

This is a repository copy of *The Return–Volatility Relation in Commodity Futures Markets*.

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/113572/>

Version: Accepted Version

Article:

Chiarella, Carl, Kang, Boda orcid.org/0000-0002-0012-0964, Nikitopoulos, Christina Sklibosios et al. (1 more author) (2016) *The Return–Volatility Relation in Commodity Futures Markets*. *The Journal of Futures Markets*. pp. 127-152. ISSN 1096-9934

<https://doi.org/10.1002/fut.21717>

Reuse

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.



The return-volatility relation in commodity futures markets

Journal:	<i>Journal of Futures Markets</i>
Manuscript ID:	Futures-14-5177.R2
Wiley - Manuscript type:	Research Article
Key Words:	Return-volatility relation, Gold futures volatility, Crude oil futures volatility, Commodity futures returns, Normal/inverted futures markets

SCHOLARONE™
Manuscripts

Review

The return-volatility relation in commodity futures markets

Carl Chiarella^a, Boda Kang^b, Christina Sklibosios Nikitopoulos^{a,*}, Thuy-Duong Tô^c

^a*University of Technology, Sydney,
Finance Discipline Group, UTS Business School,
PO Box 123 Broadway NSW 2007, Australia*

^b*University of York,*

Department of Mathematics,

Heslington, York YO10 5DD, United Kingdom

^c*The University of New South Wales,*

*UNSW Business School, School of Banking and Finance
Sydney NSW 2052, Australia*

Abstract

By employing a continuous time multi-factor stochastic volatility model, the dynamic relation between returns and volatility in the commodity futures markets is analysed. The model is estimated by using an extensive database of gold and crude oil futures and futures options. A positive relation in the gold futures market and a negative relation in the crude oil futures market subsist, especially over periods of high volatility principally driven by market-wide shocks. The opposite relation holds over quiet periods typically driven by commodity-specific effects. According to the proposed convenience yield effect, normal (inverted) commodity futures markets entail a negative (positive) relation.

Keywords: Return-volatility relation; Commodity futures returns; Gold futures volatility; Crude oil futures volatility; Normal futures markets; Inverted futures markets;

JEL: G13, E32, Q40

[☆]The authors would like to thank an anonymous reviewer and the editor Robert Webb for valuable comments. Thanks are also extended to seminar participants at the 2014 Bachelier Finance Society World Congress, the 2014 Conference on the Performance of Financial Markets and Credit Derivatives, the FIRN 2013 conference and the UTS Finance Discipline Group internal research seminars for fruitful discussions and helpful suggestions. Financial support from the Australian Research Council is also acknowledged (DP 1095177, The Modelling and Estimation of Volatility in Energy Markets).

^{☆☆}Professor Carl Chiarella, Dr Boda Kang, Dr Christina Sklibosios Nikitopoulos and Dr Thuy-Duong Tô.

*Correspondence author, Finance Discipline Group, UTS Business School, University of Technology, Sydney, PO Box 123 Broadway NSW 2007, Australia. Tel: +61 2 9514 7768, Fax: +61 2 9514 7722, e-mail: christina.nikitopoulos@uts.edu.au

1. Introduction

One of the fundamental measures to analyze the behavior and the attribute of financial markets is the nature of the relation between returns and innovations in volatility. As nowadays commodity derivatives play a vital role in investment and risk management, a comprehensive understanding of the relation between price returns and volatility changes in commodity futures markets is of critical importance.

Asymmetric volatility is a well known empirical phenomenon in equity markets describing the negative relation between stock returns and stock return (conditional) volatility with the effect being more robust during periods of market crashes where high volatility is combined with low returns. An extensive literature has been dedicated to explain this relation. The two main accounts put forward are the leverage effect postulated by Black (1976b) and the volatility feedback effect proposed by Campbell and Hentschel (1992), while more recently, a new explanation via the behavioral effect has been introduced by Hibbert, Daigler and Dupoyet (2008).¹ The return-volatility relation has also been studied in the commodity markets. The empirical phenomenon of a positive return–volatility relation (referred to as *inverted asymmetric* volatility) in commodity markets has been explained by Ng and Pirrong (1994) via the inventory effect that is linked to the theory of storage. Tully and Lucey (2007) and Baur (2012) study the return–volatility relation in the gold market and use the safe haven property of gold to explain the prevailed positive relation. Hassan (2011) and Salisu and Fasanya (2013) have found a negative return–volatility relation in the crude oil market while Carpentier and Samkharadze (2013) demonstrated the importance of accommodating the asymmetric volatility feature in commodity hedging strategies. However, most of these studies are based on GARCH models that preserve the limitation of allowing only for V or U shaped volatility reactions (as they restrict volatility to be a deterministic function of

¹The leverage effect implies that negative stock return shocks cause an increase in volatility due to the increase in the firm's leverage. A similar reaction in the commodity market is also termed as the leverage effect, where negative futures (or spot) return shocks tend to increase volatility. The volatility feedback effect suggests that any change in volatility, but in particular an increase in volatility, will decrease stock returns. Alternatively based on the behavioral concepts of representativeness, affect and extrapolation bias, individuals prefer positions of high return and low risk as these represent a good investment.

1
2
3
4
5
6
7
8 the return shock). Moreover, these models investigate the relation only in spot commodity
9 markets as they cannot model sufficiently the entire volatility term structure of futures
10 returns. Nevertheless, as futures prices emerge with greater transparency (compared to spot
11 commodity prices, which depend heavily on grade and location), an analysis of futures prices
12 would provide a more comprehensive understanding of the nature of the relation between
13 the price dynamics in commodity markets and their volatility.
14
15
16
17
18

19 The key contribution of this paper is that it provides both a qualitative and a quan-
20 titative analysis of the return–volatility relation in commodity futures markets by using a
21 stochastic volatility model within the Heath, Jarrow and Morton (1992) framework. The
22 proposed continuous time model is well suited to analyze and measure the entire term struc-
23 ture of the commodity futures return volatility. It accommodates multiple volatility factors
24 with flexible volatility term structures ranging from exponentially decaying to hump-shaped.
25 Moreover, the volatility factors feature the empirically observed characteristics of unspanned
26 components in commodity futures volatility (Trolle and Schwartz (2009)) and potential asym-
27 metries in the dynamics of the volatility, (Ng and Pirrong (1994)). These two features can
28 be captured by the correlation between the innovations of the futures price returns and the
29 innovations of its volatility.
30
31
32
33
34
35
36
37
38

39 The proposed stochastic volatility model possesses finite-dimensional affine realizations
40 for commodity futures prices and quasi-analytical prices for options on commodity futures.
41 Subsequently, the model is estimated by fitting to both futures prices and options prices.
42 An extensive database of daily futures and option prices extending to 31 years for gold
43 and 21 years for crude oil is used. Thus, the estimated models integrate information from
44 both futures prices and options prices that guarantee a better fit to the observable futures
45 term structure as well as the (implied) volatility term structure. Existing literature on
46 the return–volatility relation estimates volatility by using either sample return variances or
47 implied volatilities, see for instance Dennis, Mayhew and Stivers (2006) for a study in equity
48 markets. There is limited literature that takes into account both futures prices and option
49 prices.
50
51
52
53
54
55
56
57
58

59 The gold futures market and the crude oil futures market were considered in the analysis
60 for several reasons. Both markets are amongst the most liquid commodity derivatives mar-

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

kets that impact, and are influenced by, macro-economic and financial conditions, as they are used widely for investment purposes and/or hedging and speculation. Moreover, gold is classified as an investment commodity whereas crude oil as a consumption commodity. Since these two markets are fundamentally different, the study provides insightful findings of the nature of the return–volatility relation in these two distinct futures markets. To enhance the analysis and verify the significance of different volatility market conditions, the models are estimated over the whole period (as it has been done in most literature) as well as over smaller subsamples differentiated by their volatility intensity.

The study not only identifies the nature of the return-volatility relation but also provides a justification of the results, by using long-established theories and hypotheses such as the volatility feedback effect, the safe haven property and the theory of storage. Based on our empirical results, during volatile periods which are typically driven by market-wide shocks, the gold futures return-volatility relation is positive as it can be explained by the safe haven property of gold. However, during less volatile market conditions, a negative relation in the gold futures market is present indicating that gold futures respond similarly to financial assets like equities (for instance, the volatility feedback effect is present). Conversely, during high volatility periods, the crude oil futures return-volatility relation is negative, while during low volatility periods the relation in the crude oil futures market becomes positive, as explained by the inventory effect. To explain the crude oil futures volatility reaction, a new account emerges, the so called *convenience yield effect*. This effect is more general compared to the inventory effect as it can account for non-fundamental changes of futures prices apart from the associated fundamental changes such as inventory. Based on the empirical results for crude oil, one of the key findings is that normal (inverted)² consumption commodity markets are characterised by a negative (positive) return-volatility relation. Consequently, the return–volatility relation can be linked to the convenience yield of the commodity.

It is further illustrated that for these two commodity futures markets, when the market uncertainty is high, the potential market–wide shock effects dominate, while when the market

²A futures market is normal (inverted) when futures prices of near maturities trade lower (higher) than futures prices of distant maturities, leading to an upward-sloping (downward-sloping) forward curve.

1
2
3
4
5
6
7
8 is quiet the commodity-specific shock effects dominate. This is consistent with the finding of
9
10 Dennis et al. (2006) who have shown that asymmetric (implied) equity volatility is primarily
11
12 attributed to systematic market-wide factors rather than aggregated firm-level effects. The
13
14 paper also confirms that the relation is consistent with the option-derived implied volatility
15
16 skew.

17
18 The remaining of the paper is organized as follows. Section 2 discusses the relation
19
20 of futures returns and volatility for the two distinct types of commodity markets, namely
21
22 investment commodities and consumption commodities. Section 3 presents a generalised
23
24 stochastic volatility model and introduces an empirical tool to analyse the return-volatility
25
26 relation in commodity futures markets. Section 4 describes and analyzes the data of gold
27
28 and crude oil derivatives and explains the method employed to estimate the proposed model.
29
30 Section 5 presents the estimation results and reflections on the return-volatility relation.
31
32 Section 6 concludes.

33 34 **2. The relation between returns and volatility in commodity futures markets**

35
36 This section discusses the features of the return-volatility relation in commodity futures
37
38 markets in terms of the differences between investment commodities and consumption com-
39
40 modities. For each commodity, the spot prices as well as the futures prices are determined
41
42 by regular supply and demand forces driven by commodity specific fundamentals such as in-
43
44 ventories, production and consumption. Apart from the regular supply and demand forces,
45
46 commodity derivative prices, returns and volatility are also affected by market-wide shock
47
48 factors such as investment growth, interest rates, exchange rates, market contractions and
49
50 weather. Consequently, we argue that the relation between futures price returns and volatil-
51
52 ity is influenced by two main sources, namely, commodity-specific effects and market-wide
53
54 shock effects. These two effects impact differently the two distinctive commodity markets,
55
56 namely, investment commodities and consumption commodities. These effects are further
57
58 confirmed in Section 5 for the gold futures market and the crude oil futures market.
59
60

2.1. *Investment commodities*

Investment commodities such as the monetary metals gold and silver are treated by the majority of the market participants as financial assets. In the leading financial market, the equity market, there is compelling empirical evidence that the volatility feedback effect and/or the leverage effect typically generates an asymmetric volatility reaction. For investment commodities accordingly, a similar volatility reaction should prevail where falling commodity price returns are associated with an increase in conditional volatility. As the spot commodity price returns are historically positively correlated to the futures price returns (for instance for gold, the correlation coefficient between the 12-month futures returns and 1-month futures returns over thirty years is 0.991), this effect can be transmitted to the futures prices, implying that increasing futures price volatility is associated with negative futures price returns. Additionally, if a volatility shock is anticipated then futures traders are not willing to trade, thus futures prices drop to balance buying and selling volumes. Thus negative futures return shocks are associated with increasing volatility. Overall due to commodity specific effects, for investment commodities, a negative relation between futures price returns and volatility should be pertinent.

Alternatively, most investment commodities, such as gold, have the property of a safe haven investment. That is, investment commodities are sought by investors during periods of uncertainty driven by market-wide shocks, see Baur and McDermott (2010) and Reboredo (2013). Positive commodity price changes that are principally associated with safe haven purchases, are signals for increasing risk or uncertainty in macroeconomic and financial conditions. This introduces uncertainty in the market, thus increasing volatility. This effect is consistent with the empirical studies of Giamouridis and Tamvakis (2001) and Baur (2012) in the gold market. Subsequently, as futures commodity prices are historically positively correlated with spot commodity prices, the same reaction would be anticipated in the futures markets. Apart from the safe haven property, according to the Market Pressure Theory (Cootner (1960)), while futures prices increase, large long speculative trading activity in futures markets would be taken that will lead to further futures price increases enforcing a positive return-volatility relation. Additionally, the connection between inventory and volatility (as low inventory signals high future volatility) can also potentially produce inverted

1
2
3
4
5
6
7
8 asymmetries in futures volatility.
9

10 2.2. *Consumption commodities*

11
12 For consumption commodities, such as the crude oil, supply and demand forces primarily
13 affect the market via inventory. As explained by Ng and Pirrong (1994), in accordance to the
14 Theory of Storage, the inventory effect implies that a low inventory (among other reasons
15 due to a shortage in the commodity) will drive commodity prices to rise and the market
16 volatility to increase and vice versa. Carpentier and Dufays (2013) have confirmed that
17 the inventory effect is generally significant for a variety of commodities, especially when the
18 unconditional variance is time-varying. Inventory and convenience yield, according to the
19 Theory of Storage (Kaldor (1939)), are negatively correlated. Pindyck (2001) explains that
20 as volatility increases, the convenience yield increases as a result of an increasing demand for
21 storage; market participants will increase their inventories in order to absorb the anticipated
22 shocks in production and consumption. This drives the commodity spot price to rise more
23 than futures prices, leading to an inverted futures market (which is a typical market condition
24 for consumption commodities especially over commodity scarcity periods). At the same
25 time, the higher the convenience yield, the stronger the pressure for a rise in, preliminarily,
26 the spot and subsequently the futures commodity prices is anticipated. The volatility of
27 the spot price returns, the volatility of the futures price returns and the volatility of the
28 convenience yield are also all increasing. Thus positive futures return shocks are associated
29 with increasing volatility. However, if inventory is high, the convenience yield is relatively
30 low, commodity prices tend to decrease and the volatility is getting lower, reflecting the
31 decreasing risk of the exhaustion of inventories. Hence negative return shocks would signal
32 lower future volatility. Overall for consumption commodities, due to commodity specific
33 effects including the inventory effect, a positive relation between futures price returns and
34 volatility should be pertinent.
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54

55 The impact of severe market-wide shocks on the futures prices of consumption commodi-
56 ties and consequently the return–volatility relation is not always definite and straightfor-
57 ward.³ In the last decade, as a result of the 9/11 terrorist attacks, the US invasion to Iraq
58
59
60

³Note that, all consumption commodities do not have the same reactions to extreme market conditions of

1
2
3
4
5
6
7
8 and Global Financial Crisis (GFC), see Kilian (2009) and Tokic (2010), the crude oil mar-
9
10 ket has experienced excessive volatility, not only in terms of volatility of spot prices and
11
12 futures price returns but also in terms of volatility of adjusted spreads (adjusted spreads are
13
14 considered as a measure of convenience yield, see Geman and Ohana (2009)). The 2008 oil
15
16 bubble was attributed to the increasing oil demand amidst stagnant oil production to meet
17
18 the strong global economic growth that occurred up to 2008, see Kilian and Hicks (2013), as
19
20 well as increasing speculative trading activity.⁴ These factors have been causing oil prices to
21
22 rise until mid of 2008 where intense economic contraction led by the GFC caused oil prices
23
24 to plunge by 80% between July 2008 and December 2008. While the increasing volatility was
25
26 predominantly attributed to economic contractions that impacted negatively on the demand
27
28 for crude oil, that led to a noticeable decrease of the spot and futures commodity prices and
29
30 returns (as observed in the second half of 2008) implying a negative return-volatility relation
31
32 and a considerable decrease of the convenience yield.⁵ Thus the considerably lower (mostly
33
34 negative) convenience yield was reflected by the persistence of normal market conditions in
35
36 these markets. Traditionally inverted consumption commodity markets were *normal* and
37
38 more specifically, the crude oil market underwent an extended period of normal market con-
39
40 ditions over the last four years following the economic contraction of the GFC.⁶ This reverse
41
42 flow of risk premium may be the result of financial investors seeking portfolio diversification
43

44 extensive volatility. For instance as a result of the significant market-wide shock of the GFC, the traditionally
45
46 inverted crude oil market has been normal for an extended period of 4 years, while the sugar futures market
47
48 was strongly inverted. The fall of sugar production in 2009–2010 due to poor weather conditions in major
49
50 production areas combined with the scarcity of investment capital to increase production as a result of the
51
52 GFC and an increasing demand for sugar from food industries have led to a very strong convenience yield.

