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# A generalised model of electrical energy demand from small household appliances

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## Abstract

Accurate forecasting of residential energy loads is highly influenced by the use of electrical appliances, which not only affect electrical energy use but also internal heat gains, which in turn affects thermal energy use. It is therefore important to accurately understand the characteristics of appliance use and to embed this understanding into predictive models to support load forecast and building design decisions. Bottom-up techniques that account for the variability in socio-demographic characteristics of the occupants and their behaviour patterns constitute a powerful tool to this end, and are potentially able to inform the design of Demand Side Management strategies in homes.

To this end, this paper presents a comparison of alternative strategies to stochastically model the temporal energy use of low-load appliances (meaning those whose annual energy share is individually small but significant when considered as a group). In particular, discrete-time Markov processes and survival analysis have been explored. Rigorous mathematical procedures, including cluster analysis, have been employed to identify a parsimonious strategy for the modelling of variations in energy demand over time of the four principle categories of small appliances: audio-visual, computing, kitchen and other small appliances. From this it is concluded that a model of the duration for which appliances survive in discrete states expressed as bins in fraction of maximum power demand performs best. This general solution may be integrated with relative ease with dynamic simulation programs, to complement existing models of relatively large load appliances for the comprehensive simulation of household appliance use.

*Keywords:* Electrical Appliances, Stochastic Modelling, Markov Chain, Occupant Behaviour,

### 3 **1. Introduction**

4 In the UK approximately 20% of energy use in households is due to electrical appliances  
5 [1], and this proportion is higher in better insulated homes. Residential electrical appliance  
6 use has direct implications for local Low Voltage (LV) networks, the loads on them and their  
7 integrity; and indirect implications for thermal energy demands, since electrical energy is  
8 ultimately dissipated as heat, most of which is emitted within the building envelope. It is  
9 therefore important to be able to reliably predict electrical appliance use, in particular the  
10 magnitude and temporal variation of the energy use and power demand profiles arising from  
11 the aggregation of individual appliances, to support design and regulation of LV networks  
12 serving communities of buildings and of building's thermal systems.

13 But this is a complicated task, for the ownership and use of different types of appliance  
14 significantly varies from house to house, and between users. Addressing this diversity requires  
15 that we have an appropriate basis for allocating appliances to households depending on their  
16 composition and socio-economic characteristics and for predicting their subsequent use. This  
17 in turn implies the use of stochastic simulation and bottom-up approaches that may also  
18 facilitate the future testing of Demand Side Management (DSM) strategies.

19 So far, bottom-up approaches have focused on the modelling of high-load appliances:  
20 those that are commonly owned and which contribute significantly to total annual electricity  
21 use. Examples include cold (fridge and freezer), wet (washing machine and dishwasher) and  
22 cooking appliances. For example, the model of Jaboob et al.[2] predicts when the appliances  
23 are switched on, the duration for which they will remain on and their fluctuating power  
24 demands whilst on. But in our everyday lives we also use myriad low-load appliances. Their  
25 individual share of energy use may be small, in some cases even negligible, but it is significant  
26 when considering them as a group (or groups).

27 The objective of this paper is to find a parsimonious strategy for modelling four cate-  
28 gories of low-load appliances: audio-visual, computing, kitchen and other appliances, which  
29 collectively account for those that are not represented by current device specific models. The

1 work presented here extends and further develops that introduced in [3].

2 The remainder of the paper is organised as follows: section 2 introduces former work in  
3 the field; in section 3 the mathematical methods employed in the modelling are described,  
4 with their application presented in section 4, together with simulation results and evaluations  
5 of performance; section 5 concludes the paper.

## 6 2. Background

7 *Bottom-up* approaches describe the dynamics of a system by explicitly modelling the be-  
8 haviour of the individual parts of that system. For the case of energy use, they consider the  
9 individual modelling of every end-use, or aggregates of them, in order to obtain aggregate  
10 profiles. These approaches are particularly promising given their potential for a) improving  
11 predictions of energy use of individual buildings or neighbourhoods when integrated with  
12 building energy simulation, b) sizing decentralised generation and storage devices, and c)  
13 testing Demand Side Management (DSM) strategies and rules for load management. More-  
14 over, bottom-up approaches have the potential to explicitly include the effects of household  
15 composition and individuals' behavioural diversity.

16 Regarding appliance modelling, bottom-up approaches can be configured at different ag-  
17 gregation levels: from a pure microsimulation where each device is explicitly modelled, to  
18 strategies that consider aggregations of device for typologies of them.

19 Detailed microsimulation approaches are considered in probabilistic empirical models (as  
20 defined in [4]), which tend to model appliances one-by-one. Collected data, information on  
21 dwelling and household (occupants) characteristics, technical properties of appliances and  
22 aggregated values of energy use are combined in such approaches, and probabilistic methods  
23 are applied to generate results with profile diversity. Stokes' model [5] generates profiles at  
24 three aggregation levels: 30-minute-resolution average household, 30-minute-resolution spe-  
25 cific household (with occupancy considerations) and 1-minute resolution specific household  
26 (including information relating to appliances' cycles). It considers 14 appliances plus mis-  
27 cellaneous, although only 9 different input monitored power cycles are taken into account,  
28 resulting in a limitation on the diversity of profiles generated, which in turn leads to poor

1 results estimating the energy demand in the validation of the models. Paatero and Lund [6]  
2 introduce a social random factor (supposed to capture the social variety of the demand) that  
3 improves the diversity of patterns obtained; however, only yearly consumption data for the  
4 16 end-uses is used for the generation of the models, together with other aggregate statistics,  
5 restricting the resolution to hourly time steps. In general, these approaches do not describe in  
6 terms of model parameters the dynamic behaviour of appliances, but they generate empirical  
7 profiles of power demand as a function of time.

8 Relatively more aggregated methods are models based on time-use-survey (TUS) datasets.  
9 In TUS datasets, the respondents fill in diaries of their activities during the day usually for  
10 one week periods, such as cooking, sleeping, travelling to work, etc. This data provides  
11 a powerful input to bottom-up models, since it encapsulates highly detailed information  
12 describing occupants' activities, that can be related to the use of appliances. To this end,  
13 Capasso [7] presents a first strategy linking occupants' activities with appliance use, using  
14 TUS data. The model produces 15-minute profiles of electricity use, considering aggregations  
15 of appliance that correspond to just four type of activities: cooking, housework, leisure and  
16 hygiene; each associated with a blend of large and small appliances, which are allocated by  
17 considering the average range of appliances present in the simulated household. The relation  
18 between performing an activity and using an appliance is described with a single coefficient  
19  $\alpha$  (defined as a human resources). Tanimoto [8] combines TUS with statistical data of  
20 ownership of appliances and its peak and stand-by powers. 31 activities are considered in  
21 this case, so that the level of aggregation is low, but this is contrasted by a small dataset size  
22 (58 households over 2 days).

23 In a similar vein, Widén [9] uses Swedish TUS data to model electricity use by assigning  
24 appliances to related activities (9 different categories in this case) and imposing five standard  
25 end-use profiles based on the type of their demand profile: demand disconnected from activity,  
26 power demand constant during activity, power demand constant after activity (with and  
27 without addition of temporal constraint) and fluctuating power demand (only applied to  
28 lighting). This approach is further developed in [10], where inhomogeneous Markov chains  
29 generate sequences of domestic activities that have an impact on power demand (5 minute and

1 1 hour granularity), including dependencies with the number of occupants performing these  
2 activities. A yet finer temporal resolution of 1 minute is achieved in the work developed by  
3 Richardson et al.[11]. Based on 7 different activities, a load curve for the appliances is created  
4 using the probability of switching on an appliance when an activity is being performed, and  
5 applying a fixed power conversion scheme. Using a calibration procedure based on the total  
6 time of use of an appliance, they obtain annual energy predictions. Although this tuning  
7 ensures a good overall match in annual energy demand, this does not imply the absence of  
8 compensating errors in the modelling of different appliance typologies, or that the dynamic  
9 characteristics of appliance use are well represented.

10 Although activity modelling is a promising method to obtain accurate energy demand  
11 profiles, this activity-appliance pairing approach does not facilitate the modelling of the range  
12 of appliances, because the activities that are recorded in time use surveys is insufficiently  
13 detailed, limiting the applicability of this approach to the modelling of either relatively high-  
14 load appliances or aggregations of small and large appliances for which there is weak empirical  
15 evidence. There has been no rigorous validation of those bottom-up modelling strategies to  
16 date, whether these are based on TUS data or not, to demonstrate their ability to faithfully  
17 capture energy use/power demand dynamics. These methods also have no rigorous basis  
18 for modelling the dependency of appliance ownership and related use characteristics as a  
19 function of household socio-demographic composition.

20 In partial response to these shortcomings, Jaboob [2] assigns (exclusively large) appli-  
21 ances to households as a function of their socio-demographic characteristics. The activities  
22 of the members of these households are then predicted, from which the conditional likelihood  
23 that related appliances will be switch on is modelled, as is the corresponding duration that  
24 they will remain on and their time-varying mean power demands whilst on. Thus, this mod-  
25 elling chain rigorously resolves for dynamic variations in mean power demand, in contrast to  
26 static power conversion schemes. Moreover, it presents the possibility of being used together  
27 with explicit models of low-load appliances, in order to obtain accurate values of the total  
28 electricity use of a house.

29 To this end and informed by these past endeavours, our task is develop a parsimonious

1 strategy for the use of relatively low-load appliances, in complement to Jaboob’s model of  
2 high-load appliances.

### 3 **3. Methods**

4 In this work we are interested in modelling the energy and power demands of low-load  
5 appliances to support building, systems and network design. In order to contribute to ac-  
6 curate predictions of residential energy use, we need to address the diversity in dwelling  
7 characteristics and human behaviours. Thus, we identify the following modelling tasks:

8 I Perform low-load appliance allocation, using cumulative distribution functions describ-  
9 ing the peak power demand of aggregates of appliances. Devices are categorised into  
10 four groups: audio-visual, computing, small kitchen and other (miscellaneous house-  
11 work, garden and personal care appliances).

12 II Model the characteristic use of these appliances in individual households. To this end,  
13 we utilise the fractional energy use  $f$ : the ratio of the actual to the maximum energy  
14  $E_{max}$  that an appliance can use, determined by its rated power. Modelling  $f$ , we can  
15 distinguish between:

- 16 • Switching on/off events.
- 17 • Fluctuating demands whilst the appliances are in use.

18 Two considerations need to be taken into account in carrying out these tasks. Firstly,  
19 stochastic methods are required, as we are interested in describing the underlying randomness  
20 in households’ appliance use and investment decisions. These methods rely on the definition  
21 of coefficients that represent the system as a probability distribution, which can be dependent  
22 on different variables such as time of the day, number of occupants, weather, etc. Secondly,  
23 using the normalized fractional energy  $f$  instead of absolute energy allows us to evaluate load  
24 profiles from different appliances of a similar type, but that do not necessarily have the same  
25 magnitude. In this way, appliances can be classified into groups and modelled as a category.

1 Candidate techniques that have been used to good effect in the modelling of occupants'  
2 behaviours include Bernoulli processes (activities [2]), discrete-time random or Markov pro-  
3 cesses (presence [12], blinds [13], windows [14]) and continuous-time random processes (blinds  
4 [13], windows [14]): the latter being a hybrid between discrete and continuous time random  
5 process models.

6 Furthermore, It has been previously shown [2, 9, 11] that stochastic methods are suc-  
7 cessful in describing energy demands and the information listed in Task II. In the work here  
8 presented, two of these statistical approaches have been exploited:

- 9 • *Discrete-time Markov processes* can model the probability of transitions occurring be-  
10 tween energy states  $s_i(t)$ , with or without time dependency. Energy states are the  
11 result of discretising the range of fractional energy values. This discretization process  
12 can be more efficiently achieved if complemented with clustering techniques.
- 13 • *Survival analysis* can model the switching-on/off of appliances, as well as the duration  
14 an appliance remains in different energy states.

15 In the methodology presented here we have tested a range of strategies in order to find the  
16 most parsimonious approach. In this, we have ensured that the number of subjective decisions  
17 needed for modelling have been minimised, so that the methodology can be appropriately  
18 applied independently of the data set employed to estimate the models' coefficients.

### 19 3.1. Modelling fractional energy

#### 20 3.1.1. Discrete-time Markov processes

21 A Markov process is a stochastic process that fulfils the Markov property, by which a  
22 future state depends on the most recent state, and not on any prior history [15]. A stochastic  
23 process  $X(t)$  is therefore a Markov process if for every  $n$  and  $t_1 < t_2 < \dots < t_n$ :

$$P[X(t_n) = x_n | X(t_{n-1}) = x_{n-1}, \dots, X(t_1) = x_1] = P[X(t_n) = x_n | X(t_{n-1}) = x_{n-1}]. \quad (1)$$

24 Markov chains describe the process of making transitions between a present state  $i$  to a future  
25 state  $j$ , according to a probability distribution, described by a state transition probability

1 matrix (or Markov matrix) as follows:

$$P_{ij} = \begin{pmatrix} p_{11}(t) & p_{12}(t) & \dots & p_{1m}(t) \\ p_{21}(t) & p_{22}(t) & \dots & p_{2m}(t) \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1}(t) & p_{m2}(t) & \dots & p_{mm}(t) \end{pmatrix}, \quad (2)$$

2 where

$$p_{ij}(t) = \frac{n_{ij}(t)}{n_i(t)} = \frac{n_{ij}(t)}{\sum_j n_{ij}(t)} \quad (3)$$

3 is the probability that a transition from  $i$  to  $j$  takes place, given by the ratio of transitions  
4 that occur to state  $j$  from  $i$  to the total number of transitions occurring from  $i$ .

5 The dimensions of a Markov matrix  $m \times m$  are given by the number of states  $m$  defined in  
6 the system. At the same time, the coefficients in the matrix may or may not depend on time.  
7 In the first instance, a time-homogeneous Markov process is considered, where the system  
8 can be described using a single matrix. We then consider a time-inhomogeneous Markov  
9 chain, in which the number of matrices  $r$  is given by the number of time slots considered to  
10 have different transition probabilities. For instance, if it is assumed that the probabilities are  
11 different for each hour of a day, then  $r = 24$  (considering a single-day). This means that the  
12 probability distribution is given by a matrix of dimension  $r \times m \times m$ .

13 Appropriate dimensioning of the Markov matrices is not a trivial task: if  $m$  and  $r$  are  
14 set too low or even equal to 1, the dynamics or temporal variation of the system may not  
15 be suitably described by the model. On the other hand, if  $m$  and  $r$  are set too high, there  
16 is a risk of performing redundant calculations, adding unnecessary computing complexity, as  
17 well as a risk of overfitting the model.

