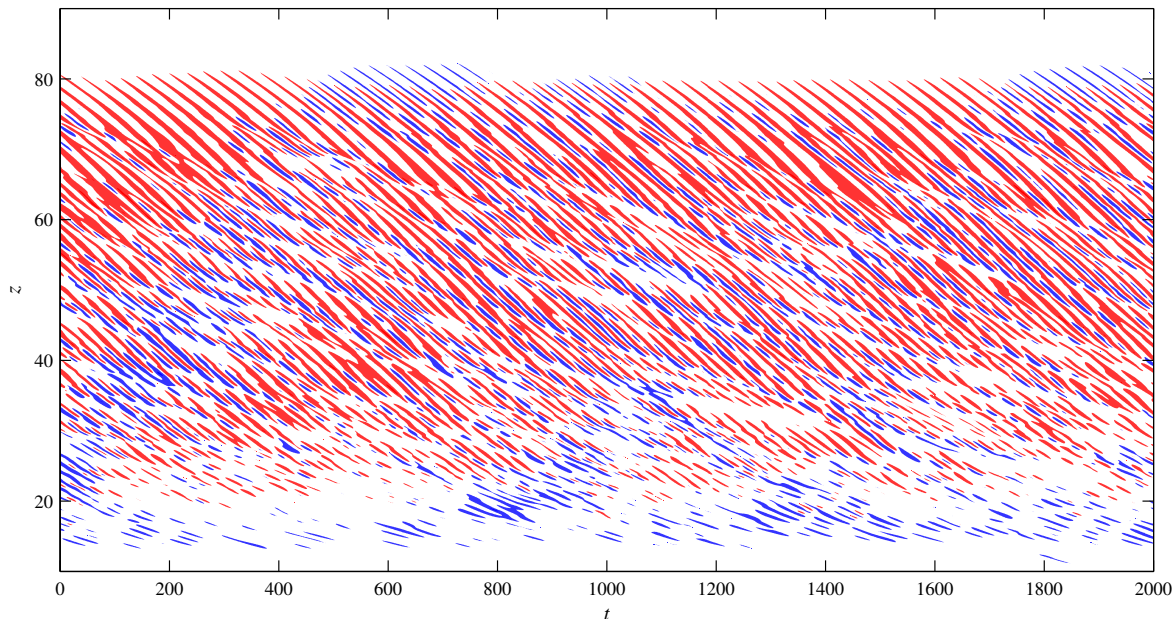


Figure 3. As in fig. 2, but for $Ha = 240$ and $Re = 5000$, and only for \mathbf{B}_a with $h = 200$. Note also the slightly reduced range $z \in [10, 90]$, but the longer time interval in comparison with fig. 2, to show the increased temporal variability.

to the waves, whereas the other is an absolute instability with definite beginning and ending locations.

It is worthwhile also to further quantify the ‘starting points’ of the absolute instabilities, occurring at $z \approx 38, 80$ and 165 for $h = 100, 200$ and 400 , respectively. The local values of the axial field, $\delta \sin(\kappa z)$, are then given by $0.068, 0.059$ and 0.052 respectively. For comparison, if we fix $\Omega_o/\Omega_i = 0.26$, $Ha = 120$ and $Re = 2500$, and instead consider δ as the control parameter, the critical value for the onset of the *convective* instability for \mathbf{B}_c is $\delta = 0.041$. That is, the absolute instabilities are indeed starting in portions of the cylinders where the field locally is convectively unstable.

The last point to note in fig. 2 is the existence of clearly defined phase and group velocities for the wave patterns. Each individual streak represents a single Taylor vortex drifting along the cylinders, so a simple measurement of the typical slopes yields a phase velocity of around 0.06 . Beyond that, we see how individual streaks come and go in groups, whose edges yield a different, smaller slope, with typical values around 0.02 . These – fully nonlinear – results will be compared with linear values below.

Fig. 3 shows the effect of doubling the Hartmann and Reynolds numbers from those in fig. 2. The same general pattern is still observed, including even similar phase and group velocities. The waves are noticeably less regular though. The transition from almost periodic to more irregular behaviour happens almost immediately after their formation. Additionally, the entire overall pattern exhibits a greater variability, with fluctuations occurring on larger length and time scales than seen in fig. 2.

