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# Analysis of Dynamic Spectrum Leasing for Coded Bi-Directional Communication

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**Abstract**—In this paper, we aim to present a cooperative relaying based two way wireless communication scheme which can provide both spectral and energy efficiency in future wireless networks. To this end, we propose a novel network coding based Dynamic Spectrum Leasing (DSL) technique in which the cognitive secondary users cooperatively relay the primary data for two-way primary communication. In exchange for the relaying services, the primary grants exclusive access to the secondary users for their own activity. We model the random geometry of the ad hoc secondary users using a Poisson point process. We devise a game theoretic framework for the division of leasing time between the primary cooperation and secondary activity phases. We demonstrate that under these considerations and employing network coding, DSL can improve the number of bits that are successfully transmitted by 54% as compared to un-coded direct two way primary communication. Also the energy costs of the proposed DSL scheme are more than 10 times lower. Employing DSL also enables the cognitive users to get reasonable time for their own transmission after increasing the primary spectral and energy efficiency.

**Index Terms**—Dynamic Spectrum Leasing, Nash Bargaining, Cognitive Radio, Cooperative Relaying, Network Geometry.

## I. INTRODUCTION

Cognitive Radios (CRs) are envisioned to be a possible solution to the problem of *spectrum scarcity* that has emerged as a result of stringent spectrum allocations and under-utilization of the in-use spectrum [1]. This inefficient utilization/allocation of bandwidth causes the spectrum extinction and introduces difficulties in deploying new wireless networks and enhancing the capacity of the existing ones. CRs co-exist with licensed networks and enable optimum utilization of their spectrum across both geographical and temporal domains [2]. This dynamic exploitation of the spectral resources of the legacy (primary) network is allowed for the CRs as long as they do not interfere with the primary network operations.

Apart from solutions like CR networks to improve the spectral utilization in existing wireless networks, the *energy costs* of future wireless systems has become a serious concern. This is due to the fact that the current energy consumption trends indicate that if the communication systems continue to develop and spread at the same pace as today's, a significant portion of the total energy production of any country would be needed to meet the requirements of future communication systems [3], [4]. At this juncture, an ideal future wireless system would; 1) maximize the utilization of the existing bandwidth, 2) minimize the power consumption while supporting a high quality of communication.

Dynamic Spectrum Leasing (DSL) [5] has been suggested as one of the many different approaches [2] to realize CR network operation. In DSL, the primary network willingly leases the spectrum to the CRs, also called the secondary users. The secondary

users cooperatively relay the primary data to its destination during a part of the leased time. These relaying services of CRs are the incentive for the primary network to lease the spectrum to the CR network. The secondary nodes offer these services to the primary in order to get some share in the primary owned spectrum for their own activity as a reimbursement to its relaying services.

In this paper, we propose a network coding (NC) [6] based DSL scheme in which the CRs assist the primary network in two-way communication and enhance its data transmission at low energy costs. So far, DSL has not been investigated as a tool for two way communication providing greater spectral and energy efficiency as compared to direct communication. For an accurate analysis and quantification of the viability of DSL, the division of leasing time between the cooperative relaying and secondary activity duration is very important since both primary and secondary wish to maximize their share of exclusive spectral access time. In most of the existing studies, the decision on this division is influenced more by the primary users. Moreover, it is of prime importance that the random geometry of the ad hoc secondary nodes present in the network is analyzed since the relaying performance is directly dependent on the inter node distances between the CRs and the primary nodes. Unfortunately, most of the studies do not consider the exact geometry and its effects on DSL performance.

In this paper, we have modeled one way communication using DSL considering the geometry of the ad hoc CR network. Motivated by the potential gains of DSL, in this work we model two way DSL communication to attain spectral and energy efficiency considering the random geometry of the CR nodes present in the network. The main contributions of this paper are:

- 1) We consider a realistic network topology for both primary and secondary networks for topological considerations and the efficient selection of the cooperation areas. In this paper, we formulate a geometry based framework for the analysis of DSL and subsequent cooperation-area selection mechanism. These considerations help us demonstrate that successful data transmission can be increased while maintaining the quality of service (QoS) of primary communication by leasing the spectrum to CR nodes that are at a spatially suitable location.
- 2) We use a primitive network coding and relaying technique to realize and enhance the two-way communication rate between two primary nodes present in the network with the help of relaying services of the CRs. A simple scheme that we use in our work is proposed in [7] and is as follows. Suppose two nodes  $N_1$  and  $N_2$  want to send packets  $D_1$  and  $D_2$  to each other respectively using a relay node. In relay assisted half duplex transmission, during the first time

slot,  $N_1$  transmits its data to the wireless relay. Similarly,  $N_2$  also transmits its data to the relay in the second slot. The relay performs simple symbol level XOR operation to combine the two packets as follows.

$$D = D_1 \oplus D_2. \quad (1)$$

The combined packet is transmitted to both the nodes simultaneously in the third slot. Having knowledge of their own packet, each node can extract the packet sent by the other node to it. Without network coding, the two packets take four time slots to get to their destinations using the relays. With network coding, the same has been achieved in three time slots.

In our work, we extend this network coding and relaying technique for DSL communication which has not been done so far. We propose that a set of cooperating secondary nodes perform NC to facilitate two-way communication between the two primary sources. The time slot saved by using NC is used such that geographically suitable relays facilitate the communication between the primary nodes for a longer time hence attaining greater throughput.

- 3) In order to ensure fairness and mutual satisfaction, it is important to divide the leasing time in a way that both primary and CRs agree to their respective share of time for spectral access. In previous studies [5],[8], this division has been influenced more by the decision of the primary network that needs to be aware of the channel state information (CSI) of the secondary network to make the decision. The secondary network needs to observe the primary action and only decides in reaction to primary decision. Unlike previous studies, we propose a mutual agreement based division in our work that provides proportional fairness. Also, the primary is not required to have CSI knowledge of the secondary transmitter-receiver pairs. Out of the total DSL operational time, from 25% to 35% of the time is reserved for exclusive CRN communication which is otherwise dormant.

Due to the appropriate geometrical relay selection, intelligent division of leasing time and application of network coding, our analysis of DSL shows that there is up to 54% improvement in the number of bits successfully exchanged between the two primary nodes during the same time as compared to direct two-way communication between the two primary nodes.

- 4) As mentioned earlier, the energy requirements of the design of any communication system has become a key concern due to the rapid growth in energy consumption. This motivates us to formally carry out the modeling and analysis of the energy efficiency of leasing for two-way communication to measure its viability as an effective low power communication infrastructure for future wireless networks. Our results indicate that the proposed DSL scheme can be more than 10 times more energy efficient as compared to the dedicated primary link.

To follow a systematic approach, we discuss the state of the art DSL based approaches in Section II. The network setup that we consider to model network coding based DSL is laid out in Sec. III. In Sec. IV we formalize the average capacity of the primary to primary and DSL based two-way communication that

satisfies the network QoS requirements considering the random geometry of the ad hoc CR network involved in DSL. Further in the section, we study the problem of division of the leasing time between different phases of DSL using a game theoretic framework. We formulate a Nash Bargaining game in which both the primary and the secondary bargain over their share of time and try to maximize it. The equilibrium time division and average capacity analysis is used in Sec. V to evaluate the performance of DSL. After the throughput modeling and analysis of DSL, we model and study the energy efficiency of DSL in Sec. VI. Performance gains in terms of the throughput and energy are shown and discussed in detail in Section V and VI. We finally conclude our work in Section VII. It is important to mention that we intentionally exclude studying and comparing our scheme with only two-way relaying without DSL (or without any reimbursement mechanism for the secondary users). Such a scheme would essentially mean that dedicated wireless nodes are available in the network willing to spend their battery for the primary node without any remuneration, which is not the focus of this paper. Here we propose and compare a method of cooperation between a licensed and a few unlicensed spectrum users in a way that the cooperation leads to spectral and energy efficiency for both types of users.

## II. PREVIOUS WORK

Our work addresses three research areas in wireless communications (specifically CRNs); exploiting cooperative diversity, characterization of spectral leasing models and energy efficiency of the architecture. Energy efficiency has been explored in the context of cognitive radios by using adaptive modulation techniques [9] and optimal transmission duration estimation [10] in order to achieve power/bandwidth efficiency. The studies regarding the energy efficiency of CRNs mostly consider a generic scenario where spectrum sensing is employed.

An overview of various possible ways of exploiting cooperative diversity in cognitive radio networks has been suggested in [11]. The existing literature on dynamic spectrum leasing can be characterized into three main types; 1) in which the incentive for leasing is based on monetary rewards, [12]-[13], 2) where leasing is allowed as long as the interference from the CRs is below an ‘interference cap’ [14]-[15], 3) where the incentive for leasing is based on service rewards [5], [8], [16], which is the model on which this study is based. For the first two types, numerous literary contributions exist, however, its survey is out of the scope of this paper. Our focus is based on the third framework which was first explored by [5] where an analytical study of service based DSL for one way communication is provided and cooperative diversity of the secondary relays has been exploited. In [8] the same framework is carried forward and applied in an ARQ based model where a portion of the retransmission slot is leased by the legacy network to the relays for their traffic in exchange for cooperative retransmission by the relays. In [16], the authors consider an infrastructure hierarchical spectrum leasing approach. In their work, they consider multiple primary nodes that select their respective individual relays for cooperation. However, these studies do not consider DSL communication with a geometric modeling of the network.