18 In our case, fractional energy  $f$  is a continuous variable with values between 0 and 1,  
19 that is discretized into  $m$  energy states  $s$ . The time variable  $t$  is discrete, and it takes  
20 values every 10 minutes, but it can also be divided into  $r$  temporal states. In this sense, the  
21 subdivision chosen of the two-dimensional space  $\{t, f\}$  generated by the time of the day and  
22 the fractional energy of a category of appliances will set the values of  $m$  and  $r$ , that determine  
23 the dimension of the transition matrix. Consequently, estimating an adequate and efficient  
24 subdivision of  $\{t, f\}$  is key in the formulation of a parsimonious model. In our search for an

1 objective methodology, clustering techniques were identified as good candidates to evaluate  
2 the partitioning of this space.

### 3 3.1.2. Matrix dimensioning: Density based clustering

4 Cluster analysis techniques provide a powerful and systematic mechanism for identifying  
5 groups or common features of a database  $D$  of  $n$  objects. There are a large number of  
6 clustering algorithms, two of the main being hierarchical and partitional [16] algorithms.  
7 The former decomposes  $D$  into a nested hierarchy of clusters, represented by a dendrogram,  
8 i.e. a tree diagram that splits the database into subsets of smaller size, until each object  
9 belongs to one subset. The process can be agglomerative or divisive, depending on whether  
10 the structure is made from the leaves towards the root or from the root to the leaves. The  
11 latter creates a single-level partition of  $D$  into  $k$  clusters based on similarity and distance  
12 measures. The parameter  $k$  is required as an input, even though it is not generally known *a*  
13 *priori*.

14 A third type of clustering method is density-based clustering algorithms, which apply  
15 local cluster criteria [17] in order to classify  $D$ . They identify regions of high density that are  
16 separated from other clusters by regions of a low density of points, which can be classified  
17 as noise. Each object of the database is evaluated in terms of density in the neighbour-  
18 hood, which has to exceed some threshold. Density-based clustering algorithms present some  
19 advantages over other types of clustering:

20 (i) They are suitable for large data sets.

21 (ii) Clusters may have irregular shapes.

22 (iii) Although distance metrics are employed, clusters are identified based on density esti-  
23 mations of areas of the data set. The advantage of this is the identification of points  
24 that do not belong to any cluster, allowing for the treatment of unstructured (noise)  
25 points.

26 DBSCAN is a typical density-based clustering algorithm that was developed in 1996 [18].  
27 The core idea behind DBSCAN is that for each object in a cluster, the neighbourhood of

1 radius  $\epsilon$  has to be populated with a minimum number of points  $MinPts$ .  $\epsilon$  and  $MinPts$  are  
 2 the two only parameters required.

3 However, the cluster structure of a real data set cannot usually be identified with a single  
 4 global density parameter, but rather by clusters of different density, as well as their intrinsic  
 5 structure. The OPTICS algorithm [17] is a generalization of DBSCAN. Instead of a clustering  
 6 division, OPTICS outputs an ordering of the database relative to its density-based clustering  
 7 structure, containing information for every density level up to a "generating distance"  $\epsilon_0$ , that  
 8 allows for analysis of the grouping structure (hierarchy). A graphical interpretation of the  
 9 ordering is available through a reachability plot [17], where clusters are identified as "dents"  
 10 in the plot. The authors of this algorithm provide a method for automatically determining  
 11 the cluster hierarchy using the information extracted from the reachability plot. However,  
 12 a simpler alternative method for automatic extraction of the clusters is described in [19], in  
 13 which the most significant clusters are simultaneously selected from different density levels.  
 14 Interestingly the authors also show that reachability plots are equivalent to the dendrograms  
 15 of single-link clustering methods.

### 16 3.1.3. Survival analysis

17 Survival analysis [20] models the waiting time until a given event occurs, also referred  
 18 to as *survival time*. Let  $T$  be a non-negative continuous random variable representing the  
 19 survival time until an on-appliance is switched off (or an off-appliance is switched on); with  
 20 probability density function (p.d.f.)  $f(t)$  given by a Weibull distribution:

$$f(t) = \begin{cases} \frac{k}{\lambda} \left(\frac{t-\gamma}{\lambda}\right)^{k-1} e^{-\left(\frac{t-\gamma}{\lambda}\right)^k} & t > \gamma \\ 0 & t < \gamma \end{cases} \quad (4)$$

21 where  $k > 0$ ,  $\lambda > 0$  and  $\gamma > 0$  are the shape, scale and location parameters of the Weibull  
 22 distribution [20]. Thus, the cumulative distribution function (c.d.f.)  $F(t) = P\{T < t\}$   
 23 gives the probability of the event to have occurred by duration  $t$ . The *survival function*  
 24  $S(t) = 1 - F(t) = P\{T \geq t\}$  is then defined as the complement of the c.d.f, and describes  
 25 the probability to remain in a given state before  $t$ :

$$S(t) = e^{-\left(\frac{t-\gamma}{\lambda}\right)^k}. \quad (5)$$

1 By inverting equation (5), it is possible to obtain directly the duration for which an appliance  
2 will continue (survive) in a specific energy state  $s$  as:

$$t_s = \gamma + \lambda [-\ln(w)]^{1/k}, \quad (6)$$

3 given a number  $w \in [0, 1)$  drawn randomly from a uniform distribution.

4 Two cases have been studied in this work, either defining:

5 • Two energy states,  $s_0$  for  $f = 0$  and  $s_1$  for  $f \in (0, 1]$ , corresponding to *on/off* states.

6 Thus, switching on/off events are explicitly modelled.

7 • Eleven energy states, following an arbitrary division of 10 equidistant fractional energy  
8 states, plus the *off* state:  $s_i$  for  $f = \{0; 0-0.1; 0.1-0.2; \dots; 0.9-1\}$ , respectively. Such  
9 a division of  $f$  allows us to test the added value of refined characterisation of energy  
10 states.

11 Therefore, occurrences of each event and durations are first extracted from the data, and  
12 used to fit Weibull distributions, obtaining scale, shape and location parameters  $\lambda$  and  $k$ .  
13 These distributions are then used to calculate survival times in a simulation using equation  
14 (6).

#### 15 3.1.4. Monte Carlo simulation

16 Monte Carlo methods may be defined as *the representation of a mathematical system by a*  
17 *sampling procedure which satisfies the same probability laws* [15]. They provide a method to  
18 artificially represent a stochastic process by a sampling procedure, which will be determined  
19 by the particular underlying probability distribution of the given process.

20 For the specific problem posed here, the probability structure is given by either the  
21 parameters of the Markov chain or the survival analysis. In the former, a sequence of energy  
22 states  $\mathbf{s} = \{s_0, s_1, \dots, s_n\}$  is simulated, employing an inverse function method. For time  $t$   
23 a random number is drawn from a continuous uniform distribution over the interval  $[0, 1)$   
24 and the corresponding interval in the c.d.f. is selected as the state for time step  $t + 1$ . This  
25 process is repeated for each time step in the simulation.

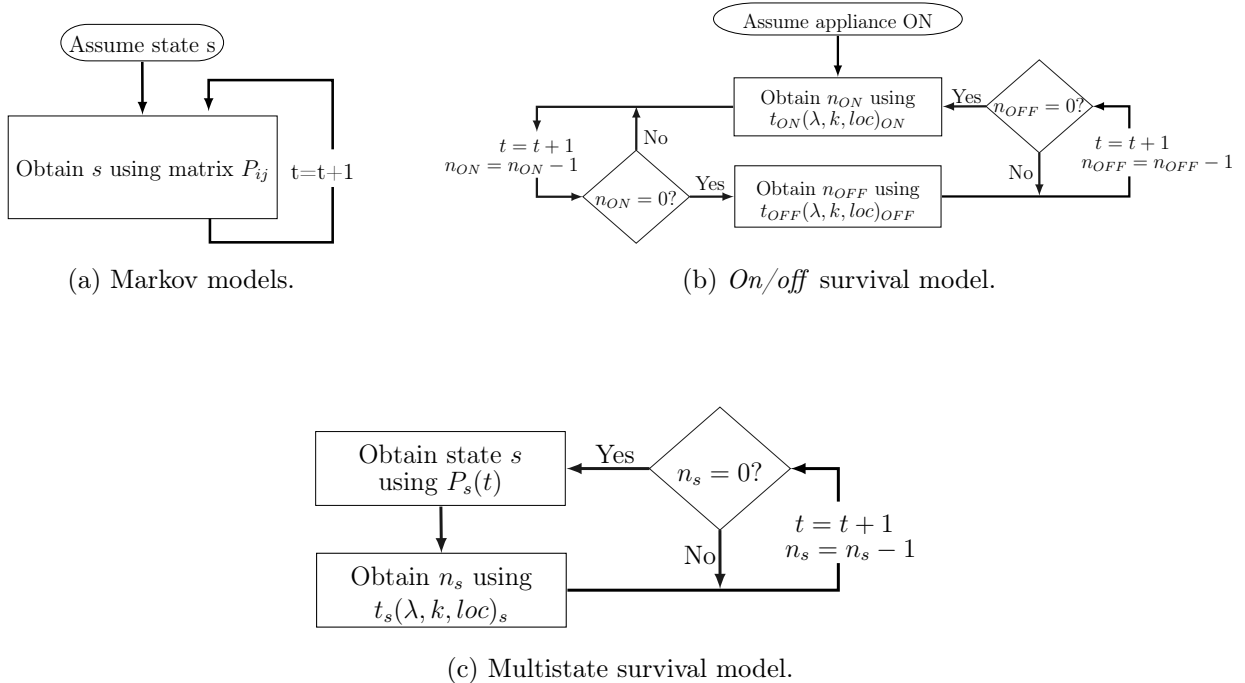


Figure 1: Simulation flowcharts.

1 For the latter case, equation (6) is employed -for an extracted random number  $w$ - to  
 2 obtain the survival time  $t_s$  until a change of state occurs, covering a number  $n_s$  time steps.  
 3 Temporal probabilities  $P_s(t)$ , extracted as the hourly likelihood of finding each of the states,  
 4 are used to simulate the following state (this step is unnecessary for the *on/off* model, as  
 5 there are only two states available:  $s_{ON}$  and  $s_{OFF}$ ). Thus, times and transitions between  
 6 states are successively calculated for the simulation period, as outlined in figure 1. The key  
 7 advantage of this approach is that it does not require calculations for the  $n_s$  time steps while  
 8 the devices remain in the same state.

9 *From energy states to an energy profile.* The sequence of energy states  $\mathbf{s}$  now needs to be  
 10 transformed back into a fractional energy profile  $\mathbf{f}_{sim}$ . Thus, each energy-temporal state  
 11 is multiplied by its corresponding mean or median fractional energy  $\tilde{F}$ , depending on the  
 12 strategy employed, leading to a simulated fractional energy profile

$$\mathbf{f}_{sim} = \mathbf{s} \cdot \tilde{F}(\mathbf{s}). \quad (7)$$

1 One final step transforms these profiles from fractional to actual energy values:

$$\mathbf{E}_{sim} = \mathbf{f}_{sim} \cdot \tilde{E}_{max}, \quad (8)$$

2 where  $\tilde{E}_{max}$  is a statistical measure of the maximum energy (or power  $\tilde{P}$  as required) for all  
3 instances in the category. Thus, the estimation of  $\tilde{E}_{max}$  values becomes critical to calculating  
4 accurate aggregate energy profiles. Assignment of the maximum energy can be performed  
5 using c.d.f.'s of the relevant appliance categories, or else using simple mean or median mea-  
6 sures.

7 Although assignment of  $\tilde{E}_{max}$  is important, it is also trivially complicated. In what follows  
8 then, we focus on testing the underlying hypothesis in our modelling strategies rather than  
9 in the fidelity of predictions of aggregate energy profiles that require a random assignment  
10 process.

### 11 *3.2. Household Electricity Survey data set*

12 The Household Electricity Survey [21] is an extensive monitoring survey of 250 house-  
13 holds in the UK, carried out during 2010 and 2011. Apart from detailed socio-demographic  
14 information, it contains data describing the appliances present in every monitored household  
15 and their temporal electrical energy use during 1 or 2 months, with records every 2 minutes.  
16 Of the 250 households, 26 of them were additionally monitored for a whole year, with 10 min-  
17 utes resolution. Since the one-month data was not measured during the same month for all  
18 households, only the data recorded for the 26 houses during a whole year was utilised in the  
19 analysis presented here, in order to avoid possible seasonal effects on the use of appliances.

20 The relevant low-load appliances found in the dataset were classified into four categories,  
21 following the type of activity that involves their use:

- 22 - audio-visual (excluding TVs, that are considered as high-load appliances given their  
23 extensive use),
- 24 - computing,
- 25 - small kitchen appliances (excluding cookers, microwaves and ovens),

1 - other small appliances.

2 Figure 2 shows the types of device available in the data set and their contribution to annual  
3 energy use, with categories depicted in different colours. The height of the bars represents  
4 the mean value of annual energy use of the corresponding type of appliance, whereas the  
5 width is proportional to the number of instances observed in the 26 households for the given  
6 device. Thus, the area of the bar indicates the total energy use of that appliance throughout  
7 the stock of houses surveyed.

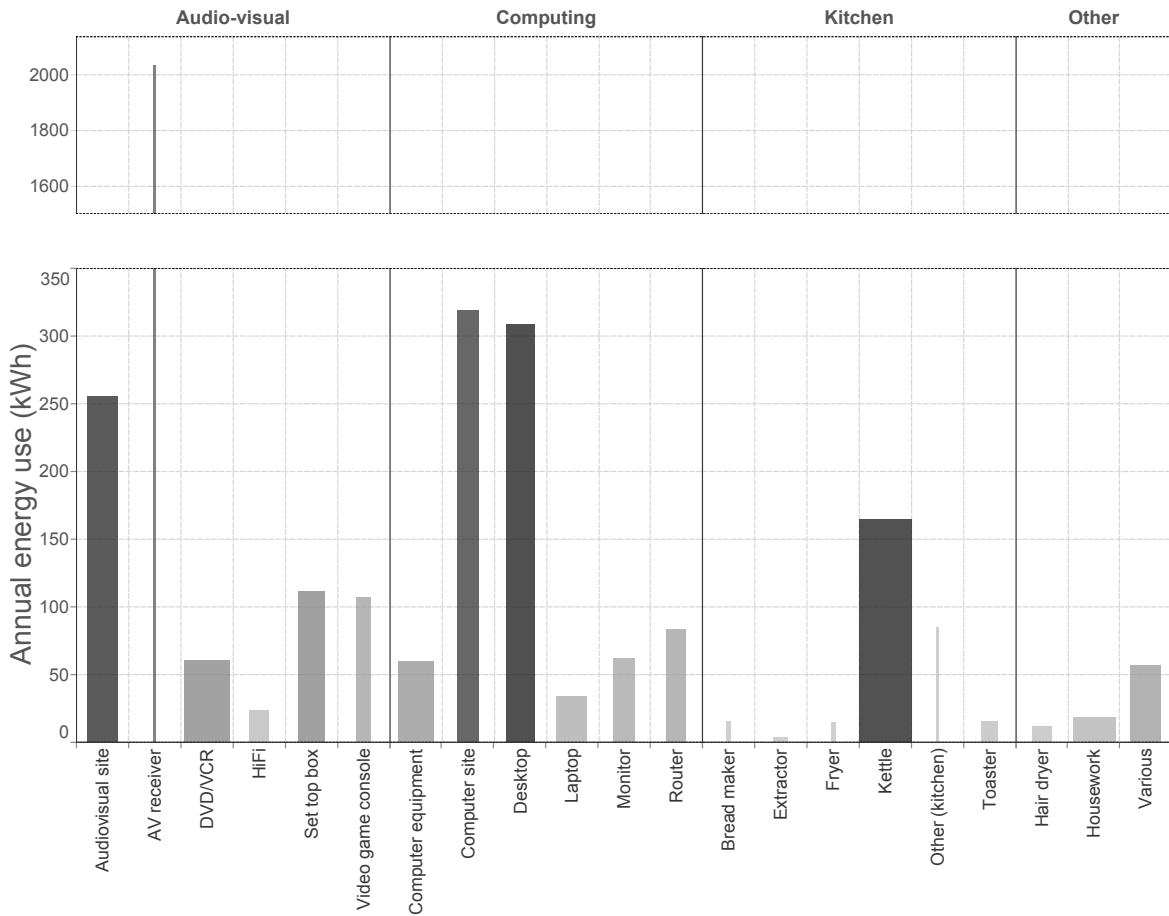


Figure 2: Annual energy use of the types of appliances considered in the modelling, divided in four categories: audio-visual, computing, kitchen and other. The height of the bars corresponds to the mean annual energy use, while the width is proportional to the number of instances recorded in the 26 houses. Combining this information, darker bars identify the dominant types of appliance for the category.

