Fuzzy chronic poverty: A proposed response to Measurement Error for Intertemporal Poverty Measurement

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Abstract

A number of chronic poverty measures are now being used to quantify the prevalence and intensity of chronic poverty, vis-a-vis transient experiences; relying on panel datasets. Welfare trajectories over time are assessed in order to identify the chronically poor and distinguish them from the non-poor, or the transiently poor, and assess the extent and intensity of intertemporal poverty. We examine the implications of measurement error for some popular discontinuous chronic poverty measures, and propose corrections to these measures that seeks to minimize the consequences of measurement error. The approach is based on a novel criterion for the identification of chronic poverty that draws on fuzzy set theory. We offer an empirical illustration using a panel dataset from rural Ethiopia to show the relevance of the approach.

JEL Codes: I32, D63
Keywords: Intertemporal poverty, poverty measurement, measurement error, fuzzy sets theory.

1 Introduction

Measuring poverty over time is a subject that has grown in academic and policy interest over the past ten years in particular, not least due to the increasing availability of panel datasets. Several intertemporal poverty measures have now been proposed and are in use in empirical applications around the world. While no one measure has yet become the standard, several extensions of the Foster-Greer-Thorbecke set of static measures (Foster et al., 1984) are currently proposed. Porter and Quinn (2013) review the intertemporal poverty measurement literature, and show that some of the well-established properties of static poverty measurement are not easy to extend to the intertemporal context. We do not review all of them here, but we note that several options are available to the poverty analyst. Intertemporal poverty measures have been proposed, inter alia, by Jalan and Ravallion (2000); Porter and Quinn (2008); Hoy and Zheng (2011); Calvo and Dercon (2009); Foster (2009); Bossert et al. (2012); Gradin et al. (2012); Foster and Santos (2013); Dutta et al. (2013).

In particular, there has been a policy interest in trying to capture duration of poverty, and identify those who may be said to be "chronically" poor as opposed to transiently poor. In parallel, chronicity is a concept which many authors in the intertemporal poverty literature have sought to incorporate. This is an appealing concept in the intertemporal context: all other things equal, the length of time spent in poverty may have a more than one-for-one impact on the underlying wellbeing of a person.[1] There is also a direct analogy to the unemployment literature, which shows that spending longer time in unemployment may also decrease the chances of exiting unemployment. The same may be posited regarding poverty.

The literature has not yet managed to design a *continuous* measure of intertemporal poverty that incorporates an appropriate concept of *duration sensitivity*, even though these two properties are not incompatible in theory. Continuity is an important and desirable property of any poverty measure, given that any discontinuity would render the

[1] See for example several qualitative research papers and summaries from the Chronic Poverty Research Centre, www.chronicpoverty.org.

measure excessively sensitive to small changes in the wellbeing indicator being used. Of particular concern, the measure would also be sensitive to measurement error generating spurious fluctuations around poverty lines; in turn leading to misclassifications of people as either non (chronic poor) or poor. In the static and multidimensional context, similar concerns have motivated the incorporation of insights from fuzzy set theory (see e.g. Lemmi and Betti, 2006), in order to better identify the poor, and to avoid the problem of setting a poverty line that then classifies people as poor or non-poor, with nothing in between. Marano et al. (2015) introduce an approach for measuring longitudinal poverty using fuzzy set theory, but based on a latent variable approach.

Our focus in this paper is somewhat pragmatic, building on these insights in particular for application with widely used monetary measures of chronic poverty that speak to the policy literature. We create a "thick" poverty line enabling us to mitigate the potentially excessive sensitivity of discontinuous intertemporal poverty measures to spurious transitions across the poverty line. Specifically we propose a generalization of two popular intertemporal poverty measures: the measure of Foster (2009) and the more recent measure of Gradin et al. (2012). The two new proposals are characterized by a lower sensitivity to transitions around the poverty lines. In accordance with fuzzy set theory applied to poverty measurement, our measures allow *some* people to have a fuzzy poverty status, somewhere between being poor and non-poor.

We explore these measures' empirical implications with the Ethiopian Panel Household Survey. In our specific application, we find suggestive evidence that the Foster measure may *overestimate* chronic poverty in the presence of measurement error. By contrast, we find that the Gradin et al (2012) measure may *underestimate* chronic poverty (with respect to each measure's distinct definition of poverty).

The rest of the paper proceeds as follows. Firstly, we briefly introduce a few ideas about intertemporal poverty measurement, followed by a basic notion of poverty identification with fuzzy sets. Then we dedicate two sections, respectively, for the new proposals generalizing the measures of Foster (2009) and Gradin et al. (2012). The empirical illustration follows; and, finally, the paper ends with some concluding remarks.

2 Intertemporal Poverty Measurement

Consider a matrix X, whose N rows have information on the wellbeing attainments of N individuals across a time span. Each column, therefore, hosts the distribution of the attainment across the population in a specific time period. The number of columns/periods is T.

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1T} \\ x_{21} & x_{22} & \cdots & x_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nT} \end{pmatrix} \tag{1}$$

A typical attainment element of the matrix is: $x_{nt}\,(\in \mathbb{R}_+)$, that is, the attainment of individual n in period t. The poverty lines, specific to each period, are denoted by z_t (from a vector of poverty lines, $Z : (z_1, \ldots, z_t, \ldots, z_T)$), and a person is deemed poor in period t if: $x_{nt} < z_t$. When conceptualising poverty over time, it is useful to think about the **trajectory** of wellbeing attainments experienced by an individual n (Porter and Quinn, 2008), that is, the nth row of the data matrix $\mathbf{x}_n = (x_{n1}, x_{n2}, \ldots, x_{nT})$.

Some earlier measures developed in the literature (e.g. Jalan and Ravallion (1998) which is an extension of Rodgers and Rodgers (1993)) are also extensions of the Foster et al. (1984) measures, based on an averaging of the income stream over time, meant to capture the notion of permanent income. The methodology is straightforward, and intuitively appealing: if a person's average income (over the time period under consideration) lies below the poverty line, then they can be deemed chronically poor. Their poverty gap (and further poverty measures) can be calculated also using this average, and the FGT formula above can be applied to mean consumption or income. The methodology has been criticised partly because it allows a period of high income to compensate for a period in severe poverty. This led several authors (Calvo and Dercon, 2009; Foster, 2009) to propose an extension of the static FOCUS axiom (Foster and Shorrocks, 1991) to the intertemporal context. They propose that the principle of STRONG FOCUS should apply to any chronic poverty measure; that is, the poverty measure should not be sensitive to changes in wellbeing, *in any time period when wellbeing is above the poverty line.*[2]

The concept of STRONG FOCUS is not sufficient to distinguish between the group of static poor and intertemporal poor, which led Foster (2009) to introduce the concept of DURATION SENSITIVITY, which is at the heart of the identification strategy in his measure: only those people who are poor for at least a certain proportion of time qualify as chronic poor. Porter and Quinn (2008) show that this property is incompatible with another property, that penalises depth of poverty and allows a non-zero elasticity of substitution of wellbeing between periods (INTERTEMPORAL TRANSFER). Which of these properties one wishes to incorporate in the analysis is a normative choice, and depends on the policy context and the data under consideration.