Fig. 4 shows time-averaged power spectra of the solutions in fig. 2. The convective

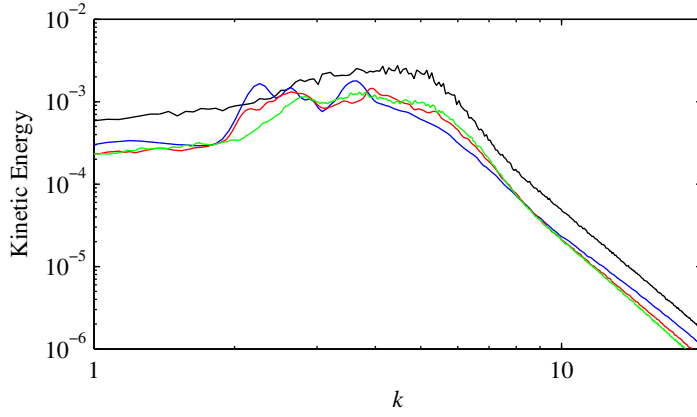


Figure 4. Time-averaged axial power spectra of the solutions in fig. 2. The black line shows \mathbf{B}_c with $h = 100$; blue, red, green respectively denote \mathbf{B}_a with $h = 100, 200$ and 400 . Note also that only the range $k \in [1, 20]$ is shown here, but the full computations include both smaller k , corresponding to the longest lengthscales up to h , and larger k , corresponding to dissipative lengthscales much shorter than the typical Taylor vortex size.

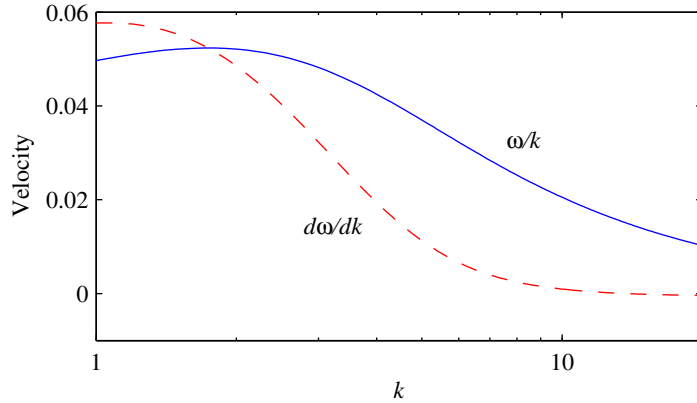


Figure 5. Linear eigenvalue calculations at $\Omega_o/\Omega_i = 0.26$, $\delta = 0.1$, $Ha = 120$ and $Re = 2500$. The resulting $\omega(k)$ is then used to construct phase and group velocities ω/k and $d\omega/dk$.

and absolute instability results are again remarkably similar, all exhibiting a broad peak around $k \approx 2-6$, indicative of the fact that the Taylor vortices are always much the same size. The main difference is that the absolute results are slightly below the convective ones. This is easily explained by the fact that for \mathbf{B}_c the waves exist throughout the entire extent of the cylinder, whereas for \mathbf{B}_a they only exist in $\sim 60\%$ of the domain.

Finally, returning to the phase and group velocities extracted from fig. 2, let us consider how well they might compare with equivalent linear results. Fig. 5 shows the results of fixing $\Omega_o/\Omega_i = 0.26$, $\delta = 0.1$, $Ha = 120$ and $Re = 2500$ in the \mathbf{B}_c linear eigenvalue formulation, solving for the frequency ω (the imaginary part of the eigenvalue) as a function of wavenumber k , and then plotting ω/k and $d\omega/dk$. Both

quantities vary considerably with k , but in the range $k \approx 2 - 6$, where fig. 4 indicates that the power is concentrated, the phase velocity is $O(0.04)$, and the group velocity $O(0.02)$. That these numbers are similar to the slopes extracted from fig. 2 suggests that even in the fully nonlinear regime in figs. 2 and 3, there are still aspects of the solutions that are not so different from a simple linear analysis of these waves. See also Rüdiger *et al* (2006), where linear wave speeds like these are compared with the experimental results.

4. Conclusion

In this work we have shown how the question of convective versus absolute instabilities can be addressed in magnetic Taylor-Couette flows even in unbounded periodic cylinders, by imposing magnetic fields that cause the waves to travel in different directions in different parts of the domain. We found that for the helical magnetorotational instability both linear and nonlinear results are broadly similar for the convective and absolute cases, illustrating the robust nature of the HMRI. These results are also of interest in general fluid dynamics and pattern formation (Hoyle 2006) contexts.

Finally, it would be worthwhile to extend this work to fully three-dimensional solutions. For the Hartmann numbers considered here axisymmetric modes are strongly preferred over non-axisymmetric ones, but for only slightly larger values even purely azimuthal fields become unstable to non-axisymmetric instabilities, in what is now known as the azimuthal magnetorotational instability, or AMRI (Hollerbach *et al* 2010, Seilmayer *et al* 2014). One might anticipate extremely complicated mode interactions then, with the axisymmetric HMRI dominating in regions where $|\sin(\kappa z)| \approx 1$, as in this work, but the non-axisymmetric AMRI dominating in regions where $|\sin(\kappa z)| \ll 1$. A three-dimensional, parallelized code has recently been developed (Guseva *et al* 2015), so further calculations in this direction are planned.

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