1 One shortfall encountered in the data set is that there is no information describing the  
 2 rated power of the appliances, posing a challenge to the accurate estimation of  $E_{max}$ . Con-  
 3 sequences derived from this and the solution proposed are discussed in section 3.2.2.

4 The procedure adopted in our work was to test a range of strategies to model one appliance  
 5 category, the audio-visual category, in order to identify the most parsimonious approach, and  
 6 then to deploy this to other categories of appliance.

### 7 3.2.1. Audio-visual category

8 In this section the nature of the data used to test the modelling techniques is presented.  
 9 Table 1 displays the total number of instances of each subcategory of appliance considered  
 10 in the audio-visual category, present in the 26 houses, leading to a total of 102 instances for  
 11 the category. Our first step was to extract fractional energy values from the electricity use  
 12 records, as

$$f = \frac{E}{E_{max}}. \quad (9)$$

Type of appliance	Number instances	$\tilde{E}_{raw}$ (Wh)	$\tilde{E}_{SH}$ (Wh)
AV receiver	1	86.4	86.4
Audio-visual site	33	42.4	34.9
DVD/VCR	28	8.2	5.41
HiFi	14	33.1	29.4
Set top box	17	5.2	5.2
Video-game console	9	19.9	16.7

Table 1: Types of appliances available in the audio-visual category: number of instances (each device in each house) and mean maximum energy for the subcategory, before ( $\tilde{E}_{raw}$ ) and after ( $\tilde{E}_{SH}$ ) applying outlier filters.

13 Given an estimate of  $\tilde{E}_{max}$  this transformation outputs a normalised profile for each  
 14 appliance in each house with values in the interval  $f \in [0, 1]$ , that can be combined now  
 15 with other instances or other types of device, allowing the category to be modelled. It is also  
 16 possible to explore how these profiles vary during the period of a day, in order to identify

1 patterns or dominant behaviours. Interesting characteristics of the data set include that:

2 i. The data set contains over 4.2 million data points.

3 ii. 33.9% of the data are zero values ( $f = 0$ ), suggesting that appliances are off for around  
4 a third of the time.

5 iii. The off-state exhibits temporal dependency, reaching maximum values in the early  
6 hours of the morning (39%) when most people are sleeping, and a minimum (29%)  
7 between 20h and 22h, when most people are present and awake.

8 iv. 15% of the entries have fractional energy lower than 0.1, which likely corresponds to a  
9 stand-by state, a common feature of audio-visual devices.

10 v. As with [iii.](#), a concentration of stand-by states is found during the early hours of the  
11 day, whereas appliances are most often used at maximum power during the late hours  
12 of the evening.

13 A preliminary visualization of the two-dimensional space created by the time period of a day  
14 and the fractional energy values  $\{t, f\}$ , is depicted in [figure 3](#). Dark areas represent denser  
15 regions of the data set, showing common values recorded during certain times of the day.  
16 Values of  $f = 0$  were excluded to help with the interpretation.

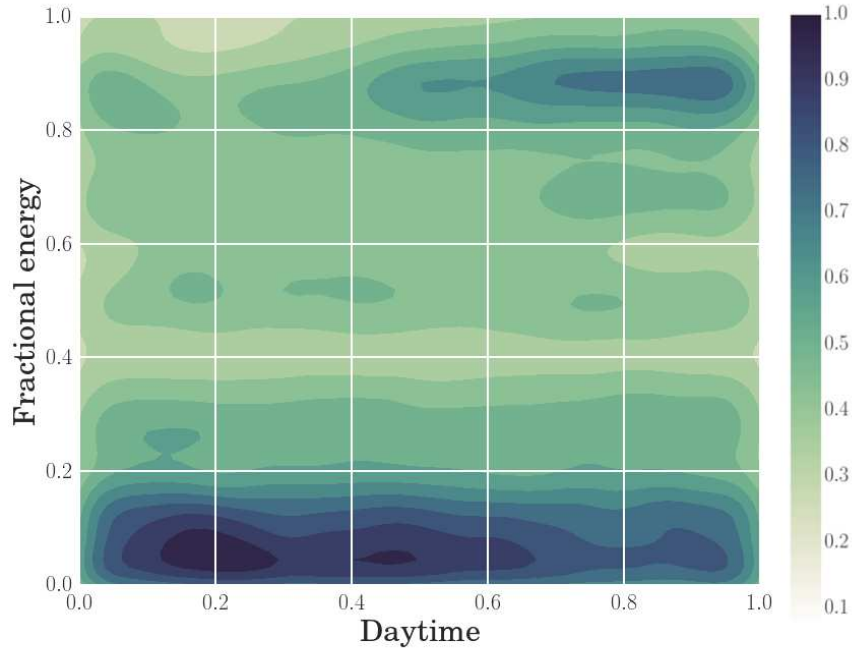


Figure 3: Distribution of audio-visual category data, for the values of fractional energy over a day (normalised values). A sample of 50000 entries is plotted. Since the data set contains a large amount of values with  $f = 0$ , these data were excluded to facilitate comparison between other values.

### 1 3.2.2. Data preprocessing: outliers and maximum energy estimation

2 As previously mentioned, our data set does not include appliance name plate (power)  
 3 ratings. The fractional energy modelling approach, however, is dependent on the values  
 4 of  $E_{max}$  and requires this input at two specific stages. Firstly in using equation (9) to  
 5 extract fractional energy profiles for each instance. Secondly after the simulations have been  
 6 performed, to compute an energy profile from a simulated fractional energy time-series for  
 7 the category.

8 In order to estimate  $E_{max}$  from the data, the maximum energy record for each profile  
 9 was used. The existence of discrepant entries for the same type of appliance suggested that  
 10 a data cleaning process was necessary. This could be due to the fact that each data point  
 11 represents the energy corresponding to the mean power drawn by an appliance over a period

1 of ten minutes. Since this may fluctuate between 0 and the nameplate rating it could be  
2 that the calculated value of  $E_{max}$  results from an appliance that has been working at higher  
3 power during a shorter period of time (*e.g.* a kettle that never takes 10 minutes to boil).  
4 This problem was overcome by obtaining maximum energy values from the 2-minute data  
5 (also subjected to a cleaning pre-process), with the purpose of selecting consistent entries.

6 A Seasonal Hybrid Extreme Studentized Deviate test (S-H-ESD) [22] was employed to  
7 detect anomalies. S-H-ESD is based on the generalized ESD algorithm to detect one or  
8 more outliers in a univariate data set that follows an approximately normal distribution,  
9 and is applicable to time-series data. Its main feature is that it is able to predict both  
10 local and global anomalies, taking into account long-term trends on the temporal profile to  
11 minimize the number of false positives. In other words, the conditions to detect an outlier  
12 vary depending on local temporal windows. When no trend is identified, the algorithm works  
13 as an ordinary outlier filter. The algorithm is part of the *AnomalyDetection* package in R  
14 [23].

15 The result of applying the outlier filter is presented in table 1 as the mean maximum  
16 energy per subcategory of appliances in the audio-visual data set before and after the outlier  
17 test. For some categories the exclusion of outliers leads to a discrepancy on the estimation  
18 of  $E_{max}$  as large as 300%.

### 19 3.3. Validation methods

#### 20 3.3.1. Cross validation

21 In statistical modelling, cross-validation processes are used to assess how effectively the  
22 results will generalize to a different data set [24]. Cross-validation computes the average error  
23 obtained from evaluation measures of different partitions of the data set. There are several  
24 methods for cross-validation, such as random sub-sampling, leave-one-out cross validation  
25 and  $K$ -fold cross validation. In our work we favour  $K$ -fold cross validation in which the data  
26 set is partitioned in to  $K$  subsamples. A single subsample is used as the validation set and  
27 the remaining ( $K-1$ ) subsamples are used as the training set. This process is then iteratively  
28 repeated  $K$  times (*folds*), until each partition has been used once as a validation set. A mean

1 performance error can then be computed as the average error:

$$e = \frac{1}{K} \sum_{i=1}^K e_i, \quad (10)$$

2 where  $e_i$  represents some error between prediction  $\hat{y}_i$  and observation  $y_i$ .  $K$ -fold cross vali-  
3 dation is a computationally expensive method, but produces an accurate estimation of the  
4 goodness of fit.

### 5 3.3.2. Time series analysis

6 Selecting an adequate strategy for the modelling of fractional energy requires a comparison  
7 of performance between simulation and observation data sets during the validation period.  
8 Time series analysis provides a powerful method to compare and understand internal struc-  
9 ture on both temporal profiles, extracting meaningful statistical information. The objective  
10 is to describe the validation time series with a set of parameters that should be replicated  
11 by the simulation time series. In particular, it is possible to decompose the fractional energy  
12 profile into trend, seasonal and irregular (or remainder) component, allowing for evaluations  
13 of each of the components at a different level. Figure 4 shows an example of a decomposed  
14 time series.

15 The following information is used from the decomposition exercise:

- 16 • *Trend component.* The observed data presents a flat trend curve. Therefore, the mean  
17 value of the trend component of both simulation and observation can be used for  
18 comparison, giving an idea of the average fractional energy expected.
- 19 • *Seasonal component.* A daily period (or seasonal component) is expected in the use of  
20 appliances. The models are expected to reproduce this periodicity correctly, and this  
21 can be studied using the cross-correlation function [25] between two signals  $(X_t, Y_t)$ ,  
22 which is defined as

$$\rho_{XY}(\tau) = \frac{1}{N-1} \frac{\sum_{t=1}^N (X_t - \mu_X)(Y_{t+\tau} - \mu_Y)}{\sigma_X \sigma_Y}, \quad (11)$$

23 where  $\mu_k, \sigma_k$  are the mean and standard deviation of process  $k = X, Y$ , respectively,  
24 and  $\tau$  is the lag or time delay between both. Equation 11 provides an insight into the

1 relationship and dependence between observed and simulated periodic components.  
2 Based on that, we examine:

3 – Pearson’s coefficient, as an index of the linear correlation at  $\tau = 0$  (considering  
4 both signals to be synchronised); ideally this should be equal to 1.

5 – Time delay of maximum correlation, in order to determine whether the signals are  
6 in phase with each other.

7 • *Irregular component.* After extracting the trend and seasonal components, a residual  
8 fluctuating variation remains.

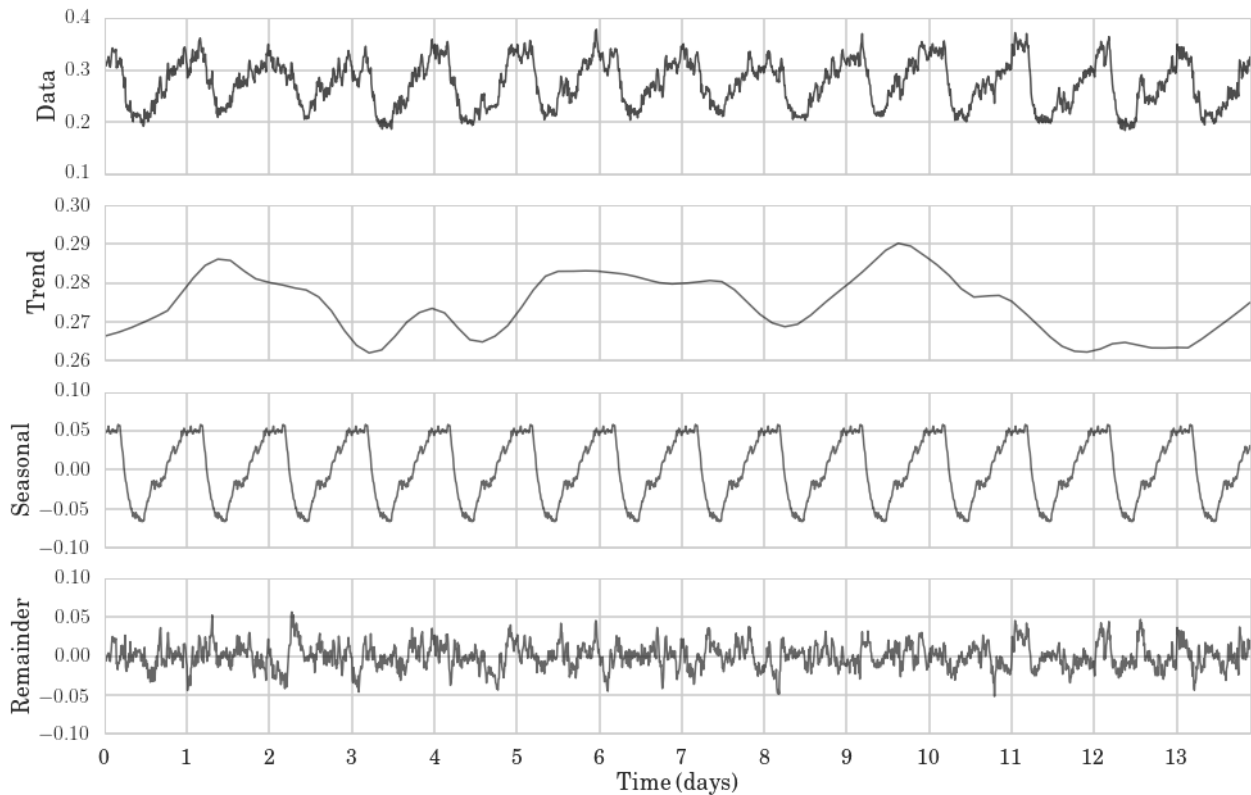


Figure 4: Example of time series decomposition into trend, seasonal and remainder component.

