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PERFORMANCE IMPROVEMENT FOR WIDEBAND DOA ESTIMATION WITH WHITE NOISE REDUCTION BASED ON UNIFORM LINEAR ARRAYS

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ABSTRACT

A method is proposed for reducing the effect of white noise in wideband uniform linear arrays via a combination of a judiciously designed transformation followed by highpass filters. The reduced noise level leads to a higher signal to noise ratio for the system, which can have a significant effect on the performance of various direction of arrival (DOA) estimation methods. As a representative example, the compressive sensing-based wideband DOA estimation method is employed here to demonstrate the improved estimation performance, this is confirmed by simulation results.

***Index Terms*—** Wideband arrays, direction of arrival estimation, white noise reduction, uniform linear arrays.

1. INTRODUCTION

Direction of arrival (DOA) estimation in array signal processing has been an active field of research for many years [1, 2, 3, 4], with a wide range of applications including radar, sonar, and wireless communications. Many methods have been proposed for both narrowband and wideband signals, and two representative ones are the MUSIC and the ESPRIT algorithms [5, 6], which were originally proposed for narrowband signals. For wideband signals, a commonly used approach is to decompose the wideband signal into different frequency bins and transform the wideband problem into a narrowband one through various focusing or interpolation algorithms [7, 8]. In addition, methods such as incoherent signal subspace method (ISSM) [9], coherent signal subspace method (CSSM) [10] and test of orthogonality of projected subspaces method (TOPS) [11] have also been proposed. Recently, with the development of compressive sensing theory [12, 13], many sparsity based DOA estimation methods were developed [14, 15, 16, 17], including various extensions to the wideband case [18, 19, 20].

In general, the performance of a DOA estimation algorithm is dependent on the level of noise in the system, and normally the lower the level of noise, the better the performance. One common assumption for noise in wideband arrays is that it is spatially (and also temporally in many cases) white, i.e. the noise at array sensor is uncorrelated with the noise at any other sensor. Under this assumption, it seems there is not much that can be done about the noise and we have to simply accept whatever noise component is left after the required processing for the signal part.

In this paper, we aim to reduce the white noise level of a wideband array system so that the performance of various DOA estimation algorithms can be improved. A novel noise reduction method is developed to reduce the white noise level of a wideband uniform linear array (ULA) using a set of judiciously designed transformations. The spatial characteristics of the white noise and the directional wideband signals received by the array are different. Based on this difference and motivated by the low-complexity subband-selective adaptive beamformer proposed in [21, 22], we first transform the received wideband sensor signals into a new domain where the directional signals are decomposed in such a way that their corresponding outputs are associated with a series of tighter and tighter highpass spectra, while the spectrum of noise still covers the full band from $-\pi$ to π in the normalised frequency domain. Then, a series of highpass filters with different cutoff frequencies are applied to remove part of the noise spectrum while keeping those of the directional signals unchanged. Finally, an inverse transformation is applied to the filtered outputs to recover the original sensor signals, where compared to the original set of received sensor signals, the directional signals are left intact while the noise power is reduced.

One condition placed on the transformation matrix is that it must be invertible. We have further assumed that it is also unitary and thus used the discrete Fourier transform (DFT) matrix as a representative example in our simulations section. Detailed analysis shows that the signal to noise ratio (SNR) of the array after the proposed processing can be improved by about 3dB in the ideal case, and this is then translated into a corresponding enhanced performance for DOA estimation. To demonstrate this improved performance, a compressive sensing based wideband DOA estimation method will be employed.

This paper is organised as follows. In Sec. 2, the proposed noise reduction method is introduced, followed by the compressive sensing based wideband DOA estimation method in Sec. 3. Simulation results are presented in Sec. 4 and conclusions are drawn in Sec. 5.