Network coding comprises a set of well studied techniques in which the messages of two communicating terminals are combined and exchanged between them by cooperating relays

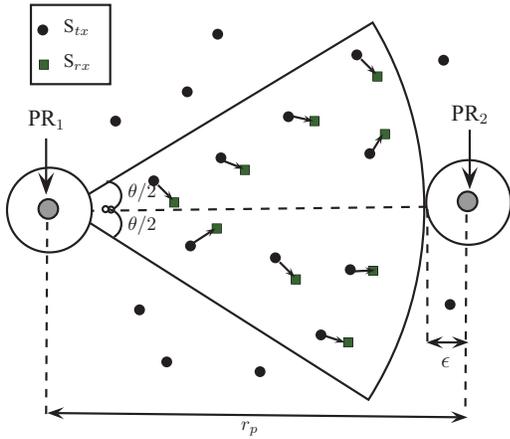


Figure 1. Network Geometry

that are aware of the messages of both the terminals. [7] and [17] are the fundamental works that introduced the concept and performance bounds of network coding. A detailed overview of various network coding based techniques can be found in [6]. However, to the best of our knowledge, this technique has not been explored in the context of DSL.

A comprehensive survey in [18] addresses the application of different games to model dynamic spectrum sharing. Previously, in [5], [16], a linear search based algorithm followed by a Stackelberg game was proposed to divide the leasing time between the primary and secondary activities. However, it does not cater for mutual agreement on leasing time division if 1) primary chooses a selfish time distribution as the leader and 2) the secondary in turn plays suboptimal strategy to hurt the interest of the primary in successive realizations of the game. Nash bargaining has been used for solving various problems of resource allocation in wireless networks e.g., [19] and it is shown to attain a Pareto optimal solution that specifically discourages selfish behavior in the network. However, it has not been exploited to model leasing time division for two way primary communication by any previous study.

To the best of our knowledge, none of the above mentioned studies consider network coding for two way DSL communication. The game theoretic modeling and setup of DSL such that the spectral and energy efficiency of the communication is improved as compared to direct transmission is our novel contribution.

### III. SYSTEM MODEL

We consider a legacy network consisting a pair of primary nodes that communicate with each other in half duplex mode. Along with the primary network, the collocated cognitive secondary users also form a wireless ad hoc network.

#### A. Physical Model

We consider one of the primary source node, say  $PR_1$  located at the origin. The second primary source node,  $PR_2$  is located at a fixed distance  $r_P$  from the origin. Each primary source communicates to the other primary node at a fixed transmit power  $P_t$  for a duration  $T$ .

In order to quantify the geometry of the collocated cognitive radios, we consider secondary nodes that constitute an infinite Stationary/homogeneous Poisson point process (PPP)  $\Phi$  [20] with intensity  $\lambda$ , in terms of the number of nodes per unit area. Considering that the secondary nodes form a homogeneous PPP, the CR nodes are uniformly distributed in space. For the purpose of relaying, the primary nodes select a sector,  $\text{sec}(r_P, \theta)$ , of radius  $r_P$  and angle  $\theta$ . Inspired by maximum forward progress based relaying strategies [21], the cognitive nodes lying within this sector become the potential DSL relays. Here, we also consider a small disk of radius  $\epsilon$  from which CRs are not selected for cooperative relaying. This constraint ensures that CRs lying very close to any one of the primary nodes do not participate in cooperation because their distance from the respective primary receiver will be nearly equal to the direct link distance. Hence, a significant gain in transmission rate cannot be obtained using these relays due to the path loss incurred. The area of the cooperation sector is given as  $\{A_c = \frac{\theta}{2} ((r_P - \epsilon)^2 - \epsilon^2)\}$  where  $\theta \in [0, \pi]; \epsilon \geq 1$ .

The selected relays also form a PPP  $\Phi_r \subset \Phi$  with a total number of nodes  $k = \lambda |A_c|$ . A graphical illustration of the operational network geometry is given in Fig. 1. The secondary nodes form a bipolar transmitter-receiver structure. Once spectrum is leased to the secondary transmitters  $S_{tx}$  for their own activity, they transmit to their respective receivers  $S_{rx}$  located at a fixed distance  $r_0$  from the respective transmitters. As shown in [22], such network model is simple and easy to follow, Also it is shown that it is easily extendible to nearest neighbor model where the receiver of a particular transmitter is assumed to be its nearest neighbor relaxing the fixed distance  $r_0$  assumption.

We consider a wireless channel that suffers from path loss and fading. For a distance  $r$  between any two nodes, the channel between them can be expressed as  $ahl(r)$  [23] where the fading power gain  $h$  between any two nodes is an independent and identical (i.i.d.) exponentially distributed random variable with unit mean.  $a$  is a frequency dependent constant. For the sake of simplicity, we consider it to be unity throughout the discussion.  $l(r) = \min(1, r^{-\alpha})$  is the distance dependent path loss upper bounded by unity for source-destination pairs located very close to each other. Also,  $\alpha \geq 2$  is the path loss exponent. The noise at the receiver front end is considered to be additive white Gaussian with power  $\sigma^2$ .

#### B. MAC Model

During direct two way communication, each primary node transmits at a power  $P_t$  achieving a rate  $R_{PP}$  for a time  $T$ . In DSL, the primary network aims to achieve the same two-way transmission rate  $R_{PP}$  during time  $2T$  using lesser transmit power.

1) *Selection of Cooperating CR relays* : In DSL, the primary signals its willingness to lease the spectrum to the secondary relays over a control channel. We assume that secondary nodes employ listening on the control channel. This beacon contains the duration  $\mathcal{T} = 2T$  and angle  $\theta$  of lease. The CRs are assumed to be aware of their location with respect to the primary transmitter and receiver. On the reception of the leasing beacon (containing  $\theta$  information), only those nodes participate in cooperation that lie within the cooperation region  $A_c$  as defined earlier.

$P_t$	Transmit Power of the primary	$P_s$	Transmit Power of the secondary
$r_P$	Distance between two primary users	$\Phi$	Point process of the secondary users
$\mathcal{T} = 2T$	Total time for DSL	$\theta$	Angle of cooperation
$\lambda$	Density of secondary users	$\epsilon$	Protective disk radius
$A_c$	Area of cooperation	$h$	Rayleigh faded channel coefficient
$r_0$	Distance between two secondary users	$\alpha$	Path loss coefficient
$\sigma^2$	Power of AWGN at receiver front	$q_{sp}$	Success probability of secondary to primary communication
$q_{ps}$	Success probability of primary to secondary communication	$t_{sp}$	Time reserved for secondary to primary communication
$t_{ss}$	Time reserved for secondary to secondary communication	$t_{ps}$	Time reserved for primary to secondary communication
$n_{PP}$	No. of successfully transmitted bits in direct communication	$n_{DSL}$	No. of successfully transmitted bits in DSL communication

Table I  
SYMBOLS USED

2) *Communication in DSL mode*: Beacon enabled signaling is adopted for DSL to initiate and agree on leasing parameters. Spectrum leasing for time  $\mathcal{T} = 2T$  takes place in the following three phases (also see Fig. 2).

- Each primary source transmits its data to be relayed to its receiver to the secondary nodes lying in the cooperation region for a time  $t_{p1s}$  and  $t_{p2s}$  at a rate  $R_{p1s}$  and  $R_{p2s}$  respectively. During this phase, the transmit power of the primary sources is  $\hat{P}_t < P_t$ . Such low power communication exploits the geometric selection of cooperating nodes in the vicinity of both primary users and maintains low energy costs.
- The cooperating secondary nodes perform network coding to combine the two primary signals as in eq. 1. The coded data  $D$  is then transmitted at the physical layer to both the primary nodes by forming a distributed antenna array employing Distributed Space Time Coding (DSTC)<sup>1</sup>. At the physical layer, this transmission is done at a rate  $R_{sp}$  and power  $P_s < P_t$  for a duration  $t_{sp}$  such that

$$t_{p1s} + t_{p2s} + t_{sp} < \mathcal{T}$$

- Finally, for the remaining time, the secondary nodes gain an exclusive access to the channel. During this time  $t_{ss}$ , they transmit to their respective receiver at a rate  $R_{SS}$  and power  $P_s$ . This time is the fare that the primary network has to pay for the relaying services of the secondary nodes.