### 9 3.3.3. Sensitivity and specificity analysis

10 Sensitivity and specificity analysis represents a strong indicator of the model’s absolute  
11 aggregate performance: its ability to correctly reproduce the time dependent properties of the

1 process being simulated. Sensitivity or true positive rate (TPR) is defined as the proportion  
2 of matching cases between simulated and observed values, i.e.:

$$TPR = \frac{TP}{TP + FN}, \quad (12)$$

3 where  $T, F, P, N$  represent True, False, Positive and Negative and TP is the total number of  
4 truly predicted positive outcomes (true positives). Specificity or true negative rate (TNR) is  
5 defined as [26]:

$$TNR = \frac{TN}{TN + FP}. \quad (13)$$

6 In an ideal case, one would have  $TPR = 1$  and  $TNR = 1$  (or  $FPR = 1 - TNR = 0$ ).  
7 Comparison of these indicators can be plotted in receiver-operating characteristic (ROC)  
8 space. This analysis can be complemented with the model accuracy

$$ACC = \frac{TP + TN}{P + N}, \quad (14)$$

9 giving an indication on the overall performance of the model. For multi-state systems (as in  
10 our case, with multiple energy states) this is a particularly exigent evaluation technique.

#### 11 3.3.4. Application of validation methods

12 In this work, 10-fold cross validation is performed for every approach suggested. For each  
13 iteration, several error measures at three different levels have been taken into account:

14 1. At the first level, we are interested in evaluating the quality of the fractional energy  
15 signal produced by our simulations. *Time series decomposition* has been performed  
16 (section 3.3.2) to extract the following comparative measures:

- 17 • Relative error of the expected value of the trend component.
- 18 • Pearson's coefficient and time delay of maximum correlation of the seasonal com-  
19 ponent.

20 2. We are also interested in evaluating the accuracy of the averaged daily profile of frac-  
21 tional energy, as well as the models' effectiveness in predicting energy states. For this  
22 we use:

- 1       • *Simulated energy states.* Sensitivity and specificity analysis is directly applied to  
2       the simulated energy states, producing ACC values and an ROC plot.
  - 3       • *Absolute state prediction.* The probability of predicting each of the states during  
4       the validation period is calculated and compared for observation and simulation.  
5       Discrepancies between both magnitudes are represented with RMSE.
  - 6       • *Temporal probability of state prediction.* The probability distribution for each state  
7       over time provides insight into the temporal variation of each state, allowing us  
8       to identify situations when some states are over or under-predicted, and even at  
9       which periods during the day. Discrepancies again are calculated with RMSE.
  - 10      • *Fractional energy daytime profile.* Once the energy states have been converted  
11      into fractional energy values, it is possible to evaluate the results for a typical day  
12      over the validation period (averaged over all the days for which fractional energy  
13      values are available for each time step). Residuals and RMSE are calculated to  
14      describe performance.
- 15    3. Finally, the selected methodology should perform well in calculating *total energy use*.  
16    Each simulated instance is converted to an energy profile using maximum energy values  
17    of the appliances present. The total energy use over the validation period is then  
18    obtained and compared for the relevant category of appliance.

19    The validation data set is a subset of the data that corresponds to 10% of the available  
20    total time range. This subset does not contain a unique time series of values, but a number  
21     $q$  equal to the number of instances in the category. The simulation of energy states was  
22    performed  $q$  times over the validation period, in order to perform the sensitivity and specificity  
23    analysis for the energy states. For the other evaluation measures, averaged values for all  
24    instances were considered for both observation and simulation.

## 25    4. Results and discussion

26    In this section we first explain the application of the techniques presented in [3.1.1](#) and  
27    [3.1.3](#), respectively, to the data set introduced in section [3.2](#). Then, simulation results are

1 described and evaluated for each of the strategies tested to model fractional energy use of  
2 audio-visual appliances, justifying the selection of one of them. Finally, the selected strategy  
3 is applied to the other appliance categories, and a final evaluation of the model is given.

#### 4 *4.1. Application of Markov model*

5 As a first approach, the  $\{t, f\}$  space was arbitrarily divided with  $m = 11$  (11 energy  
6 states: one for the off-state plus ten of 0.1 fractional energy width) and  $r = 24$  (one temporal  
7 state per hour), leading to 264 subdivisions.

8 Clustering techniques were then applied to the audio-visual appliances data set. Excluding  
9 entries when the appliances are switched off (i.e.  $f = 0.0$ ), there are over  $2.8 \cdot 10^6$  data points  
10 (from a total of over  $4 \cdot 10^6$ ), which is still large given the computational expense of the  
11 clustering algorithms used. In order to overcome this problem, a random sampling process  
12 [27] was carried out, selecting 50,000 points that roughly represent 2% of the total size of the  
13 data set.

14 Implementations of the DBSCAN and OPTICS algorithms were tested, corroborating that  
15 the unique global density parameter of DBSCAN was not effective at finding a satisfactory  
16 partition of the data set into clusters; since we are interested in finding clusters of different  
17 density.

18 *Subdivision of  $\{t, f\}$  space.* The objective of applying a density-based clustering algorithm  
19 (OPTICS) is to produce an efficient subdivision of the two-dimensional space  $\{t, f\}$ , as  
20 described in section 3.1.1. As summarised graphically in 5, the process works as follows:

- 21 1. Find parameters that produce a good clustering structure.
- 22 2. Adjust the clusters found to fit a cell of rectangular shape. In order to avoid overlapping  
23 of cells, data points between the 1st and 99th percentile are selected for each cell. Points  
24 identified by OPTICS as noise are grouped into noise cells that will fill the empty space  
25 not covered by the clusters.
- 26 3. Define the grid established by the edges of the rectangles.

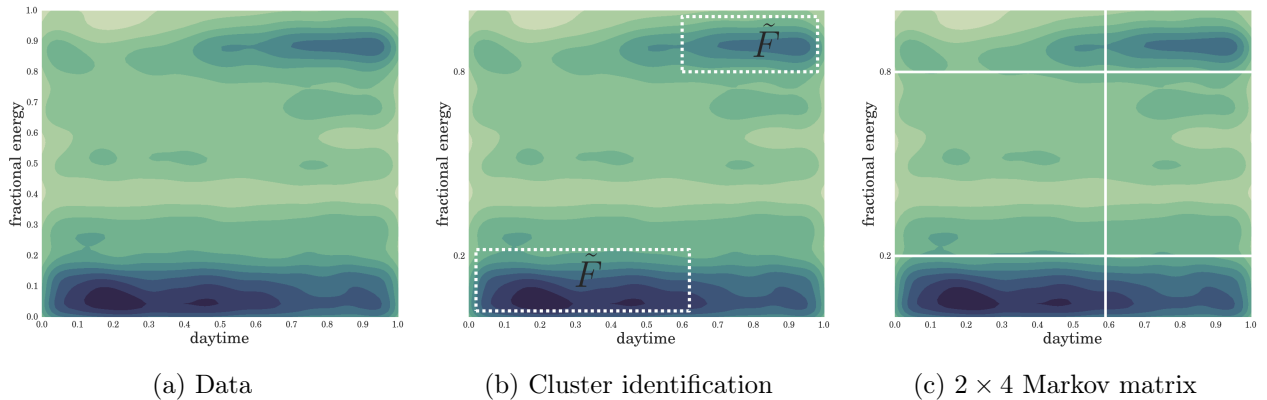


Figure 5: From the data: (a) clusters are identified by the algorithm allowing rectangles ranging from the 1st to 99th percentiles to be extracted (b); the rest of the space will be divided into noise cells. From that, a grid is defined, (c) whose partitions will be the dimension of the matrix (taking into account the off state  $f = 0$ ), associating with each cell the median value of fractional energy  $\tilde{F}$  of the data points it contains.

1 4. Associate with each cell a fractional energy value  $\tilde{F}$  corresponding to the median value  
 2 of the points it contains.

3 OPTICS requires two parameters to produce the ordering of the points: first, the gener-  
 4 ating distance  $\epsilon_0$ , referring to the largest distance considered for clustering (clusters will be  
 5 able to be extracted for all  $\epsilon_i$  such that  $0 < \epsilon_i < \epsilon_0$ ); second, the minimum number of points  
 6 that will define a cluster *MinPts*. However, the algorithm used to automatically extract the  
 7 clusters from the ordering of the points and their reachability distance makes use of a further  
 8 7 parameters [19], upon which the clustering structure obtained will vary. For this work, the  
 9 OPTICS algorithm was implemented in Python<sup>1</sup>.

10 After a systematic search for a well performing solution, a set of successful parameters  
 11 was identified<sup>2</sup>. These values lead to a hierarchical solution, with four incremental nested  
 12 partitions, from two clusters at the top level of the hierarchy, to eleven at the leaves. The four

<sup>1</sup>Aided by script provided in <https://github.com/amyxzhang/OPTICS-Automatic-Clustering.git>

<sup>2</sup>Parameters found following description in [19] are:  $\epsilon = 0.08$ ;  $MinPts = 50$ ;  $minClustSizeRatio = 0.03$ ;  $minMaximaRatio = 0.001$ ;  $significantMin = 0.003$ ;  $checkRatio = 0.8$ ;  $maximaRatio = 0.87$ ;  $rejectionRatio = 0.8$  and  $similarityThreshold = 0.6$ .

1 different levels (summarized in Table 2) lead to four different divisions of the  $\{t, f\}$  space and  
 2 four Markov matrices with different dimensions. The relative performance of these different  
 3 structures is evaluated in the following sections.

Name	Hierarchy level	Clusters	Noise cells	$\{t, f\}$ dimension
OPTICS - 5x15	IV	11	9	5x12
OPTICS - 4x14	III	10	5	4x12
OPTICS - 3x11	II	7	4	3x11
OPTICS - 1x3	I	2	None	1x3

Table 2: Hierarchical levels of clustering considered for  $\{t, f\}$  space partition, with number of clusters and number of noise cells identified.

#### 4 4.2. Application of Survival analysis

5 The two alternatives considered for the survival models are a simple two-state (on-off)  
 6 model and a multistate model with 0.1 divisions in  $f$ , so that there are eleven states in total  
 7 (*cf.* 3.1.3). This multistate model encapsulates temporal variations since the transitions  
 8 to the following state are computed based on the temporal probability of finding each of  
 9 the states. Weibull parameters (shape, scale and location) introduced in equation (4) are  
 10 estimated from the data points for each of the energy states. Once obtained, the simulation  
 11 runs as depicted in figure 1b.

#### 12 4.3. Approach selection

13 The fractional energy use of the audio-visual category of appliances was modelled using  
 14 a range of strategies. The goodness of fit of the models is evaluated from different points of  
 15 view, following the description in section 3.3.4.

##### 16 4.3.1. Fractional energy time series

17 Decomposition of the time series over the validation period allows for the extraction of  
 18 statistical information from the structure of the observed and simulated data and to compare  
 19 their different components: trend, seasonal and remainder (see section 3.3.2).

1 *Trend.* The trend component presents a constant value over the whole year of observed data,  
 2 so there is no need to fit a function. It gives an estimation of the mean value of the fractional  
 3 energy, given that the daily seasonality has been removed. The trend can be used to indicate  
 4 a relative error from the simulations, as shown in table 3.

5 *Seasonal component.* The seasonal components are shown in figure 6, plotted for several  
 6 days. There are two models, OPTICS-1x3 and Survival, for which an inadequate handling  
 7 of dynamics is clearly apparent. For the other cases, those with larger numbers of temporal  
 8 states produce an understandably more accurate seasonal profile (11x24-SHESD, OPTICS-  
 9 5x14, with 24 and 5 temporal states, respectively). Also, the Survival Multistate model  
 10 represents surprisingly well the daily seasonality, considering that the temporal dependency  
 11 is included only in the transitions between states, but not in their duration.

12 Table 3 complements those results with numerical values for Pearson’s coefficient and  
 13 temporal lag at maximum correlation. Again, the best value for Pearson’s coefficient and  
 14 time lag is achieved using the models with a larger number of temporal states 11x24-SHESD  
 15 and Survival Multistate, followed by OPTICS-4x12 and OPTICS-5x14.

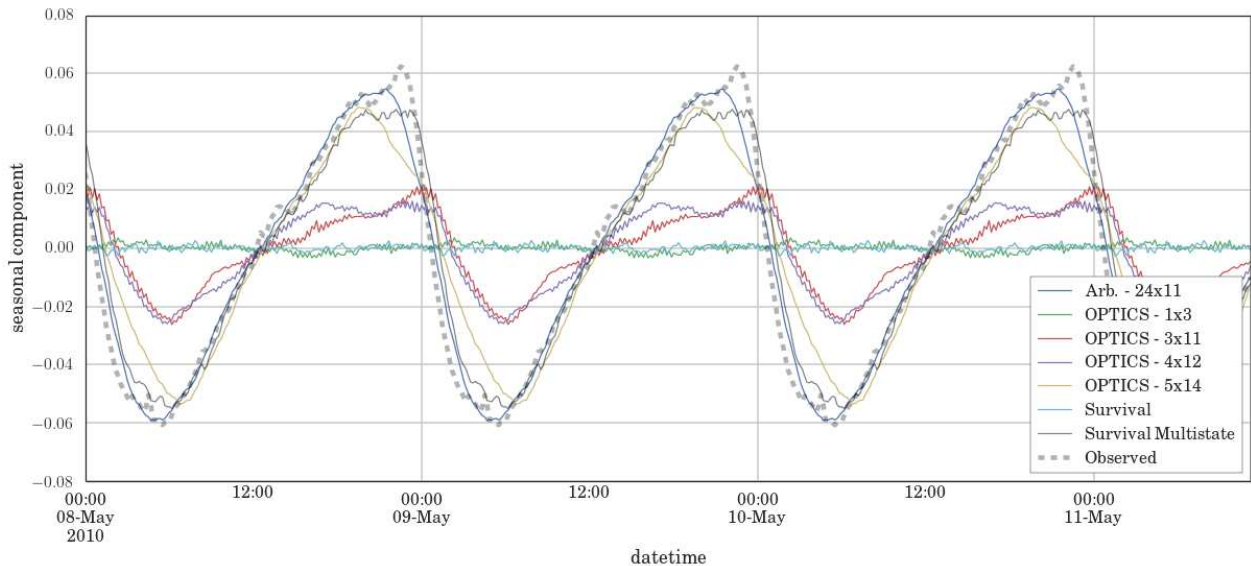


Figure 6: Comparison of seasonal components.

	<b>Rel. error trend</b>	<b>Pearson's coeff.</b>	<b>lag</b>
Arb - 24x11	2.35%	0.989	0.0
OPTICS - 5x14	5.08%	0.9501	-0.0278
OPTICS - 4x12	4.91%	0.960	-0.0486
OPTICS - 3x11	5.40%	0.854	-0.0625
OPTICS - 1x3	3.83%	-0.446	-0.424
Survival	0.831%	-0.0896	0.236
Survival multistate	5.61%	0.983	-0.0208

Table 3: Summary of validation time series decomposition. From left to right: error on the average value of the trend; Pearson's correlation coefficient and time delay (lag) of seasonal component. "Arb" refers to the arbitrary subdivision of the data into 24 time states and 11 fractional states.