53 ⁴Increasing liquidity attributed to the price discovery in futures markets has the effect of relating positive
54
55 returns with decreasing volatility (the liquidity effect). Note that non-fundamental changes to futures prices
56
57 typically are transmitted to spot prices even without inventories adjustments, see Morana (2013).

58 ⁵This is not consistent with the fundamentals of consumption commodities as presented in Pindyck (2001),
59
60 where under increasing volatile market conditions, the convenience yield increases as a result of an increasing
demand for storage.

⁶It is worth noticing that the Asian financial crisis in 1997–1998 caused a similar extended period of
normal market conditions that lasted for approximately one and half years.

1
2
3
4
5
6
7
8 ally associated with a high level of volatility, see Gorton, Hayashi and Rouwenhorst (2013),
9 or implying a structural shift in inventory management, see Morana (2013)).

10
11 In light of the above observations, a new effect emerges that has explanatory power
12 on the return–volatility relation in futures commodity markets and it is referred to as the
13 *convenience yield effect*. This effect is more relevant to consumption commodities. According
14 to this effect, as convenience yield increases (decreases) implying that the commodity market
15 becomes inverted (normal), then the increasing volatility associated with the shocks effecting
16 the convenience yield leads to increasing (decreasing) futures returns resulting from the
17 upward (downward) pressure in the spot commodity prices. Thus inverted commodity futures
18 markets should entail a positive return–volatility relation, while normal commodity futures
19 markets give rise to a negative return–volatility relation. Note that the convenience yield
20 effect is more general compared to the inventory effect as it can account for non-fundamental
21 changes of futures prices. Section 5.2 demonstrates that this effect can explain the return–
22 volatility relation especially in the crude oil market.
23
24
25
26
27
28
29
30
31
32
33

34 35 **3. Modelling the return–volatility relation**

36 37 *3.1. A stochastic volatility model for commodity futures prices*

38
39 Let $\mathbf{V} = \{\mathbf{V}_t, t \in [0, T], \}$ denote a stochastic volatility process describing the uncertainty
40 in the commodity futures market and $F(t, T, \mathbf{V}_t)$ denote the futures price, at time $t \geq 0$,
41 of a commodity futures contract with maturity T (for all maturities $T \geq t$). The spot
42 price at time t of the underlying commodity, denoted as $S(t, \mathbf{V}_t)$ satisfies the relationship
43 $S(t, \mathbf{V}_t) = F(t, t, \mathbf{V}_t)$, $t \in [0, T]$. It is well known that the commodity futures price process
44 can be described by a driftless stochastic process under a risk-neutral probability measure Q ,
45 since the futures price process is equal to the expected future commodity spot price under
46 this measure, see Duffie (2001). Thus the risk-neutral dynamics of the futures price can be
47
48
49
50
51
52
53
54
55
56
57
58
59
60

modelled as⁷

$$\frac{dF(t, T, \mathbf{V}_t)}{F(t, T, \mathbf{V}_t)} = \sum_{i=1}^n (\kappa_{0i} + \kappa_i(T-t)) e^{-\eta_i(T-t)} \sqrt{\mathbf{V}_t^i} dW_i^1(t), \quad (1)$$

$$d\mathbf{V}_t^i = \mu_i^V (\nu_i^V - \mathbf{V}_t^i) dt + \varepsilon_i^V \sqrt{\mathbf{V}_t^i} \left(\rho_i dW_i^1(t) + \sqrt{1 - \rho_i^2} dW_i^2(t) \right), \quad (2)$$

where, $\mathbf{V}_t = \{\mathbf{V}_t^1, \dots, \mathbf{V}_t^n\}$ is an n -dimensional process, $W^1(t) = \{W_1^1(t), \dots, W_n^1(t)\}$ and $W^2(t) = \{W_1^2(t), \dots, W_n^2(t)\}$, for all $t \in [0, T]$, denote independent standard Wiener processes under the risk-neutral probability measure and for $i = 1, \dots, n$, the parameters κ_{0i} , κ_i , η_i , μ_i^V , ν_i^V , ε_i^V and ρ_i are constants. These model specifications allow for a variety of shapes for the volatility structure of futures prices, including exponentially decaying and hump-shaped volatility structures. There is empirical evidence that these are typical volatility structures of commodity futures return volatility, see Trolle and Schwartz (2009) and Chiarella, Kang, Nikitopoulos and Tô (2013).

The commodity forward price model (1) and (2) admits finite dimensional realisations, see Chiarella and Kwon (2003) and Björk, Landén and Svensson (2004) and leads to exponentially affine structures for the futures price process as the following proposition presents.

Proposition 1. *The futures price $F(t, T, \mathbf{V}_t)$ at time t of a commodity futures contract with maturity T is exponentially affine and can be expressed as*

$$F(t, T, \mathbf{V}_t) = F(0, T, V_0) \exp\{-Z(t, T)\} \quad (3)$$

$$Z(t, T) = \sum_{i=1}^n \left(\frac{1}{2} (\gamma_{i1}(T-t)x_i(t) + \gamma_{i2}(T-t)y_i(t) + \gamma_{i3}(T-t)z_i(t)) + (\beta_{i1}(T-t)\phi_i(t) + \beta_{i2}(T-t)\psi_i(t)) \right), \quad (4)$$

where for $i = 1, 2, \dots, n$

$$\begin{aligned} \beta_{i1}(T-t) &= (\kappa_{0i} + \kappa_i(T-t)) e^{-\eta_i(T-t)}, \\ \beta_{i2}(T-t) &= \kappa_i e^{-\eta_i(T-t)}, \\ \gamma_{i1}(T-t) &= (\beta_{i1}(T-t))^2, \\ \gamma_{i2}(T-t) &= 2\beta_{i1}(T-t)\beta_{i2}(T-t), \\ \gamma_{i3}(T-t) &= (\beta_{i2}(T-t))^2. \end{aligned} \quad (5)$$

⁷A suitable filtered probability space $(\Omega, \mathcal{A}_T, \underline{\mathcal{A}}, P)$, $T \in (0, \infty)$ is considered and the required processes are $\underline{\mathcal{A}}$ -adapted bounded with regular and predictable drifts and diffusions thus unique strong solutions for the associated SDEs exist.

The state variables $x_i(t)$, $y_i(t)$, $z_i(t)$, $\phi_i(t)$ and $\psi_i(t)$, $i = 1, 2, \dots, n$ satisfy the stochastic differential equations

$$\begin{aligned} dx_i(t) &= (-2\eta_i x_i(t) + \mathbf{V}_t^i) dt, \\ dy_i(t) &= (-2\eta_i y_i(t) + x_i(t)) dt, \\ dz_i(t) &= (-2\eta_i z_i(t) + 2y_i(t)) dt, \\ d\phi_i(t) &= -\eta_i \phi_i(t) dt + \sqrt{\mathbf{V}_t^i} dW_i(t), \\ d\psi_i(t) &= (-\eta_i \psi_i(t) + \phi_i(t)) dt, \end{aligned} \tag{6}$$

subject to the initial conditions $x_i(0) = y_i(0) = z_i(0) = \phi_i(0) = \psi_i(0) = 0$, with the volatility processes \mathbf{V}_t^i following the dynamics (2).

Proof: Follows along the lines of Chiarella et al. (2013). ■

Duffie, Pan and Singleton (2000) and Collin-Dufresne and Goldstein (2002) have derived quasi-analytical solutions for European options on assets with their dynamics being characterized by exponentially affine models. Chiarella et al. (2013) have adjusted these results and have shown that under the commodity forward price model specifications (1) and (2), quasi-analytical prices for European options on futures contracts can be obtained, which are used in our empirical analysis. For the sake of completeness, Appendix A presents the European option pricing equations.

In addition, according to Dai and Singleton (2000) and Doran and Ronn (2008), the market price of futures price risk and the market price of volatility risk can be respectively specified by the “complete” affine representation

$$\begin{aligned} dW_i^{\mathbb{P}}(t) &= dW_i^1(t) - \lambda_i \sqrt{\mathbf{V}_t^i} dt, \\ dW_i^{\mathbb{P}^V}(t) &= dW_i^V(t) - \lambda_i^V \sqrt{\mathbf{V}_t^i} dt, \end{aligned} \tag{7}$$

for $i = 1, \dots, n$, where λ_i and λ_i^V are constants, $dW_i^V(t) = \rho_i dW_i^1(t) + \sqrt{1 - \rho_i^2} dW_i^2(t)$ and, $W_i^{\mathbb{P}}(t)$ and $W_i^{\mathbb{P}^V}(t)$ are Wiener processes under the physical measure \mathbb{P} .

Note that, the correlation structure ρ_i of the innovations driving the futures return process and the volatility process provides an indication of the level of volatility risk that can be hedged by futures contracts. For instance, when the Wiener processes are perfectly correlated, i.e. $\rho_i = 1$ for all i , then portfolios of futures contracts can completely hedge the

associated volatility risk. Otherwise, the volatility risk can be partially spanned by futures contracts alone. However, the correlation structure of the proposed stochastic volatility model is also linked to the return-volatility relation of commodity futures prices as the next section explains.

3.2. Assessing the return–volatility relation

Engle and Ng (1993) introduced the News Impact Function (NIF hereafter) as an empirical tool to analyse the return–volatility relation. Conditioning on the information available up to time t , the NIF measures the impact of the return shocks at time t , denoted as δ_t , on the future volatility $\sigma^2(t+1, T)$ at time $t+1$. Typically, ARCH type models have been employed in the literature to study this relation, see for example the asymmetric ARCH models discussed in Glosten, Jagannathan and Runkle (1993) and Baur (2012).

However this definition cannot be applied directly to stochastic volatility models, where two sources of uncertainty are present at time t , namely the return shocks δ_t and the volatility shocks, denoted as ϵ_t . For stochastic volatility models, the NIF should be generalised to account for its stochastic nature. Thus, the generalised NIF measures the impact of the return shocks δ_t at time t on the *expected* future volatility $\mathbb{E}[\sigma^2(t+1, T)]$ at time $t+1$ (when conditioning on the information available up to time t). To compute the generalised NIF, the information available up to time t is set to be constant, thus lagged volatility is fixed at the long run mean of $\sigma^2(t, T)$, denoted as $\bar{\sigma}^2$. Thus the generalised NIF is defined as

$$NIF(\delta_t) = \mathbb{E}[\sigma^2(t+1, T)|\delta_t, \sigma^2(t, T) = \bar{\sigma}^2, \sigma^2(t-1, T) = \bar{\sigma}^2, \dots], \quad (8)$$

and, accordingly for the multi-dimensional case of model (1), it can be defined as

$$NIF(\delta_{it}) = \mathbb{E}[\sigma_i^2(t+1, T)|\delta_{it}, \sigma_i^2(t, T) = \bar{\sigma}_i^2, \sigma_i^2(t-1, T) = \bar{\sigma}_i^2, \dots], \quad (9)$$

where $\bar{\sigma}_i^2$ are constants for $i = 1, 2, \dots, n$ and δ_{it} is the return shocks at time t of the corresponding factor i .

Proposition 2. *The NIF for the stochastic volatility model (1) and (2) can be expressed as*

$$NIF(\delta_{it}) = \varphi_i^2(t+1, T) \left[\mu_i^V \nu_i^V \Delta t + (1 - \mu_i^V \Delta t) \bar{V}_t^i + \rho_i \epsilon_i^V \sqrt{\bar{V}_t^i} \delta_{it} \right], \quad (10)$$

where $\bar{V}_t^i = \bar{\sigma}_i^2 / \varphi_i^2(t, T)$ and $\varphi_i(t, T) = (\kappa_{0i} + \kappa_i(T-t))e^{-\eta_i(T-t)}$.

Proof: Let $R_F(t, T)$ denote the returns of the futures price of futures contracts with maturity T , then according to (1), returns can be expressed as

$$R_F(t, T) = \sum_{i=1}^n \sigma_i(t, T, \mathbf{V}_t) dW_i^1(t), \quad (11)$$

where, by using the time deterministic functions $\varphi_i(t, T)$,

$$\begin{aligned} \sigma_i(t, T, \mathbf{V}_t) &= (\kappa_{0i} + \kappa_i(T - t))e^{-\eta_i(T-t)} \sqrt{\mathbf{V}_t^i} \\ &= \varphi_i(t, T) \sqrt{\mathbf{V}_t^i}. \end{aligned} \quad (12)$$

An Euler approximation of the stochastic volatility process (2) provides the relation⁸

$$\mathbf{V}_{t+1}^i \approx \mu_i^V \nu_i^V \Delta t + (1 - \mu_i^V \Delta t) \mathbf{V}_t^i + \varepsilon_i^V \sqrt{\mathbf{V}_t^i} \epsilon_{it}, \quad (13)$$

where the volatility shocks ϵ_{it} of the volatility factor i and the return shocks δ_{it} are iid $N(0, 1)$ with correlation structure $\text{corr}(\epsilon_{it}, \delta_{it}) = \rho_i$. By using (12) and (13) and fixing information at time t or earlier at the constant long run volatility $\bar{\sigma}_i^2$, the generalised NIF (9) can be evaluated as follows

$$\begin{aligned} NIF(\delta_{it}) &= \mathbb{E}[\sigma_i^2(t+1, T, \mathbf{V}_{t+1}^i) | \delta_{it}, \sigma_i^2(t, T, \mathbf{V}_t^i) = \bar{\sigma}_i^2, \sigma_i^2(t-1, T, \mathbf{V}_{t-1}^i) = \bar{\sigma}_i^2, \dots] \\ &= \varphi_i^2(t+1, T) \mathbb{E}[\mathbf{V}_{t+1}^i | \delta_{it}, \mathbf{V}_t^i = \bar{V}_t^i, \mathbf{V}_{t-1}^i = \bar{V}_{t-1}^i, \dots] \\ &= \varphi_i^2(t+1, T) \left[\mu_i^V \nu_i^V \Delta t + (1 - \mu_i^V \Delta t) \bar{V}_t^i + \rho_i \varepsilon_i^V \sqrt{\bar{V}_t^i} \delta_{it} \right], \end{aligned} \quad (14)$$

which derives the result. ■

The slope of the NIF identifies the return–volatility relation. It is evident from equation (14) that the slope of the NIF depends on the correlation coefficient ρ_i , thus the return–volatility relation can be assessed through the correlation coefficient ρ_i . A negative ρ_i leads

⁸The stochastic volatility process used here is the process with the correlated Wiener processes $W_i^1(t)$ and $W_i^V(t)$, such that,

$$d\mathbf{V}_t^i = \mu_i^V (\nu_i^V - \mathbf{V}_t^i) dt + \varepsilon_i^V \sqrt{\mathbf{V}_t^i} dW_i^V(t),$$

with

$$\mathbb{E}^Q[dW_k^1(t) \cdot dW_j^V(t)] = \begin{cases} \rho_k dt, & k = j; \\ 0, & k \neq j. \end{cases}$$

to a negative NIF slope which indicates that when there is a negative shock in the stochastic volatility factor i , there will be a positive shock to the futures returns and vice versa. This will imply a negative return-volatility relation, consequently, an asymmetric volatility reaction. Conversely, a positive ρ_i implies a positive relationship between return and conditional volatility, leading to an inverted asymmetric volatility reaction.