2. THE PROPOSED WHITE NOISE REDUCTION METHOD

A block diagram for the general structure of the proposed noise reduction method is shown in Fig. 1. The M received array signals $x_m[n]$, $m = 0, \ldots, M-1$, are first processed by an $M \times M$ transformation matrix $\mathbf{B}$, and then its outputs $q_m[n]$, $m = 0, \ldots, M-1$, pass through a set of highpass filters with impulse responses given by $h_m[n]$, $m = 0, \ldots, M-1$; the outputs of these filters are denoted by $z_m[n]$, $m = 0, \ldots, M-1$, and these are then transformed by $\mathbf{B}^{-1}$. For simplicity, we assume $\mathbf{B}$ is unitary, i.e. $\mathbf{B}^{-1} = \mathbf{B}^H$,

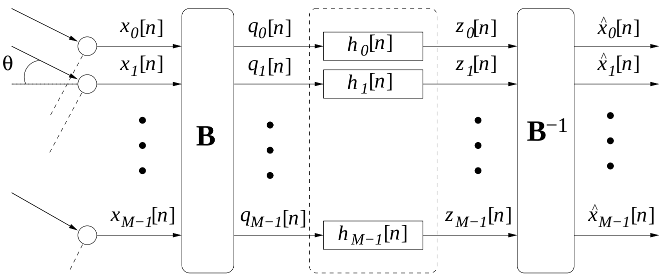


Fig. 1: Structure of the proposed white noise reduction method.

where $\{\}^H$ denotes the Hermitian transpose.

We assume there are K wideband signals $s_k(t)$ impinging on the array from different incident angles $\theta_k, k = 0, \cdots, K-1$. The received array signal $x_m(t)$ at the m-th sensor consists of these wideband signals and white noise $\bar{n}_m(t)$, i.e. we have

$$x_m(t) = \sum_{k=0}^{K-1} s_k\left[t - \tau_m(\theta_k)\right] + \bar{n}_m(t), \tag{1}$$

where $\tau_m(\theta_k)$ represents the time delay (relative to a reference sensor) of the k-th impinging signal with the incident angle θ_k arriving at the m-th sensor of the array. Taking the first sensor in the array as the reference point, we have $\tau_0(\theta_k) = 0$. So with

$$d_m(t) = \sum_{k=0}^{K-1} s_k\left[t - \tau_m(\theta_k)\right], \tag{2}$$

then (1) becomes

$$x_m(t) = d_m(t) + \bar{n}_m(t). \tag{3}$$

With a sampling frequency f_s, the discrete version of the array vector snapshot is

$$\mathbf{x}[n] = \mathbf{d}[n] + \bar{\mathbf{n}}[n], \tag{4}$$

where

$$\mathbf{x}[n] = [x_0[n], x_1[n], \cdots, x_{M-1}[n]]^T,$$
$$\mathbf{d}[n] = [d_0[n], d_1[n], \cdots, d_{M-1}[n]]^T,$$
$$\bar{\mathbf{n}}[n] = [\bar{n}_0[n], \bar{n}_1[n], \cdots, \bar{n}_{M-1}[n]]^T.$$

Thus the output of the $M \times M$ transformation matrix **B** is

$$\mathbf{q}[n] = \mathbf{B}\mathbf{x}[n] = \mathbf{B}(\mathbf{d}[n] + \bar{\mathbf{n}}[n]), \tag{5}$$

where $[\mathbf{B}]_{m,l} = b_{m,l}$ and $\mathbf{q}[n] = [q_0[n], q_1[n], \cdots, q_{M-1}[n]]^T$. Each row of the transformation matrix can be considered as a fixed beamformer with its output given by $q_m[n] = \sum_{l=0}^{M-1} b_{m,l} x_l[n]$. The corresponding beam response is

$$R_m(\Omega, \theta) = \sum_{l=0}^{M-1} b_{m,l} e^{-jl\mu\Omega\sin\theta} = B_m(\mu\Omega\sin\theta), \tag{6}$$