Two properties that capture the specific concept of chronicity, or length of time spent under the poverty line, have been proposed so far in the literature: the first relates to the total number of time periods spent in poverty, regardless of their order in time. This has been termed DURATION SENSITIVITY by Porter and Quinn (2013) and TIME MONOTONICITY by Foster (2009). The second is contiguity of poverty (introduced by Bossert et al. (2012), whose measure is generalised by Gradin et al. (2012), and recently further developed by Dutta et al. (2013)). CONTIGUITY refers to the concept that *consecutive* spells of poverty without any recovery time in between may be more damaging to wellbeing than when there is some recovery time between. So, for example in a three-period panel, a sequence [poor, poor, non-poor] would be ranked as worse off than [poor, non-poor, poor] for a poverty measure satisfying CONTIGUITY. Both these are appealing normative properties.

However, another property that is highly desirable is CONTINUITY, which is motivated by the notion that an infinitesimal change in wellbeing in any period should lead to no more than an infinitesimal change in the value of the individual trajectory measure, i.e. the evaluation of intertemporal poverty (Porter and Quinn, 2013). If the trajectory ordering is not continuous then we may find trajectories which are ordered in a perverse way (see

[2] For an alternative view see Dutta et al. (2013). See also Foster and Santos (2013) for an intermediate approach between the perfect compensation implied by the framework of Jalan and Ravallion (1998) and the null compensation implied by the strong focus axiom.

Quinn (2014) for a further illustration). For empirical applications this is also extremely important: a discontinuous measure would be excessively sensitive to measurement error, at any point of discontinuity.

3 Duration-sensitive Poverty Measures

3.1 The chronic poverty measures of Foster (2009)

We first tackle one of the most popular measures, that proposed by Foster (2009), which has increasingly been adopted in policy applications Perez-Mayo (e.g. 2009); Nunez Velasquez (e.g. 2009), in addition to being the inspiration for the family of multidimensional poverty indices proposed by Alkire and Foster (2011). Foster proposes a property of **Time Monotonicity**, whereby an additional period of poverty experienced by an already chronically poor person should lead to an increase in the poverty measure.

Foster's measure includes a 'double cutoff': 1) A poverty line indicating material deprivation in one time period, and 2) a duration cutoff denoting the minimum number, or proportion, of periods in poverty experienced by one individual, or household, that categorises them as "chronically poor". If a person is deprived for a period at least as long as the duration cutoff, then the person is considered chronically poor. This measure's focus axiom is insensitive to any deprivations from people who are not deemed chronically poor in the identification stage. Given this duration cutoff, measurement error has been shown to have quite serious consequences around the discontinuity (Porter, 2010).

We recall that Foster et al. (1984) introduced a class of measures, known as p-alpha or FGT measures:

$$P_\alpha(\mathbf{x}) = \frac{1}{N}\sum_{n=1}^{N}\left(1-\frac{x_n}{z}\right)^\alpha \mathbb{I}(x_n \leq z), \tag{2}$$

for $\alpha \geq 0$; which satisfy Focus, Anonymity, Weak Monotonicity, Weak Transfer, Subset Consistency and Population Size Neutrality. In addition they satisfy Strict Monotonicity and Continuity for $\alpha > 0$ and Strict Transfer for

$\alpha > 1$.[3] They have become very well-known and widely applied.

As outlined above, any person n is deemed poor in any period t if: $x_{nt} < z_t$. Foster's chronic poverty measure is based on a deprivation count which has a very simple and intuitive understanding. A person is chronically poor (as opposed to transiently poor, or non-poor) if they are poor for a minimum number of time periods relative to the period under consideration. A count of deprivation periods is computed weighting each deprivation period with weights w_t, from a vector of weights $W : (w_1, \ldots, w_t, \ldots, w_T)$, such that: $w_t \in]0, 1[\subset \mathbb{R}_{++} | \sum_{t=1}^{T} w_t = 1$. Hence the weighted number of deprivation periods suffered by individual n is: $c_n \equiv \sum_{t=1}^{T} w_t \mathbb{I}\left(z_t > x_{nt}\right)$.

Foster (2009) identifies the chronically poor as those individuals whose weighted count of deprivation is above (or equal to) the duration cutoff, $\tau \in [0, 1] \subset \mathbb{R}$. The poverty identification function is thus:

$$\varphi(c_n; \tau) \equiv \mathbb{I}(c_n \geq \tau) \tag{3}$$

Then, for an individual poverty function Foster proposes a weighted sum of the powered and censored normalized poverty gaps in every period, i.e. the FGT metric (Foster et al., 1984):

$$p(\mathbf{x}_n; Z, W, \tau, \alpha) \equiv \varphi(c_n; \tau) \sum_{t=1}^{T} w_t (1 - \frac{x_{nt}}{z_t})^{\alpha} \mathbb{I}(z_t > x_{nt}), \quad \alpha \geq 0 \tag{4}$$

Note that in this approach to chronic poverty measurement the sequence and timing of poverty spells does not impact the individual measure. Foster calls this property "time anonymity".

Finally, the social poverty measure, P, has a functional form satisfying desirable properties like individual anonymity, population replication invariance and additive decomposability:

$$P \equiv \frac{1}{N} \sum_{n=1}^{N} p(\mathbf{x}_n; Z, W, \tau, \alpha) \tag{5}$$

[3] Besides the FGT family, other measures satisfy these properties; e.g. that of Chakravarty (1983) and Clark et al. (1981).

3.2 The intertemporal poverty measures of Gradin et al. (2012)

The second measure under consideration is the one by Gradin et al. (2012), which is a generalisation of Bossert et al. (2012). This measure has a slightly different duration property, in that consecutive spells of poverty are weighted more heavily. Bossert et al. (2012) observe that: "[t]he negative effects of being in poverty are cumulative, hence a two-period poverty spell is much harder to handle than two one-period spells that are interrupted by one (or more) period(s) out of poverty." (p1).