Note that, when information up to time t is set at the constant long run volatility $\bar{\sigma}_i^2$, then (12) derives

$$\bar{\sigma}_i^2 = \varphi_i^2(t, T) \mathbf{V}_t^i = \varphi_i^2(t-1, T) \mathbf{V}_{t-1}^i = \dots$$

Thus the lagged long run volatility factors \mathbf{V}_t^i are specified by $\bar{V}_t^i = \bar{\sigma}_i^2 / \varphi_i^2(t, T)$, for $t \leq T$, i.e. the value of $\bar{\sigma}_i^2$ and $\varphi_i(t, T)$ at time t .

The multi-dimensional set up used in the proposed model allows to easily identify and distinguish between the asymmetric volatility factors and the inverted asymmetric volatility factors and measure their contribution. Furthermore, from the following equation

$$NIF(\delta_{1t}, \dots, \delta_{nt}) = \sum_{i=1}^n NIF(\delta_{it}), \quad (15)$$

the overall weighted average effect of these volatility factors can be computed and an aggregate assessment of the return-volatility relation can be obtained. See also Yu (2004) and Yu (2005) for applications of the generalised NIF.

4. Data and Method

A database from NYMEX⁹ of futures and futures options for two commodities, gold and crude oil, is used to estimate the stochastic volatility model of Section 3. Gold and crude oil are two of the most liquid commodity markets, whilst they are fundamentally different as gold is classified as an investment commodity, whereas crude oil as a consumption commodity. The gold database covers around 31 years, from 4 October 1982 to 16 April 2012. The crude oil database covers around 21 years, from 2 January 1990 to December 2010, as option price data were available only from 1990. While the database of the gold derivatives market is one

⁹The database was purchased from CME Group.

of the longest derivatives databases, crude oil is one of the richest as crude oil is the most active commodity derivatives market.

As the number of available futures contracts over the sample period is very large, for the estimation exercise, futures contracts are selected based on their liquidity. For the options data, the options on the selected futures contracts are considered, but maturities that are more than two years are intentionally avoided. The proposed model assumes non-stochastic interest rates, therefore the option pricing formula for long maturities is not precise. The option prices provided by the CME are American options which are converted to European prices as required for the proposed model, by using the same approach proposed by Broadie, Chernov and Johannes (2007) for equity options and by Trolle and Schwartz (2009) for commodity options.¹⁰

4.1. Gold data

Over the last 31 years, the number of available gold futures contracts with positive open interest per day has increased from 11 on 4th of October 1982 to 22 on 16th of April 2012. In the same period, the maximum maturity of futures contracts with positive open interest has also increased from 690 (calendar) days to 2,078 days.

Based on liquidity, the first three monthly contracts, near to the trade date are initially selected,¹¹ followed by the four contracts which have either February, April, June, August, October or December expiration months. Beyond that, liquidity is concentrated in June and December contracts only, hence these four contracts are followed by four semi-annual contracts which have either June or December expiration months. As a result, the total number of futures contracts to be used in our analysis is 69,684, with the number of contracts to be used on a daily basis varying between 8 and 10. Figure 1.a–Figure 1.c plot the selected futures prices on Wednesdays over the sample period.

From the options data, the options on the first six selected futures contracts are con-

¹⁰The Barone-Adesi and Whaley (1987) formula for American option prices is inverted to obtain lognormal implied volatilities that are subsequently used to compute European Black (1976) prices.

¹¹As liquidity is very low for contracts with maturity less than 14 days, so the first contract should have more than 14 days to maturity.

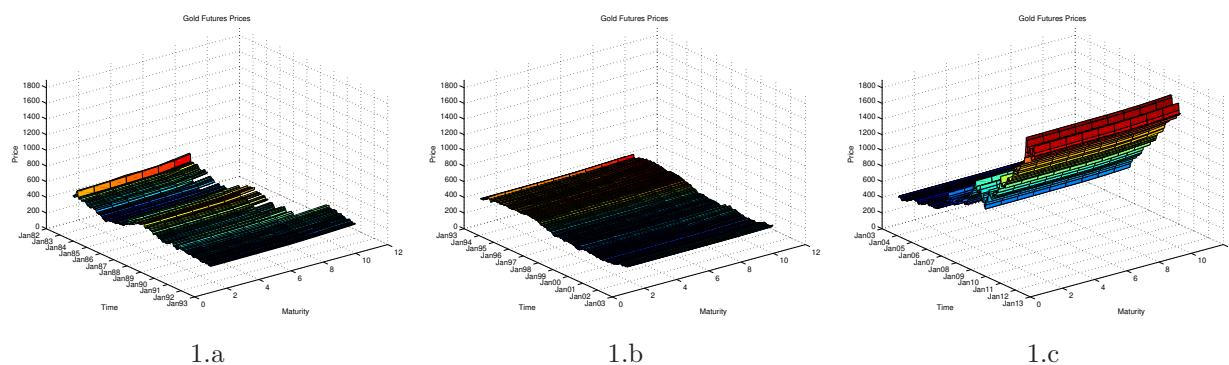


Figure 1: **Gold Futures**

The figure presents futures prices of selected gold futures contracts in three different periods, namely, October 1982 to December 1992, January 1993 to December 2002 and January 2003 to April 2012. The selected contracts are: the first three monthly contracts near to the trade date; the next four contracts which have either February, April, June, August, October or December expiration; and the next four semi-annual contracts (June or December). The data are displayed only on Wednesdays.

sidered, namely the first three monthly contracts and the next three contracts which have either February, April, June, August, October or December expiration months. For each option maturity, six moneyness¹² intervals have been used, 0.86 – 0.90, 0.91 – 0.95, 0.96 – 1.00, 1.01 – 1.05, 1.06 – 1.10, 1.11 – 1.15. In each moneyness interval, out-of-the-money and at-the-money options that are closest to the interval mean are included. Based on this selection criteria, 367,412 option contracts are considered over the 31 years, with the daily range varying between 19 and 72 contracts (per trading day). Note that the total number of trading days where both futures and options data are available is 7,427.

4.2. *Crude Oil Data*

In the 21-year crude oil database, the liquidity has consistently increased for all maturities. Based on their liquidity, the first seven monthly contracts, near to the trade date are included,¹³ followed by the three contracts which have either the March, June, September or December expiration months. Beyond that, liquidity is concentrated in December contracts

¹²We define as moneyness the ratio of the option strike and the price of the underlying futures contract.

¹³Similarly to the gold futures contracts, the first contract should have more than 14 days to maturity.

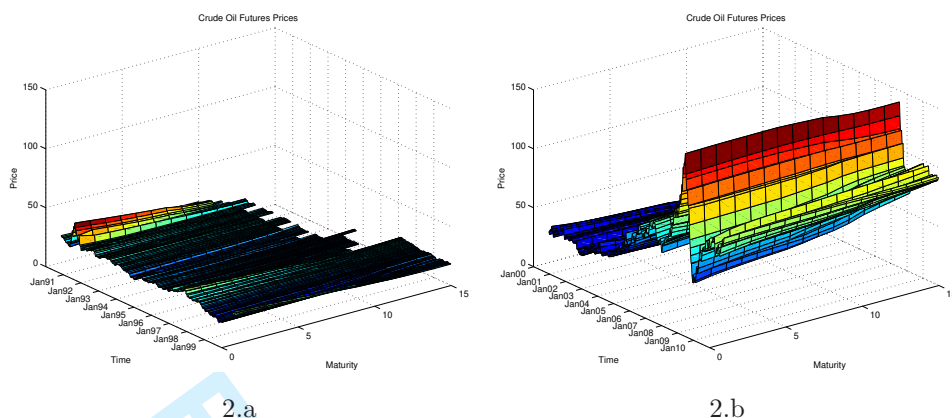


Figure 2: **Crude Oil Futures**

The figure presents prices, returns and implied volatilities of selected crude oil futures contracts in two different periods, namely, January 1990 to December 1999 and January 2000 to December 2010. The selected contracts are: the first seven monthly contracts near to the trade date (with the first contract having more than 14 days to maturity); the next three contracts which have either March, June, September or December expiration months; and the next five December contracts. The data are displayed only on Wednesdays.

only, therefore the next five December contracts are included. As a result, the total number of futures contracts to be used in the analysis is 70,735, with the number of contracts to be used on a daily basis varying between 8 and 15. Figure 2.a and Figure 2.b plot the selected futures prices on Wednesdays during the sample periods.

From the crude oil options database, the options on the first ten futures contracts are considered, namely the first seven monthly contracts and the next three quarterly contracts. For each option maturity, six moneyness intervals, $0.86 - 0.90$, $0.91 - 0.95$, $0.96 - 1.00$, $1.01 - 1.05$, $1.06 - 1.10$, $1.11 - 1.15$ are used. In each moneyness interval, out-of-the-money and at-the-money options that are closest to the interval mean are selected. Overall, 433,137 crude oil option contracts are included in the study, with the daily range varying between 29 and 100 contracts (per trading day). Note that the total number of trading days where both futures and options data are available is 5,272.

4.3. Further model specifications

The analysis is undertaken by fitting the model to the entire sample, as well as, to subperiods of an approximate length of a decade. As it can be visually detected in Figure 1

and Figure 2, as well as, from the descriptive statistics displayed in Table 1, over these decades, marked differences in futures price returns and in volatility have occurred in both commodity markets. Noteworthy events such as the gold price crisis in 1990, the Gulf War in 1991 and the GFC in 2008 have influenced the commodity markets and led to extreme market price swings and thus volatility. Moreover, for an analysis of the volatility of the futures commodity markets, it is potentially more informative to concentrate on subperiods as the trading of futures contracts is used to relatively short-lived strategies.¹⁴

Table 1: Descriptive statistics

Gold Futures								
	1982 – 1992		1993 – 2002		2003 – 2012		1982 – 2012	
Maturity	1M	13M	1M	13M	1M	13M	1M	13M
Mean	-0.000061	-0.000091	0.000027	0.000020	0.000661	0.00066	0.000194	0.000181
St. Dev.	0.010674	0.010662	0.008064	0.007884	0.012754	0.012852	0.010623	0.010578
Kurtosis	9.364871	9.285527	17.842600	16.5688	6.692487	6.682154	9.668339	9.312701
Skewness	-0.283403	-0.294973	1.064952	0.925083	-0.2806	-0.30895	-0.07089	-0.14321

Crude Oil Futures						
	1990 – 1999		2000 – 2010		1990 – 2010	
Maturity	1M	13M	1M	13M	1M	13M
Mean	0.000044	-0.000002	0.000471	-0.000569	0.000263	0.000294
St. Dev.	0.024396	0.012993	0.025635	0.017173	0.025051	0.015327
Kurtosis	36.4799	12.0566	7.073648	5.603809	19.69962	7.593329
Skewness	-1.792799	-0.66896	-0.203111	-0.204234	-0.90038	-0.32904

The table displays the descriptive statistics for daily log returns of gold futures prices between October 4, 1982 and December 28, 2012 and for daily log returns of crude oil futures prices between January 2, 1990 and April 16, 2010.

¹⁴Nevertheless, for a study on the volatility of the spot commodity markets, it might be more effective to look at the whole period as many investors tend to hold positions on the underlying commodity, especially investment commodities such as gold, over longer periods of time.

1
2
3
4
5
6
7
8 For the crude oil market, Bekiros and Diks (2008) point out that the market changes
9 significantly before and after 1999. Given the reduction in OPEC spare capacity and the
10 increase in the US and China's oil consumption and imports, there was an increase in the
11 oil price as well as its volatility. Therefore, the sample is separated into two subsamples,
12 one from 1990-1999, and one from 2000-2010. Table 1 displays the summary statistics for
13 the gold futures market. The middle period is quite different to the other two periods. In
14 the middle period, prices experience much lower volatility with positive skewness and excess
15 kurtosis. However, the gold price crisis in 1990 and the GFC in 2008 amongst other reasons
16 have triggered significant volatility in the gold futures market over the first and the third
17 sample periods, respectively. Consequently, the gold data are examined in three sample
18 periods.¹⁵

19
20
21
22
23
24
25
26
27
28 The number of stochastic volatility factors affecting the evolution of the futures curve
29 is initially assessed by performing a principal component analysis (PCA) of futures price
30 returns. According to the PCA results displayed in Table 2, in the gold futures market,
31 one factor can explain on average 98% of the variations, whereas for the crude oil market,
32 a two-factor model should be used as two factors explain 98% between 1990 and 1999 and
33 94% between 2000 and 2010. As a robustness check, a two-factor stochastic volatility model
34 for gold futures was also considered but the additional factor did not improve the model fit
35 to futures and option prices. In addition for the gold futures market, both hump-shaped
36 and exponentially decaying ($\kappa_i = 0$) specifications were investigated and it was found that
37 the exponentially decaying specifications provide a better fit.¹⁶ For the crude oil futures
38 market, hump-shaped volatility specifications perform better, as it has been demonstrated in
39 Chiarella et al. (2013). Therefore a one-factor stochastic volatility model with exponentially
40 decaying volatility functions for gold and a two-factor stochastic volatility model with hump-
41 shaped volatility functions for crude oil are estimated.

54
55
56
57
58
59
60
¹⁵The fact that the decades used in the two commodity markets do not completely overlap, does not
impair the validity of the results. The key criterion of the analysis is the relative levels of volatility over the
different sample periods, thus the break points are not of critical importance.

¹⁶Results can be provided by the authors upon request.

Table 2: Accumulated percentage of factor contribution

Gold			
Time Period	One factor	Two factors	Three factors
1983 - 1992	0.9815	0.9967	0.9983
1993 - 2002	0.9786	0.9946	0.9977
2003 - 2012	0.9922	0.9988	0.9998

Crude Oil			
Time Period	One factor	Two factors	Three factors
1990 - 1999	0.9042	0.9822	0.9972
2000 - 2010	0.8761	0.9402	0.9719

The table displays the accumulated percentage of PCA factor contribution to gold futures return variation and crude oil futures return variation. One factor is able to explain most of the variations of the gold futures returns, while at least two factors are required for the crude oil futures returns, during each of the subperiods.

4.4. Estimation method

The model is expressed in a state-space mode consisting of the system equations and the observation equations. The system equations describe the discrete evolution of the underlying state-space consisting of the (six) state variables $\mathbf{x}_i(t)$, $\mathbf{y}_i(t)$, $\mathbf{z}_i(t)$, $\phi_i(t)$, $\psi_i(t)$ and \mathbf{V}_t^i with dynamics (under the physical probability measure) driven by the stochastic differential equations (2), (6) and (7). The observation equations relate the observed options and futures prices to the state variables. Specifically, futures prices are exponentially affine in the state-space (as described in (3)) while the options prices are nonlinear functions of the state-space. Therefore an extended Kalman filter is required that applies an approximate linearization of the associated nonlinear observation equations, see Trolle and Schwartz (2009) and Chiarella et al. (2013) for similar applications. The quasi-maximum likelihood estimation algorithm is employed. Appendix B presents the details. The loglikelihood function is maximised by using the constrained optimization algorithm e04jy from the NAG library.¹⁷

¹⁷By using function values only, the quasi-Newton algorithm e04jy finds a minimum or maximum of a continuous function, subject to fixed upper and lower bounds of the independent variables. The e04jy algorithm is available in the NAG toolbox of Matlab.