where $\mu = d/cT_s$ and $\Omega = \omega T_s$, with d being the inter-element spacing of the uniform linear array (ULA), c the wave propagation speed, T_s the sampling period, and ω is angular frequency. With $\hat{\Omega} = \mu\Omega\sin\theta$, we have

$$B_m(\hat{\Omega}) = \sum_{l=0}^{M-1} b_{m,l} e^{-jl\hat{\Omega}}, \tag{7}$$

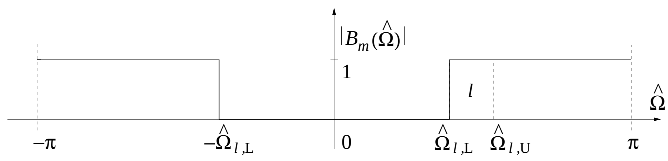


Fig. 2: Frequency response of the l^{th} row vector followed by the corresponding highpass filter in the ideal case.

where $B_m(\hat{\Omega})$ is the frequency response of the m-th row vector of the transformation matrix **B** (where we consider each row vector as the impulse response of a finite impulse response filter).

By assuming that the sampling frequency is twice the highest frequency component of the wideband signal and that the array spacing is half the wavelength of the highest frequency component, we have $\mu = 1$ [4]. The frequency responses $B_m(\hat{\Omega})$ of the row vectors are arranged to be bandpass with bandwidths of $2\pi/M$ and all together they cover the whole frequency band $[-\pi; \pi]$ [22]. Taking the *l*th row vector as example, its frequency response is

$$\left|B_l(\hat{\Omega})\right| = \begin{cases} 1, & \text{for } \hat{\Omega} \in [\hat{\Omega}_{l,L}; \hat{\Omega}_{l,U}] \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

as shown in Fig. 2. Considering the above frequency response, the received array signal components with frequency $\Omega \in (-\hat{\Omega}_{l,L}; \hat{\Omega}_{l,L})$ will not be able to pass through this row vector, since $\hat{\Omega} = \Omega\sin\theta$ does not fall into the passband of $[\hat{\Omega}_{l,L}; \hat{\Omega}_{l,U}]$ no matter what the direction of arrival θ is. Therefore, the frequency range of the output is $|\Omega| \geq \hat{\Omega}_{l,L}$ and the lower bound is determined by $\hat{\Omega}_{l,L}$ when $\hat{\Omega}_{l,L} > 0$. On the contrary the lower bound is determined by $|\hat{\Omega}_{l,U}|$ when $\hat{\Omega}_{l,L} < \hat{\Omega}_{l,U} < 0$.

Therefore, the output spectrum of the signal part of $q_l[n]$ corresponding to the l-th row vector will then be highpass filtered as illustrated in Fig. 2. Since the noise at the array sensors is spatially white, the output noise spectrum of the row vector is still a constant, covering the whole spectrum. More importantly, since **B** is unitary, there is no change to the total noise power after the transformation.

As shown in Fig. 1, each $q_l[n], l = 0, \cdots, M-1$ is the input to the corresponding highpass filter $h_l[n], l = 0, \cdots, M-1$, and that filter should cover the whole bandwidth of the signal part, i.e. having the same highpass frequency response as shown in Fig. 2. As a result, in the ideal case, the highpass filters will not have any effect on the signal part, and the signal part should pass through the highpass filter without any distortion. On the contrary, frequency components of the white noise which fall into the stopband of the highpass filters will be removed. The output of these highpass filters is given by

$$\mathbf{z}[n] = \begin{bmatrix} z_0[n] \\ z_1[n] \\ \vdots \\ z_{M-1}[n] \end{bmatrix} = \begin{bmatrix} q_0[n] * h_0[n] \\ q_1[n] * h_1[n] \\ \vdots \\ q_{M-1}[n] * h_{M-1}[n] \end{bmatrix}, \tag{9}$$

where * denotes the convolution operator.