3) *Bargaining game*: During the process of leasing, the most crucial factor is the division of leasing time between the above three phases. It is important that each operational element of the network gets enough share of time to meet its transmission throughput requirements. To ensure such time division, we formulate a network level game where each of the player, i.e., primary network (player 1) and the secondary network (player 2) engage themselves in the arbitration for the time division over a control channel. In our case, the primary user initiates the leasing process. In response, the secondary users determine their demand and adopt a strategy according to the primary offer. If the offer is acceptable, the game is concluded and leasing is successful. If the CRs want to bargain further, another round of offer and respective response is played. In case the negotiations are unsuccessful, the game ends and the leasing is not done. It is further assumed that the CRs form a homogeneous network in terms of the hardware platform and leasing time demand and they do not show malicious

<sup>1</sup>Ideal orthogonal codes are considered here where the details of DSTC codebook and operational parameters can be found in [5],[24].

behavior. We will come back to such division of time in Section IV-B.

### C. Assumptions

For simplicity and tractability of the analysis, we assume that the primary and secondary network are aware of the CSI within their respective networks. A practical implementation of such information exchange can be found in [25]. Also, both the primary and secondary are assumed to be aware of the average fading characteristics of their link with each other (phase I:  $PR_{1,2}$  to  $S_{tx}$  and phase II:  $S_{tx}$  to  $PR_{1,2}$ ). These characteristics are assumed to remain constant over a significant number of transmission blocks due to quasi-static geometry and slow fading in the channel. The CRs are aware of their location with respect to the primary transmitter and receiver. Moreover, the primary and the secondary users are considered to be in perfect time synchronization with each other. Cost effective methodologies for implementing time synchronization in ad hoc networks have been suggested in [26] hence encouraging us to propose a time sharing based communication scheme. The control beacon signal by the primary user to initiate spectrum leasing can be used for synchronization between the primary and the secondary nodes. Most of the important symbols used in the paper have been gathered in tab. I.

## IV. GEOMETRIC ANALYSIS OF DYNAMIC SPECTRUM LEASING

In order to accurately model DSL, we carry out a detailed analysis of its performance determinants like average link capacities and time division in the discussion to follow. Our determination of the DSL transmission rate and time is strictly driven by the actual CR densities and inter-node distances following a Poisson geometry.

### A. Average Link Capacities $R_{PP}$ , $\bar{R}_{p1s}$ , $\bar{R}_{p2s}$ , $\bar{R}_{sp}$ and $\bar{R}_{SS}$

In conventional two-way communication, the data transmission rate  $R$  between each primary source destination pair is given by

$$R = \log_2(1 + \text{SNR}) \quad (b/s/Hz), \quad (2)$$

where signal to noise ratio (SNR) measured at the receiver is  $\frac{P_t h_p l(r_P)}{\sigma^2}$  and  $h_p$  is the channel power gain between the source and the destination,  $P_t$  is the transmit power and  $l(r_P)$  is the distance dependent path loss<sup>2</sup> between the nodes. Note that since

<sup>2</sup>Due to the assumption that there is no transmitter within  $\epsilon \geq 1$  distance from both  $P_{tx}$  and  $P_{rx}$ ,  $l(r_P)$  is assumed to be  $\frac{1}{r_P^\alpha}$  unless stated otherwise.

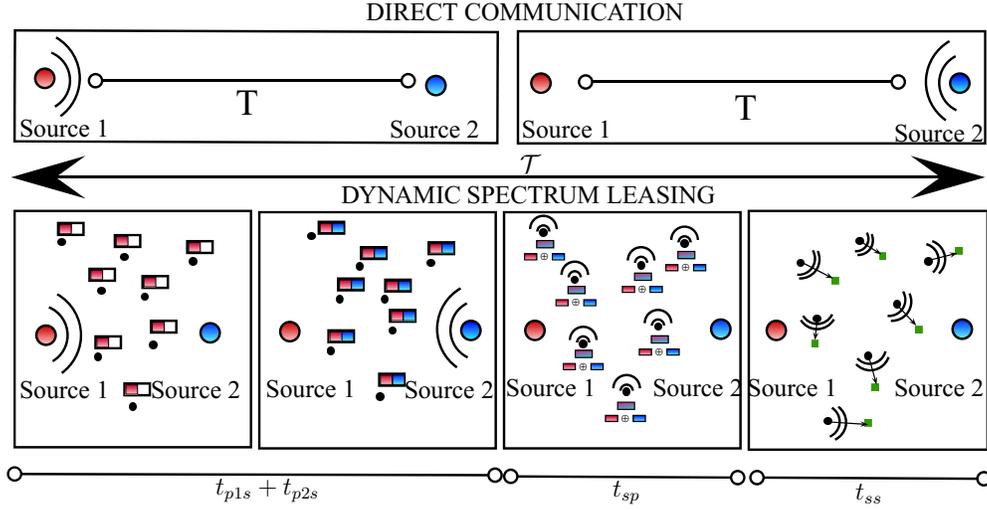


Figure 2. Direct communication and Dynamic Spectrum Leasing

the distance between the two nodes is fixed and we consider the channel gain  $h_p$  to be symmetric, it is safely assumed that the communication rate from  $PR_1$  to  $PR_2$ , is the same as the transmission rate from  $PR_2$  to  $PR_1$ . In practical networks, the primary maintains a certain QoS for its communication. Here we define this QoS  $\rho$ -outage rate,  $R_{QoS}$ , as the largest rate of transmission  $R$  such that the outage probability on this link is less than  $\rho$ .

**Lemma 1.** *The  $\rho$ -outage rate,  $R_{PP}$ , for the link  $(PR_1, PR_2)$  is given as*

$$R_{PP} = \log_2 \left[ 1 - \left( \frac{P_t l(r_P)}{\sigma^2} \right) \ln(1 - \rho) \right] \quad (\text{b/s/Hz}), \quad (3)$$

*Proof:* According to eq. 2, the instantaneous data transmission rate from  $PR_1$  to  $PR_2$  depends upon the channel gain  $h_p$  between the source and the destination. However, to maintain the transmission quality of the link, i.e.,  $R \geq R_{QoS}$ , we calculate the probability of outage on this link. Mathematically

$$\begin{aligned} p_{out} &= \Pr \{ \log_2(1 + \text{SNR}) < R_{QoS} \}, \\ &= \Pr \left\{ h_p < (2^{R_{QoS}} - 1) \left( \frac{\sigma^2}{P_t l(r_P)} \right) \right\}, \end{aligned}$$

Using the exponential distribution of the channel power gain, the  $\rho$ -outage probability becomes

$$\rho = 1 - \exp \left( - (2^{R_{QoS}} - 1) \left( \frac{\sigma^2}{P_t l(r_P)} \right) \right).$$

The transmission rate  $R_{PP}$  achieved between a typical primary source-destination pair for a given quality of service constraint  $\rho$  is

$$R_{PP} = \log_2 \left[ 1 - \left( \frac{P_t l(r_P)}{\sigma^2} \right) \ln(1 - \rho) \right], \quad (4)$$

which is the rate that satisfies the primary QoS ( $p_{out} < \rho$ ). ■

When the spectrum is leased to the secondary users, the cooperative link performance is dictated by the cooperative capacity. As mentioned earlier, nodes centered only in the effective area of communication,  $A_c$ , are considered for cooperation.

The cooperative capacity depends upon the transmission rates  $R_{p1s}$ ,  $R_{p2s}$  and  $R_{sp}$  achieved in the first two leasing phases. It is important to mention that the overall performance of the cooperative communication link will be bounded by the minimum transmission rate of all the relays during the first phase. For simplicity of analysis, we consider that the relay at the maximum distance from the primary transmitter will result in the worst rate performance. We bound the rate by the worst case performance by considering the distance between and primary transmitter,  $PR_{1,2}$ , and the relay node to be maximum. Also, assuming the point process  $\Phi$  to be stationary, the average worst rate  $\bar{R}_{p1s}$  from  $PR_1$  to its farthest relay is equal to the rate  $\bar{R}_{p2s}$  from  $PR_2$  to the relay at the farthest distance from it considering a static geometry and average channel effects. Hence we denote the minimum rate of the first phase as  $R_{ps}$ .

The effective DSL capacity  $R_{DSL}$  is then given as [27],

$$R_{DSL} = \min\{q_{ps}\bar{R}_{ps}, q_{sp}\bar{R}_{sp}\} \quad (\text{b/s/Hz}) \quad (5)$$

where,  $q_{ps}$  and  $q_{sp}$  are the probabilities that the communication in the first phase at rate  $\bar{R}_{ps}$  and in the second phase at rate  $\bar{R}_{sp}$  are successful.