#### 1 4.3.2. Evaluation of average daily profile

2 In the previous section the signal simulated over the whole period was compared; but we  
3 are also concerned with how well the averaged daily profile is represented, in terms of the  
4 predictive power of simulated energy states and the consequent fractional energy profile.

5 *Fractional energy states prediction.* Figure 7 shows the dependency of the RMSE (calculated  
6 for every 10-minute timeslot) with time for the probability of finding the system in each of  
7 the defined energy states. The *on/off* Survival approach gives RMSE values an order of  
8 magnitude larger than for the Markov models, indicating a poor overall estimation of the  
9 two states considered. Since there are only two states defined, their probabilities of being  
10 simulated are complementary,  $P_{s=0}(t) = 1 - P_{s=1}(t)$ ; therefore, a poor estimation of  $P_{s=0}(t)$   
11 implies a poor estimation of  $P_{s=1}(t)$ .

12 Furthermore, the shape of the curves for Survival and OPTICS-1x3 models implies that  
13 the temporal dependency of the system is not well encapsulated. The former exhibits an  
14 increase in error during the late hours, suggesting a worse prediction of the on-state; while  
15 the RMSE in the latter increases both in the evening and during the night, revealing an  
16 under performance for both the off state and the maximum energy state.

1 Temporal dependency is well represented in the Survival Multistate model, although the  
 2 overall error in energy state prediction is slightly higher than with the Markov approaches.  
 3 This could suggest that the specific energy states are better represented when clustered energy  
 4 values have been considered.

5 The total RMSE for temporal and absolute (not temporal) daily state predictions are  
 6 presented in table 5. In both cases, OPTICS-5x14 outperforms the other strategies, sug-  
 7 gesting that the larger the number of energy states (14 in this case), the more accurate the  
 8 probability prediction.

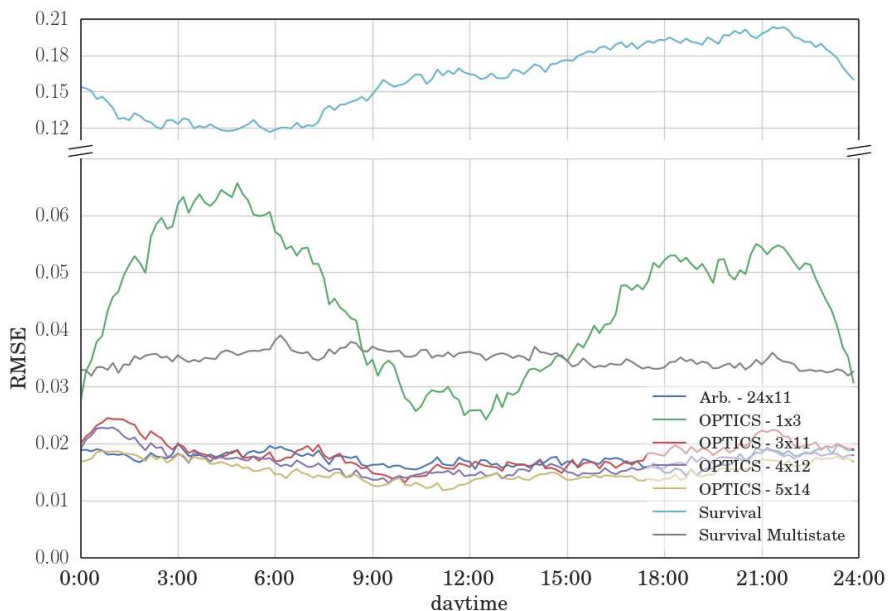


Figure 7: Temporal dependency of RMSE, calculated for each 10-minute time slot, for all states of each approach.

9 As noted in section 3.3 the accuracy of the modelling of states can also be evaluated  
 10 using ROC parameters, as shown in table 4; although this is a particularly onerous test when  
 11 applied to multi-state systems, so that TPR is not expected to be high. Once again the  
 12 OPTICS 5x14 and Survival Multistate models outperform their counterparts.

13 *Fractional energy averaged daily profile.* The residuals in fractional energy for an average day  
 14 tend to increase towards the boundaries of the day (Figure 8), where users are more active

	TPR	TNR	ACC
Arb - 24x11	0.173	0.917	0.483
OPTICS - 5x14	0.160	0.935	0.487
OPTICS - 4x12	0.172	0.925	0.485
OPTICS - 3x11	0.178	0.918	0.483
OPTICS - 1x3	0.339	0.669	0.456
Survival	0.499	0.499	0.450
Survival multistate	0.189	0.919	0.484

Table 4: Accuracy of model.

1 in switching devices and regulating them. Nevertheless, the results suggest that even with  
2 temporally crude models, dynamics are well encapsulated (with the exception of OPTICS-  
3 1x3 and Survival); particularly in the case of the model with the largest number of temporal  
4 states, Arb.-24x11, as reflected in table 5.

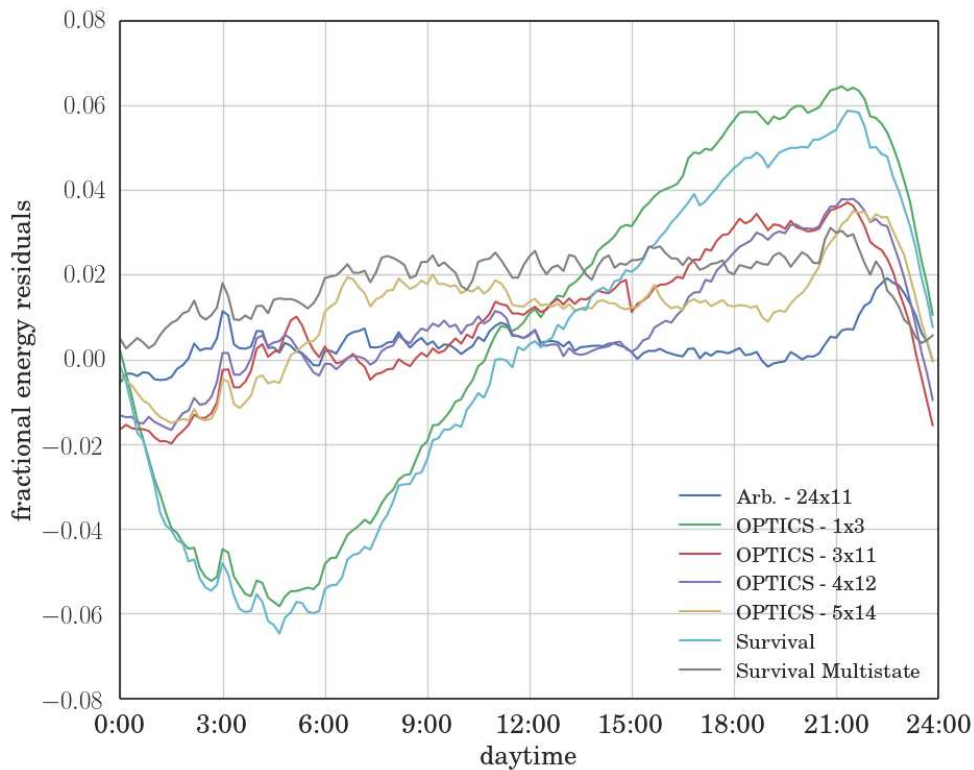


Figure 8: Daily profile residuals of fractional energy.

	<b>Fractional energy (<math>f</math>)</b>	<b>Absolute state prediction</b>	<b>Temporal state prediction</b>
Arb - 24x11	$2.056 \cdot 10^{-2}$	$2.00 \cdot 10^{-2}$	$1.76 \cdot 10^{-2}$
OPTICS - 5x14	$2.32 \cdot 10^{-2}$	$1.86 \cdot 10^{-2}$	$1.56 \cdot 10^{-2}$
OPTICS - 4x12	$2.54 \cdot 10^{-2}$	$2.06 \cdot 10^{-2}$	$1.73 \cdot 10^{-2}$
OPTICS - 3x11	$2.70 \cdot 10^{-2}$	$2.23 \cdot 10^{-2}$	$1.89 \cdot 10^{-2}$
OPTICS - 1x3	$4.66 \cdot 10^{-2}$	$1.84 \cdot 10^{-2}$	$4.85 \cdot 10^{-2}$
Survival	$4.22 \cdot 10^{-2}$	$1.13 \cdot 10^{-1}$	$1.63 \cdot 10^{-1}$
Survival multistate	$2.53 \cdot 10^{-2}$	$3.55 \cdot 10^{-2}$	$3.50 \cdot 10^{-2}$

Table 5: RMSE values of the daily profile results, in terms of the fractional energy profile, absolute state prediction (without temporal dependency), and temporal state prediction.

### 1 4.3.3. Total energy prediction

2 For the total energy prediction over the validation period, a random assignment process of  
3 maximum energy values is performed for each of the instances' fractional energy simulations,  
4 as explained in section 3.1.4.

5 In order to compare the results, a box plot is presented in figure 9, and the residual error  
6 in energy use prediction is presented in table 6. Whilst the median residual error is in all  
7 cases relatively low, the simulated values are consistently positively skewed, overestimating  
8 the upper quartile in total energy use. This is caused by a loss of information during the  
9 modelling process. Errors compound from the modelling of fractional states, through the  
10 assignment of maximum energy values to the subsequent prediction of energy use for the  
11 relevant appliance category. Thus, even though each task in our modelling process faithfully  
12 reproduces reality, errors inevitably arise when using models estimated from aggregate data  
13 of the four typologies of appliance to the prediction of specific device behaviours; errors  
14 that will reduce in magnitude as the size of the stock of appliances simulated increases.  
15 This is reasonable considering that our goal is to estimate communities of buildings and the

1 appliances contained within them.

	Mean Residual (kWh)	Median Residual (kWh)
Arb - 24x11	-13.5	-1.83
OPTICS - 5x14	-13.5	-1.76
OPTICS - 4x12	-12.7	-1.43
OPTICS - 3x11	-12.5	-1.48
OPTICS - 1x3	-11.8	-1.68
Survival	+1.13	+1.80
Survival multistate	-11.6	-1.44

Table 6: Residual error between observation and simulation, for mean and median of the total energy use over the validation period.

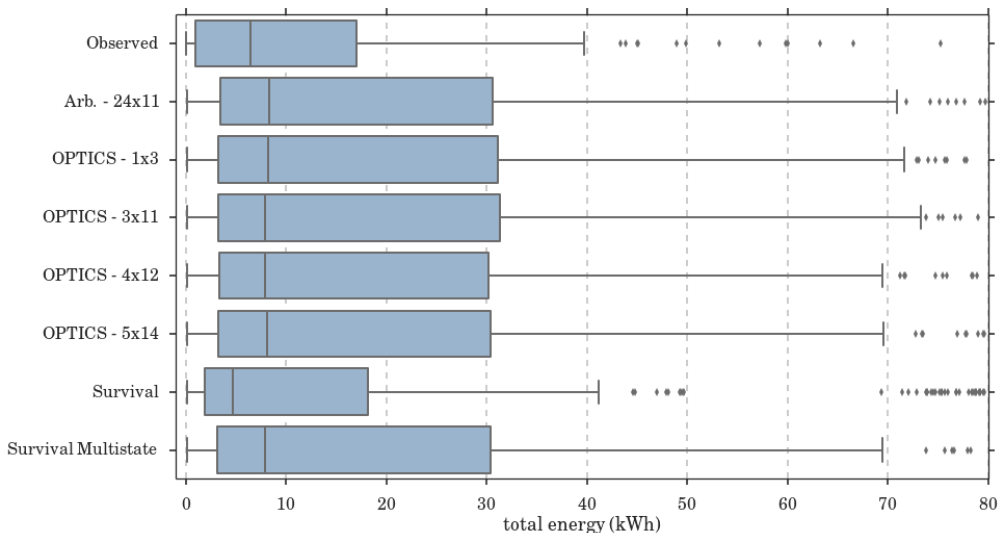


Figure 9: Boxplot comparing observed and predicted total energy use over the validation period.

#### 2 4.3.4. Summary

3 The complexity of the different approaches can also be used for comparison, based on  
 4 the type and number of parameters that the models need. They are summarised in table 7.

1 For the Markov based approaches, the parameters needed are those that build the Markov  
 2 matrix, and are dependent on its dimension. Additionally, the clustering process requires 9  
 3 extra parameter values, which are not easily extracted, as the clustering algorithm requires  
 4 a trial and error process which is complicated and time-consuming. The parameters needed  
 5 in the survival models are those that describe the Weibull distribution, plus the hourly  
 6 probability distribution of each state to occur (trivial to obtain and which can be simplified  
 7 using less time slots.)

	<b>Number of parameters</b>	<b>Type</b>
Arb - 24x11	$24 \times 11$	Markov matrix
OPTICS - 5x14	$5 \times 14 + 9$	Markov matrix and clustering
OPTICS - 4x12	$4 \times 12 + 9$	Markov matrix and clustering
OPTICS - 3x11	$3 \times 11 + 9$	Markov matrix and clustering
OPTICS - 1x3	$1 \times 3 + 9$	Markov matrix and clustering
Survival	$3 \times 2$	Weibull parameters
Survival multistate	$3 \times 11 + 24 \times 11$	Weibull and states' probability

Table 7: Number and type of parameters needed for the different type of model.

8 To inform our selection of the most parsimonious modelling strategy the relative perfor-  
 9 mance of each of the models tested is qualitatively summarised in figure 10, using a color  
 10 coded diagram. From this it is apparent that the Survival Multistate strategy outperforms its  
 11 counterparts: its predictive power is comparable to that of the more refined Markov models,  
 12 but is considerably simpler in formulation, both in the estimation of its coefficients and in  
 13 subsequent implementation. It performs well in the time series analysis, temporal state pre-  
 14 diction and fractional energy profile, acceptably well in absolute state prediction, accuracy  
 15 and total energy use. For these reasons, the Survival Multistate approach has been deployed  
 16 to model the other categories.