Gradin et al. (2012) take a similar approach to that of Foster, in that the measure is an intertemporal sum of FGT per-period poverty measures. However, they do not incorporate the duration cutoff for identification. This means that anyone with any period of poverty at all is included in the set of the intertemporally poor (in the poverty identification literature this would be deemed a *union* identification approach). In order to penalise contiguous periods of poverty the authors introduce a weight multiplying the FGT normalized poverty gap. This weight, w_{nt}, depends on the length of a contiguous poverty spell, denoted by s_{nt}. Thereby the same poverty shortfall gets weighted more heavily if it belongs in a longer experience of uninterrupted poverty:

$$p_{\text{G}}(\mathbf{x}_n; Z, S, \alpha) = \frac{1}{T}\sum_{t=1}^{T}(1 - \frac{x_{nt}}{z})^{\alpha}\mathbb{I}(z > x_{nt})w_{nt} \quad \alpha \geq 0; \tag{6}$$

where

$$w_{nt} = \left(\frac{s_{nt}}{T}\right)^{\beta}, \quad \beta > 0 \tag{7}$$

and S is the vector of poverty spells, s_{nt}. So, for example, a single period in poverty enters with a weight of $(1/T)^{\beta}$; whereas both periods in a two-period spell would be weighted by $(2/T)^{\beta}$ as in (7). As noted by Porter and Quinn (2013), the Gradin *et al.* measure satisfies WEAK IDENTIFICATION, GENERAL FOCUS, WEAK MONOTONICITY, STRONG FOCUS, RESTRICTED STRICT MONOTONICITY (if $\alpha > 0$) and CONTIGUITY; but not STRICT MONOTONICITY, CONTINUITY, NON-DECREASING COMPENSATION or TIME SYMMETRY. Its discontinuities mean that it does not satisfy INTERTEMPORAL TRANSFER or DURATION SENSITIVITY although it does satisfy each of these for certain poverty trajectories. Gradin et al. (2012) note that Foster (2009) is a special case of their measure if $\beta = 0$ and $\tau = 0$.[4] Finally, the social poverty measure, P, can be constructed by inserting (6) into the general form (5).

4 Poverty identification with fuzzy sets

In order to compensate for the potential effects of measurement error on duration-sensitive chronic poverty measures, we propose a generalization of the two measures outlined above, building on the fuzzy set literature. Fuzzy set theory has been used extensively in the social sciences for some time (e.g. see Ragin, 2000; Smithson and Verkuilen, 2006). In the poverty literature, fuzzy set theory was introduced as an alternative identification criterion by researchers who were unhappy with the blunt dichotomy posed by traditional poverty lines for the identification of the poor. Instead they opted for the membership functions used in fuzzy set theory (see e.g. Lemmi and Betti, 2006). While we do not intend to contest the practice of setting a poverty line for identification purposes, we do worry about the consequences of using a traditional poverty line in chronic poverty assessments based on duration-sensitive measures, when transitions across the line may be taking place spuriously due to measurement error. Since traditional measurement error corrections are usually not readily available (for a comprehensive treatment, see Bound et al. (2001)), we propose a fuzzy-style adjustment to the period-specific poverty lines, and then to the identification criteria of both the time-specific poor and the chronically poor. This adjustment smooths out the impact of (potentially spurious) transitions that take place across, and in close proximity to, the poverty lines. Thereby we generalize some of the proposed duration-sensitive measures of chronic poverty.

An illustration of our proposed identification adjustment is in Figure 1 where a traditional poverty line, z, is compared against a "thick" poverty line bounded by z_1 and z_2 such that $z_1 < z < z_2$. This is the general family of fuzzy poverty lines introduced by Chakravarty (2006). In a traditional identification approach, a person is deemed poor

[4] Although this is strictly true only if the time-period weights are all equal to $\frac{1}{T}$ in the framework proposed by Foster (2009).

if his/her income is below z, and non-poor otherwise. Under a fuzzy approach, poverty status ceases to be dichotomic if a person's income is in the interval $[z_1, z_2]$; for example, in the proposal by Chakravarty (2006) the membership function in that interval is given by $\pi_{nt} = (\frac{z_2 - x_{nt}}{z_2 - z_1})^\theta$.

Two important features of our application of a fuzzy identification approach to chronic poverty measurement stand out: (1) transitions across z, in its vicinity, do not generate abrupt changes in poverty status when the "thick" poverty line is used. For big changes in poverty status to happen, the magnitude of the transition has to be large enough to cross from z_1 to z_2 (or the other way around). In those cases we assume that the transition is less likely to be spurious (e.g. driven by measurement error). (2) Our fuzzy identification approach can be fine-tuned by either changing the values of $[z_1, z_2]$ or by changing the parameters that control the shape of the membership function.

As it is clear from (3), a change in x_{nt} that modifies the deprivation status in period t, i.e. a transit across z_t, increases, or reduces, c_n in the amount w_t. In turn such a perturbation may or may not change $\varphi(c_n; \tau)$ from 1 to 0 (or viceversa), in the case of measures like Foster's. As long as there is transit across z_t, a change in individual poverty status is possible, *irrespective of the magnitude of the change in* x_{nt} *that caused the transit*. However we do not want small, and potentially spurious, changes around z_t to have a significant effect on chronic poverty status. In order to reduce the likelihood of such occurrence, we propose an alternative poverty identification function, which is very similar to (3), with the exception that now deprivation in one particular period is determined by the fuzzy poverty line introduced by Dombi (1990):

$$\pi_{nt} = \left\{ \begin{array}{ccc} 1 & if & x_{nt} < z_{1t} \\ \frac{(z_{2t} - x_{nt})^2}{(x_{nt} - z_{1t})^2 + (z_{2t} - x_{nt})^2} & if & z_{1t} \leq x_{nt} \leq z_{2t} \\ 0 & if & x_{nt} > z_{2t} \end{array} \right\} \tag{8}$$

where $z_{1t} \leq z_t \leq z_{2t}$, i.e. there is now a "thick" poverty line. We note here also, that if we are particularly concerned with errors of exclusion, rather than those of inclusion, we may wish to set the lower bound of the thick poverty line at z, and an upper bound somewhere above it. The choice of the bounds for the "thick" poverty line are discussed further in the empirical section below.