**Lemma 2.** *The average transmission rate from any primary transmitter  $PR_{1,2}$  to the farthest secondary relay,  $\bar{R}_{ps}$ , is given as*

$$\bar{R}_{ps} = \log_2 \left[ 1 + \left( \frac{1 - \exp(-\lambda \frac{\theta}{2} (r_P - \epsilon)^2)}{(r_P - \epsilon) - C} \right)^\alpha \frac{\hat{P}_t}{\sigma^2} \right] \quad (\text{b/s/Hz}) \quad (6)$$

*Proof:* At any secondary relay  $i$  at a distance  $r_i$ , the achieved communication rate is,

$$R_{ps_i} = \log_2 \left[ 1 + \frac{\hat{P}_t h_{ps_i} l(r_i)}{\sigma^2} \right], \quad (7)$$

where  $h_{ps_i}$  is the channel gain between  $PR_{1,2}$  and the arbitrary relay  $i$  and  $l(r_i)$  is the path loss and  $\hat{P}_t \ll P_t$  is the transmit power. The average rate  $R_{ps_i}$  at any typical relay can be found using a similar approach as in lemma 1. As mentioned earlier, here we bound the rate by the worst case performance by considering

the distance between  $PR_{1,2}$  and the relay node to be maximum. The average distance  $\mathbb{E}[r_n]$  from a node to its farthest neighbor within a sector with angle  $\theta$  and radius  $r_P$  i.e.,  $\sec(r_P, \theta)$  in a 2-dimensional PPP can be found out on the same lines as in [28] to be

$$\mathbb{E}[r_n] = \frac{(r_P - \epsilon) - C}{1 - \exp\left(-\lambda \frac{\theta}{2} (r_P - \epsilon)^2\right)} \quad (8)$$

where  $C = \sqrt{\frac{\pi}{2\lambda\theta}} \exp\left(-\lambda \frac{\theta}{2} (r_P - \epsilon)^2\right) \operatorname{erfi}\left(\sqrt{\frac{\lambda\theta}{2}} (r_P - \epsilon)\right)$  and  $\operatorname{erfi}(x)$  is the imaginary error function such that,

$$\operatorname{erfi}(x) = \frac{2}{\sqrt{\pi}} \int_{t=0}^x \exp(-t^2) dt.$$

Using the above relation, the rate of transmission from  $PR_{1,2}$  to the farthest neighbor ( $n = k$ ) can be found. Jensen's inequality can be applied to find the average value of the transmission rate using the fact that  $\mathbb{E}[h_{ps_n}] = 1^4$ . From eq. 8, the average path loss can be calculated if the secondary node density and the area of cooperation are known. Hence, as stated before, assuming the stationarity of the point process  $\Phi$ , the average rate of the furthest relay in the first phase  $\bar{R}_{ps}$  is upper bounded by,

$$\bar{R}_{ps} \leq \log_2 \left[ 1 + \left( \frac{1 - \exp\left(-\lambda \frac{\theta}{2} (r_P - \epsilon)^2\right)}{(r_P - \epsilon) - C} \right)^\alpha \frac{\hat{P}_t}{\sigma^2} \right]. \quad (9)$$

and the other relays can achieve a better rate than eq. 9. ■

In the second phase of cooperation, the selected secondary relays form a  $k$  antenna array and perform DSTC to send the data to the receiver with a rate  $R_{sp}$ . The primary data is relayed by the set of CRs located towards the center of the sector of cooperation. Such geometric considerations allow us to assume that the same rate  $R_{sp}$  is achieved between the relay set of CRs and both the primary nodes.

The rate of communication when DSTC is employed for multiple relay transmission to a common destination has been evaluated in [5], [29], [24]. In the context of the geometric modeling of dynamic spectrum leasing, we use the DSTC communication rate and determine its mean value considering the geometric parameters.

**Lemma 3.** *The average transmission rate,  $\bar{R}_{sp}$ , when  $k$  secondary relays, i.e.,  $k \in \Phi_r$  form an antenna array, where secondary relay  $i$  is located at a distance  $\hat{r}_i$  from  $PR_1$  or  $PR_2$  is given by*

$$\bar{R}_{sp} = \log_2 \left[ 1 + \frac{\lambda \theta P_s}{\sigma^2} \left( \frac{(r_P - \epsilon)^{2-\alpha} - \epsilon^{2-\alpha}}{2-\alpha} \right) \right] \quad (b/s/Hz), \quad (10)$$

where, the secondary transmits with a power  $P_s$ , the channel gain between  $S_{tx}$  and  $PR_{1,2}$  is  $h_{sp_i}$ .

*Proof:* The rate of the DSTC communication with  $k$  relay nodes is given as,

$$R_{sp} = \log_2 \left[ 1 + \sum_{i \in \Phi_r} \frac{P_s h_{sp_i} l(\hat{r}_i)}{\sigma^2} \right].$$

<sup>3</sup>Due to the stationarity of the point process, we safely assume that the distance  $r_n$  can be measured while considering the primary transmitter to be at the origin.

<sup>4</sup>Log functions are considered to be convex and Jensen's inequality which is only applicable to convex functions is used to bound the communication rate in eq.7. Throughout the rest of the paper, Jensen's inequality is applied on convex functions.

Similar to the previous discussion, we again apply Jensen's inequality to find the average secondary to primary transmission rate  $\bar{R}_{sp}$ . In this case, the aggregate contribution to the received power due to the channel gains and the distances of all the relays  $\sum_{i \in \Phi_r} \frac{h_{sp_i}}{\hat{r}_i^\alpha}$  from the primary receiver can be calculated using Campbell's theorem for stationary Poisson point process [20]. In our case, the region of interest is the two dimensional area bounded by the sector of radius  $r_P - 2\epsilon$  and angle  $\theta$  in radians. Using the Campbell's theorem for this area, the expectation results in

$$\mathbb{E} \left[ \sum_{i \in \Phi_r} \frac{h_{sp_i}}{\hat{r}_i^\alpha} \right] = \lambda \theta \left( \frac{(r_P - \epsilon)^{2-\alpha} - \epsilon^{2-\alpha}}{2-\alpha} \right).$$

Using the above value for  $\mathbb{E} \left[ \sum_{i \in \Phi_r} \frac{h_{sp_i}}{\hat{r}_i^\alpha} \right]$ , the average rate comes out to be as in eq. 10. ■

In the third phase of spectrum leasing, the secondary transmitters of the cooperation region communicate with their respective receivers. We consider a bi-polar model of the secondary source-destination pairs [30] as shown in Fig. 1. We are interested in knowing the average transmission capacity of the  $(S_{tx}, S_{rx})$  link,  $\bar{R}_{SS}$ . In this case, all the secondary transmitters in the cooperation region simultaneously communicate with their receivers in order to utilize the leased bandwidth for their own transmission. In this phase, similar to the direct communication, we consider a realistic situation under which the secondary network also operates under a fixed QoS constraint  $\rho_s$ .

**Lemma 4.** *The average  $\rho_s$ -outage rate,  $\bar{R}_{SS}$ , for the link  $(S_{tx_i}, S_{rx_i})$ , where the channel gain between the source  $i$  and its destination  $h_{ss_i}$  is exponential, the transmit power is  $P_s$  and the distance between them is  $r_0$  is given as*

$$\bar{R}_{SS} = \log_2 \left[ 1 - \left( \frac{l(r_0)}{\frac{\sigma^2}{P_s} + \kappa_1} \right) \ln(1 - \rho_s) \right] \quad (b/s/Hz), \quad (11)$$

where  $\kappa_1$  is the average aggregate interference.

*Proof:* The Signal to Interference plus Noise Ratio (SINR) at the secondary receiver can be quantified as,

$$\text{SINR} = \frac{P_s h_{ss_i} l(r_0)}{\sigma^2 + \sum_{j \in \Phi_r} P_s h_{ss_j} l(r_j)}$$

where,  $h_{ss_j}$  is the channel gain between the secondary transmitter  $j$  causing interference at  $S_{rx_i}$  at a distance  $r_j$ . The aggregate interference at any receiver is denoted as  $I = \sum_{j \in \Phi_r} h_{ss_j} l(r_j)$ .