1
2
3
4
5
6
7
8 To tackle the inhomogeneous nature of the model, a time-homogeneous version of the
9 model (3) is assumed where for all T , $F(0, T) = f_o$, where f_o is a constant representing the
10 long-term futures price (at infinite maturity) and it should also be estimated. To deal with
11 identification, the long run mean of the volatility process ν_i^V is normalised to one.
12
13
14
15

16 5. Estimation results

17
18
19 The parameter estimates of the proposed model are presented next when fitted to gold
20 derivative prices and crude oil derivative prices and the prevailed return–volatility relation
21 is analyzed in terms of possible explanatory factors.
22
23
24
25

26 5.1. Gold futures market

27
28 The parameter estimates of the one-factor stochastic volatility model with exponentially
29 decaying specifications when fitted to gold derivative prices are displayed in Table 3. The
30 model is estimated over the whole sample period of approximately thirty-one years, as well
31 as over three subperiods representing regimes with different volatility intensity. The perfor-
32 mance of the model is adequate with low root mean squared errors (RMSEs) of the percentage
33 differences between actual and fitted gold futures prices as well as of the difference between
34 actual and fitted implied gold option volatilities, as it is displayed in Figure 3.
35
36
37
38
39
40

41 Substantial differences in the estimates are detected over the three subperiods. The un-
42 derlying stochastic volatility factor \mathbf{V}_t has a higher mean reversion coefficient μ^V in the
43 middle period (1993-2002) compared to the other two periods. The impact of each stochas-
44 tic volatility factor on the volatility of the gold futures market is scaled by the function
45 $\varphi(t, T) = (\kappa_0 + \kappa(T - t))e^{-\eta(T-t)}$ (see equation (12)). For all periods, the estimates of the
46 attenuation parameter η are small, implying that volatility shocks impact relatively equally
47 along different maturities of the futures contracts. In addition, the correlation coefficients ρ
48 between shocks to the stochastic volatility factors and shocks to the gold futures returns are
49 significantly different from zero and range between 14% to 23%. The gold futures market
50 volatility is therefore not completely spanned by futures contracts.
51
52
53
54
55
56
57
58
59
60

Table 3: Parameter estimates - Gold futures market

	Period 1	Period 2	Period 3	Whole sample
	1982 – 1992	1993 – 2002	2003 – 2012	1982 – 2012
κ_0	0.2737 (0.0161)	0.5683 (0.0420)	0.8842 (0.0344)	0.4925 (0.0116)
η	0.0010 (0.0001)	0.0063 (0.0001)	0.0015 (0.0001)	0.0010 (0.0001)
μ^V	0.0876 (0.0013)	0.3171 (0.0035)	0.0817 (0.0005)	0.1027 (0.0045)
ε^V	1.9898 (0.1422)	2.0000 (0.0691)	0.6933 (0.0466)	0.5200 (0.0033)
ρ	0.1820 (0.0063)	-0.1423 (0.0136)	0.2288 (0.0173)	0.6670 (0.0087)
λ^V	1.0211 (0.0511)	1.0211 (0.0431)	0.9948 (0.0521)	0.5106 (0.0096)
λ	0.3641 (0.0218)	-0.5620 (0.0391)	0.1676 (0.0154)	0.1821 (0.0037)
f_0	5.8986 (0.0231)	2.6578 (0.0101)	1.8157 (0.0032)	1.8349 (0.0621)
σ_f	0.0012 (0.0000)	0.001 (0.0000)	0.001 (0.0000)	0.001 (0.0000)
σ_o	0.06 (0.0023)	0.08 (0.0012)	0.05 (0.0013)	0.07 (0.0009)

The table displays the maximum likelihood estimates for the one-factor model specifications (the standard errors in parenthesis) over thirty years, in addition to three subperiods, namely; October, 1982 to December, 1992, January 1993 to December 2002 and January 2003 to April 2012. Here f_0 is the homogenous futures price at time 0, namely $F(0, T) = f_0$, for all T . The quantities σ_f and σ_o are the standard deviations of the log futures prices measurements errors and the option price measurement errors, respectively. The long run mean of the volatility process, ν^V , has been normalised to one for identification purposes.

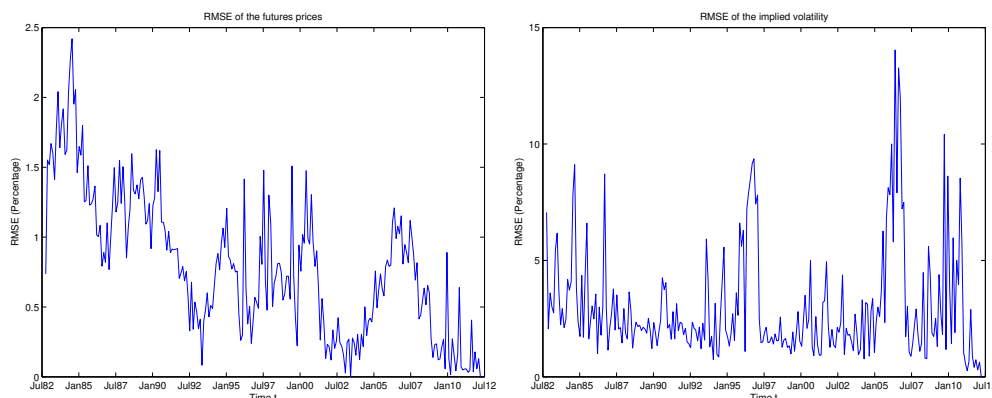


Figure 3: Model goodness of fit - gold futures market

The figure shows the RMSEs of the percentage differences between actual and fitted gold futures prices (the left-hand panel) as well as of the difference between actual and fitted implied option volatilities (the right-hand panel) for the one-factor model with exponentially decaying volatility. The model is estimated for the whole period from October 1982 to April 2012.

5.1.1. Return–volatility relation in the gold futures market

According to the estimation results in Table 3, when the model is fitted to the whole sample, the correlation estimate is positive, implying that in general there is a positive return–volatility relation in the gold futures market. Thus the gold futures volatility for all maturities has a similar response to return shocks as the spot gold volatility, see Baur (2012).

However when the model is estimated over the three subperiods, the responses clearly vary, depending on the level of the volatility. In the less volatile Period 2, the estimated correlation is negative and equal to -0.1423 , while in Period 1 and Period 3, a positive correlation is observed in both periods. The estimation results confirm the effects discussed in Section 2 regarding the return–volatility relation in investment commodity futures markets such as gold. Period 2 is characterized by low volatility and commodity futures prices are more likely to be principally determined by commodity specific effects. In the case of the gold futures market, as gold is classified as an investment asset like equities, a negative return–volatility relation should be expected, see Bekaert and Wu (2000) and Dennis et al. (2006). Conversely, in Period 1 and Period 3, the volatility was considerably higher at times. In the equity markets, there is empirical evidence that during high volatility regimes, systematic

1
2
3
4
5
6
7
8 risk factors become more influential, see Dennis et al. (2006). This is also prevalent in the
9 gold futures markets. The positive correlation of 0.182 in the period 1982-1992 and of 0.228
10 in the period 2003-2012 signify an inverted asymmetric volatility reaction that signals that
11 the safe haven property (a property that is typically instigated by market-wide shock effects)
12 was dominant over these periods.
13
14
15
16

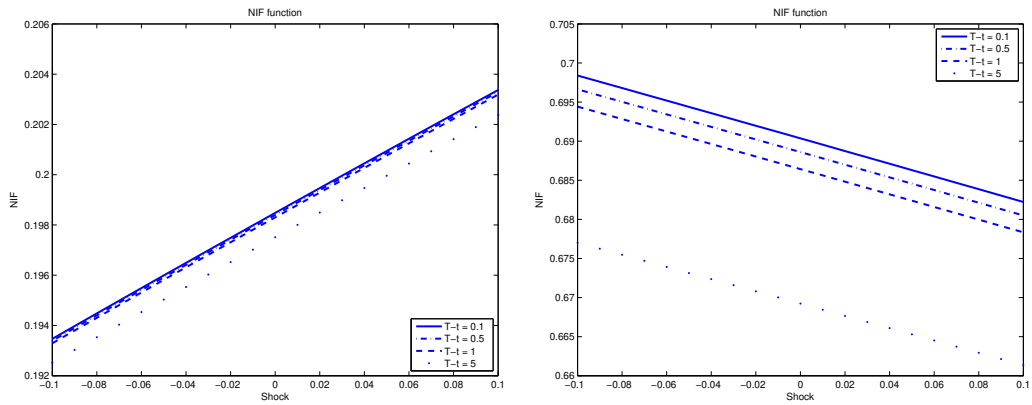
17 One noteworthy observation is that the results in Periods 2 and 3 are also consistent
18 with the convenience yield effect. The gold futures market is a market that is typically
19 normal (upward sloping forward curve) thus it should be characterised by a negative return-
20 volatility relation as it was revealed from our analysis in Period 2. However, during Period
21 3 and more specifically on 2 December 2008, gold futures market was inverted for the first
22 time in history, augmenting the empirically observed positive return-volatility relation.
23
24
25
26
27

28 Figure 4 displays the NIF over the three subperiods used in our analysis and for four dif-
29 ferent times to maturity; 0.1, 0.5, 1 and 5 years. First, the NIF is a monotonically decreasing
30 (asymmetric volatility) or an increasing (inverted asymmetric volatility) function. Second,
31 as the time to maturity increases the slope of the NIF remains essentially the same due to
32 the low parameter values for η that eliminates the impact of the time to maturity. However,
33 shocks of the same magnitude in the futures returns have a marginally stronger impact to
34 near maturity futures prices compared to longer maturity futures prices. Furthermore, this
35 effect is stronger over the less volatile periods such as the period between January 1993 to
36 January 2003, as shown in Figure 4.b.
37
38
39
40
41
42
43
44

45 5.2. Crude oil futures market

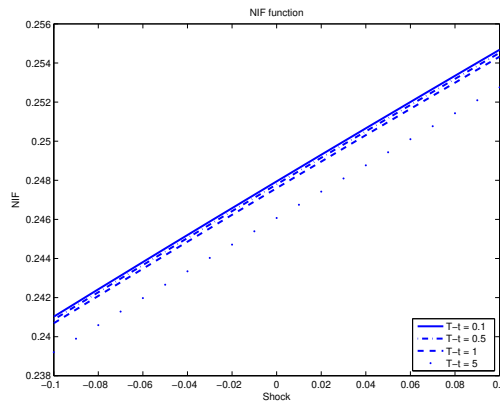
46
47 The parameter estimates of the two-factor stochastic volatility model with hump-shaped
48 specifications for crude oil futures are displayed in Table 4. Estimation is carried out over the
49 whole sample period of approximately twenty years and over two subperiods. Figure 5 shows
50 the RMSEs of the percentage differences between actual and fitted crude oil futures prices as
51 well as of the difference between actual and fitted implied crude oil option volatilities for the
52 two-factor model with hump-shaped volatility. Generally, the model performs well, except
53 a short period of the post-GFC period.
54
55
56
57
58
59
60

The crude oil market differs from the gold market in the way volatility changes as time



4.a

4.b



4.c

Figure 4: NIF for gold futures market

The figure compares the NIF for gold futures. Panels 4.a, 4.b, and 4.c present the NIF functions for the three periods, namely, October 1982 to December 1992, January 1993 to December 2002 and January 2003 to April 2012 respectively. For each period, we show the NIF for four different time to maturities; 0.1, 0.5, 1 and 5 years.

Table 4: Parameter estimates - Crude oil futures market

	Period 1: 1990 – 1999		Period 2: 2000 – 2010		Whole sample: 1990 – 2010	
	$i = 1$	$i = 2$	$i = 1$	$i = 2$	$i = 1$	$i = 2$
κ_{0i}	0.1852 (0.0116)	1.0374 (0.0768)	0.0677 (0.0072)	0.7459 (0.0316)	0.0010 (0.0100)	0.7077 (0.0436)
κ_i	1.8370 (0.0743)	0.0677 (0.0065)	1.9167 (0.0684)	0.4010 (0.0152)	1.8635 (0.0376)	0.2037 (0.0082)
η_i	1.4754 (0.0846)	0.0325 (0.0024)	0.0931 (0.0050)	0.0890 (0.0062)	0.5881 (0.0137)	0.0010 (0.0022)
μ_i^V	0.0010 (0.0001)	0.1490 (0.0057)	0.0010 (0.0004)	0.0010 (0.0005)	0.0019 (0.0003)	0.2379 (0.0133)
ε_i^V	0.7543 (0.0352)	1.2690 (0.0689)	0.5880 (0.0473)	1.2806 (0.0871)	0.8513 (0.0639)	1.4707 (0.0791)
ρ_i	-0.0668 (0.0042)	0.1225 (0.0168)	-0.0290 (0.0041)	-0.1442 (0.0109)	-0.1606 (0.0105)	-0.1681 (0.0096)
λ_i^V	-0.9521 (0.0721)	1.0619 (0.0678)	0.9974 (0.0351)	1.0423 (0.0645)	-0.9574 (0.0532)	1.1939 (0.0825)
λ_i	0.6349 (0.0328)	1.0227 (0.0637)	0.7424 (0.0449)	1.0227 (0.0623)	0.1844 (0.0050)	1.0227 (0.0661)
f_0	2.0099 (0.0132)		1.9530 (0.0211)		1.9513 (0.0257)	
σ_f	0.0010 (0.0000)		0.0010 (0.0000)		0.0010 (0.0000)	
σ_o	0.0100 (0.0021)		0.1070 (0.0053)		0.0377 (0.0015)	

The table displays the maximum-likelihood estimates for the hump-shaped two-factor stochastic volatility model specifications and the standard errors in parenthesis for twenty years as well as for two ten-year subperiods, namely, January, 1990 to December, 1999, January 2000 to December 2010. Here f_0 is the homogenous futures price at time 0, namely $F(0, T) = f_0$, for all T . The quantities σ_f and σ_o are the standard deviations of the log futures prices measurements errors and the option price measurement errors, respectively. We normalized the long run mean of the volatility process, ν_i^V , to one to achieve identification.

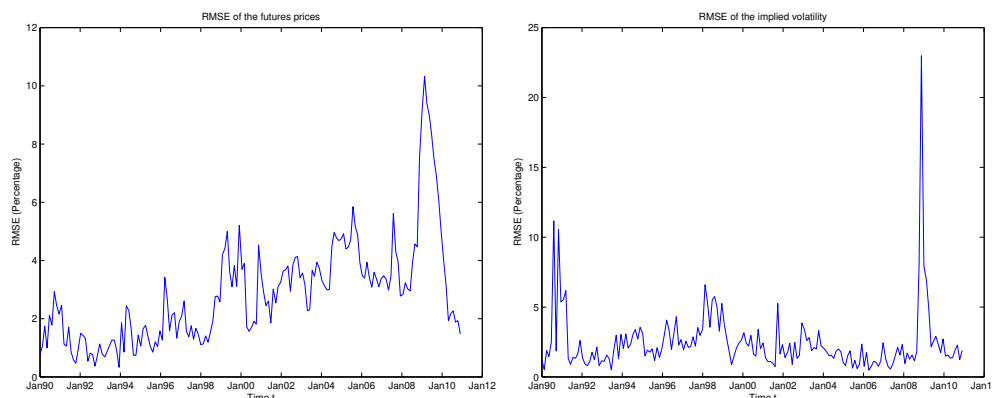


Figure 5: **Model goodness of fit - crude oil futures market**

The figure shows the RMSEs of the percentage differences between actual and fitted crude oil futures prices (the left-hand panel) as well as of the difference between actual and fitted implied option volatilities (the right-hand panel) for the two-factor model with hump-shaped volatility. The model is estimated for the whole period from January 1990 to December 2010.

to maturity changes. In the gold market, the effect dies out (though very slowly) as the time to maturity increases. In the crude oil market, the significant estimates of κ_i confirm the existence of hump-shaped volatility. Similar to the gold market, the volatility of the crude oil market is not completely spanned by futures contracts. The correlation coefficient ρ between shocks to the stochastic volatility factors and shocks to the crude oil futures returns vary in magnitude from 3% to 15%. Furthermore, the low value of μ_i^V suggest that the volatility factors are highly persistent, results that are consistent with findings by Cunado, Gil-Alana and Perez-De-Gracia (2010).