Now we consider the noise reduction effect of these filters. Each filter removes part of the noise except for the one corresponding to the row vector with a frequency response covering the zero frequency component, which should allow all the frequencies to pass. Assume that the size M of the array is an odd number. Then in the ideal case, the ratio between the total noise power after and before the processing of the M highpass filters can be expressed as

$$\frac{P_{no}}{P_{ni}} = \frac{1}{M}\left(1 + 2\left(\frac{M-1}{M} + \frac{M-3}{M} + \cdots + \frac{2}{M}\right)\right), \quad (10)$$

where P_{no} is the total noise power at the output of the filters and P_{ni} is the total noise power at their input. If the size of the array is an even number, then this ratio is given by

$$\frac{P_{no}}{P_{ni}} = \frac{1}{M}\left(1 + 2\left(\frac{M-1}{M} + \frac{M-3}{M} + \cdots + \frac{3}{M}\right) + \frac{1}{M}\right). \quad (11)$$

As a result, we have

$$r(M) = \frac{P_{no}}{P_{ni}} = \begin{cases} \frac{M^2+2M-1}{2M^2}, & \text{if } M \text{ is odd} \\ \frac{M^2+2M-2}{2M^2}, & \text{if } M \text{ is even.} \end{cases} \quad (12)$$

When $M \to \infty$, the noise power will be reduced by half. Since the filters have no effect on the signal part, the ratio between the total signal power and the total noise power will be improved by almost 3dB in the ideal case. For a finite M, the improvement will be less than 3dB. For example, when $M = 16$, it is about 2.5dB.

Applying the inverse of the transformation matrix $\mathbf{B}^{-1} = \mathbf{B}^H$ to $\mathbf{z}[n]$, we obtain the estimates of the original input sensor signals $\hat{x}_m[n]$, $m = 0, 1, \cdots, M-1$. In vector form, we have

$$\hat{\mathbf{x}}[n] = \mathbf{B}^{-1}\mathbf{z}[n], \quad (13)$$

where $\hat{\mathbf{x}}[n] = [\hat{x}_0[n], \hat{x}_1[n], \cdots, \hat{x}_M - 1[n]]^T$. After going through these processing stages, there is no change in the signal part at the final output $\hat{x}_l[n]$, $l = 0, 1, \cdots, M-1$ compared to the original signal part in $x_l[n]$, $l = 0, 1, \cdots, M-1$. On the contrary, since $\mathbf{B}^{-1}$ is also unitary, the total noise power stays the same between $\hat{\mathbf{x}}[n]$ and $\mathbf{z}[n]$, which is almost half the total noise power in $\mathbf{x}[n]$. Therefore, for a very large array size M, we have

$$\|\hat{\mathbf{x}}[n]\|_2^2 \approx \|\mathbf{d}[n]\|_2^2 + \frac{1}{2}\|\bar{\mathbf{n}}[n]\|_2^2 . \quad (14)$$

where $\|.\|_2$ is the l_2 norm

From above, in terms of the total signal power to total noise power ratio (TSNR), we can easily find the following relationship

$$TSNR_{\hat{\mathbf{x}}} \approx \frac{\|\mathbf{d}[n]\|_2^2}{\frac{1}{2}\|\bar{\mathbf{n}}[n]\|_2^2} = 2TSNR_{\mathbf{x}}. \quad (15)$$

So in the ideal case, for a very large M, the TSNR has almost been doubled using the proposed noise reduction method. This can be translated into better performance for different array processing applications such as DOA estimation.

3. COMPRESSIVE SENSING BASED DOA ESTIMATION

To demonstrate the improved array processing performance with an improved TSNR, we focus here on the wideband DOA estimation problem. The DOA estimation method adopted here is a direct adaptation and extension of the method developed in [17, 20] for co-prime arrays.

First, for the received vector signal in (4), it is divided into P non-overlapping groups with length L, and an L-point DFT is then applied. The l-th frequency bin samples of the p-th group are placed into one vector as:

$$\mathbf{X}[l,p] = [X_0[l,p], X_1[l,p], ..., X_{M-1}[l,p]]^T, \quad (16)$$

where

$$X_n[l,p] = \sum_{i=0}^{L-1} x_{1,n}[Lp+i].e^{-j\frac{2\pi}{L}il}, \quad (17)$$

with $p = 0, 1, ..., P-1$ and $l = 0, 1, ..., L-1$.