PLACE FIGURE 1 HERE

PLACE FIGURE 2 HERE

4.1 Poverty identification with fuzzy sets: the case of the measures by Foster (2009)

Drawing on the preceding section, the next step in our proposal to amend the Foster measures is to redefine the intertemporal deprivation count: $c_n^\pi \equiv \sum_{t=1}^{T} w_t \pi_{nt}$. Then the new individual poverty function is:

$$p_\pi(\mathbf{x}_n; Z_\pi, W, \tau, \alpha) \equiv \varphi(c_n^\pi; \tau) \sum_{t=1}^{T} w_t \pi_{nt} [1 - \frac{x_{nt}}{z_{t2}}]^\alpha \mathbb{I}(z_{t2} > x_{nt}), \alpha \geq 0, \tag{9}$$

where $\varphi\left(c_n^\pi; \tau\right) = \mathbb{I}(c_n^\pi \geq \tau)$, and the vector Z_π is now made of trios of poverty lines, one per time period, $Z_\pi := \{z_{11}, z_1, z_{21}; \ldots; z_{1t}, z_t, z_{2t}; \ldots, z_{1T}, z_T, z_{2T}\}$.

Finally, the new social poverty function is:

$$P_\pi \equiv \frac{1}{N} \sum_{n=1}^{N} p_\pi\left(x_{n.}; Z_\pi, W, \tau, \alpha\right) \tag{10}$$

For the rest of the paper, especially in the empirical application, we focus on the measure with $\alpha = 0$. Two interesting differences between the families of measures in 9 and the original one by Foster (2009) are worth highlighting. Firstly, our proposal fulfills the original properties of Foster measures, in addition, now, to continuity. Hence, for instance, a transit across z_t is less likely to change c_n^π by a full amount of w_t. The change, Δc_n^π depends now on the magnitude of the change in x_{nt}, Δx_{nt} :

$$\Delta c_n^\pi = \left[\pi_{nt}\left(x_{nt} - \Delta x_{nt}\right) - \pi_{nt}\left(x_{nt}\right)\right] w_t \tag{11}$$

The lower sensitivity of c_n^π to the same change in x_{nt}, as reflected in (11), is the main feature rendering P_π better protected from drastic changes in deprivation status, and chronic poverty status, due to small and potentially spurious transits across z_t.

However this new specification has other consequences. A second, expectable, difference is that the baseline number of chronically poor people according to P_π in 10 need not coincide with that according to P in 5. For example, in the case of deprived people in period t, the following condition, for continuous variables, establishes the circumstances under which P_π *overstates* the proportion of deprived people in period t:

$$\int_{z_{1t}}^{z_t} [1-\pi(x)]\, dF(x) < \int_{z_t}^{z_{2t}} \pi(x)\, dF(x) \tag{12}$$

where $F(x)$ is the cumulative distribution function of x and $\pi(x)$ is the membership function with support in the range $[z_{1t}, z_{2t}]$. The left-hand side of (12) measures the loss in full deprivation status experienced by those who still have partial deprivation status, i.e. individuals for whom $z_{1t} \leq x_{nt} \leq z_t$. The right-hand side measures the acquired partial deprivation status among individuals who, otherwise, would not be considered deprived in period t, i.e. people for whom $z_t \leq x_{nt} \leq z_{2t}$. Whenever the latter is greater than the former, the social poverty headcount is greater according to P_π.

The case of $\alpha = 0$

When $\alpha = 0$, the individual poverty function in 9 reduces to:

$$p_\pi(\mathbf{x}_n; Z_\pi, W, \tau, 0) = \varphi(c_n^\pi; \tau) \sum_{t=1}^{T} w_t \pi_{nt}, \tag{13}$$

Then, following Foster (2009), the social poverty function can be expressed as the product of the chronic poverty headcount times the average proportion of poverty periods among the chronically poor (hence why it is also known as a *duration-adjusted headcount ratio*):

$$P_{\pi;}(0) \equiv \frac{1}{N} \sum_{n=1}^{N} p_\pi\left(x_{n.}; Z_\pi, W, \tau, 0\right) = H_\pi D_\pi, \tag{14}$$

where:

$$H_\pi \equiv \frac{1}{N}\sum_{n=1}^{N}\varphi\left(c_n^\pi;\tau\right) \quad (15)$$

and:

$$D_\pi \equiv \frac{1}{HN}\sum_{n=1}^{N}\varphi(c_n^\pi;\tau)c_n^\pi = \frac{P_{\pi;}(0)}{H}. \quad (16)$$

Since P with $\alpha = 0$ (i.e. the duration-adjusted headcount ratio)in the original Foster formulation (i.e. in 5) can also be expressed in terms of a chronic poverty headcount (H) multiplied by an average proportion of poverty periods among the chronically poor (D), then it is clear, from comparing against 14, 15 and 16, that a fuzzy identification function can change not only the chronic poverty headcount, but also the average duration statistic.

4.2 Poverty identification with fuzzy sets: the case of the measures by Gradin et al. (2012)

In the case of the measures by Gradin et al. (2012), the concern with a small perturbation generating a transit across z_t, and changing the deprivation status in period t, is not that the individual chronic poverty status may be affected, since in these measures a union approach to identification is considered, i.e. $\tau = 0$. However, as is clear from 7, the small perturbations just described can produce significant changes in the spell variables, i.e. s_{nt}, which in turn affect the weights. This becomes apparent by examining the formula for s_{nt}:

$$s_{nt} = [\sum_{i=t-m}^{t+n}\mathbb{I}(z_i > x_{ni})][\prod_{i=t-m}^{t+n}\mathbb{I}(z_i > x_{ni})]\mathbb{I}(z_i \leq x_{n,t-m-1})\mathbb{I}(z_i \leq x_{n,t+n+1}) \quad (17)$$

As is clear in (17), changes in period poverty status, both within $t-m$ and $t+n$, as well as in the immediately adjacent periods ($t-m-1$, $t+n+1$), can generate discontinuous changes in s_{nt}. Our proposal seeks to reduce this sensitivity to small changes in x_{nt} generating transit across z_t, by introducing π_{nt}, from (8), into (17), thereby "thickening" the poverty lines. This yields the following spell value function:

$$s^{f}_{nt} = [\sum_{i=t-m}^{t+n} \pi_{ni}][\prod_{i=t-m}^{t+n} \pi_{ni}]\mathbb{I}(z_{2i} \leq x_{n,t-m-1})\mathbb{I}(z_{2i} \leq x_{n,t+n+1}) \tag{18}$$