5.2.1. Return–volatility relation in the crude oil futures market

When the model is fitted to the whole sample, the estimated correlations between futures returns and innovations in the volatility for the two factors capturing the evolution of the volatility structure are -0.1606 and -0.1681 , respectively. These negative correlations suggest a negative return–volatility relation, which is a similar response as in the spot crude oil market. Hassan (2011) and Salisu and Fasanya (2013) found that the leverage effects postulate asymmetric volatility in crude oil markets. This is also a typical response in the equity markets, essentially implying that negative returns in the crude oil futures market

would potentially increase volatility in the oil futures price more than positive returns do.

Table 5: **Contribution of volatility factors**

	1990 – 1999		2000 – 2010	
	Contribution	Return–Vol relation	Contribution	Return–Vol relation
σ_1	34.97 %	negative	64.48 %	negative
σ_2	65.03 %	positive	35.52%	negative

The table reports the contribution of each volatility factor to the total variance of the crude oil futures price returns for the two-factor model.

A more thorough analysis over the two subsamples though reveals that there is more to the crude oil futures return–volatility relation. The descriptive statistics on Table 1 suggest that Period 2 (2000–2010) is far more volatile compared to Period 1 (1990–1999), especially around the 2003 Gulf War and the GFC in 2008. In the less volatile Period 1, the connection between inventories and volatility, as explained by the Theory of Storage and more specifically the inventory effect, justifies a positive return–volatility relation, as discussed in Section 2.2. Furthermore in Period 1, the crude oil futures market was mostly inverted (77.39% of the time in weak backwardation¹⁸) that consequently implied a high convenience yield. Thus the positive correlation estimate (0.1225) for one of the volatility factors also verifies the argument that inverted commodity markets give rise to a positive return–volatility relation, as the convenience yield effect suggests in Section 2.

Building on the results by Dennis et al. (2006) and Elder, Miao and Ramchander (2012), it is assumed that volatility can be decomposed into two main components (similarly to returns); one component could be postulated by systematic market-wide shocks, while the other component could be controlled by commodity specific shocks. Table 5 displays the relative contribution of each volatility factor to the total volatility, and the type of volatility reaction. Accordingly, the dominant factor that accounts for 65% of the market variation is

¹⁸Weak backwardation is defined as the case of “discounted” futures prices being below spot commodity prices. We have computed the percentage of weak backwardation for the 13-month futures contracts.

1
2
3
4
5
6
7
8 the positively correlated volatility factor. This finding supports the Dennis et al. (2006) no-
9 tion asserting that when the market is quiet then commodity specific shock effects dominate
10 such as inventory. On the contrary, the second (less-contributing) volatility factor displays a
11 negative correlation of -0.0668 indicating an asymmetric volatility reaction and potentially
12 captures the impact of other market related shocks.
13
14
15

16
17 In the more volatile Period 2, the correlation estimates for the two volatility factors
18 are both negative implying a negative return-volatility relation for both factors, a reaction
19 that has been explained by the leverage effect, see Salisu and Fasanya (2013). Taking into
20 account also the contribution of each factor to the total variance, see Table 5, the more
21 dominant factor with a contribution of 64.48% displays a correlation of -0.0290 , while
22 the other factor has a correlation of -0.1442 . Thus the dominant volatility factor has an
23 inverse reaction compared to the Period 1, a reaction that the convenience yield effect can
24 also account for. The crude oil market was normal for over four years and these correlation
25 estimates support our notion of the convenience yield effect claiming that normal commodity
26 markets should yield a positive return–volatility relation.¹⁹ Additionally, it is apparent that
27 this extended period of normal market conditions (in the crude oil futures market that is
28 traditionally inverted) was triggered primarily by the significant market-wide shocks that the
29 crude oil market experienced over this period such as GFC, global economic contraction and
30 currency depreciation/appreciation. Thus, when market uncertainty is high, market–wide
31 shock effects dominate which is consistent with Dennis et al. (2006) conclusions in equity
32 markets.
33
34
35
36
37
38
39
40
41
42
43
44
45
46

47 The NIF for the crude oil futures market is a surface, rather than a line as in the gold
48 futures market, due to the use of two volatility factors, see Figure 6. In the second period,
49 namely 2000–2010, both volatility factors have negative correlation with shocks to the futures
50 returns, therefore the NIF surface clearly tilts down as volatility shocks increase. Even
51
52
53

54
55 ¹⁹The magnitude of the correlation of this volatility factor is marginally negative probably due to the fact
56 that we fit the model into a ten-year dataset and the market was normal for only four years. When the
57 model was fitted into the 5-year dataset from 2006 to 2010 where the market was mostly normal then the
58 correlations are -0.2130 and -0.0614 respectively. The results are not presented here for the sake of saving
59 space but they are available upon request.
60

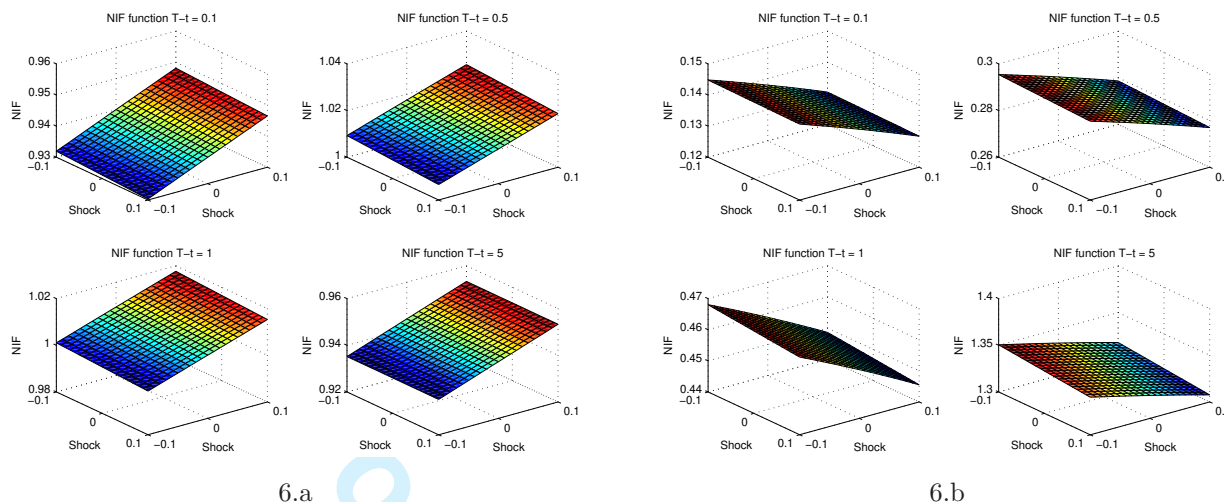


Figure 6: NIF for crude oil futures market

Panel 6.a is for the first period 1990 – 1999, while Panel 6.b is for the second period 2000 – 2010

though the volatility factors are hump-shaped, the hump for both factors is beyond five years time to maturity. As a result, in the practical range of futures contract volatility, it is observed a monotonic increase in volatility impact (when there is a shock in the futures market) as the time to maturity increases. In Period 1, on the other hand, only the hump for the first volatility is beyond 5 years, whereas the hump for the second volatility is around 5 months. The two volatility factors have an opposite relationship with the shocks to futures return (negative ρ_1 and positive ρ_2), however, the second volatility factor is stronger than the first one (see Table 6), resulting in an upward sloping aggregate NIF.

5.3. Skewness of futures returns

The ability of the futures price volatility skewness to relate to the return-volatility relation in the two commodity futures markets is also assessed. Table 1 presents the descriptive statistics for one-month and thirteen-month gold futures prices and crude oil futures prices. Generally over volatile market conditions, a negative skewness has been detected in the commodity (spot and futures) markets. This negative skewness is also evident in equity markets and has been linked to their asymmetric volatility feature, (Harvey and Siddique (2000)). Two noteworthy observations can be made in the commodity futures markets; (a) high volatility in the gold futures market is related to a negative skewness of the gold futures

1
2
3
4
5
6
7
8 returns, while low volatility is related to a positive skewness of the gold futures returns and
9
10 (b) high volatility in the crude oil futures market is related to a negative skewness of the
11 crude oil futures returns, while low volatility is related to a more negative skewness. During
12 volatile periods, the convenience yield effect linked to futures market conditions (normal
13 or inverted) is critical in explaining the negative return–volatility relation in the crude oil
14 futures market, while the safe haven property of gold prices can explain the positive return–
15 volatility relation in the gold futures market. However, under low volatility regimes, their
16 importance becomes secondary as does their link to the asymmetric behavior of volatility.²⁰
17
18 Under stable market conditions, gold traders, acting as rational investors, tend to prefer
19 positive return feedbacks (more than the negative ones) while crude oil traders respond with
20 more negative return feedbacks than positive ones. A tentative explanation is that under
21 stable market conditions, the contribution of the convenience yield increases, causing crude
22 oil futures prices to drop, thus more negative futures return feedbacks are more likely to
23 occur rather than positive ones.
24
25
26
27
28
29
30
31
32
33

34 35 **6. Conclusion**

36
37
38 This paper examines the relation between futures returns and volatility changes in two
39 major commodity futures markets, gold and crude oil. By using an extensive database
40 on futures and options, a continuous time stochastic volatility model is estimated. The
41 correlation structure between the futures returns and the innovations of its volatility gauges
42 the nature of the futures return-volatility relation.
43
44
45
46

47 The empirical investigations have led to four main findings regarding the return-volatility
48 relation in futures markets. First, the proposed model is consistent with results from existing
49 literature that claims that the gold volatility is inverted asymmetric, mostly due to safe haven
50 property and the crude oil futures volatility is asymmetric, due to the volatility feedback
51 effect and/or the leverage effect. Nevertheless, the approach used in the paper is different to
52 prior research in two respects, a) a continuous time stochastic volatility model is used while
53
54
55
56
57
58

59 ²⁰Wu (2001) empirically demonstrated that over stable market conditions the volatility feedback effect has
60 a secondary role on explaining volatility asymmetry in equity markets.

1
2
3
4
5
6
7
8 most literature employs GARCH type models and b) it is demonstrated that this relation
9 holds similarly in the commodity futures markets, rather than the spot commodity markets
10 to which most literature is dedicated.
11

12
13 Second, a new effect has been proposed, namely the convenience yield effect, that links the
14 state of the commodity futures market (normal or inverted) to the return–volatility relation
15 (negative or positive). More specifically, as convenience yield increases (decreases) implying
16 that the commodity futures market become inverted (normal), then the increasing volatility
17 associated with the effects that impact on the convenience yield will lead to increasing
18 (decreasing) futures returns. Thus inverted (normal) commodity futures markets incite a
19 positive (negative) return–volatility relation. This effect becomes an important factor in
20 explaining the futures return–volatility relation, especially for crude oil.
21
22

23
24 Third, the intensity of the volatility also plays an important role as the reaction over
25 periods of low volatility is typically different to the reaction over periods of high volatility
26 in both the gold and the crude oil futures markets. Over quiet periods, a negative return-
27 volatility relation is prevailed in the gold futures market, while during volatile periods where
28 market-wide effects dominate, a positive relation is present. Thus, tail events are very
29 important determinants of the return–volatility relation in the commodity futures markets.
30
31

32
33 Fourth, the model allows for several volatility factors, each one of which can potentially
34 capture different volatility effects. Indeed for crude oil, the two driving volatility factors have
35 different reactions. In a low volatility regime, the most contributing volatility factor holds a
36 positive relation with returns as the convenience yield effect claims (crude oil futures market
37 was predominantly inverted), while the less contributing volatility factor holds a negative
38 relation potentially due to the volatility feedback effect. However in a high volatility regime,
39 the most contributing volatility factor holds a negative relation due to the convenience yield
40 effect (as crude oil futures market was mainly normal).
41
42

43
44 These findings are of critical importance for practical applications related to trading and
45 pricing derivatives and play an important role in investment management and risk man-
46 agement. This paper, by employing an alternative approach via continuous time stochastic
47 volatility models, has successfully captured stylised facts about the return-volatility relation
48 in the crude oil futures market and the gold futures market. The results induce the pursuit of
49
50
51
52
53
54
55
56
57
58
59
60

a sophisticated economic analysis to statistically test the results in terms of specifying true break points for volatility and exploring the direct impact of commodity-specific effects such as inventories or market-wide shocks, such as investment growth, in the return–volatility relation. This type of economic analysis is beyond the purpose of the current study.

Appendix A. The European Option Pricing Equations

The price at time t of a European put option with strike K and maturity T_o on a futures contract with maturity T , is expressed as

$$\begin{aligned} \mathcal{P}(t, T_o, T, K) &= \mathbb{E}_t^Q[e^{-\int_t^{T_o} r_s ds} (K - F(T_o, T, \mathbf{V}_{\mathbf{T}_o}))^+] \\ &= P(t, T_o)[KG_{0,1}(\log(K)) - G_{1,1}(\log(K))] \end{aligned} \quad (\text{A.1})$$

where

$$G_{a,b}(y) = \frac{\phi(t; a, T_o, T)}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im}[\phi(t; a + \mathbf{i}bu, T_o, T)e^{-iuy}]}{u} du, \quad (\text{A.2})$$

and $P(t, T_o)$ is the price at time t of a zero-coupon bond with maturity T_o . Note that $\mathbf{i}^2 = -1$ and for $t \leq T_o \leq T$, the characteristic function $\phi(t; v, T_o, T) =: \mathbb{E}_t^Q[\exp\{v \ln F(T_o, T, \mathbf{V}_{\mathbf{T}_o})\}]$ is expressed as

$$\phi(t; v, T_o, T) = \exp\{M(t; v, T_o) + \sum_{i=1}^n N_i(t; v, T_o) \mathbf{V}_t^i + v \ln F(t, T, \mathbf{V}_t)\}. \quad (\text{A.3})$$

$M(t) = M(t; v, T_o)$ and $N_i(t) = N_i(t; v, T_o)$ (for $i = 1, \dots, n$) satisfy the Ricatti ordinary differential equations

$$\frac{dM(t)}{dt} = - \sum_{i=1}^n \mu_i^V \nu_i^V N_i(t), \quad (\text{A.4})$$

$$\frac{dN_i(t)}{dt} = - \frac{v^2 - v}{2} (\varphi_i)^2 - (\varepsilon_i^V v \rho_i \varphi_i - \mu_i^V) N_i(t) - \frac{1}{2} \varepsilon_i^{V^2} N_i^2(t), \quad (\text{A.5})$$

subject to the terminal conditions $M(T_o) = N_i(T_o) = 0$, where $\varphi_i = (\kappa_{0i} + \kappa_i(T-t))e^{-\eta_i(T-t)}$.