Define $S_k[l,p]$ and $\bar{N}_m[l,p]$ as the DFT of the p-th group impinging signals $s_k[n]$ and noise $\bar{n}_m[n]$, respectively. $\mathbf{S}[l,p] = [S_0[l,p], ..., S_{K-1}[l,p]]^T$ is a column vector holding signals from the l-th frequency bin, and $\bar{\mathbf{N}}[l,p] = [\bar{N}_1[l,p], ..., \bar{N}_{M-1}[l,p]]^T$ is the column noise vector of the array. Then the output signal model in the DFT domain can be expressed as:

$$\mathbf{X}[l,p] = \mathbf{A}(l,\theta)\mathbf{S}[l,p] + \bar{\mathbf{N}}[l,p], \quad (18)$$

where $\mathbf{A}(l,\theta) = [\mathbf{a}(l,\theta), ..., \mathbf{a}(l,\theta_{K-1})]$ is the steering matrix at frequency f_l corresponding to the l-th frequency bin. The column vector $\mathbf{a}(l,\theta_k)$ is the steering vector at frequency f_l and angle θ_k. The auto-correlation matrix of the observed array vector $\mathbf{X}[l,p]$ is:

$$\begin{aligned} \mathbf{R_{xx}}[l] &= E\left\{\mathbf{X}[l,p]\mathbf{X}^H[l,p]\right\} \\ &= \sum_{k=0}^{K-1}\sigma_k^2[l]\mathbf{a}(l,\theta_k)\mathbf{a}^H(l,\theta_k) + \sigma_n^2[l]\mathbf{I}_M, \end{aligned} \quad (19)$$

where $E\{\}$ denotes the statistical expectation, and $\mathbf{I}_M$ is the $M \times M$ identity matrix. $\sigma_k^2[l]$ represents the power of the k-th impinging signal of the l-th frequency bin and $\sigma_n^2[l]$ is the corresponding noise power. Vectorizing $\mathbf{R_{xx}}[l]$ yields:

$$\mathbf{u}[l] = \text{vec}\{\mathbf{R_{xx}}\} = \mathbf{A}\tilde{[}l]\tilde{\mathbf{s}}[l] + \sigma_n^2\tilde{\mathbf{I}}_M, \quad (20)$$

where $\tilde{\mathbf{A}}[l] = [\tilde{\mathbf{a}}(l,\theta_0), ..., \tilde{\mathbf{a}}(l,\theta_K - 1)]$, $\tilde{\mathbf{a}}(l,\theta_k) = \mathbf{a}^*(l,\theta_k) \otimes \mathbf{a}(l,\theta_k)$ ($\otimes$ is the Kronecker product), and $\tilde{\mathbf{s}}[l] = \{\sigma_0^2[l], \sigma_1^2[l], ..., \sigma_{K-1}^2[l]\}^T$. $\tilde{\mathbf{I}}_M$ is an $M^2 \times 1$ column vector obtained by vectorizing the identity matrix $\mathbf{I}_M$. Eq. (20) characterizes a virtual array with $\tilde{\mathbf{A}}[l]$ and $\tilde{\mathbf{s}}[l]$ as its steering matrix and impinging signal vector, respectively. Eq. (20) can be reduced to:

$$\mathbf{u}[l] = \tilde{\mathbf{A}}^\circ[l]\tilde{\mathbf{s}}^\circ[l], \quad (21)$$

where $\tilde{\mathbf{A}}^\circ[l] = \left[\tilde{\mathbf{A}}[l], \tilde{\mathbf{I}}_M\right]$ and $\tilde{\mathbf{s}}^\circ = \left[\tilde{\mathbf{s}}^T[l], \sigma_n^2[l]\right]^T$. For the l-th frequency, with a search grid of K_g potential arriving angles $(\theta_{g,0}, ..., \theta_{g,K-1})$, the steering matrix is

$$\tilde{\mathbf{A}}_g[l] = \left[\tilde{\mathbf{a}}(l,\theta_{g,0}), ..., \tilde{\mathbf{a}}(l,\theta_{g,K_g-1})\right] .$$