An illustration

In this section we provide one illustration of the impact of "thickening" the poverty lines in the context of the Gradin et al. poverty measures. The four panels of Figure 3 show the income profiles of an individual over three periods. According to the top left panel, the individual is poor in periods 1 and 3 if poverty line z is used. In the top right panel, the individual's income in period 2 is lower enough to render him/her poor. Comparing the poverty spells of the two top panels it turns out that: $S^{tl} := (1, 0, 1)$, while $S^{tr} := (3, 3, 3)$ (where "tl" and "tr" denote, respectively, the top left and the top right panels). Let g^{α}_{t} be the (FGT) normalized poverty gap in period t. Then $p^{tl}_{\text{G}} = (g^{\alpha}_{1} + g^{\alpha}_{3})(\frac{1}{3})^{\beta}$, $p^{tr}_{\text{G}} = (\sum_{t=1}^{3} g^{\alpha}_{t})(\frac{3}{3})^{\beta}$; and the difference between the two is:

$$\Delta p^{top}_{\text{G}} \equiv p^{tr}_{\text{G}} - p^{tl}_{\text{G}} = 3^{-\beta}[g^{\alpha}_{2}(3)^{\beta} + (g^{\alpha}_{1} + g^{\alpha}_{3})(3^{\beta} - 1^{\beta})] \tag{19}$$

PLACE FIGURE 3 HERE

By contrast, the two bottom panels perform the same comparison but using a "thick" poverty line, between z_1 and z_2, for period poverty identification, and z for the normalized poverty gaps. Using π_{nt} with the membership function proposed by Chakravarty (2006), it turns out that: $s^{bl}_{t} = 2 + (\frac{z_2 - x_2}{z_2 - z_1})^{\theta}$ $\forall t = 1, 2, 3$, while $s^{br}_{t} = 2 + (\frac{z_2 - x_2 + \epsilon}{z_2 - z_1})^{\theta}$ $\forall t = 1, 2, 3$ (where "bl" and "br" denote, respectively, the bottom left and the bottom right panels, and ϵ represents the drop in income on the right-half panels). Then $p^{bl}_{\text{G}} = 3^{-\beta} \sum_{t=1}^{3} g^{\alpha}_{t}(2+$

$[\frac{z_2-x_2}{z_2-z_1}]^\theta)^\beta$, $p^{br}_{\text{G}} = 3^{-\beta}\sum_{t=1}^{3} g^\alpha_t(2+[\frac{z_2-x_2+\epsilon}{z_2-z_1}]^\theta)^\beta$; and the difference between the two is:

$$\Delta p^{bot}_{\text{G}} \equiv p^{br}_{\text{G}} - p^{bl}_{\text{G}} = 3^{-\beta}\sum_{t=1}^{3} g^\alpha_t[(2+[\frac{z_2-x_2+\epsilon}{z_2-z_1}]^\theta)^\beta - (2+[\frac{z_2-x_2}{z_2-z_1}]^\theta)^\beta] \qquad (20)$$

Comparing (19) against (20), it is clear that the impact of ϵ should be milder on $\Delta p^{bot}_{\text{G}}$ than on $\Delta p^{top}_{\text{G}}$ as long as: $\theta > 0$, $\beta \geq 1$, $x_2 - \epsilon > z_1$ and $z_2 > x_2$. For instance, when $\beta = \theta = 1$, as in (21):

$$\Delta p^{bot}_{\text{G}}(\beta=\theta=1) = \frac{\epsilon}{3(z_2-z_1)}\sum_{t=1}^{3} g^\alpha_t < \frac{2}{3}(g^\alpha_1 + g^\alpha_3) + g^\alpha_2 = \Delta p^{top}_{\text{G}}(\beta=\theta=1) \qquad (21)$$

5 Empirical application

We explore the empirical implications of these generalizations using the Ethiopian Rural Household Survey (ERHS). The ERHS is a well-known panel dataset from a developing country that has been extensively used for poverty and mobility analysis (Baulch and Hoddinott, 2000; Dercon and Shapiro, 2007; Dercon et al., 2012). The ERHS contains data on just over 1100 households in 15 villages, observed at six points in time over a fifteen year period, 1994 – 2009. The timing of the rounds is not even, with fieldwork in 1994, 1995, 1997, 1999 , 2004 and 2009.[5] We use information on household consumption, that households were asked to recall for the week prior to the survey, including food that was home grown, bought at market, and received as a gift or benefit from government. In this way, we can assume that any consumption smoothing that the household intended, and was able to implement, would have been implemented. Below, we note the likely measurement error that this method may incorporate.

The poverty line is village-specific, and represents the amount needed to consume just over 2000 calories per day per adult equivalent, plus some very basic non-food items (such as firewood to cook). It is thus an extremely austere poverty line, around one-third of the commonly used "dollar a day" international poverty line. In each round we also deflated consumption and the poverty line by a village-specific food price index based on prices

[5] Two rounds were actually fielded in 1994, but only six months apart, so we drop the second one.

collected at the community level, and thus we construct a measure of consumption per adult equivalent. For more details on this survey and the calculation of the, by now, quite widely used consumption basket, see Dercon and Krishnan (1998). The poverty line is on average 43 Ethiopian Birr (1994 prices) per adult equivalent in the household.

PLACE TABLE 1 HERE

Several authors have analysed wellbeing based on consumption measures in the ERHS, including most recently Baulch (2011) and Dercon et al. (2012). Table 1 shows that cross sectional, or 'snapshot', poverty fell in the study villages between 1994 and 2004, with the headcount (P_0) falling from just under 43% to just under 20%, but then the headcount rate increased between 2004 and 2009 back to 35%. The other two measures, the average poverty gap (P_1) and the poverty severity index (P_2), followed a similar trend.

Table 2 shows the tabulation of number of periods spent in poverty. Looking at households over time, there is a lot of movement in and out of poverty, and fewer than a third of all households have never experienced any poverty at all. However, only 2% recorded consumption below the poverty line in every visit over the ten-year period. Hence we are faced with exactly the kind of exercise that was outlined in the theory section above. Some households have longer periods in poverty, but do not fall very much below the poverty line; some have fewer episodes of poverty but some of those are very severe.

PLACE TABLE 2 HERE

We now calculate the "fuzzy" poverty measures outlined in the theory section above, the chronic poverty measure of Foster (2009) and the intertemporal poverty measure of Gradin et al. (2012) by taking an upper and lower bound around the poverty line. As discussed above, both of these measures are chosen for illustration, as they incorporate discontinuities in their design, whereby a reclassification of a household-time-period from poor to non-poor (and vice versa) could more than proportionally affect the measure.