Appendix B. The Extended Kalman Filter

The system equation describing the dynamics of the six state variables of the proposed model is expressed as

$$X_{t+1} = \Phi_0 + \Phi_X X_t + w_{t+1}, \quad w_{t+1} \sim iid N(0, Q_t), \quad (\text{B.1})$$

where $X_t = (x_i(t), y_i(t), z_i(t), \phi_i(t), \psi_i(t), \mathbf{V}_t^i)$ are driven by the stochastic differential equations (2), (6) and (7). The observation equation that links the state variables with the market-observable variables (in our case futures prices and options prices) is of the form

$$z_t = h(X_t) + u_t \quad u_t \sim iid N(0, \Omega). \quad (\text{B.2})$$

Note that, the h function is nonlinear due to the nonlinear functional forms of the option prices, see (A.1) and (A.2). Let $\hat{X}_t = \mathbb{E}_t[X_t]$ and $\hat{X}_{t|t-1} = \mathbb{E}_{t-1}[X_t]$ denote the expectations of X_t at t and $t - 1$ respectively and let \mathcal{C}_t and $\mathcal{C}_{t|t-1}$ denote the corresponding estimation error covariance matrices. The h function is linearized around $\hat{X}_{t|t-1}$, as

$$z_t = (h(\hat{X}_{t|t-1}) - H_t' \hat{X}_{t|t-1}) + H_t' X_t + u_t, \quad u_t \sim iid N(0, \Omega), \quad (\text{B.3})$$

where

$$H_t' = \left. \frac{\partial h(X_t)}{\partial X_t'} \right|_{X_t = \hat{X}_{t|t-1}}.$$

Then the Kalman filter yields

$$\begin{aligned} \hat{X}_{t+1|t} &= \Phi_0 + \Phi_X \hat{X}_t, \\ \mathcal{C}_{t+1|t} &= \Phi_X \mathcal{C}_t \Phi_X' + Q_t, \end{aligned}$$

and

$$\begin{aligned} \hat{X}_{t+1} &= \hat{X}_{t+1|t} + \mathcal{C}_{t+1|t} H_t' F_t^{-1} \epsilon_t, \\ \mathcal{C}_{t+1} &= \mathcal{C}_{t+1|t} - \mathcal{C}_{t+1|t} H_t' F_t^{-1} H_t \mathcal{C}_{t+1|t}, \end{aligned}$$

with

$$\begin{aligned} \epsilon_t &= z_{t+1} - h(\hat{X}_{t+1|t}), \\ F_t &= H_t \mathcal{C}_{t+1|t} H_t' + \Omega. \end{aligned}$$

Finally, the log-likelihood function is constructed as

$$\log L = -\frac{1}{2} \log(2\pi) \sum_{t=1}^T N_t - \frac{1}{2} \sum_{t=1}^T \log |F_t| - \frac{1}{2} \sum_{t=1}^T \epsilon_t' F_t^{-1} \epsilon_t. \quad (\text{B.4})$$

References

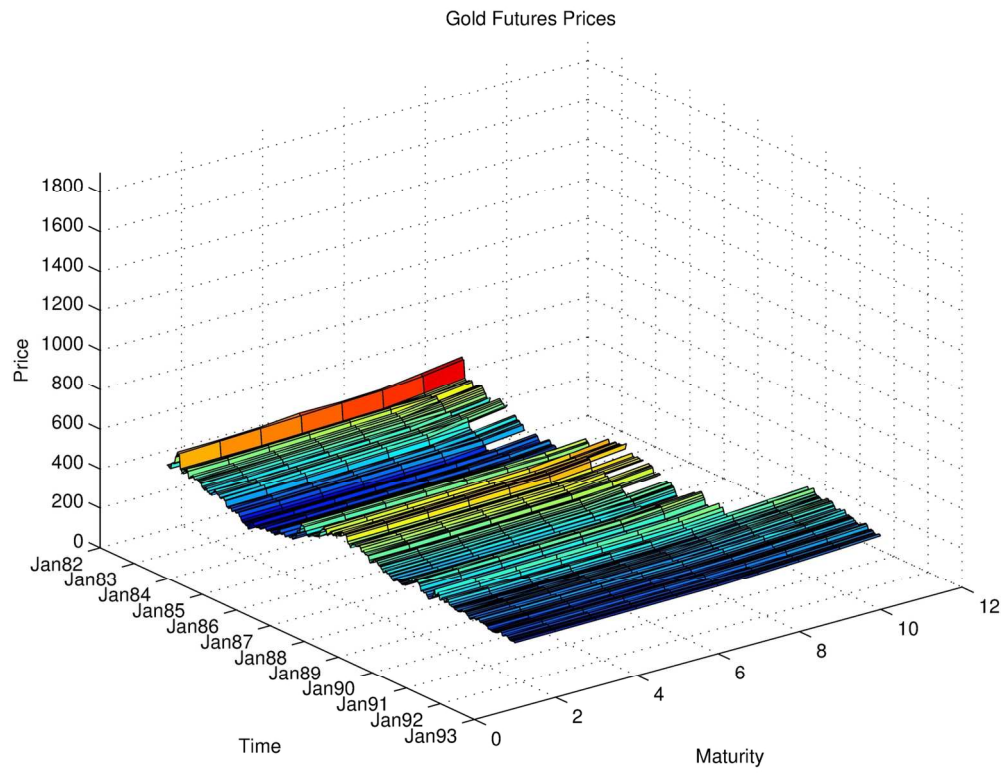
- Barone-Adesi, G. and Whaley, R. (1987), 'Efficient analytic approximation of American option values', *Journal of Finance* **42**, 301–320.
- Baur, D. G. (2012), 'Asymmetric volatility in the gold market', *The Journal of Alternative Investments* **14**(4), 26–38.
- Baur, D. G. and McDermott, T. K. (2010), 'Is gold a safe haven? International evidence', *Journal of Banking and Finance* **34**, 1886–1898.
- Bekaert, G. and Wu, G. (2000), 'Asymmetric volatility and risk in equity markets', *The Review of Financial Studies* **13**(1), 1–42.
- Bekiros, S. D. and Diks, C. G. H. (2008), 'The relationship between crude oil spot and futures prices: Cointegration, linear and nonlinear causality', *Energy Economics* **30**, 2673–2685.
- Björk, T., Landén, C. and Svensson, L. (2004), 'Finite dimensional markovian realizations for stochastic volatility forward rate models', *Proceedings of the Royal Society* **460**(Series A), 53–83.
- Black, F. (1976), 'The pricing of commodity contracts', *Journal of Financial Economics* **3**(1-2), 167–179.
- Black, F. (1976b), Studies in stock price volatility changes, in 'Proceedings of the 1976 Business Meeting of the Business and Economic Statistics Section, American Statistical Association', pp. 177–181.
- Broadie, M., Chernov, M. and Johannes, M. S. (2007), 'Understanding index option returns', *Review of Financial Studies* **22**(11), 4493–4529.
- Campbell, J. Y. and Hentschel, L. (1992), 'No news is good news: An asymmetric model of changing volatility in stock returns', *Journal of Financial Economics* **31**, 281–318.
- Carpantier, J. F. and Dufays, A. (2013), 'Commodity Inventory Effect', *University of Luxembourg, CREA Discussion Papers, 2013-07*.
- Carpantier, J. F. and Samkharadze, B. (2013), 'The Asymmetric Commodity Inventory Effect on the Optimal Hedge Ratio', *The Journal of Futures Markets* **33**(9), 868–888.

- 1
2
3
4
5
6
7
8 Chiarella, C., Kang, B., Nikitopoulos, C. S. and Tô, T. (2013), 'Humps in the volatility structure
9 of the crude oil futures market: New evidence', *Energy Economics* **40**, 989–1000.
10
11 Chiarella, C. and Kwon, O. K. (2003), 'Finite Dimensional Affine Realisations of HJM Models in
12 Terms of Forward Rates and Yields', *Review of Derivatives Research* **6**(2), 129–155.
13
14 Collin-Dufresne, P. and Goldstein, R. S. (2002), 'Do bonds span the fixed income markets? Theory
15 and evidence for unspanned stochastic volatility', *Journal of Finance* **57**(4), 1685–1730.
16
17 Cootner, P. H. (1960), 'Returns to speculators: Telser vs. Keynes', *Journal of Political Economy*
18 **68**, 396–404.
19
20 Cunado, J., Gil-Alana, L. and Perez-De-Gracia, F. (2010), 'Persistence in Some Energy Futures
21 Markets', *Journal of Futures Markets* **30**(5), 490–507.
22
23 Dai, Q. and Singleton, K. J. (2000), 'Specification analysis of affine term structure models', *Journal*
24 *of Finance* **55**(5), 1943–1978.
25
26 Dennis, P., Mayhew, S. and Stivers, C. (2006), 'Stock returns, implied volatility innovations, and the
27 asymmetric volatility phenomenon', *Journal of Financial and Quantitative Analysis* **41**(2), 381–
28 406.
29
30 Doran, J. S. and Ronn, E. I. (2008), 'Computing the market price of volatility risk in the energy
31 commodity market', *Journal of Banking and Finance* **32**, 2541–2552.
32
33 Duffie, D. (2001), *Dynamic asset pricing theory*, Princeton, NJ:Princeton University Press.
34
35 Duffie, D., Pan, J. and Singleton, K. (2000), 'Transform analysis and asset pricing for affine jump-
36 diffusions', *Econometrica* **68**(6), 1343–1376.
37
38 Elder, J., Miao, H. and Ramchander, S. (2012), 'Impact of macroeconomic news on metal futures',
39 *Journal of Banking and Finance* **36**, 51–65.
40
41 Engle, R. and Ng, V. (1993), 'Measuring and testing the impact of news in volatility', *Journal of*
42 *Finance* **43**, 1749–1778.
43
44 Geman, H. and Ohana, S. (2009), 'Forward curves, scarcity and price volatility in oil and natural
45 gas markets', *Energy Economics* **31**(4), 576–585.
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

- 1
2
3
4
5
6
7
8 Giamouridis, D. G. and Tamvakis, M. N. (2001), 'The relation between return and volatility in the
9 commodity markets', *Journal of Alternative Investments* **4**(1), 54–62.
- 10
11
12 Glostén, L. R., Jagannathan, R. and Runkle, D. E. (1993), 'On the relation between the expected
13 value and the volatility of the nominal excess return on stocks', *Journal of Finance* **48**, 1779–1801.
- 14
15
16
17 Gorton, G. B., Hayashi, F. and Rouwenhorst, K. G. (2013), 'The fundamentals of commodity
18 futures returns', *Review of Finance* **17**(1), 35–105.
- 19
20
21 Hamilton, J. D. and Wu, J. (2014), 'Risk premia in crude oil futures prices', *Journal of International*
22 *Money and Finance* **42**(C), 9–37.
- 23
24
25
26
27 Harvey, C. and Siddique, A. (2000), 'Conditional skewness in asset pricing tests', *Journal of Finance*
28 **55**(3), 1263–1296.
- 29
30
31 Hassan, S. A. (2011), 'Modelling asymmetric volatility in oil prices', *Journal of Applied Business*
32 *Research* **27**(3), 7178.
- 33
34
35
36
37 Heath, D., Jarrow, R. and Morton, A. (1992), 'Bond pricing and the term structure of interest
38 rates: A new methodology for contingent claims valuation', *Econometrica* **60**(1), 77–105.
- 39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
195
196
197
198
199
200
201
202
203
204
205
206
207
208
209
210
211
212
213
214
215
216
217
218
219
220
221
222
223
224
225
226
227
228
229
230
231
232
233
234
235
236
237
238
239
240
241
242
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289
290
291
292
293
294
295
296
297
298
299
300
301
302
303
304
305
306
307
308
309
310
311
312
313
314
315
316
317
318
319
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376
377
378
379
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403
404
405
406
407
408
409
410
411
412
413
414
415
416
417
418
419
420
421
422
423
424
425
426
427
428
429
430
431
432
433
434
435
436
437
438
439
440
441
442
443
444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517
518
519
520
521
522
523
524
525
526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579
580
581
582
583
584
585
586
587
588
589
590
591
592
593
594
595
596
597
598
599
600
601
602
603
604
605
606
607
608
609
610
611
612
613
614
615
616
617
618
619
620
621
622
623
624
625
626
627
628
629
630
631
632
633
634
635
636
637
638
639
640
641
642
643
644
645
646
647
648
649
650
651
652
653
654
655
656
657
658
659
660
661
662
663
664
665
666
667
668
669
670
671
672
673
674
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689
690
691
692
693
694
695
696
697
698
699
700
701
702
703
704
705
706
707
708
709
710
711
712
713
714
715
716
717
718
719
720
721
722
723
724
725
726
727
728
729
730
731
732
733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805
806
807
808
809
810
811
812
813
814
815
816
817
818
819
820
821
822
823
824
825
826
827
828
829
830
831
832
833
834
835
836
837
838
839
840
841
842
843
844
845
846
847
848
849
850
851
852
853
854
855
856
857
858
859
860
861
862
863
864
865
866
867
868
869
870
871
872
873
874
875
876
877
878
879
880
881
882
883
884
885
886
887
888
889
890
891
892
893
894
895
896
897
898
899
900
901
902
903
904
905
906
907
908
909
910
911
912
913
914
915
916
917
918
919
920
921
922
923
924
925
926
927
928
929
930
931
932
933
934
935
936
937
938
939
940
941
942
943
944
945
946
947
948
949
950
951
952
953
954
955
956
957
958
959
960
961
962
963
964
965
966
967
968
969
970
971
972
973
974
975
976
977
978
979
980
981
982
983
984
985
986
987
988
989
990
991
992
993
994
995
996
997
998
999
1000
- Kaldor, N. (1939), 'Speculation and economic stability', *Review of Economic Studies* **7**, 1–27.
- Kilian, L. (2009), 'Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market', *American Economic Review* **99**, 1053–1069.
- Kilian, L. and Hicks, B. (2013), 'Did unexpected strong economic growth caused the oil shock of 2003–2008?', *Journal of Forecasting* **32**(5), 385–394.
- Morana, C. (2013), 'Oil price dynamics, macro-finance interactions and the role of financial speculation', *Journal of Banking and Finance* **37**, 206–226.
- Ng, V. K. and Pirrong, S. C. (1994), 'Fundamentals and Volatility: Storage, Spreads, and the Dynamics of Metals Prices', *Journal of Business* **67**(2), 203–230.

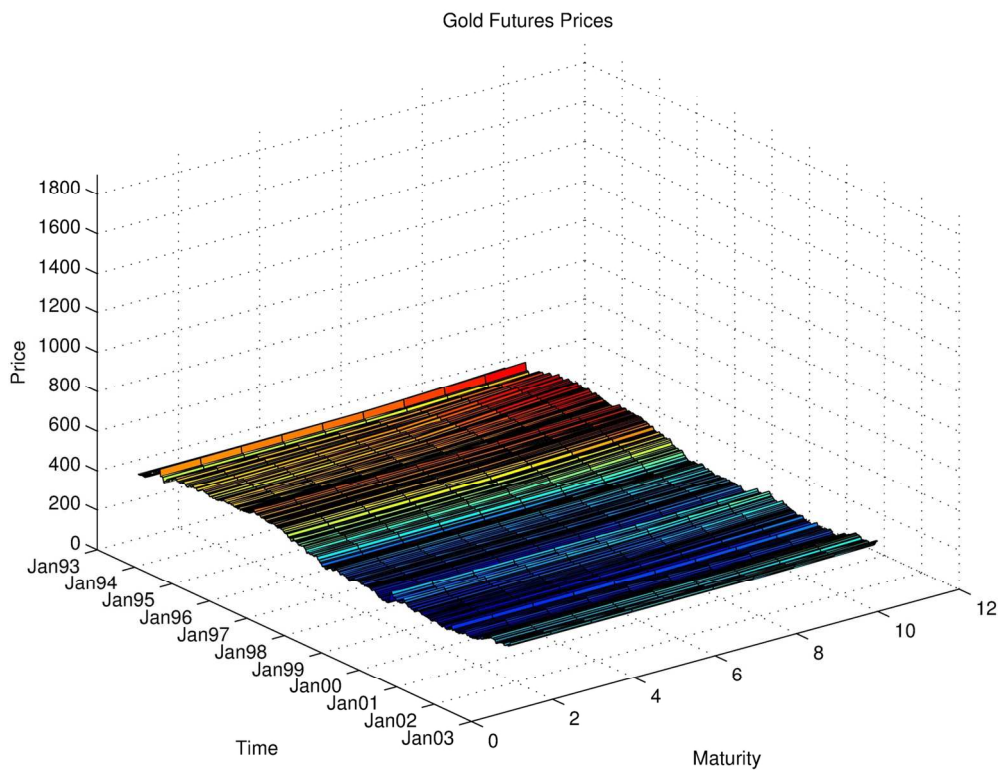
- 1
2
3
4
5
6
7
8 Pindyck, R. (2001), 'The dynamics of commodity spot and futures markets: A primer', *The Energy*
9 *Journal* **22**(3), 1–16.
10
11
12 Reboredo, J. C. (2013), 'Is gold a safe haven or a hedge for the US dollar? Implications for risk
13 management', *Journal of Banking and Finance* **37**, 2665–2676.
14
15
16 Salisu, A. and Fasanya, I. (2013), 'Modelling oil price volatility with structural breaks', *Energy*
17 *Policy* **52**(C), 554–562.
18
19
20
21 Tokic, D. (2010), 'The 2008 oil bubble: Causes and consequences', *Energy Policy* **38**, 6009–6015.
22
23
24 Trolle, A. B. and Schwartz, E. (2009), 'Unspanned stochastic volatility and the pricing of commodity
25 derivatives', *Review of Financial Studies* **22**(11), 4423–4461.
26
27
28 Tully, E. and Lucey, B. M. (2007), 'A power GARCH examination of the gold market', *Research*
29 *in International Business and Finance* **21**(2), 316–325.
30
31
32 Wu, G. (2001), 'The determinants of asymmetric volatility', *The Review of Financial Studies*
33 **14**(3), 837–859.
34
35
36 Yu, J. (2004), 'Asymmetric response of volatility: Evidence from stochastic volatility models and
37 realised volatility', *Research Collection School of Economics (Open Access), Paper 820* .
38
39
40 Yu, J. (2005), 'On Leverage in a Stochastic Volatility Model', *Journal of Econometrics* **127**, 165–
41 178.
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