Constructing a column vector $\tilde{\mathbf{s}_g}[l]$ consisting of K_g elements, with each representing a potential arriving signal, yields

$$\tilde{\mathbf{A}}_g^\circ[l] = \left[\tilde{\mathbf{A}}_g[l], \tilde{\mathbf{I}}_M\right]; \quad \tilde{\mathbf{s}}_g^\circ[l] = \left[\tilde{\mathbf{s}}_g[l], \sigma_n^2[l]\right]^T . \quad (22)$$

Now we construct two matrices, a block diagonal matrix $\tilde{\mathbf{D}}_g$ using $\tilde{\mathbf{A}}_g^\circ[l_q]$

$$\tilde{\mathbf{D}}_g = \text{blkdiag}\{\tilde{\mathbf{A}}_g^\circ[l_1], \tilde{\mathbf{A}}_g^\circ[l_2], \cdots, \tilde{\mathbf{A}}_g^\circ[l_Q]\} \quad (23)$$

and a $(K_g+1) \times Q$ matrix $\mathbf{R}$ using $\tilde{\mathbf{s}}_g^\circ[l_q]$ with

$$\mathbf{R} = \left[\tilde{\mathbf{s}}_g^\circ[l_1], \tilde{\mathbf{s}}_g^\circ[l_2], \cdots, \tilde{\mathbf{s}}_g^\circ[l_Q]\right] .$$

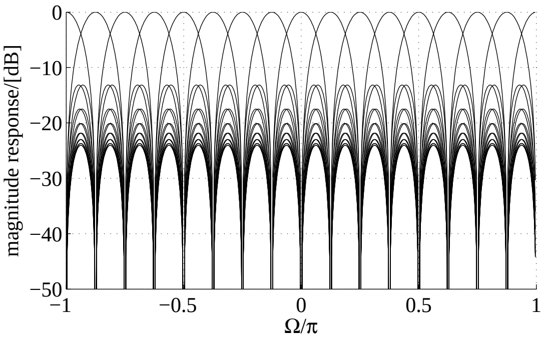


Fig. 3: Magnitude response of the 16×16 DFT matrix.

Then we can obtain the following wideband virtual array model $\tilde{\mathbf{u}} = \tilde{\mathbf{D}}_g \tilde{\mathbf{r}}$ where $\tilde{\mathbf{u}} = \left[\mathbf{u}^T[l_1], \cdots, \mathbf{u}^T[l_Q]\right]^T$ and $\tilde{\mathbf{r}} = \text{vec}(\mathbf{R})$ is a $(K_g+1) \cdot Q \times 1$ column vector by vectorizing $\mathbf{R}$. We use the row vector $\mathbf{r}_k$, $1 \leq k \leq K_g+1$, to represent k-th row of matrix $\mathbf{R}$. Then we form a new $(K_g+1) \times 1$ column vector $\hat{\mathbf{r}}$ based on the l_2 norm of $\mathbf{r}_k$, $1 \leq k \leq K_g+1$, as given below

$$\hat{\mathbf{r}} = \left[\|\mathbf{r}_1\|_2, \|\mathbf{r}_2\|_2, \cdots, \left\|\mathbf{r}_{K_g+1}\right\|_2\right]^T . \tag{24}$$

Finally, the group sparsity based wideband DOA estimation is formulated as follows

$$\min_{\tilde{\mathbf{r}}} \quad \|\hat{\mathbf{r}}\|_1 \text{ subject to } \quad \left\|\tilde{\mathbf{u}} - \tilde{\mathbf{D}}_g \tilde{\mathbf{r}}\right\|_2 \leq \epsilon . \tag{25}$$

The problem in (25) can be solved using CVX, a software package for specifying and solving convex programs [23, 24].