Followed by the proof of lemma 1, the transmission rate achieved between a typical secondary source-destination pair for a given quality of service constraint  $\rho_s$  can be stated. However, here we are interested in the average rate. From Jensen's inequality, the average  $\rho_s$ -outage rate  $\bar{R}_{SS}$  is

$$\bar{R}_{SS} \geq \log_2 \left[ 1 - \left( \frac{l(r_0)}{\frac{\sigma^2}{P_s} + \mathbb{E}[I]} \right) \ln(1 - \rho_s) \right]. \quad (12)$$

In order to find the average rate  $\bar{R}_{SS}$ , it can be noted that aggregate interference at a secondary receiver directly affects the average transmission rate. In the scenario under consideration where multiple transmitters gain access to the channel during the same time interval, interference between these simultaneous transmissions is a crucial issue. In order to determine the average

rate  $\bar{R}_{SS}$ , it is essential to first quantify the average amount of interference,

$$\mathbb{E}[I] = \mathbb{E} \left[ \sum_{j \in \Phi_r} h_{ss_j} l(r_j) \right].$$

Using the definition of Laplace Transform and taking expectation over both the point process and fading in the above integral,

$$\mathbb{E}[\exp^{-sI}] = \mathbb{E}_{\Phi} \left[ \prod_{j \in \Phi_r} \mathbb{E}_h \left[ \exp(-sh_j l(r_j)) \right] \right].$$

From the definition of probability generating functional [20], the integral takes the form,

$$\mathbb{E}[\exp^{-sI}] = \exp\left(-\mathbb{E}_h \left[ \int_{\mathbb{R}^d} (1 - \exp(-sh_l(r))) \lambda dx \right] \right). \quad (13)$$

Cumulants of a probability distribution can be defined in terms of the moment generation function (MGF)

$$\kappa_n = \frac{d^n}{ds^n} \ln(\mathbb{E}[\exp^{sI}]) \Big|_{s=0}. \quad (14)$$

We use eq. 13 and the definition in eq. 14 to find the cumulants of interference for  $d$ -dimensional network as

$$\kappa_n = -\lambda \mathbb{E}_h \left[ h^n \int_{\mathbb{R}^2 \cap \text{sec}(\theta, (r_P - \epsilon))} r^{-\alpha n} \exp(shr^{-\alpha}) dr^{d-1} dr \right]. \quad (15)$$

For any positive value of  $n$ , eq. 15 evaluated at  $s = 0$  gives the  $n^{\text{th}}$  cumulant  $\kappa_n$  of the interference distribution. The first cumulant of the distribution for a two dimensional network, i.e.,  $d = 2$ , can be calculated as,

$$\kappa_1 = \mathbb{E}[I] = \lambda \theta \left( \frac{(r_P - \epsilon)^{2-\alpha} - \epsilon^{2-\alpha}}{2-\alpha} \right). \quad (16)$$

The average aggregate interference can be used to determine the  $\rho_s$ -outage rate following eq. 12. It is worth mentioning here that the average interference calculated using the first cumulant results in the same equation as for the average aggregate signal coming from various secondary relays to the primary receiver in phase II. Hence cumulant based approach is an alternative way of calculating the aggregate signal from spatially distributed transmitters under Rayleigh faded channel. ■

After computing the individual link transmission rates, we are interested in knowing the overall transmission rate achieved in the DSL operational mode. We assume that a decode and forward type single hop relaying mechanism is used in the cooperation phase. The effective DSL capacity  $R_{DSL}$  is then given as eq. 5. The probability of successful DSL transmission ( $p_{suc} = 1 - p_{out}$ ) is dependent upon the probability of successful  $PR_1$  and  $PR_2$  to  $S_{tx}$  (phase I) and  $S_{tx}$  to  $PR_1$  and  $PR_2$  (phase II) transmission. These probabilities can be denoted as  $q_{ps}$  and  $q_{sp}$  respectively as in eq. 5. In order to ensure successful transmission and get the same outage capacity via the cooperative link,  $p_{out}$  in both the phases of DSL should be less than or equal to the maximum acceptable outage  $\rho$ , of the direct primary link. For the first phase, the probability  $q_{ps} = \Pr\{\bar{R}_{ps} > \bar{R}_{PP}\}$  can be expressed as,

$$q_{ps} = \exp\left(-\left(2\bar{R}_{PP} - 1\right) \frac{r_n^\alpha \sigma^2}{\hat{P}_t}\right). \quad (17)$$

As in eq. 9, the average value of this probability can be estimated using eq. 8 depending upon the cooperation area and user density. Similarly, the probability of successful transmission in the cooperative relaying phase  $q_{sp}$  can be written as

$$q_{sp} = \Pr\left\{X > \left(2\bar{R}_{PP} - 1\right) \frac{\sigma^2}{P_s}\right\}, \quad (18)$$

where  $X = \sum_{i \in \Phi_r} \frac{h_{spi}}{\hat{r}_i^\alpha}$  follows the same distribution as that of interference in eq. 11. By the definition of Chebyshev's bound, the above inequality can be approximated to

$$\Pr\left\{X > \frac{\left(2\bar{R}_{PP} - 1\right) \sigma^2}{P_s}\right\} \leq \frac{\kappa_2}{\left(\left(2\bar{R}_{PP} - 1\right) \frac{\sigma^2}{P_s} - \kappa_1\right)^2}. \quad (19)$$

Utilizing the fact that cumulants of the interference are related to the parameters of the distribution as  $\kappa_1 = \mathbb{E}[I]$  and  $\kappa_2 = \text{Var}[I]$ , the probability  $q_{ps}$  takes the form,

$$q_{sp} = \frac{\lambda \theta \left( \frac{(r_P - \epsilon)^{2-2\alpha} - (\epsilon)^{2-2\alpha}}{2-2\alpha} \right)}{\left(\left(2\bar{R}_{PP} - 1\right) \frac{\sigma^2}{P_s} - \lambda \theta \left( \frac{(r_P - \epsilon)^{2-\alpha} - (\epsilon)^{2-\alpha}}{2-\alpha} \right)\right)^2}, \quad (20)$$

where  $\kappa_1$  and  $\kappa_2$  can be found using 15.

### B. Division of Leasing Time between Phase I, II and III

The most critical factor in the operation of spectrum leasing is the optimal division of the total leased time  $\mathcal{T}$  between the time  $t_{p1s}$ ,  $t_{p2s}$ ,  $t_{sp}$  and  $t_{ss}$  reserved for the primary and secondary communication. Since our geometrical evaluation of the achievable rate from both primary transmitters to the secondary relays is shown to be bounded by equal average minimum transmission rate  $\bar{R}_{ps}$ , we denote  $t_{p1s} = t_{p2s} = t_{ps}$ . The goal of the primary node is to ensure that its time-rate product of communication  $t_{ps}R_{DSL}$  and  $t_{sp}R_{DSL}$  at a certain  $\rho$ -outage probability is maximized i.e.,  $t_{ps}R_{DSL} > TR_{PP}$  by maximizing the time  $t_{ps}$  and  $t_{sp}$  for which the primary sources 1 and 2 transmit the data to be relayed to the cooperating secondary nodes respectively and the time for which the CRs relay the data to the primary nodes after applying network coding.

The goal of the secondary nodes is to maximize their share in time so that they get reimbursed for their network coding and cooperative relaying services by getting maximum time  $t_{ss}$  to communicate with  $S_{rx}$  at a target rate  $\bar{R}_{SS}$ . However, the secondary network must cooperate in relaying primary data for a time  $t_{sp}$  long enough so that the primary network can increase its throughput while maintaining its communication standards. Long  $t_{ps}$ ,  $t_{sp}$  versus a very small fraction of  $t_{ss}$  will discourage the secondary to cooperate while prolonged  $t_{ss}$  will degrade the performance of the legacy network in terms of its bandwidth efficiency. Hence an intelligent division of time is very crucial for the operation of the network.

1) *Game Formulation:* This problem can be conveniently studied in a game theoretic framework which is ideal to model such situations where each entity tries to maximize its own utility. We model the situation as a two player game using the Nash bargaining framework from cooperative game theory [31]. We consider bargaining as a two player game because every primary and secondary node is representative of the utility of all the other primary and secondary nodes as we are

considering only the average rate values and equal transmit powers for all DSL phases. The Nash bargaining framework is employed to model a situation in which the players negotiate for their agreement on a particular point out of a set of joint feasible payoffs  $\mathcal{G}$ . In a two player Nash Bargaining game,  $\mathcal{G} \equiv \{g = (g_1, g_2) : g_i = f_i(\mathbf{S}), i = 1, 2; \mathbf{S} \in S_1 \times S_2\}$ , where the functions  $f_i(\cdot)$  represent the individual utilities of the two players.  $\mathbf{S}$  is the strategy of the  $i^{th}$  player from the strategy profile  $S_i$ . In Nash Bargaining, in case the negotiations render unsuccessful, the outcome of the game becomes  $\mathcal{G}_0 = (g_{01}, g_{02})$ . It is a fixed vector known as the disagreement vector where  $g_{01}, g_{02}$  are the utilities of the two players in case of disagreement. The whole bargaining problem can be described conveniently by the pair  $(\mathcal{G}, g_0)$ . A pair of payoffs  $(g_1^*, g_2^*)$  is a Nash Bargaining solution if it solves the following optimization problem

$$\begin{aligned} & \max_{g_1, g_2} (g_1 - g_{01})(g_2 - g_{02}), & (21) \\ & \text{subject to } \begin{cases} (g_1, g_2) \in \mathcal{G} \\ (g_1, g_2) \geq \mathcal{G}_0 \end{cases} \end{aligned}$$

If the set  $\mathcal{G}$  is compact and convex, and there exists at least one  $g \in \mathcal{G}$  such that  $g > g_0$ , then the unique solution to the bargaining problem  $(\mathcal{G}, g_0)$  corresponds to the unique solution of the optimization problem [19],[31].