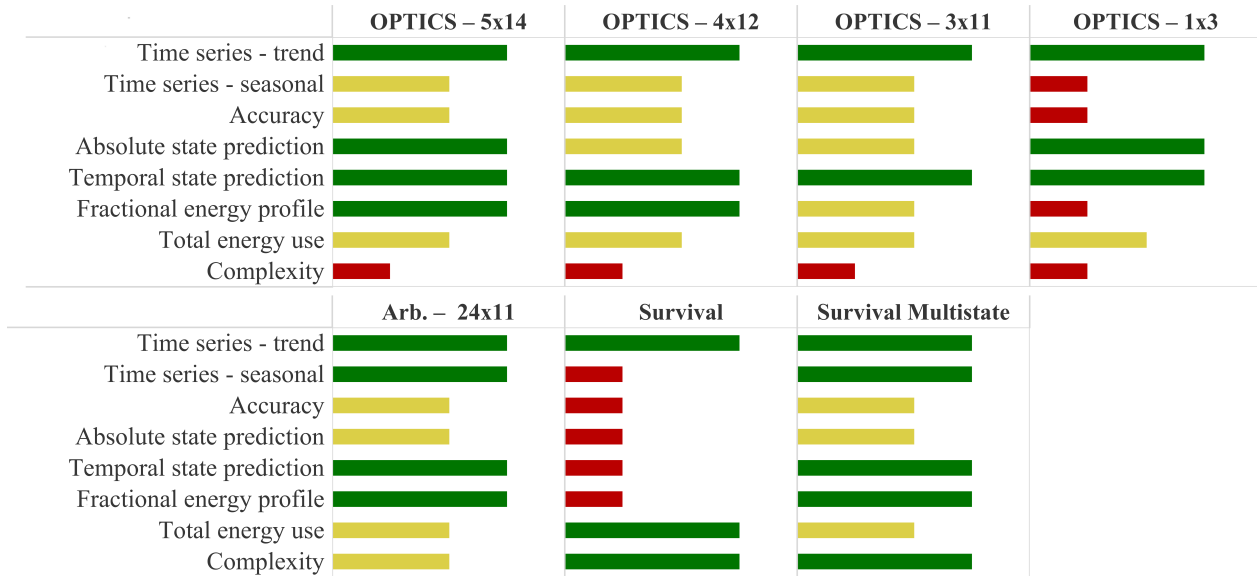


Figure 10: Summary of validation results between the approaches tested, qualitatively represented as *good* (long green bar), *average* (yellow medium bar) and *poor* (short red bar).

#### 1 4.4. Application of Survival Multistate approach to other categories

2 In this section results for the simulation of the energy use of computing devices, small  
3 kitchen appliances and a category of "other" appliances is presented, following the Survival  
4 Multistate approach.

#### 5 Discussion on modelling a diversity of appliances

6 Modelling categories of appliances prevents from the analysis of different behaviours from  
7 specific devices, which are in a large range of total time of use (between commonly-used and  
8 seldom-used appliances) and peak demand values (low-rated and high-rated appliances). At  
9 the extremes of this range two types of behaviours have been identified: dominant appliances  
10 (commonly-used and high-rated) and infrequent appliances (very rarely used over the course  
11 of a year, independently of their rated power). In both cases, these behaviours are undetected  
12 by our modelling approach, with corresponding implications for predictive accuracy.

13 In the case of the kitchen category, preliminary results as described in 2 led to the elim-  
14 ination of the kettle as part of the category. As a high-rated appliance that is commonly  
15 owned and used, its behaviour is dominant, misleading the extraction of parameters of the  
16 model. Figure 11 shows the results for the survival multistate model applied to the kitchen

1 category with and without the kettle. In this particular case, the total energy use was under-  
 2 estimated by the model, due to its inability to discriminate between the power use pattern of  
 3 this particular appliance and the other small kitchen appliances. Once removed, the result  
 4 shows a very good fit with the observed data.

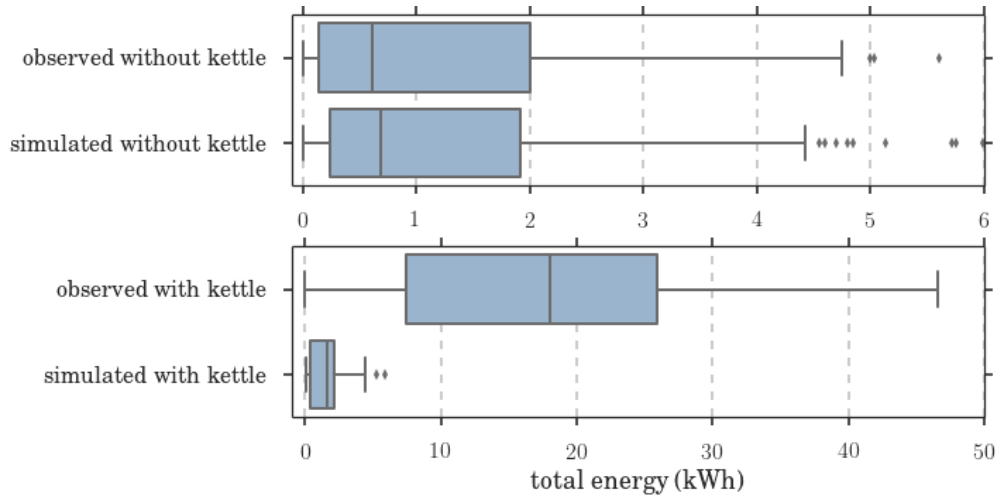


Figure 11: Effect of a dominant appliance (kettle) on the observed and simulated data for the kitchen category.

5 The category of *other* appliances, on the other hand, is biased by the effect of infrequent  
 6 appliances, which were monitored in the survey but are very rarely used: several being used  
 7 only for less than 1% of the total recorded time. Consequently, the total energy use predicted  
 8 was overestimated.

### 9 *Performance of the model*

10 The performance of the model has been evaluated in a similar fashion to that for the audio-  
 11 visual category, and is summarized in table 8. In general, the model performs comparably  
 12 to that of the modelling of audio-visual appliances. The results are remarkably good for the  
 13 case of the kitchen devices, once the kettle was removed, proving that the strategy is very  
 14 powerful for modelling relatively homogeneous type of appliances. Larger errors in energy  
 15 prediction are found for the other two cases, related again to the diversity of behaviours  
 16 present on the dataset, as explained in 4.3.3. Notwithstanding this, the average fractional  
 17 energy use is well predicted, as are the states (table 8).

Error measure		Computing	Kitchen	Other
<b>Time series analysis</b>	Relative error trend	15.7%	0.429%	8,49%
	Pearson's coefficient	0.967	0.946	0.927
	Lag	-0.014	-0.0005	-0.014
<b>ROC curve</b>	TPR	0.205	0.842	0.783
	TNR	0.920	0.984	0.978
	ACC	0.485	0.517	0.514
<b>Daily profile (RMSE)</b>	Fractional energy ( $f$ )	$4.44 \cdot 10^{-2}$	$2.23 \cdot 10^{-3}$	$1.27 \cdot 10^{-2}$
	Absolute state prediction	$5.48 \cdot 10^{-2}$	$6.16 \cdot 10^{-3}$	$1.77 \cdot 10^{-2}$
	Temporal state prediction	$4.08 \cdot 10^{-2}$	$1.06 \cdot 10^{-2}$	$1.70 \cdot 10^{-2}$
<b>Total Energy (kWh)</b>	Mean Residual	-26.4	-0.318	17.7
	Median Residual	-19.0	-0.160	-20.6

Table 8: Summary of results on application of Survival Multistate approach to other categories for: time series analysis, sensitivity and specificity, RMSE of daily profile and total energy.

#### 1 4.5. Global performance and application of the model

2 The application of the model is shown in this section in two ways: the first involves a single  
3 day simulation for a specific household (labelled in the dataset as "103028"), presented in  
4 figure 12. It contains 13 different low-load appliances, which are described in table 9. Figure  
5 12 displays the output of the model, for the four categories, when using the listed appliances.  
6 As expected, the model predicts usages of different duration and it is able to capture the spikes  
7 in the profiles. But, as expected, the model does not resolve for the specific characteristics  
8 of the individual appliances, and it does not represent different behaviours between them.  
9 However, the more common usage of audio-visual and computing appliances is well captured.

	<b>Appliance type</b>	<b>Rated Power (W)</b>
<b>Audio-visual</b>	Set top box	30
	Audio-visual site	43.2
	DVD/VCR	12.6
<b>Kitchen</b>	Bread maker	97.2
	Toaster	720
	Extractor	16.8
<b>Computing</b>	Laptop	48.6
	Computer equipement	3.6
	Desktop	108
	Monitor	30
	Router	7.2
<b>Other</b>	Housework	1248
	Various	54.6

Table 9: Available appliances in house "103028" and their rated values, extracted from the data set.

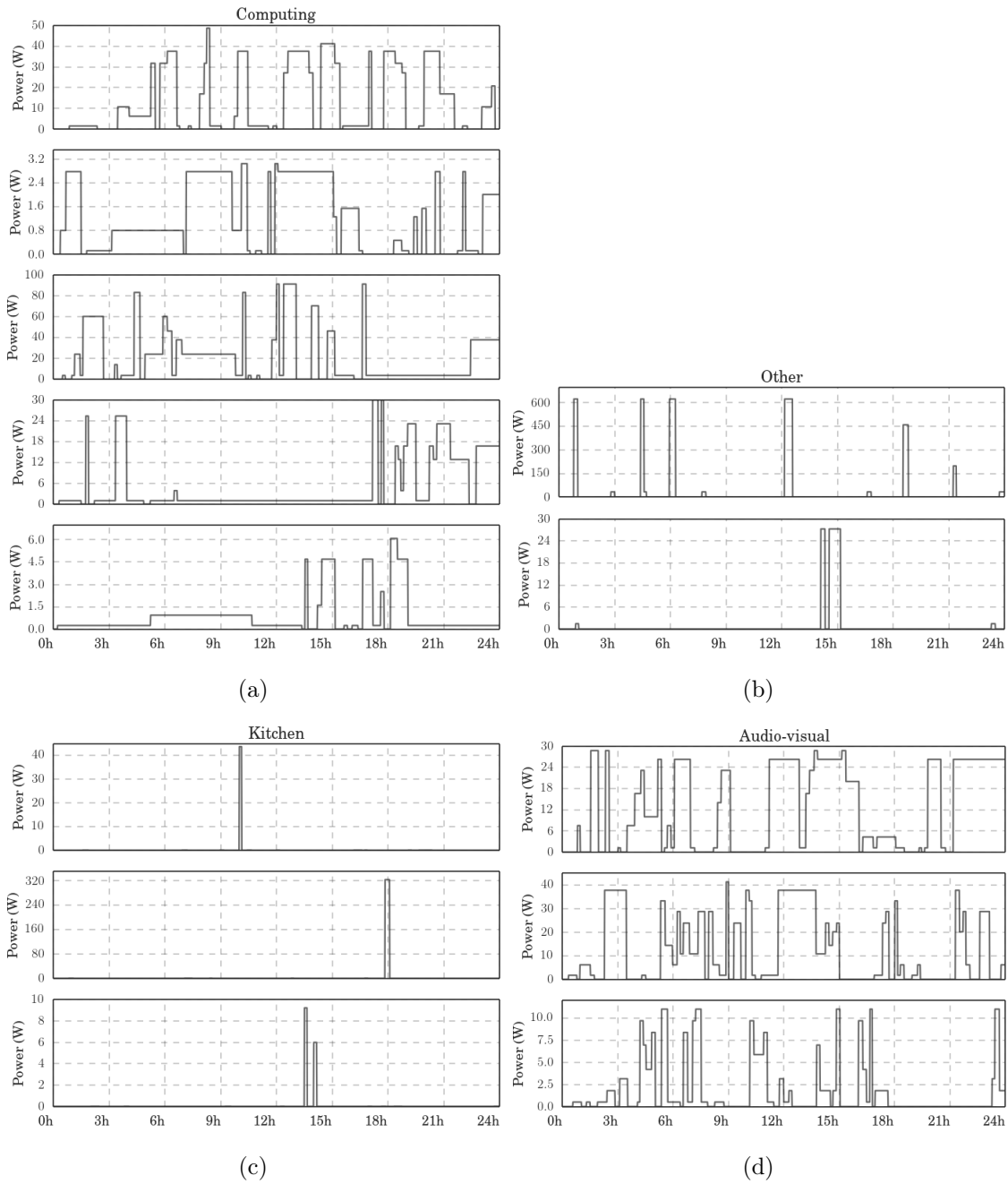


Figure 12: Example of one-day simulation for low-load appliances, by category: (a) computing, (b) other, (c) kitchen, (d) audio-visual. They correspond from upper to lower as indicated in table 9.

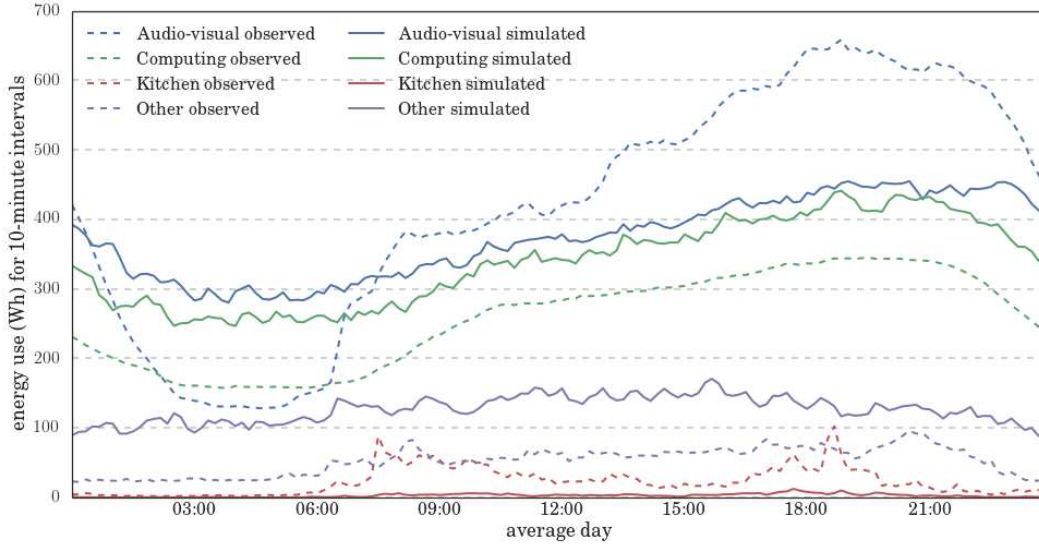


Figure 13: Averaged daily total energy usage from the different categories.

1 The second includes the averaged daily energy usage arising from all the devices in the  
2 different categories of appliances over the year, when aggregated to a community of 20 house-  
3 holds (figure 13). This situation is much more representative of the intended usage of the  
4 model than for the modelling of individual appliances in a single household. In this case, the  
5 total energy use for each category (adding up all the available devices in each household)  
6 is averaged in order to create a typical day profile for the community. Then observed and  
7 simulated data are compared. Simulated audio-visual appliances describe temporal variabil-  
8 ity, although its dependency is not as strong as in reality. The use of computing appliances  
9 is consistently slightly overestimated, but still captures variations during the day. The use  
10 of kitchen devices is slightly underestimated, and the opposite happens with the other cate-  
11 gory; this could be related to the amount and type of devices of available for this particular  
12 group of households, given that their use is reduced. We can conclude that despite of mod-  
13 elling appliances in a generalised way, where devices are not considered individually, realistic  
14 magnitudes for the electrical energy use with respect to time can still be obtained.