4. SIMULATION RESULTS

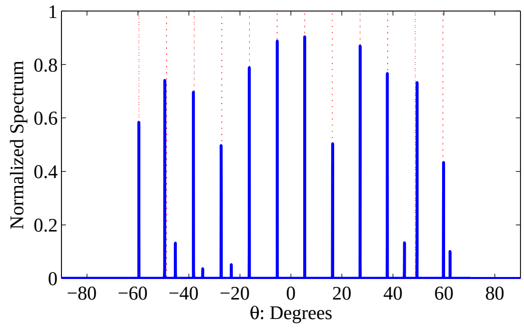
Our simulations are based on a ULA with $M = 16$ sensors. The transformation matrix $\mathbf{B}$ is a 16×16 DFT matrix with its frequency responses shown in Fig. 3. There are 12 bandlimited impinging signals with a normalized frequency range from 0.5π to π and 0dB SNR, and their DOAs are uniformly distributed between -60 to 60 degrees. The number of signals sampled in the time domain at each sensor is 32768, and a DFT of $L = 64$ points is applied. The number of data blocks used for estimating $\mathbf{R}_{\mathbf{xx}}[l]$ in (19) at each frequency bin is $P = 512$. The search grid is formed to cover the full DOA range with a step size of $0.05°$. By applying the proposed noise reduction method, 2.41dB improvement in SNR is achieved.

The DOA estimation results with and without noise reduction are shown in Fig. 4, where we can see that in Fig. 4a there are a few false directions detected and the results are not as accurate as Fig. 4b when the proposed noise reduction method is used.

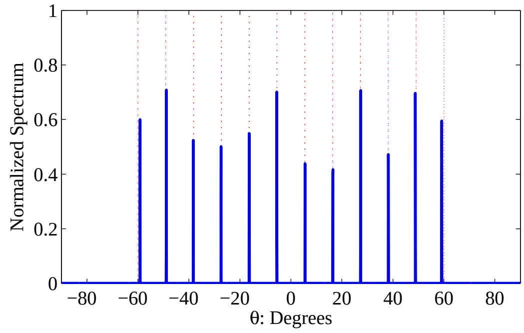
Finally, the root mean square error (RMSE) for the DOA estimation result with respect to different SNRs is shown in Fig. 5. The results are averaged from running the simulations 100 times. It can be clearly seen that using the developed noise reduction method, the DOA estimation result has been improved significantly over the whole considered input SNR range.

5. CONCLUSIONS

A method for reducing the white noise level in wideband ULAs has been introduced. With the proposed method, a maximum 3dB improvement in total signal power to total noise power ratio (TSNR) can be achieved. This increased TSNR can be translated into performance improvement in various array signal processing applications and as an example, its effect on DOA estimation was studied. As demonstrated by simulation results obtained by a simple compressive sensing based method, a significant improvement in DOA estimation accuracy has been achieved.



(a) Without white noise reduction.



(b) With white noise reduction.

Fig. 4: DOA estimation results with/without the proposed noise reduction.

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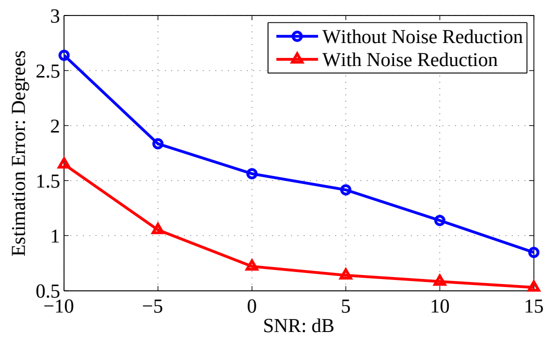

Fig. 5: RMSE vs input SNR.

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