In our case, the primary transmitter is the first player whose utility is directly dependent upon the transmission time  $t_{ps}$  and cooperation time  $t_{sp}$  and increases as it increases. For simplicity, we define the utility of the primary and the secondary node as;

$$f_1(t) = \begin{cases} t_{ps} & \text{phase 1} \\ t_{sp} & \text{phase 2} \end{cases} \quad \text{and} \quad f_2(t) = t_{ss} \quad (22)$$

respectively<sup>5</sup>, where  $2t_{ps} + t_{sp} + t_{ss} = \mathcal{T}$ .

The time demand of each player i.e.,  $t_{ps}$  and  $t_{sp}$  for the primary node and  $t_{ss}$  for the secondary node are the strategies chosen from their respective strategy profiles. In this case, the fraction of leased time should be large enough to ensure that the time-rate product of broadcast phase  $t_{ps}\bar{R}_{ps}$  and the cooperation phase  $t_{sp}\bar{R}_{sp}$  is greater than the direct communication time  $T$  and rate  $R_{PP}$  product. During the second sub-interval, a secondary node must have enough time to at least overcome its cooperation cost  $cP_s t_{0sp}$  given its average transmission rate  $\bar{R}_{SS}$ . Here  $c$  measures the bits transmitted per Watt of power consumed. Hence, the disagreement vector of our Nash Bargaining game becomes  $(\frac{\bar{R}_{PP}T}{q_{ps}\bar{R}_{ps}}, \frac{\bar{R}_{PP}T}{q_{sp}\bar{R}_{sp}}, \frac{cP_s t_{0sp}}{\bar{R}_{SS}})$  where  $g_{01}$  is the joint utility of the first two phases  $\frac{\bar{R}_{PP}T}{q_{ps}\bar{R}_{ps}}$  and  $\frac{\bar{R}_{PP}T}{q_{sp}\bar{R}_{sp}}$  and  $g_{02}$  is  $\frac{cP_s t_{0sp}}{\bar{R}_{SS}}$ .

**Lemma 5.** *The optimal proportion of time for cooperative relaying is a solution to the following maximization problem*

$$\begin{aligned} & \max (\log(t_{ps} - t_{0ps}) + \log(t_{sp} - t_{0sp}) + \log(t_{ss} - t_{0ss})), & (23) \\ & \text{subject to } \begin{cases} t_{ps}q_{ps}\bar{R}_{ps} = t_{sp}q_{sp}\bar{R}_{sp} \\ \mathcal{T} = 2t_{ps} + t_{sp} + t_{ss} \end{cases} \end{aligned}$$

where the disagreement vector is  $(t_{0ps}, t_{0sp}, t_{0ss}) = (\frac{\bar{R}_{PP}T}{q_{ps}\bar{R}_{ps}}, \frac{\bar{R}_{PP}T}{q_{sp}\bar{R}_{sp}}, \frac{cP_s t_{0sp}}{\bar{R}_{SS}})$ .

<sup>5</sup>from eqs. 22, the compactness and convexity of  $\mathcal{G}$  is straightforward.

*Proof:* The first constraint ensures a division of time such that equal time rate product is attained in the first and the second phase. This results in a unique time fraction for which both primary source nodes can receive the coded data at equal transmission rate. The second constraint ensures that the time fractions reserved for each phase do not exceed the time required in ordinary two-way communication.

From the definition of Nash Bargaining solution, the time division problem for a 2-player game can be written in a logarithmic form. Such representation of the maximization problem ensures proportional fairness of the solution for both the players. The corresponding Lagrangian for the above optimization problem can be written as follows

$$\begin{aligned} L(t_{ps}, t_{sp}, \lambda_1, \lambda_2) = & \log(t_{ps} - t_{0ps}) + \log(t_{sp} - t_{0sp}) \\ & + \log(t_{ss} - t_{0ss}) + \lambda_1 (t_{ps}q_{ps}\bar{R}_{ps} - t_{sp}q_{sp}\bar{R}_{sp}) \\ & + \lambda_2 (\mathcal{T} - 2t_{ps} - t_{sp} - t_{ss}) \end{aligned} \quad (24)$$

We simplify the Lagrangian by eliminating  $t_{ss}$  and replacing it by  $t_{ss} = \mathcal{T} - 2t_{ps} - t_{sp}$ . The maximization problem can be solved by using the Karush-Kuhn-Tucker (KKT) first order necessary conditions [21],

$$\frac{\delta L}{\delta t_{ps}} = \frac{1}{t_{ps} - t_{0ps}} + \frac{2}{2t_{ps} - \mathcal{T} + t_{0ss} + t_{sp}} + \lambda_1 (q_{ps}\bar{R}_{ps}) - 2\lambda_2 = 0. \quad (25)$$

$$\frac{\delta L}{\delta t_{sp}} = \frac{1}{t_{sp} - t_{0sp}} + \frac{1}{t_{sp} - \mathcal{T} + t_{0ss} + 2t_{ps}} - \lambda_1 (q_{sp}\bar{R}_{sp}) - \lambda_2 = 0 \quad (26)$$

and

$$\begin{aligned} \lambda_1 (t_{ps}q_{ps}\bar{R}_{ps} - t_{sp}q_{sp}\bar{R}_{sp}) &= 0, & \lambda_1 &\geq 0, \\ \lambda (\mathcal{T} - 2t_{ps} - t_{sp} - t_{ss}) &= 0, & \lambda_2 &\geq 0. \end{aligned}$$

Here we assume that  $\lambda_2 = 0$ . From the definition of the Nash Bargaining problem there exists a vector  $\mathbf{S}$  such that the optimal value of the optimization problem is strictly positive. From the constraint on the operational rate-time product, it can be seen that  $t_{ps} = \frac{t_{sp}q_{sp}\bar{R}_{sp}}{q_{ps}\bar{R}_{ps}}$ . Using this relation to solve for  $\lambda_1$  by using simple algebra, we get a quadratic equation in  $t_{sp}$  as given in eq. 27.

Using eq. 27, the values of  $t_{ps}$  and  $t_{ss}$  can be obtained. It is important to mention that only positive root of eq. 27 is considered because it maximizes the utilities of all three phases for both players. As a consequence of the first constraint in 23, the effective number of bits that get transmitted to both the primary sources in DSL is  $t_{ps}R_{ps}$  or equivalently  $t_{sp}R_{sp}$ . ■

## V. ANALYTIC EVALUATION OF DSL

Having developed the complete analytical model of DSL under geometric considerations, in this section we want to evaluate the performance of DSL based on the developed model. The analytic evaluation of DSL under varying wireless channel conditions is carried out on the basis of the derived results. For the verification of the accuracy of the analysis and the validity of the assumptions made throughout, we also simulate a practical network in which DSL is operational. Poisson distributed CR nodes with mean  $\lambda$  are considered in a network of radius 200 meters. For each realization of the Poisson network, a Rayleigh distributed channel coefficient

$$\begin{aligned}
& t_{sp}^2 \left( 3 + \frac{6q_{sp}\bar{R}_{sp}}{q_{ps}\bar{R}_{ps}} \right) + \left( 2 \left( t_{0ss} - t_{0ps} - \frac{q_{ps}\bar{R}_{ps}}{q_{sp}\bar{R}_{sp}} t_{0ps} - \mathcal{T} \right) - 4 \left( t_{0ps} + t_{0sp} \frac{q_{sp}\bar{R}_{sp}}{q_{ps}\bar{R}_{ps}} - T \right) \right) t_{sp} \\
& + t_{0ps}t_{0sp} \left( \frac{q_{ps}\bar{R}_{ps}}{q_{sp}\bar{R}_{sp}} + 2 \right) - t_{0sp}t_{0ss} + t_{0ps}t_{0ss} \frac{q_{ps}\bar{R}_{ps}}{q_{sp}\bar{R}_{sp}} + \mathcal{T} \left( t_{0sp} + t_{0ps} \frac{q_{ps}\bar{R}_{ps}}{q_{sp}\bar{R}_{sp}} \right) = 0
\end{aligned} \tag{27}$$

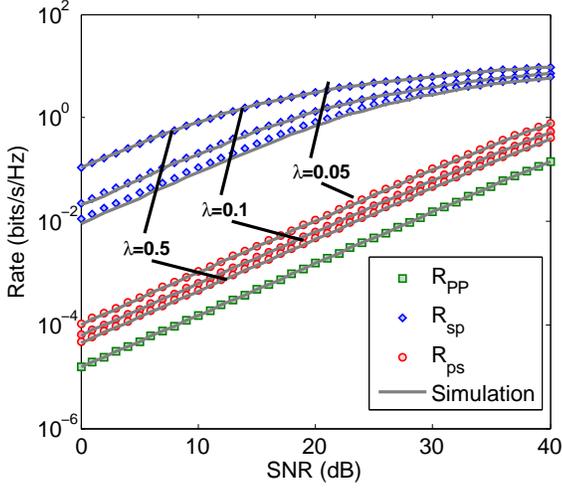


Figure 3. Transmission rates in direct and DSL communication,  $r_P = 10$ ,  $\theta = \frac{\pi}{2}$ ,  $\epsilon = 1$ ,  $P_t = 1$ ,  $\hat{P}_t = 0.2$ ,  $P_s = 0.2$ .

is generated. The transmission rate at the receiver for each spatial instance of the network is averaged for  $10^4$  different channel coefficients. This process is in turn repeated for  $10^4$  realizations of Poisson distributed CR network with intensity  $\lambda$ . Secondary network communication under interference considerations is also studied in a similar fashion. All the simulations are carried out in MATLAB.