15 The parameters used in both cases are those detailed in [Appendix A](#). Profiles for the  
16 energy use of the four categories of low-load appliances are thus obtained and compared

1 against the profiles of a selected household .

## 2 **5. Conclusion**

3 As the integrity of the envelope of both new and existing houses improves, so the pro-  
4 portion of energy that is used by household electrical appliances, which are becoming in-  
5 creasingly ubiquitous, is likely to increase. It is important then that modellers have at their  
6 disposal reliable models of appliance energy use, if they are to accurately predict the thermal  
7 performance and energy use of future homes. Furthermore, there is increasing interest in  
8 the concept of smart grids, to better regulate the distributed supply, storage and demand  
9 of electrical energy. This places increasing onus on the ability to predict the dynamic be-  
10 haviour of household electrical appliances. Whilst good progress has recently been made in  
11 the modelling of relatively large appliances: those whose prevalence and cumulative energy  
12 use supports the estimation of device-specific models. Poor progress has been made in the  
13 modelling of relatively small appliances: those whose cumulative energy use is individually  
14 small, but significant when considered as aggregates by typology. To this end we have tested  
15 a range of strategies for the modelling of small appliance categories; first predicting discrete  
16 states in fractional energy demand, then converting into absolute energy demand, given an  
17 estimate of the corresponding maximum power demand.

18 In this we deploy both discrete (Markov) and continuous (survival) time random processes;  
19 and for the former we also deploy cluster analysis to effectively partition the state transition  
20 probability space.

21 From this we draw the following conclusions:

- 22 • Modelling appliances by their typologies presents many advantages: it provides a  
23 straightforward solution for modelling the range of types of appliances, it reduces the  
24 amount of input data needed to estimate the model and the risk of overfitting, and  
25 it avoids the time-consuming process of modelling appliances individually, simplifying  
26 dynamic energy simulation.
- 27 • The model predicting time varying fractional power demands is surprisingly robust,  
28 given that it is modelling aggregates. We find that:

- 1           – Finer discretisation of temporal states improved predictive power, but these im-  
2           provements are modest beyond 5 temporal states.
- 3           – Appropriate estimation of the number of fractional energy states is not as influ-  
4           ential as the number of temporal states.
- 5           • Clustering techniques have been effectively deployed to objectively search for a parsimonious form of model: minimising the size and number of state transition probability  
6           matrices. The methods presented can be used for many other areas of research.
- 7
- 8           • Based on three types of evaluation measure (time series analysis, model accuracy and  
9           aggregated energy use), the survival multistate approach, in which survival times are  
10          estimated for selected bins of fractional energy demand, clearly outperforms its Markov  
11          process counterparts.
- 12          • However, analysing categories can compromise the fidelity of predictions of aggregate  
13          energy use, particularly if modelling small numbers of households. In our case, a suc-  
14          cessful strategy consisted of allocating maximum energy values with a random assign-  
15          ment process. The survival multistate approach has been effectively deployed to model  
16          low-load appliances in four categories: audio-visual, computing kitchen and other. The  
17          profiles output by the model have been satisfactorily compared to those of a community  
18          of households.

19 This work forms part of a larger programme of research to reliably predict appliance energy  
20 demand using bottom-up techniques for communities of households, and to test strategies  
21 for the management of these appliance demands to improve community energy autonomy.  
22 The testing and evaluation of these Demand Side Management strategies will be reported in  
23 a future paper.

## 24 **Acknowledgements**

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## 1 References

- 2 [1] Department of Energy and Climate Change. Energy consumption in the UK (2015).  
3 Technical report.
- 4 [2] S. Jaboob, D. Robinson, and O. Beckett. On the ownership and use of residential  
5 electrical appliances.
- 6 [3] A. Sancho Tomás, M. Sumner, S. Lamparter, and D. Robinson. On the energy demands  
7 of small appliances in homes. *Energy Procedia*, 78:3384 – 3390, 2015. 6th International  
8 Building Physics Conference, IBPC 2015.
- 9 [4] A. Grandjean, J. Adnot, and G. Binet. A review and an analysis of the residential electric  
10 load curve models. *Renewable and Sustainable Energy Reviews*, 16(9):6539–6565, 2012.
- 11 [5] M. Stokes. *Removing barriers to embedded generation: a fine-grained load model to*  
12 *support low voltage network performance analysis*. PhD thesis, De Montfort University,  
13 Leicester, UK, 2005.
- 14 [6] J. V. Paatero and P. D. Lund. A model for generating household electricity load profiles.  
15 *International Journal of Energy Research*, 30(5):273–290, April 2006.
- 16 [7] A. Capasso, W. Grattieri, R. Lamedica, and a. Prudenzi. A bottom-up approach to  
17 residential load modeling. *IEEE Transactions on Power Systems*, 9(2):957–964, May  
18 1994.
- 19 [8] J. Tanimoto, A. Hagishima, and H. Sagara. Validation of probabilistic methodology for  
20 generating actual inhabitants’ behavior schedules for accurate prediction of maximum  
21 energy requirements. *Energy and Buildings*, 40(3):316–322, January 2008.
- 22 [9] J. Widén, M. Lundh, I. Vassileva, E. Dahlquist, K. Ellegård, and E. Wäckelgård.  
23 Constructing load profiles for household electricity and hot water from time-use data-  
24 modelling approach and validation. *Energy and Buildings*, 41(7-1):753 – 768, July 2009.
- 25 [10] J. Widén and E. Wäckelgård. A high-resolution stochastic model of domestic activity  
26 patterns and electricity demand. *Applied Energy*, 87(6):1880–1892, June 2010.

- 1 [11] I. Richardson, M. Thomson, D. Infield, and C. Clifford. Domestic electricity use: A  
2 high-resolution energy demand model. *Energy and Buildings*, 42(10):1878–1887, October  
3 2010.
- 4 [12] J. Page, D. Robinson, N. Morel, and J.-L. Scartezzini. A generalised stochastic model  
5 for the simulation of occupant presence. *Energy and Buildings*, 40(2):83–98, January  
6 2008.
- 7 [13] F. Haldi and D. Robinson. Adaptive actions on shading devices in response to local  
8 visual stimuli. *Journal of Building Performance Simulation*, 3(2):135–153, June 2010.
- 9 [14] F. Haldi and D. Robinson. Interactions with window openings by office occupants.  
10 *Building and Environment*, 44(12):2378–2395, December 2009.
- 11 [15] A. T. Bharucha-Reid. *Elements of the Theory of Markov Processes and their Applica-*  
12 *tions*. Courier Corporation, North Chelmsford , MA (US), 2012.
- 13 [16] A. K. Jain, M. N. Murty, and P. J. Flynn. Data clustering: A review. *ACM Computing*  
14 *Surveys*, 31(3):264–323, September 1999.
- 15 [17] M. Ankerst, M. M. Breunig, H.-P. Kriegel, and J. Sander. OPTICS: Ordering points to  
16 identify the clustering structure. In *Proceedings of the 1999 ACM SIGMOD International*  
17 *Conference on Management of Data*, SIGMOD '99, pages 49–60, New York, NY, USA,  
18 1999. ACM.
- 19 [18] M. Ester, H.-P. Kriegel, J. Sander, and X. Xu. A density-based algorithm for discov-  
20 ering clusters in large spatial databases with noise. In *Proceedings of 2nd International*  
21 *Conference on Knowledge Discovery and Data Mining*, volume 96, pages 226–231, 1996.
- 22 [19] J. Sander, X. Qin, Z. Lu, N. Niu, and A. Kovarsky. Automatic extraction of clusters  
23 from hierarchical clustering representations. In *Advances in Knowledge Discovery and*  
24 *Data Mining*, volume 2637 of *Lecture Notes in Computer Science*, pages 75–87. Springer  
25 Berlin Heidelberg, 2003.

- 1 [20] D. R. Cox and D. Oakes. *Analysis of survival data*, volume 21. CRC Press, Boca Raton,  
2 USA, 1984.
- 3 [21] Department of Energy and Climate Change and Intertek Testing and Certification Ltd.  
4 Household Electricity Survey.
- 5 [22] O. Vallis, J. Hochenbaum, and A. Kejariwal. A novel technique for long-term anomaly  
6 detection in the cloud. In *6th USENIX Workshop on Hot Topics in Cloud Computing*  
7 (*HotCloud 14*), Philadelphia, PA, June 2014. USENIX Association.
- 8 [23] R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation  
9 for Statistical Computing, Vienna, Austria, 2015.
- 10 [24] R. Kohavi. A study of cross-validation and bootstrap for accuracy estimation and model  
11 selection. In *Proceedings of the 14th International Joint Conference on Artificial Intel-*  
12 *ligence - Volume 2*, IJCAI'95, pages 1137–1143, San Francisco, CA, USA, 1995. Morgan  
13 Kaufmann Publishers Inc.
- 14 [25] G.E.P. Box and G. M. Jenkins. *Time series analysis: forecasting and control*. John  
15 Wiley & Sons, Hoboken, New Jersey, 2015.
- 16 [26] Chao-Ying Joanne Peng and Tak-Shing Harry So. Logistic regression analysis and re-  
17 porting: A primer. *Understanding Statistics: Statistical Issues in Psychology, Education,*  
18 *and the Social Sciences*, 1(1):31–70, 2002.
- 19 [27] D.S. Starnes, J. Tabor, D. Yates, and D.S. Moore. *The Practice of Statistics*. W. H.  
20 Freeman, 2014.

## 1 Appendix A.

2 For purposes of implementation of the model, tables 10, 11, 12, 13 and 14 contain the val-  
3 ues of the parameters obtained from the dataset. The survival multistate model is presented  
4 in Algorithm 1 as pseudo-code.

---

**Algorithm 1** Simulate Small Appliance Usage ( $([\lambda, k, \gamma]_s, P(t, \mathbf{s}))$ )

---

```
1:  $s = 0$  ▷ Assume initial state  $s_0$ .
2:  $t = t_{START}$ 
3: while  $t < t_{END}$  do
    % Calculate duration at state  $s$ 
4:  $R1 = random(0, 1)$ 
5:  $\lambda, k, \gamma = \lambda_s, k_s, \gamma_s$  ▷ Use table 10.
6:  $t_s = \gamma + \lambda [-\ln(R1)]^{1/k}$ 
7:  $t = t + t_s$ 
    % Calculate next state
8:  $R2 = random(0, 1)$ 
9:  $s = MinIdx [(cdfP(t) - R2) > 0]$  ▷ Use table table 12 and 13.
10: Append  $s$  to  $s_{arr}$ 
11: end while
12: % Transform into fractional energy array.
13:  $f_{arr} = \tilde{F}_s \times s_{arr}$  ▷ Use table 11.
14: % Transform into energy use array.
15:  $E_{arr} = \tilde{E}_{max} \times f_{arr}$  ▷ Use table 14.
```

---

Energy state	Audio-visual			Computing			Kitchen			Other		
	$\gamma$	$k$	$\lambda$	$\gamma$	$k$	$\lambda$	$\gamma$	$k$	$\lambda$	$\gamma$	$k$	$\lambda$
$s_0$	8.92	0.743	10.74	7.52	1.529	12.52	7.80	1.37	4.29	7.46	0.930	9.61
$s_1$	8.29	0.916	7.52	7.29	1.110	4.87	8.93	1.17	6.57	8.25	1.148	5.20
$s_2$	8.38	1.096	9.82	8.03	0.889	5.37	8.39	1.25	4.29	8.83	1.147	7.16
$s_3$	8.33	0.965	6.95	8.83	0.607	20.14	8.13	1.06	5.35	9.34	1.201	9.03
$s_4$	8.95	0.648	12.20	8.52	0.977	6.19	8.02	1.26	4.50	8.86	0.872	7.34
$s_5$	9.45	0.980	15.59	8.13	0.870	6.36	8.34	1.27	4.20	9.03	1.148	13.35
$s_6$	9.02	0.747	13.06	8.68	0.790	7.76	8.72	1.12	5.21	8.89	1.070	6.92
$s_7$	9.03	1.065	11.72	8.76	0.854	10.97	8.22	1.33	4.01	8.23	1.046	5.32
$s_8$	8.40	1.005	7.25	8.95	0.657	13.82	7.63	1.28	4.58	8.39	1.214	4.47
$s_9$	8.79	0.805	13.93	8.70	0.872	15.63	8.01	1.24	4.36	8.64	0.989	6.30
$s_{10}$	8.24	1.051	5.56	8.30	1.093	5.72	8.73	1.16	7.12	8.65	0.963	8.62

Table 10: Survival distribution parameters ( $\gamma$ : location,  $k$ : shape,  $\lambda$ : scale) for four categories.

Energy state	$\tilde{F}_{Audio-visual}$	$\tilde{F}_{Computing}$	$\tilde{F}_{Kitchen}$	$\tilde{F}_{Other}$
$s_0$	0.0000	0.0000	0.0000	0.0000
$s_1$	0.0402	0.0333	0.0007	0.0265
$s_2$	0.1429	0.1297	0.1382	0.1587
$s_3$	0.2500	0.2222	0.2473	0.2513
$s_4$	0.3333	0.3500	0.3570	0.3684
$s_5$	0.4667	0.4286	0.4494	0.5000
$s_6$	0.5525	0.5581	0.5495	0.5450
$s_7$	0.6660	0.6526	0.6597	0.6565
$s_8$	0.7708	0.7717	0.7463	0.7500
$s_9$	0.8750	0.8462	0.8405	0.8333
$s_{10}$	0.9571	1.0000	0.9684	0.9167

Table 11: Median fractional energy for transforming energy states into fractional energy profiles.