How to choose the bounds for the "thick" poverty line? Traditional applications of the "totally fuzzy relative approach" effectively give each and every individual a non-zero value for the fuzzy poverty measure (Cheli and Lemmi, 1995). We do not proceed in this way. In an earlier contribution, Cerioli and Zani (1990) propose that a fuzzy poverty measure (based on FGT) could have a subsistence poverty line, z, as the minimum bound, and mean income as the maximum. For our purpose, we are interested in errors of inclusion and exclusion brought about through measurement error. We therefore seek empirical evidence on what the extent of measurement error is likely to be in a consumption survey. There is relatively little information on this, however Beegle et al. (2012) recently conducted a randomized control trial of consumption in the context of a household survey in Tanzania. The authors compare several methods to elicit recall of food consumption. The benchmark is a daily visit to the household with individual diary for each day. The method used by the ERHS survey, 7-day recall at the household level, is also included. The results show that the 7-day recall method is subject to underestimating the level of consumption, by approximately 20%. Using the same experiment, Gibson et al. (2015) conclude that measurement error is thus mean-reverting, and substantial (as shown by a higher variance of consumption relative to the benchmark). Given this information, we provide results varying the bandwidth of the thick poverty line by 10, 20 and 30% of the original poverty line.

Hence we set the upper bound for the fuzzy set (z_2) at 10% above the poverty line, and symmetrically with the lower bound (i.e. $z_1 = 0.9z$, $z_2 = 1.1z$,). We note that this means that $\pi_{nt} = 1$ only if consumption is below the lower bound poverty line z_1, and $\pi_{nt} = 0$ for consumption above the upper bound z_2. Between z_1 and z_2 π_{nt} receives a value between zero and one. We choose the fuzzy membership function proposed by Dombi (1990), though as with the setting of the line, this is an arbitrary choice; so we show results for alternative specifications based on the proposal by Chakravarty (2006) (in Annex 6 below). The results do not change substantially.

How many households have recorded consumption levels that are between the upper and lower bounds of the "thick" poverty lines as discussed above? To understand how much impact the fuzzying of poverty identification will have, we note that 545 observations (just under 8% of all observations) lie between z_1 and z_2 at $+-10\%$ fuzzy set. If we increase the set to $+-20\%$ the number doubles, and at $+-30\%$ just over a fifth of the observations (1373) are included. We conclude, then, that the fuzzy identification has the potential to affect poverty measures that incorporate a strict cutoff.

We begin our analysis with the Foster measure. For the calculations, we note from table 3 that the measure depends crucially on the value of choice parameter τ, i.e. the duration cutoff. If $\tau = 0.66$, or 4/6 periods (those poor in 4 or more of the 6 periods are classified as poor), then 16.5% of the sample will be classified as poor. If we increase the required number of periods in poverty to 5/6 for chronic classification, then only 8% are defined as chronic poor. Recall that the other measures in the Foster class of indices are calculated based on this identification step (duration, poverty gap, squared poverty gap).

We also note that, *for "crisp" poverty identification* in our setting of discrete time periods (or rounds of survey data) defining those who are chronically poor if the deprivation score is higher than or equal to 0.66 is empirically equivalent to chronic poverty identification when the deprivation score is strictly above 0.5 (see table 3, "crisp" column). In other words, in our setting, the members of the chronic poor set are the same whether we defined them as having spent four or more periods in poverty OR strictly more than three periods in poverty. This is to be expected with "crisp" (i.e. non-fuzzied) measures. However, as the results in table 3 show, the difference in the way τ is defined, can and does matter when calculating fuzzy poverty measures.

PLACE TABLE 3 HERE

We illustrate the effect of increasing the bandwidth of the fuzzy line on the headcount (H and H_π) and the duration-adjusted headcount measure (P and P_π) in table 3. In the first row, the first column shows the headcount, or proportion of households classified as chronically poor, at 16.5%. Increasing the thick poverty line at 10% reduces this to 13.5%. At 20% bandwidth, 12% of households are considered chronic poor, and at 30% bandwidth

11% of households are chronically poor. The second row incorporates the fuzzyness into a duration cutoff based on a strict inequality, with $\tau_a = 0.5$ (i.e. the deprivation score has to be strictly higher than 0.5 to identify the household as chronically poor). In this case, the proportion deemed chronically poor is increasing in the bandwidth of the thick poverty line, classifying just over 20% of households as chronically poor for the highest of our chosen illustrative bandwidths. The next two columns use the identification choices mentioned above in order to calculate the corresponding duration-adjusted headcounts, which behave similarly.

We also discussed above that, under different scenarios, the Foster measure with "crisp" poverty lines may overestimate, or underestimate, chronic poverty (vis-a-vis "fuzzy" alternatives) in the presence of measurement error, when there are transitions of a small amount above and below the poverty line that may be spurious due to measurement error. We may now wish to make a slightly different normative choice, which is to set the thick poverty line at z as a minimum, and allow periods in which consumption is just above the poverty line to still be considered poor. The assumption here would be that we care more about measurement error that misclassifies a household just above the poverty line, rather than just below it as we wish to penalise errors of exclusion more heavily than those of inclusion. Table 5 in the Annex below shows headcount measures for all of our different assumptions. The last two rows consider the poverty line z as the lower bound. By design, this would increase the poverty measures; e.g. the headcount from under 21% to just under 24%. The change may seem minor, but it could be important in terms of targeting.

Next we show our calculation of the Gradin et al "fuzzy measures". In this case, there is no second duration cutoff, due to the union approach to poverty membership. However as outlined above, in calculating the individual poverty measure, each poverty episode is weighted by the length of the "spell" to which it belongs (See equation 17). For simplicity, we again calculate only the headcount measure (i.e. $\alpha = 0$), and we use a value of $\beta = 1$ to illustrate the change in the measure when we "thicken" the poverty line.

Table 4 shows the results. As the bandwidth widens, the measure increases, by approximately 1% for every 10% of poverty-line widening. This increase is due to the net lengthening in spell duration; e.g. the effect of periods being reclassified from 0 (in terms of poverty status) to a non-zero amount on spell lengthening overtaking the spell-shortening effect of periods being reclassified from 1 to a lesser amount (see equations (17) and (18)). For the 10% bandwidth this represents a change for 448 of the people-year observations, and for the 30% bandwidth the change affects 1369 people-year observations. (By contrast, in the case of Foster measure an increase in the headcount, or lack thereof, was conditional upon the choice of the duration cutoff).

4

The magnitude of change for the fuzzy measures is of course proportional to the bandwidth, and the choice of this we would see as a pragmatic issue, depending on the perceived level of measurement error. We replicated our results using alternative definitions of the fuzzy set, and they are not substantially different. Finally, one may be concerned with errors of exclusion, rather than inclusion in the case of poverty targeting; in which case, we would recommend setting the lower bound of the thick poverty line to the original poverty line, and creating a bandwidth above it.