141x109mm (300 x 300 DPI)

review

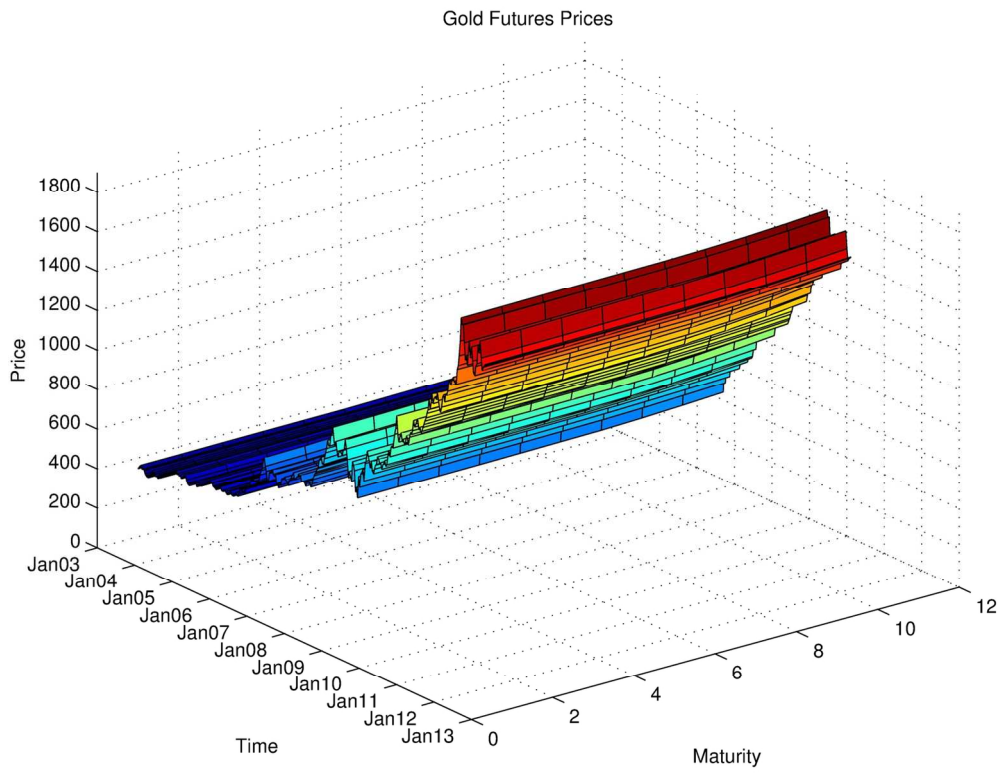


141x109mm (300 x 300 DPI)

review

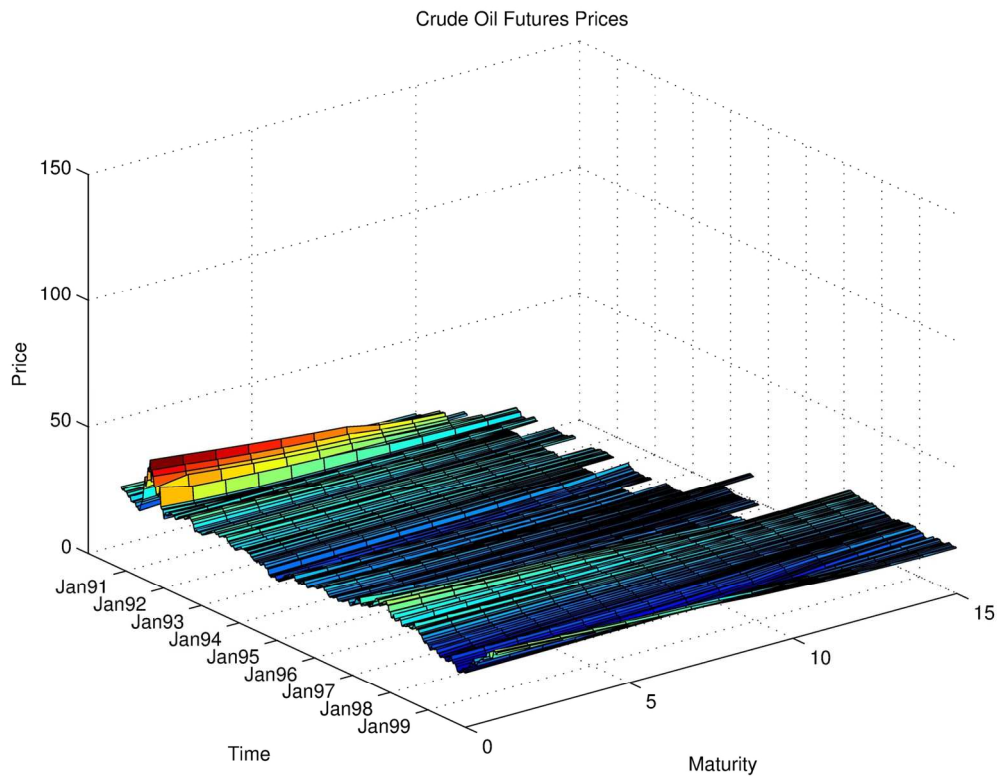
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



141x109mm (300 x 300 DPI)

Review

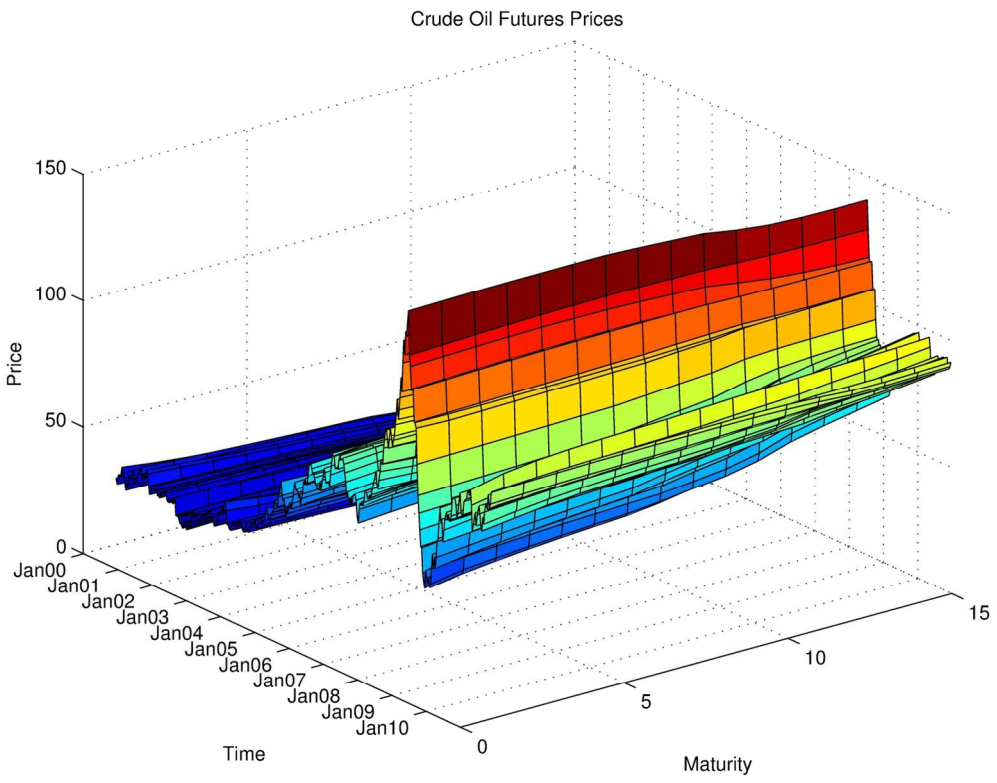


141x110mm (300 x 300 DPI)

Review

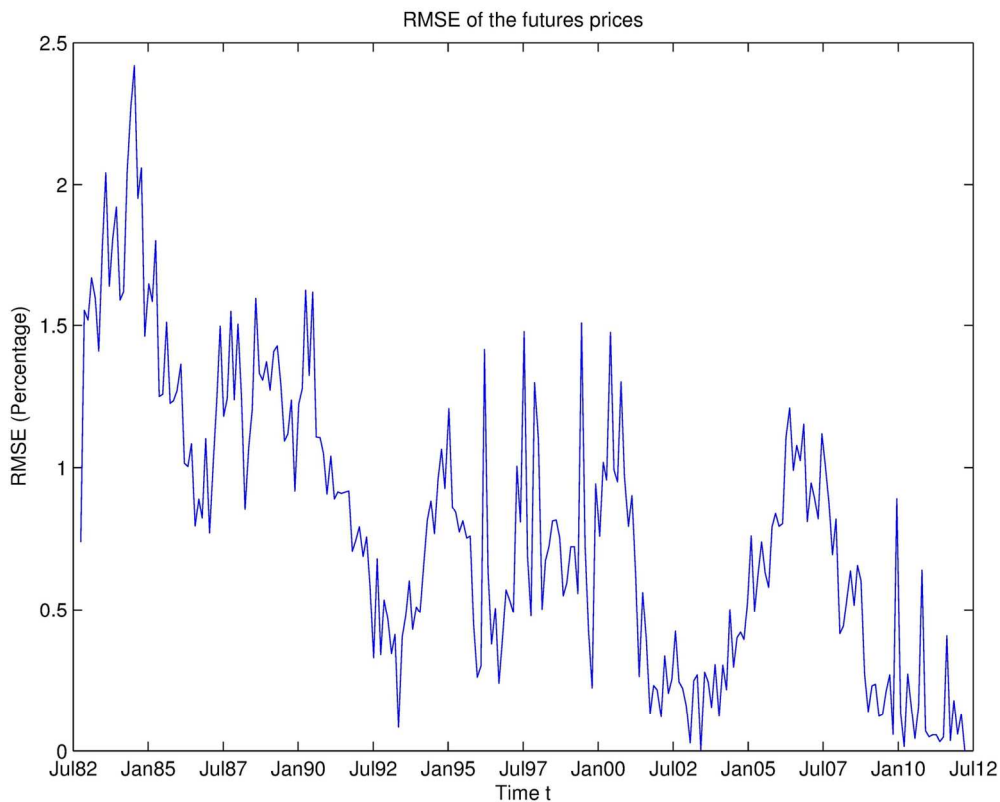
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



141x110mm (300 x 300 DPI)

Review

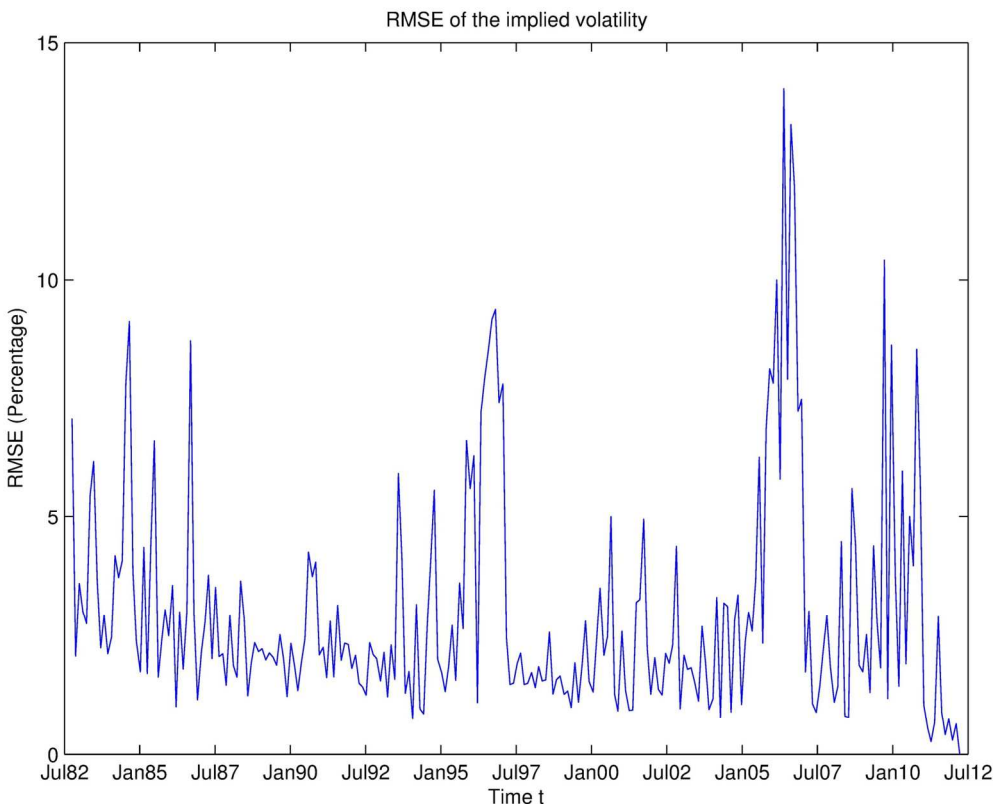


143x117mm (300 x 300 DPI)

view

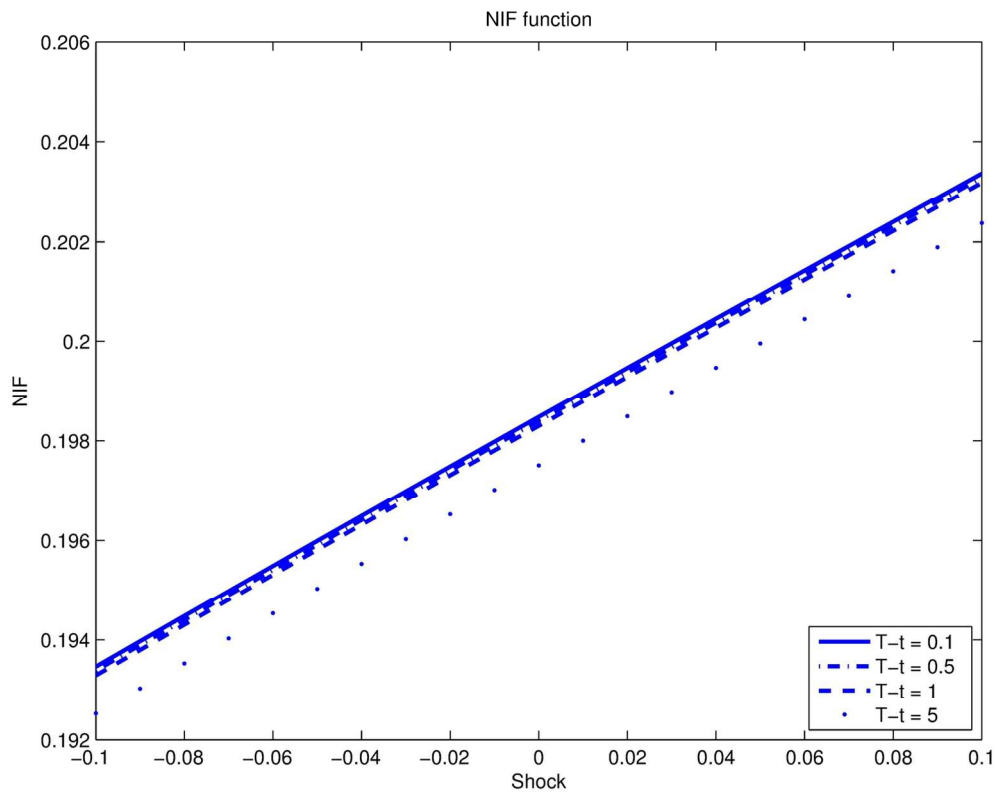
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