Hour of day	Audio-visual										
	$P(s = s_0)$	$P(s = s_1)$	$P(s = s_2)$	$P(s = s_3)$	$P(s = s_4)$	$P(s = s_5)$	$P(s = s_6)$	$P(s = s_7)$	$P(s = s_8)$	$P(s = s_9)$	$P(s = s_{10})$
0h	0.3587	0.1437	0.1174	0.0792	0.0406	0.0392	0.0293	0.0443	0.0470	0.0710	0.0295
1h	0.3799	0.1566	0.1172	0.0800	0.0353	0.0378	0.0305	0.0446	0.0459	0.0517	0.0205
2h	0.3844	0.1667	0.1171	0.0803	0.0332	0.0372	0.0384	0.0423	0.0442	0.0408	0.0153
3h	0.3859	0.1715	0.1157	0.0820	0.0302	0.0375	0.0441	0.0403	0.0441	0.0366	0.0122
4h	0.3883	0.1756	0.1128	0.0813	0.0298	0.0398	0.0440	0.0400	0.0436	0.0336	0.0113
5h	0.3879	0.1797	0.1088	0.0805	0.0298	0.0398	0.0448	0.0412	0.0420	0.0342	0.0114
6h	0.3853	0.1719	0.1115	0.0785	0.0303	0.0379	0.0419	0.0424	0.0449	0.0404	0.0151
7h	0.3781	0.1638	0.1117	0.0799	0.0336	0.0391	0.0456	0.0404	0.0451	0.0436	0.0193
8h	0.3607	0.1704	0.1020	0.0805	0.0368	0.0412	0.0506	0.0412	0.0460	0.0481	0.0223
9h	0.3437	0.1709	0.1029	0.0845	0.0390	0.0423	0.0509	0.0427	0.0481	0.0499	0.0251
10h	0.3311	0.1682	0.1057	0.0860	0.0397	0.0438	0.0521	0.0426	0.0475	0.0542	0.0291
11h	0.3283	0.1657	0.1044	0.0859	0.0377	0.0419	0.0518	0.0414	0.0502	0.0595	0.0333
12h	0.3287	0.1610	0.1022	0.0876	0.0332	0.0413	0.0502	0.0410	0.0542	0.0648	0.0358
13h	0.3255	0.1554	0.1032	0.0872	0.0318	0.0434	0.0485	0.0400	0.0549	0.0701	0.0400
14h	0.3202	0.1540	0.1049	0.0860	0.0322	0.0436	0.0467	0.0417	0.0567	0.0718	0.0421
15h	0.3144	0.1496	0.1027	0.0865	0.0346	0.0480	0.0441	0.0465	0.0566	0.0730	0.0440
16h	0.3073	0.1408	0.1035	0.0879	0.0374	0.0505	0.0426	0.0494	0.0560	0.0753	0.0492
17h	0.3021	0.1319	0.1039	0.0905	0.0387	0.0512	0.0443	0.0515	0.0564	0.0786	0.0510
18h	0.2994	0.1280	0.1020	0.0888	0.0399	0.0497	0.0454	0.0539	0.0565	0.0825	0.0539
19h	0.3001	0.1267	0.1022	0.0892	0.0416	0.0518	0.0417	0.0500	0.0563	0.0880	0.0523
20h	0.2976	0.1279	0.1068	0.0902	0.0425	0.0471	0.0400	0.0492	0.0555	0.0887	0.0545
21h	0.2953	0.1276	0.1054	0.0904	0.0438	0.0461	0.0409	0.0470	0.0565	0.0921	0.0550
22h	0.3008	0.1289	0.1046	0.0854	0.0450	0.0475	0.0418	0.0458	0.0550	0.0933	0.0519
23h	0.3222	0.1340	0.1097	0.0817	0.0449	0.0457	0.0383	0.0450	0.0512	0.0862	0.0411

Hour of day	Computing										
	$P(s = s_0)$	$P(s = s_1)$	$P(s = s_2)$	$P(s = s_3)$	$P(s = s_4)$	$P(s = s_5)$	$P(s = s_6)$	$P(s = s_7)$	$P(s = s_8)$	$P(s = s_9)$	$P(s = s_{10})$
0h	0.4084	0.2563	0.0450	0.0370	0.0266	0.0376	0.0405	0.0410	0.0534	0.0394	0.0148
1h	0.4232	0.2626	0.0479	0.0373	0.0237	0.0341	0.0349	0.0348	0.0520	0.0361	0.0134
2h	0.4306	0.2672	0.0544	0.0382	0.0229	0.0326	0.0273	0.0324	0.0496	0.0325	0.0124
3h	0.4348	0.2679	0.0575	0.0379	0.0225	0.0322	0.0237	0.0307	0.0480	0.0325	0.0122
4h	0.4361	0.2676	0.0578	0.0379	0.0226	0.0317	0.0240	0.0300	0.0388	0.0411	0.0123
5h	0.4364	0.2664	0.0575	0.0379	0.0230	0.0319	0.0238	0.0334	0.0474	0.0311	0.0113
6h	0.4331	0.2653	0.0565	0.0376	0.0234	0.0324	0.0257	0.0373	0.0486	0.0296	0.0105
7h	0.4259	0.2589	0.0526	0.0376	0.0247	0.0334	0.0323	0.0428	0.0499	0.0307	0.0113
8h	0.4138	0.2484	0.0461	0.0379	0.0265	0.0352	0.0409	0.0514	0.0544	0.0333	0.0122
9h	0.4001	0.2394	0.0432	0.0382	0.0303	0.0357	0.0445	0.0563	0.0607	0.0376	0.0141
10h	0.3884	0.2328	0.0417	0.0391	0.0322	0.0376	0.0487	0.0589	0.0627	0.0419	0.0161
11h	0.3818	0.2295	0.0405	0.0381	0.0322	0.0403	0.0517	0.0603	0.0629	0.0447	0.0181
12h	0.3773	0.2268	0.0389	0.0392	0.0327	0.0407	0.0539	0.0614	0.0642	0.0458	0.0191
13h	0.3723	0.2259	0.0376	0.0397	0.0327	0.0425	0.0548	0.0621	0.0642	0.0482	0.0201
14h	0.3660	0.2249	0.0376	0.0403	0.0329	0.0428	0.0558	0.0636	0.0655	0.0493	0.0214
15h	0.3548	0.2228	0.0376	0.0407	0.0340	0.0436	0.0559	0.0663	0.0691	0.0524	0.0227
16h	0.3428	0.2223	0.0375	0.0409	0.0367	0.0411	0.0587	0.0675	0.0719	0.0555	0.0251
17h	0.3336	0.2179	0.0386	0.0411	0.0385	0.0417	0.0604	0.0700	0.0736	0.0579	0.0266
18h	0.3252	0.2141	0.0396	0.0411	0.0394	0.0451	0.0619	0.0708	0.0765	0.0600	0.0263
19h	0.3196	0.2118	0.0395	0.0430	0.0380	0.0495	0.0633	0.0706	0.0784	0.0599	0.0266
20h	0.3162	0.2156	0.0391	0.0439	0.0384	0.0511	0.0627	0.0683	0.0775	0.0602	0.0270
21h	0.3192	0.2213	0.0391	0.0426	0.0359	0.0536	0.0615	0.0674	0.0732	0.0585	0.0276
22h	0.3430	0.2349	0.0413	0.0403	0.0310	0.0490	0.0555	0.0622	0.0666	0.0512	0.0250
23h	0.3785	0.2498	0.0433	0.0366	0.0286	0.0434	0.0469	0.0519	0.0581	0.0434	0.0195

Table 12: Hourly state probability for Audio-visual and Computing categories.

Hour of day	Kitchen										
	$P(s = s_0)$	$P(s = s_1)$	$P(s = s_2)$	$P(s = s_3)$	$P(s = s_4)$	$P(s = s_5)$	$P(s = s_6)$	$P(s = s_7)$	$P(s = s_8)$	$P(s = s_9)$	$P(s = s_{10})$
0h	0.9403	0.0567	0.0006	0.0007	0.0006	0.0005	0.0004	0.0002	0.0000	0.0000	0.0000
1h	0.9418	0.0561	0.0003	0.0004	0.0005	0.0004	0.0003	0.0001	0.0001	0.0001	0.0000
2h	0.9408	0.0560	0.0004	0.0005	0.0007	0.0007	0.0005	0.0002	0.0001	0.0000	0.0000
3h	0.9357	0.0580	0.0008	0.0009	0.0017	0.0016	0.0008	0.0003	0.0001	0.0001	0.0001
4h	0.9328	0.0595	0.0016	0.0012	0.0022	0.0017	0.0004	0.0003	0.0002	0.0002	0.0001
5h	0.9209	0.0624	0.0033	0.0025	0.0049	0.0034	0.0017	0.0005	0.0002	0.0001	0.0001
6h	0.9041	0.0719	0.0061	0.0034	0.0054	0.0048	0.0026	0.0010	0.0003	0.0002	0.0001
7h	0.8768	0.0836	0.0090	0.0068	0.0086	0.0079	0.0038	0.0023	0.0007	0.0004	0.0001
8h	0.8683	0.0862	0.0114	0.0084	0.0100	0.0081	0.0047	0.0015	0.0006	0.0005	0.0004
9h	0.8656	0.0924	0.0096	0.0082	0.0084	0.0073	0.0041	0.0023	0.0009	0.0008	0.0004
10h	0.8740	0.0913	0.0077	0.0077	0.0072	0.0057	0.0030	0.0015	0.0006	0.0007	0.0005
11h	0.8759	0.0910	0.0069	0.0075	0.0075	0.0056	0.0028	0.0013	0.0006	0.0005	0.0004
12h	0.8854	0.0861	0.0056	0.0068	0.0070	0.0045	0.0026	0.0010	0.0005	0.0003	0.0002
13h	0.8822	0.0870	0.0056	0.0076	0.0076	0.0048	0.0028	0.0013	0.0006	0.0004	0.0002
14h	0.8839	0.0879	0.0053	0.0076	0.0072	0.0040	0.0022	0.0010	0.0004	0.0004	0.0002
15h	0.8858	0.0847	0.0056	0.0073	0.0077	0.0045	0.0021	0.0010	0.0005	0.0004	0.0004
16h	0.8863	0.0839	0.0058	0.0069	0.0066	0.0046	0.0029	0.0012	0.0011	0.0005	0.0003
17h	0.8839	0.0845	0.0062	0.0077	0.0065	0.0043	0.0025	0.0014	0.0017	0.0005	0.0007
18h	0.8932	0.0792	0.0055	0.0063	0.0059	0.0043	0.0021	0.0012	0.0012	0.0006	0.0004
19h	0.9016	0.0760	0.0045	0.0053	0.0049	0.0035	0.0020	0.0010	0.0006	0.0004	0.0002
20h	0.9072	0.0719	0.0039	0.0048	0.0052	0.0036	0.0015	0.0009	0.0005	0.0003	0.0002
21h	0.9171	0.0672	0.0026	0.0037	0.0044	0.0024	0.0012	0.0007	0.0004	0.0001	0.0002
22h	0.9248	0.0634	0.0021	0.0033	0.0030	0.0017	0.0008	0.0005	0.0002	0.0001	0.0000
23h	0.9323	0.0605	0.0017	0.0021	0.0012	0.0010	0.0006	0.0003	0.0001	0.0000	0.0001

Hour of day	Other										
	$P(s = s_0)$	$P(s = s_1)$	$P(s = s_2)$	$P(s = s_3)$	$P(s = s_4)$	$P(s = s_5)$	$P(s = s_6)$	$P(s = s_7)$	$P(s = s_8)$	$P(s = s_9)$	$P(s = s_{10})$
0h	0.8724	0.0841	0.0012	0.0029	0.0097	0.0182	0.0001	0.0000	0.0112	0.0000	0.0001
1h	0.8730	0.0835	0.0013	0.0027	0.0096	0.0200	0.0000	0.0000	0.0097	0.0000	0.0000
2h	0.8722	0.0830	0.0013	0.0026	0.0101	0.0201	0.0001	0.0000	0.0105	0.0000	0.0001
3h	0.8720	0.0821	0.0014	0.0025	0.0098	0.0212	0.0002	0.0000	0.0107	0.0000	0.0001
4h	0.8725	0.0817	0.0013	0.0026	0.0097	0.0219	0.0002	0.0000	0.0101	0.0000	0.0001
5h	0.8718	0.0788	0.0015	0.0027	0.0109	0.0245	0.0004	0.0001	0.0092	0.0000	0.0001
6h	0.8692	0.0765	0.0022	0.0032	0.0120	0.0284	0.0004	0.0002	0.0077	0.0000	0.0001
7h	0.8674	0.0757	0.0026	0.0036	0.0127	0.0301	0.0008	0.0003	0.0066	0.0001	0.0001
8h	0.8644	0.0731	0.0029	0.0038	0.0136	0.0331	0.0011	0.0005	0.0069	0.0003	0.0003
9h	0.8631	0.0741	0.0026	0.0036	0.0141	0.0329	0.0010	0.0007	0.0071	0.0004	0.0004
10h	0.8577	0.0743	0.0034	0.0044	0.0143	0.0335	0.0015	0.0014	0.0084	0.0008	0.0003
11h	0.8556	0.0737	0.0040	0.0046	0.0149	0.0347	0.0013	0.0015	0.0086	0.0008	0.0003
12h	0.8571	0.0729	0.0038	0.0044	0.0146	0.0342	0.0014	0.0012	0.0089	0.0013	0.0003
13h	0.8565	0.0744	0.0037	0.0039	0.0154	0.0334	0.0013	0.0015	0.0085	0.0011	0.0003
14h	0.8579	0.0654	0.0130	0.0031	0.0151	0.0322	0.0015	0.0011	0.0094	0.0011	0.0003
15h	0.8600	0.0596	0.0197	0.0020	0.0143	0.0321	0.0019	0.0011	0.0084	0.0006	0.0002
16h	0.8614	0.0622	0.0199	0.0022	0.0136	0.0293	0.0015	0.0007	0.0073	0.0007	0.0011
17h	0.8615	0.0642	0.0198	0.0022	0.0131	0.0278	0.0015	0.0007	0.0070	0.0010	0.0010
18h	0.8622	0.0659	0.0201	0.0024	0.0126	0.0254	0.0014	0.0005	0.0074	0.0011	0.0010
19h	0.8639	0.0664	0.0204	0.0016	0.0125	0.0242	0.0011	0.0005	0.0075	0.0008	0.0012
20h	0.8639	0.0658	0.0203	0.0018	0.0127	0.0246	0.0010	0.0005	0.0075	0.0006	0.0013
21h	0.8639	0.0681	0.0195	0.0020	0.0111	0.0220	0.0009	0.0003	0.0092	0.0007	0.0022
22h	0.8624	0.0804	0.0088	0.0031	0.0103	0.0192	0.0007	0.0003	0.0114	0.0003	0.0030
23h	0.8701	0.0847	0.0018	0.0032	0.0098	0.0177	0.0002	0.0001	0.0119	0.0001	0.0003

Table 13: Hourly state probability for Kitchen and Other categories.

<b>Audio-visual</b>	<b>Computing</b>	<b>Kitchen</b>	<b>Other</b>
1.8	0.0	0.0	0.0
18.0	0.0	0.0	0.0
27.0	0.0	0.0	0.0
30.0	0.0	0.0	0.0
43.2	6.0	0.0	0.0
50.4	58.8	0.0	0.0
54.0	63.0	0.0	0.0
85.8	99.0	0.0	0.0
97.2	105.0	0.0	0.0
98.4	135.0	0.0	0.0
145.2	138.0	0.0	0.0
148.8	139.2	0.0	0.0
153.6	143.4	9.6	0.0
210.0	150.0	91.8	0.0
297.6	151.2	258.0	0.0
312.6	156.6	267.6	0.0
430.8	197.4	580.8	0.0
459.0	228.6	580.8	2.4
518.4	417.0	691.2	30.0
657.0	424.2	723.0	45.6
754.2	628.8	834.0	241.2
892.2	816.6	1416.0	915.0
1037.4	1119.6	1662.6	1213.8
1499.4	1495.8	1755.6	2861.4
2035.2	2625.0	2850.0	3811.8

Table 14: Sum of rated power (W) for all the corresponding low-load appliances in each household, for the 26 houses considered in the dataset. Cases where power is 0.0W are households with devices that have been removed from the modelling (for reasons specified in the text) or household that do not own any of these appliances.