#### A. Transmission Rates

Firstly, we study the average achievable transmission rates under both the normal and leasing mode of network operation as shown in Fig. 3. The rate under direct primary communication at a transmit power  $P_t$  increases with improving channel conditions. Here, the reliability in terms of the probability of success ( $p_{suc}$ ) of direct communication is assumed to be 90%. The outage capacity for such quality of service,  $\bar{R}_{PP}$ , defines the target capacity for communication in the primary network,  $R_{QoS}$ , for all operational modes i.e., direct and DSL. Under identical channel realizations, a demand for higher service quality (smaller  $\rho$ ) straightforwardly results in lower  $\bar{R}_{PP}$ . The rate when  $PR_2$  transmits to  $PR_1$ , is the same as achieved when  $PR_1$  transmits to  $PR_2$  due to similar average channel characteristics and constant link distance.

For the capacity analysis of DSL, we study the average achievable transmission rates in the three phases of leasing. The capacity of the primary to secondary communication in the first phase is strongly dependent upon the number of secondary nodes present in the area of cooperation. As mentioned earlier, in our analysis, we consider the lower bound to this rate by considering the average transmission rate between the primary transmitter and the furthest relay. In Fig. 3, the rate from one primary source to the furthest CR node is shown. The same rate is achieved by the other primary source. For very low secondary density, e.g.,

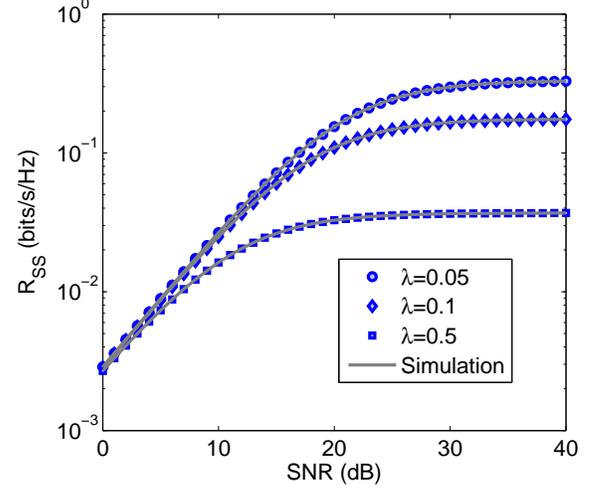


Figure 4. Communication rate during  $S_{tx}$ - $S_{rx}$  transmission,  $r_0 = 1$ ,  $\rho_s = 0.1$ ,  $P_s = 0.2$ .

$\lambda \lll 0.05$ , the probability of finding a neighbor in the region of cooperation is extremely low. For this reason, the capacity analysis for very sparse secondary network is not possible since the transmission rates from  $PR_{1,2}$  to the CRs are nearly zero. For higher  $\lambda$ , it can be seen from Fig. 3 that the average transmission rate  $\bar{R}_{ps}$  is greater than direct communication at lower transmit power  $\hat{P}_t$ . This phenomenon is a consequence of such cooperation region selection that relays are located in a close proximity to both  $PR_1$  and  $PR_2$ . Hence greater rate at lower transmit power is attained due to shorter distance between the relay and  $PR_1$ . However, if the number of secondary users increases in the cooperation region, the average distance between the transmitter  $PR_{1,2}$  and the farthest node increases which follows from the average distance quantification in eq. 8. Hence  $\bar{R}_{ps}$  decreases when  $\lambda$  increases (lower line in Fig.3). However, the cooperative relaying rate  $\bar{R}_{sp}$  increases with increasing relay density due to the diversity gain (upper line in Fig.3). Increasing  $\lambda$  increases the number of cooperating nodes, consequently, the rate  $\bar{R}_{sp} \gg \bar{R}_{PP}$  for increasing values of  $\lambda$ .

We now show that the  $\rho_s$ -outage communication rate  $\bar{R}_{SS}$  also increases with improving SINR values in Fig. 4 however, it is interference limited in higher SNR regions (here the desired QoS of the secondary network in terms of the outage probability  $\rho_s$  is  $10^{-1}$ ). This is a consequence of the improved signal strength at the receiver. However, as the density of the secondary nodes increases, the average transmission rate decreases due to the increased interference. It is clear that increasing the outage constraint from  $10^{-1}$  to  $10^{-2}$ , causes the average rate to decrease because the decoding threshold at  $S_{rx}$  is raised. Hence, a graphical illustration of this result is intentionally skipped.

Along with the analytically drawn results, achievable transmission rates under a practical Poisson network are also shown in Fig.

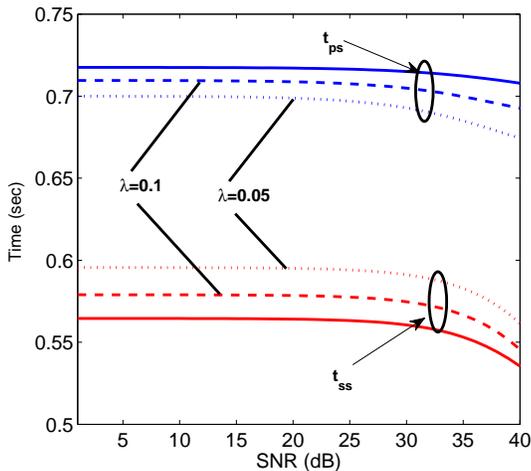


Figure 5. Nash Bargaining based division of  $\mathcal{T} = 2T = 2$  between the 1<sup>st</sup> and 3<sup>rd</sup> DSL phase,  $\lambda = 0.5$ .

3. The practical simulation results are indicated by the solid lines running over the analytic results (analytic results are indicated by lines with markers). It can be seen that the practical simulations closely match the analytic evaluation results. It validates our analytic formulation of DSL and the simplifying assumptions and bounds used for the simplification of the analysis. The practical simulation results of  $\bar{R}_{SS}$  are also shown in Fig. 4. It is to be noted that the rest of the results are based upon the communication rates of direct and DSL communication, which we have shown to be in close agreement with each other. Therefore, the practical simulations of the remaining results are skipped for the sake of brevity. Similarly, smaller sector sizes reduce the number of effective relays in the cooperation region and greater transmit powers enhance the communication rates in all phases. Hence, these parametric variations and their detailed study is intentionally skipped.

### B. Division of Leasing Time

After analyzing the transmission rates, we now study the equilibrium division of leasing time between the three phases of DSL. Fig. 5. shows the proportion of time allocated for the first and last phase. It can be seen that under all signal propagation conditions, the time required for the first phase is the highest. This is because as seen in the previous discussion (Fig.3),  $\bar{R}_{ps}$  is the lowest of all other DSL rates. In order to maximize the gain in primary data transmission,  $t_{ps}$  is higher in order to meet the condition  $t_{ps}q_{ps}\bar{R}_{ps} > T\bar{R}_{PP}$ . At higher CR densities, the time required by the CRs in the first phase also increases due to lower achievable transmission rate. In total  $2t_{ps}$  time is spent in the first phase by both the primary sources in transmitting their data to the secondaries. On the other side, since  $\bar{R}_{sp} > \bar{R}_{PP}$ ,  $\bar{R}_{ps}$ , the second phase is allocated lesser time. It can be seen in Fig. 6. However, the division of the time is such that  $t_{ps}\bar{R}_{ps} = t_{sp}\bar{R}_{sp} > T\bar{R}_{PP}$ . For sparse CR network, the time  $t_{sp}$  is greater as the number of relays are fewer. The difference  $\Delta t$  between the minimum time required by the CRs in the second phase ( $t_{0sp}$ ) and the time division output of the bargaining game ( $t_{sp}$ ) is also shown in the fig. 6. Positive values of  $\Delta t$  show that

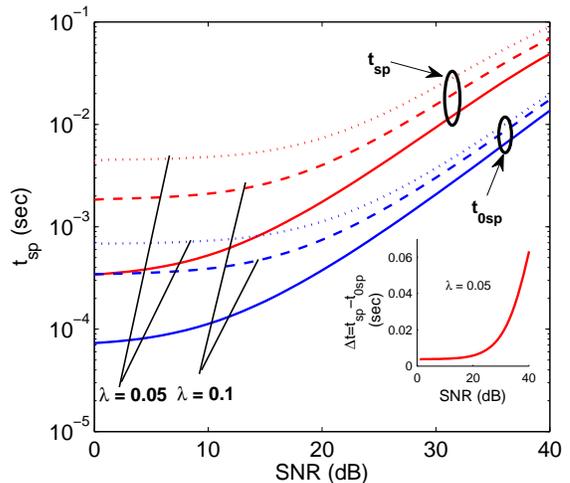


Figure 6. Reserved time for the second phase,  $\lambda = 0.5$ .

the bargaining solution provides enough flexibility to incorporate the practical time required for network coding and distributed STC is considered.