143x118mm (300 x 300 DPI)

view

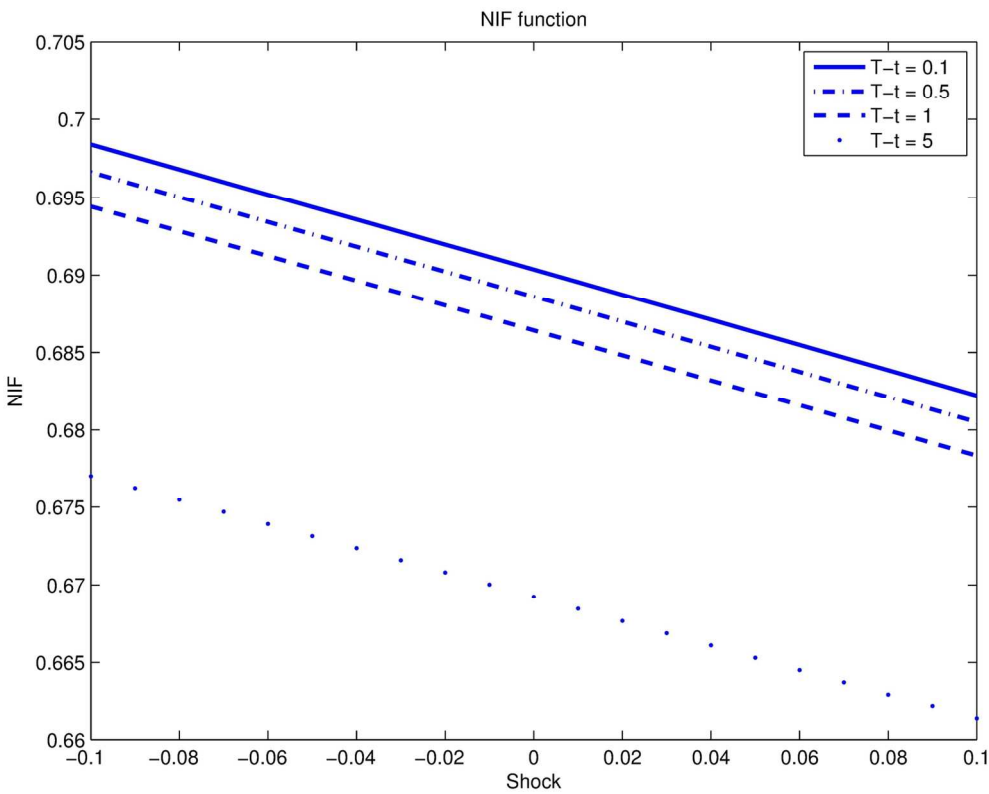


142x114mm (300 x 300 DPI)

Review

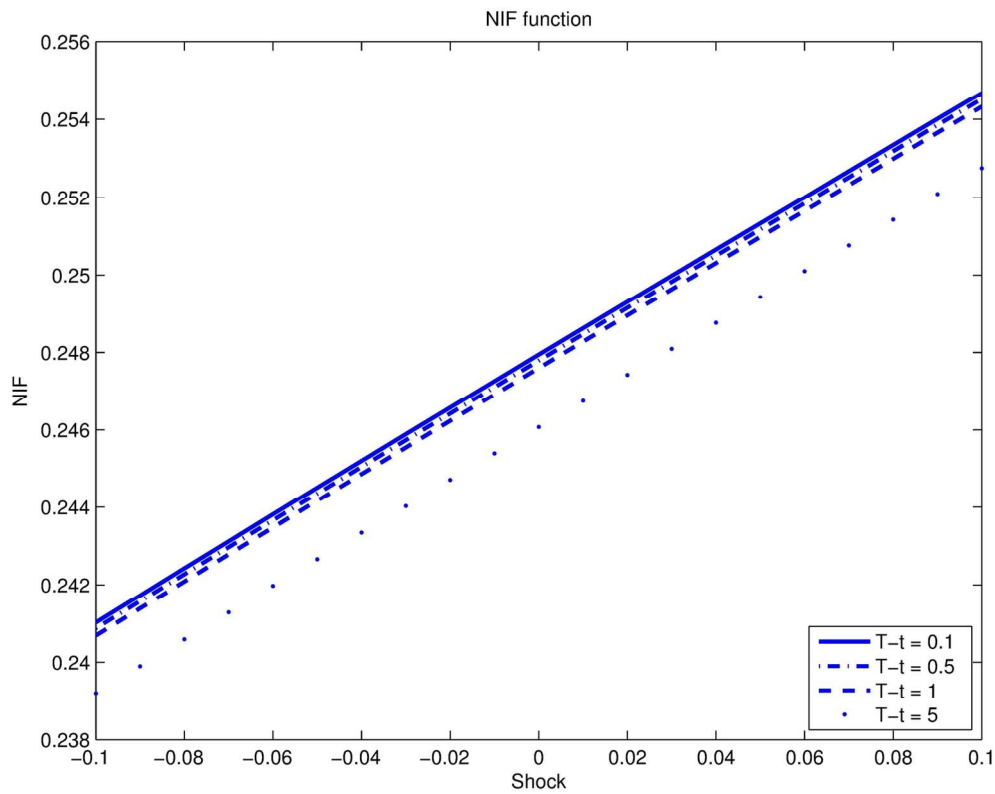
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



142x114mm (300 x 300 DPI)

Review

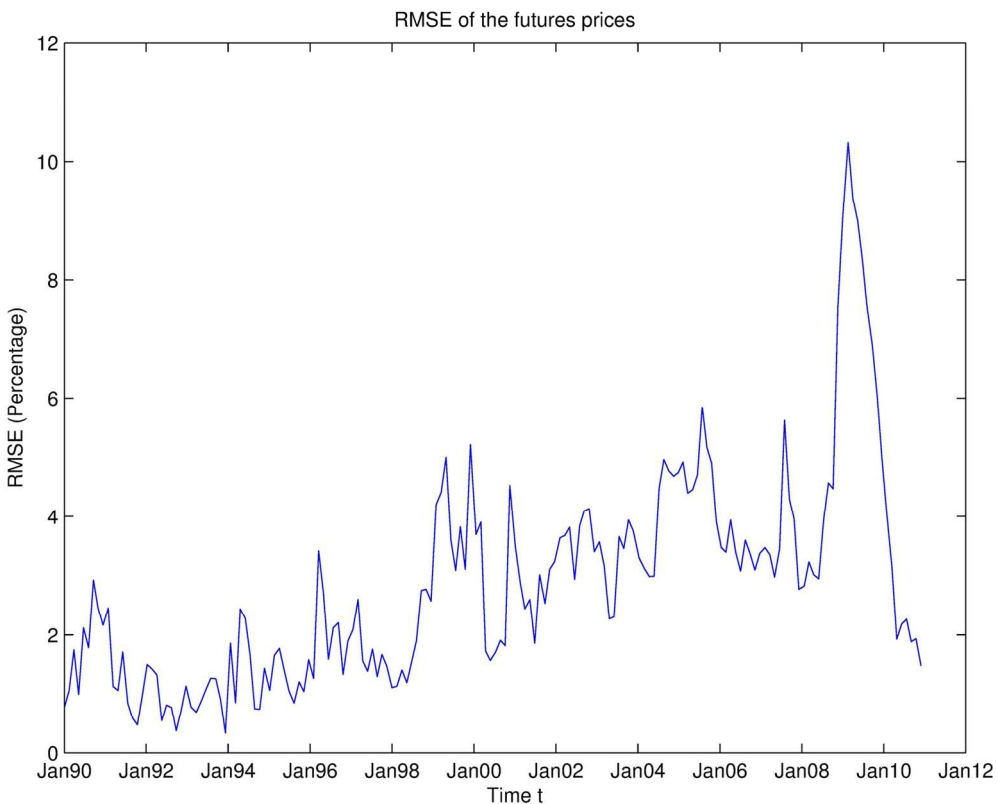


142x114mm (300 x 300 DPI)

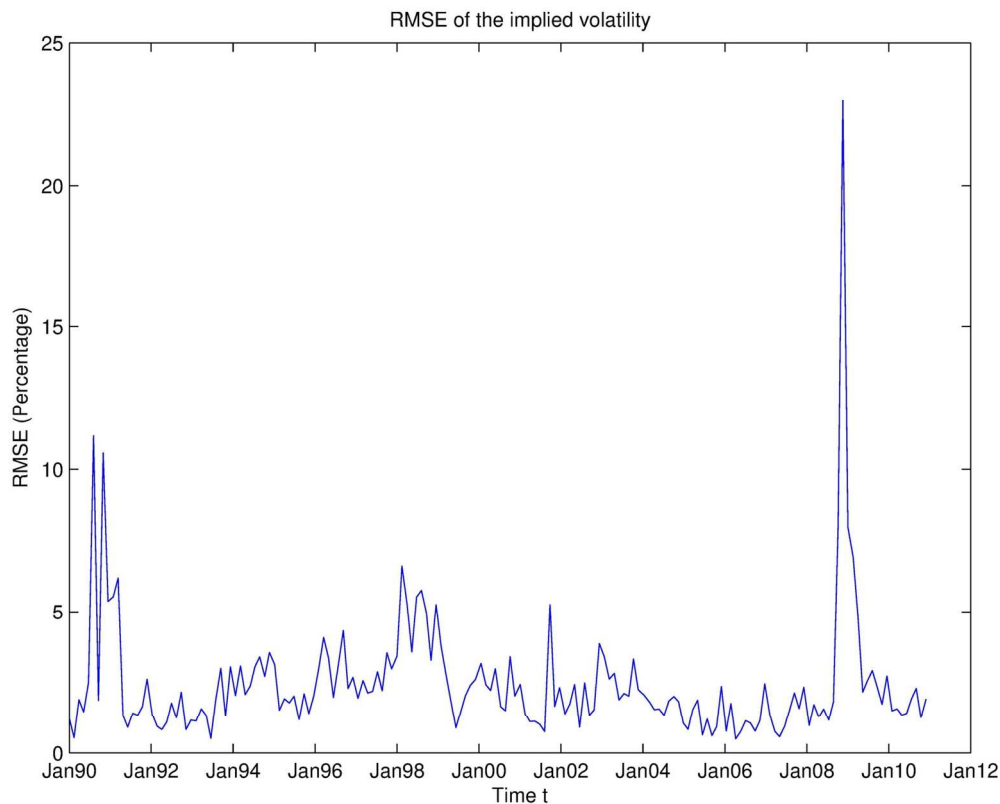
Review

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



143x117mm (300 x 300 DPI)

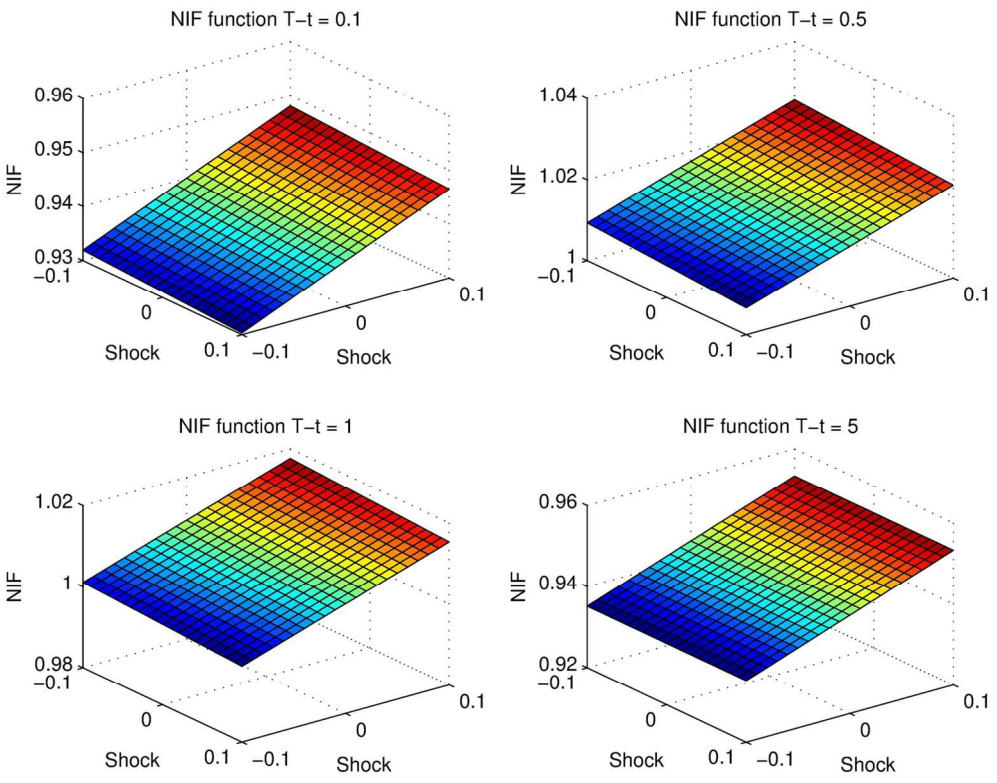


143x117mm (300 x 300 DPI)

view

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

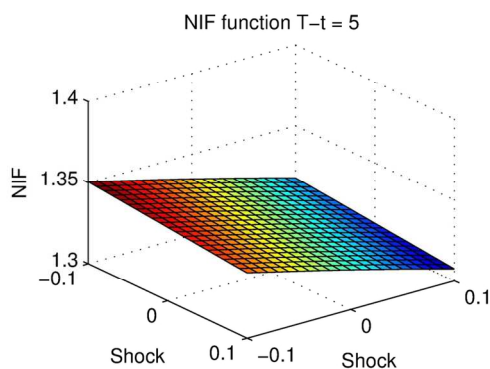
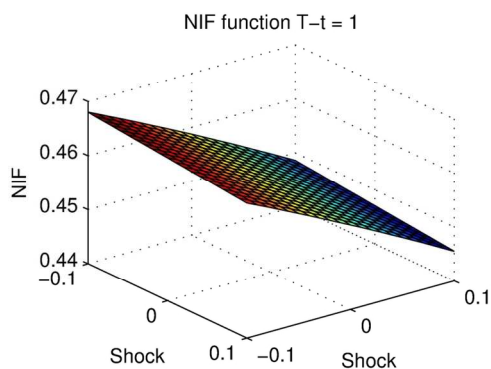
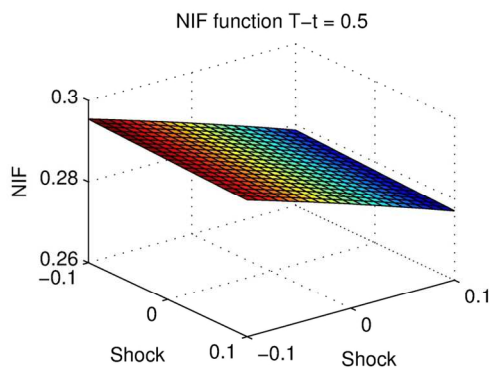
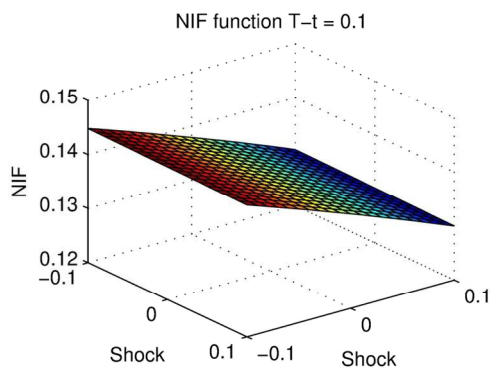
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



138x108mm (300 x 300 DPI)

Review

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



138x108mm (300 x 300 DPI)

Review