6 Conclusions

This paper presented an empirical adjustment for some recently proposed chronic or longitudinal poverty measures which show desirable normative properties, but may be excessively sensitive to measurement error, due to the discontinuity inherent in their calculation. The adjustment is fairly simple and empirically practical. Drawing on fuzzy set theory, we construct a "thick" poverty line that enters into the poverty identification step of the poverty measures in each and every time period. This thickening of the poverty line allows us to remove the discontinuity in the measures, without affecting any of their other properties.

The empirical section presents some results for rural Ethiopia, showing that, in this case, the choice of functional form for the fuzzy poverty identification method is less important than the size of the bandwidth (which determines the poverty line's "thickness").

The fuzzy adjustment around the poverty line shows that, in our application, the Foster (2009) measure may overestimate chronic poverty, while the Gradin et al (2012) measure may underestimate chronic poverty (as defined by each measure respectively) in the presence of measurement error. It is possible that, with a longer time series, the potential for measurement error to affect the results would be greater. The poverty analyst should make choices on these sensitivity tests based on the appropriate objectives of the measurement exercise (e.g. if there is higher concern over exclusion errors vis-a-vis others). We hope that our proposed adjustment method adds to the toolkit suitable for these purposes.

Tables

Table 1: **FGT Poverty, by year**

Year	P_0	P_1	P_2
1994	0.346	0.140	0.078
1995	0.378	0.151	0.081
1997	0.213	0.067	0.031
1999	0.232	0.073	0.033
2004	0.199	0.066	0.031
2009	0.347	0.127	0.064

Table 2: **Number of periods in poverty**

Item	Number	Per cent
Never Poor	343	30.19
Once	264	23.24
Twice	194	17.08
Three times	147	12.94
Four times	107	9.42
Five times	59	5.19
In every period	22	1.94
Total	1,136	100

Source: ERHS Data

Table 3: Fuzzy Foster measures

Measure	**Crisp**	**Fuzzy 10**	**Fuzzy 20**	**Fuzzy 30**
Headcount				
$\tau = 0.67$	.165	.137	.118	.108
$\tau_a = 0.5$	.165	.189	.196	.203
Duration-adjusted Headcount				
$\tau = 0.67$	.125	.106	.093	.086
$\tau_a = 0.5$	.125	.136	.139	.142
N	1136			

Notes: Fuzzy poverty defined as the S-shaped membership function (Dombi, 1990).

Table 4: Gradin et. al. Headcount measures (s-convex)

Variable	$\beta = 1$	$\beta = 0.5$
Crisp	0.108	0.168
Fuzzy 10 percent	0.115	0.173
Fuzzy 20 percent	0.123	0.178
Fuzzy 30 percent	0.132	0.185
N	1136	

Gradin et al (2012) measures. The thick poverty line is defined as in the above tables.

FIGURES

Figure 1: Fuzzy identification of deprivation status in period t (Chakravarty, 2006)

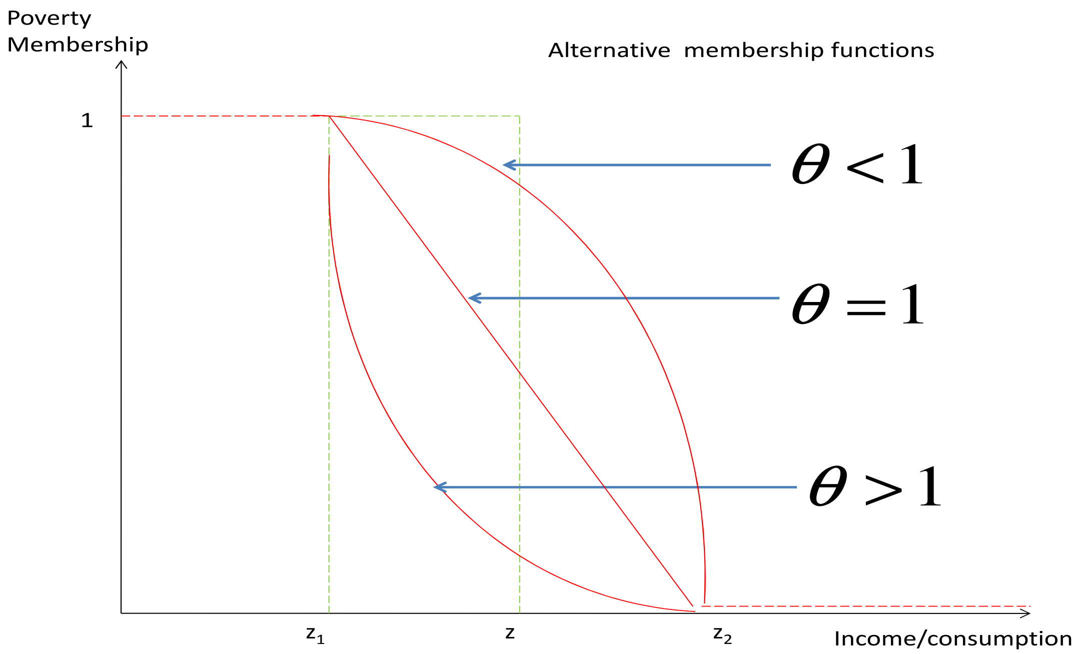


Figure 2: Fuzzy identification of deprivation status in period t (Dombi, 1990)