The time reserved for secondary activity  $t_{ss}$  in the third phase is also shown (Fig. 5). To compensate for their energy costs in the second phase and deteriorated rate performance due to interference in the third phase, the CRs are given a reasonably high time for their activity specifically at low CR densities. It is important to emphasize again that at densities much lower than 0.05, the transmission rates become so low that successful bargaining can not be established for a fair time division. Hence, analysis of lower  $\lambda$  values is not possible.

In Fig. 7, we show the increase in the time-rate product achieved by using DSL under a geometric and Nash Bargaining setup. The results indicate that DSL provides significant gain in the number of bits that are successfully transmitted in DSL as compared to the number of bits ( $2TR_{PP}$ ) in direct two way communication. This happens because the geometric vicinity and network coding services of the CR nodes provide higher transmission rates. Such enhanced performance is attained only when enough incentive is available for the secondary nodes to cooperate with the primary network. It can be seen that very high CR node densities, the first phase rate  $\bar{R}_{ps}$  gets down nearly equal to  $\bar{R}_{PP}$ . As a result, the bargaining game only results in such division that the time demands of all the players are merely satisfied. However, when the secondary network is sparse, the gains in the time rate product are up to 54%. Hence DSL under sparse secondary network maximizes the number of bits communicated successfully between the two primary networks.

## VI. ENERGY EFFICIENCY OF DYNAMIC SPECTRUM LEASING

In this section, we define and quantify the energy efficiency (EE) of the spectrum leasing model for cognitive radio networks. We define energy efficiency as the number of bits transmitted successfully across the channel per unit of energy consumed, given as,

$$EE = \frac{n_B}{J}. \quad (\text{bits/Joule}) \quad (28)$$

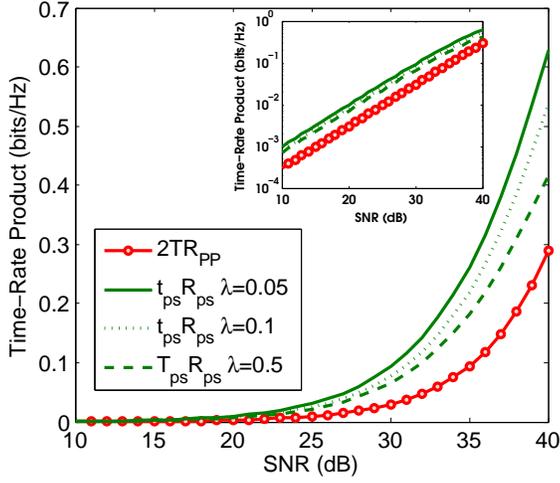


Figure 7. Time-Rate Product of un-coded Direct vs. NC based DSL based two way communication.  $P_t = 1, \hat{P}_t = 0.5, P_s = 0.2$ .

where  $n_B$  is the number of bits transmitted successfully and  $J$  is the energy consumed in Joules.

**Theorem 1.** *The energy efficiency of a licensed primary network employing direct communication  $EE_{PP}$  and while employing DSL,  $EE_{DSL}$  in terms of the number of successfully transmitted bits per unit energy can be given as*

$$EE_{PP} = \frac{n_{PP}}{TP_t}, \quad \text{and} \quad EE_{DSL} = \frac{n_{DSL}}{2t_{ps}\hat{P}_t + P_s t_{sp}k} \quad (29)$$

respectively, where  $n_{PP}$  is the number of successfully transmitted bits in direct communication,  $n_{DSL}$  are the successfully transmitted bits over the cooperative link.

*Proof:* The number of bits successfully transmitted in the transmission duration of the direct two way link  $n_{PP}$  is given as [23];

$$n_{PP} = 2R_{PP}T, \quad (30)$$

where  $R_{PP}$  follows from the result in lemma 1.

In case the primary decides to lease the spectrum, the number of bits successfully transmitted in spectrum leasing are given as

$$n_{DSL} = 2t_{ps}q_{ps}\bar{R}_{ps} = t_{sp}q_{sp}\bar{R}_{sp}, \quad (31)$$

where,  $\bar{R}_{ps}$  and  $\bar{R}_{sp}$  have been determined in eqs. 9 and 10, respectively<sup>6</sup>.

The total energy consumed during direct two way communication is  $2TP_t$  and that during DSL based cooperation is  $2t_{ps}\hat{P}_t + P_s t_{sp}k$  where the first term accounts for the energy consumed in  $PR_{1,2}$  to  $S_{tx}$  communication during the first DSL phase for a time  $t_{ps}$  and the later for the energy consumption when  $k$  secondary transmitters cooperatively relay the data to  $PR_{1,2}$  for a duration equal to the leased time  $t_{sp}$ .

Now, we look at the trends of the energy efficiency established above for direct and DSL communication. We are interested in knowing whether the improved time rate products of DSL as seen

<sup>6</sup>Here we focus on the cost of attaining the higher throughput in DSL and do not consider the additional implementation costs of DSTC and network coding at the CR and primary transmitters respectively.

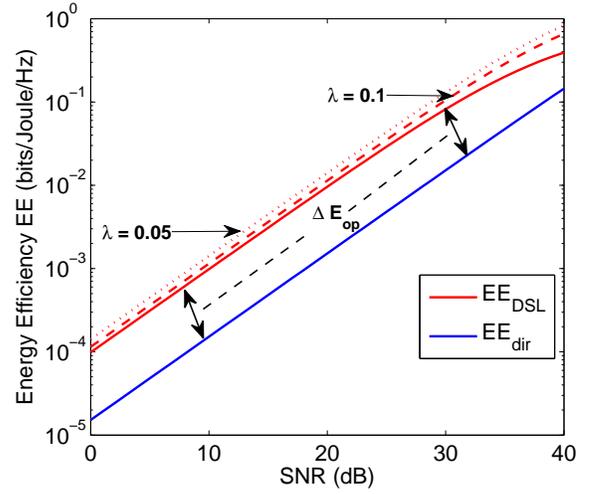


Figure 8. Energy Efficiency of Direct vs. DSL transmission,  $\lambda = 0.5$ .

in Fig. 7, come with high energy costs or DSL is also efficient on the energy front. ■

It is clearly evident that the energy efficiency of DSL is greater than that of direct communication for smaller values of  $\lambda$  (see Fig. 8). This is because the transmit power of the primary and secondary in DSL mode is low. The selection of relays which are geographically closer to both  $PR_1$  and  $PR_2$  help in achieving the same transmission rate in lesser time and hence lesser power. Also, the cooperative relaying based diversity benefits significantly increase the throughput at the primary receiver while maintaining a low transmit power. As  $\lambda$  increases, the EE of DSL decreases mainly due to two reasons;

- 1) The throughput of the cooperative DSL communication decreases as the average primary to secondary rate  $\bar{R}_{ps}$  decreases with increasing  $\lambda$  (see Fig. 3). The energy consumed in the first phase of DSL grows as the primary to secondary link operation time  $t_{ps}$  increases.
- 2) Also, in the second phase of DSL, aggregate transmit energy is higher due to increased number of relays.

It can be seen that the bargaining based leasing time division results in significantly more energy efficient communication via DSL as compared to direct communication when the secondary network is relatively sparse (i.e.,  $\lambda = 0.05$ ). Moreover, the difference  $\Delta E_{op}$  between the energy efficiency of direct and DSL based communication is also shown in the figure. The positive values of  $\Delta E_{op}$  indicate the available margin of miscellaneous circuitry and implementation energy costs. A future study that extends the DSL operation presented in this paper by discussing a specific hardware platform of the CRs and PUs may benefit from the indicated energy margins and compare their operational system energy efficiency against these results.

## VII. CONCLUSION

In this paper, we quantify and present a novel network coding based DSL approach as a spectral and energy efficient alternative to two way direct communication. We model DSL considering the random geometry of the CR nodes present in the network. We

propose a Nash Bargaining based division of leasing time between various phases of DSL so that all the entities of the network maximize their utilities. Geometry based selection of cooperating CR nodes and intelligent division of time enables up to 54% more bits to be successfully transmitted between the two primary sources at more than 10 times lower energy cost. These gains are attained only when the secondary network is relatively sparse. Hence network coding aided DSL under geometry and intelligent time division is a promising technique for future spectral and energy efficient two way wireless communication.

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