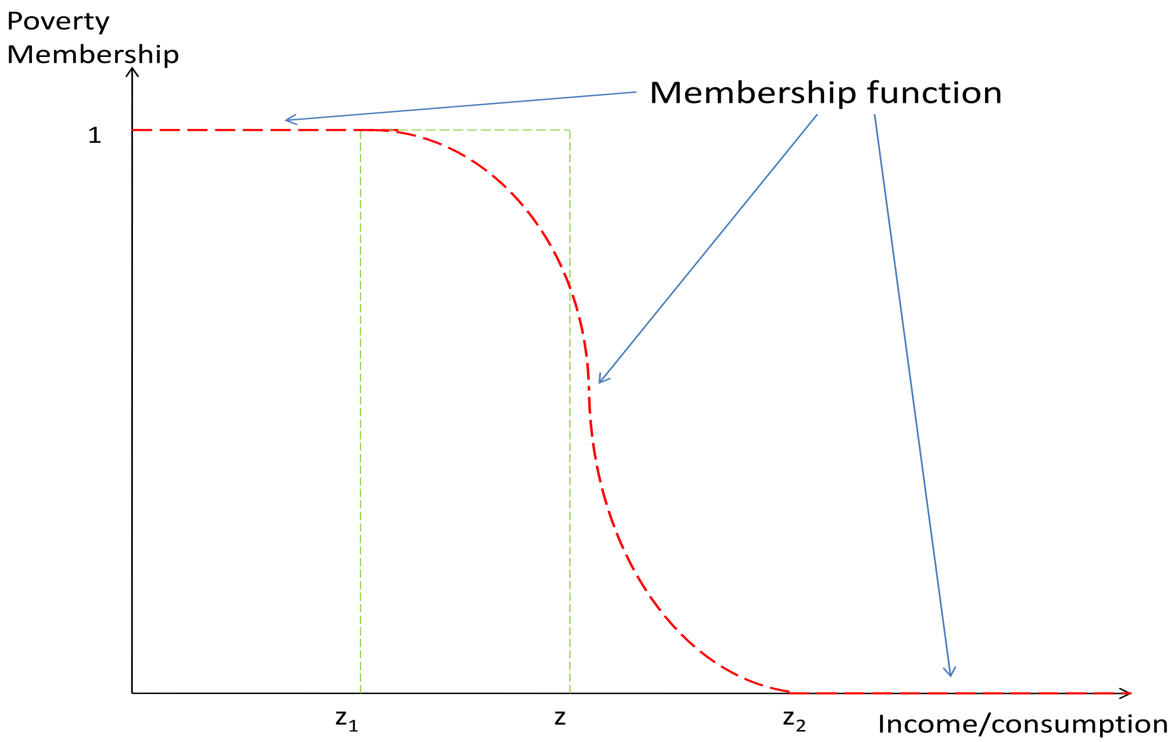
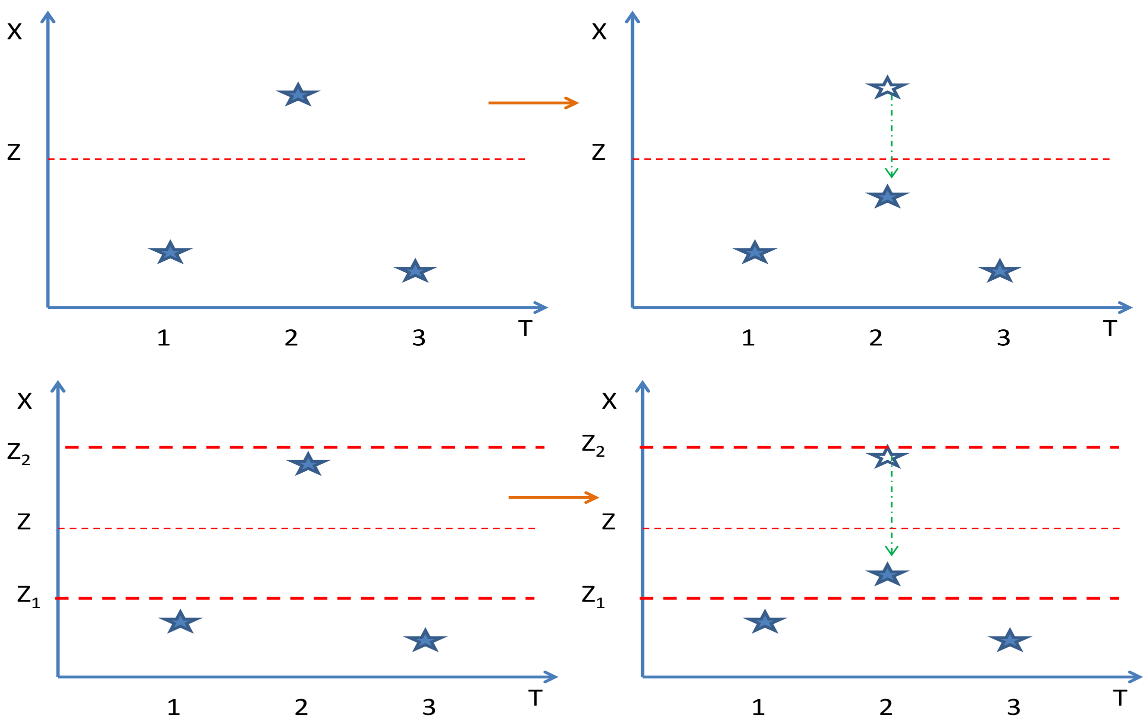


Figure 3: Fuzzy poverty spells



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Annex: Additional Tables based on alternative assumptions for the membership function

Table 5: Poverty headcounts, Foster measures, alternative membership function

Variable	Mean	Std. Dev.
Crisp	0.216	0.412
Fuzzy(10%), $\theta = 1$)	0.181	0.385
Fuzzy(20%), $\theta = 1$)	0.154	0.361
Fuzzy(30%), $\theta = 1$)	0.142	0.349
Fuzzy(10%), $\theta = 0.5$)	0.185	0.388
Fuzzy(20%), $\theta = 0.5$)	0.168	0.374
Fuzzy(30%), $\theta = 0.5$)	0.165	0.371
Fuzzy (z=min, +20%), $\theta = 1$)	0.229	0.42
Fuzzy (z=min, +20%), $\theta = 0.5$)	0.237	0.426
N	1179	

Notes: As in the main tables, Fuzzy 10% means that the upper bound of the "thick" poverty line is at $1.1z$ and the lower bound at $0.9z$, similarly with 20 and 30% respectively. θ is the parameter referred to in figure 4, for the fuzzy membership function as proposed by Dombi (1990). The bottom two lines include the poverty line z as the lower bound of the thick poverty line, with the upper bound set at $1.2z$.

Table 6: Gradin et al measures, alternative membership function

Variable	Mean	Std. Dev.
Fuzzy(10%), $\theta = 1$)	0.115	0.184
Fuzzy(20%), $\theta = 1$)	0.122	0.191
Fuzzy(30%), $\theta = 1$)	0.131	0.199
Fuzzy(10%), $\theta = 0.5$)	0.119	0.187
Fuzzy(20%), $\theta = 0.5$)	0.131	0.198
Fuzzy(30%), $\theta = 0.5$)	0.146	0.211
N	1136	

Notes: As in the main tables, Fuzzy 10% means that the upper bound of the "thick" poverty line is at $1.1z$ and the lower bound at $0.9z$, similarly with 20 and 30% respectively. θ is the parameter referred to in figure 4, for the fuzzy membership function as proposed by Dombi (1990).