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An Open-Economy Macro-Finance Model of International Interdependence: The OECD, US and the UK

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Abstract

This paper develops a multi-country macro-finance model to study international economic and financial linkages. This approach models the economy and financial markets jointly using both types of data to throw light on such issues. The world economy is modelled using data for the US and aggregate OECD economies as well as the US Treasury bond market, using latent variables to represent a common inflation trend and a US real interest rate factor. We find strong evidence of OECD effects on the US, calling into question the standard closed economy macro-finance specification. The two global latent variables also affect the UK economy, together with two additional UK-specific latent variables. These economic linkages also help to explain the comovement of yields in the US and UK Treasury bond markets.

JEL Classification: C12 E43 F41 G12

Keywords: macroeconomics, spillover effects, common shocks, macro-finance model, the term structure of interest rates

1 Introduction

As recent developments in commodity and credit markets demonstrate, the global economy is becoming increasingly integrated. The spillovers from the United States to smaller economies have been extensively studied. Event studies show that U.S. recessions usually coincide with significant reductions in global growth. Empirical studies based on panel growth regression analysis (e.g. Arora and Vamvakidis (2006)) also find evidence of large spillovers from the US to other economies. Comovement in economic activity across countries may also reflect common shocks, such as changes in oil prices or asset prices. Kose, Otrok, and Whiteman (2003) show that the 'global factor' generally plays an important role in explaining business cycles for the industrial countries. A related empirical study of the G-7 countries (Kose, Otrok, and Whiteman (2005)) finds that the 'common factor' among these countries explains a large share of output fluctuations. Both of these studies are based on dynamic factor models which extract the latent variables representing these global or common factors from the macroeconomic time series for different countries.

This paper develops a multi-country macro-finance modelling framework to study these international effects. The macro-finance approach was pioneered by Ang and Piazzesi (2003). As the name suggests, this allows bond yields to reflect macroeconomic variables. In turn, the behavior of bond yields helps inform the specification of the macroeconomy, yielding new insights into its behavior. In particular, early macro-finance studies showed that although macroeconomic variables provide a good description of the behavior of short rates they do not provide an adequate description of long term yields. This observation has prompted the use of Kalman filters in these models to reflect the changes in long run inflation expectations revealed by surprises in nominal variables, allowing the model to be used in a global setting in which there are both latent and observable macroeconomic factors. However, macro-finance models have so far focussed on the

US, assuming that it is a closed economy. Macro-finance models of countries such as the UK (Joyce, Lildholdt, and Sorensen (2008)) have also been modelled in this way despite their open trade and financial structures.

This paper adapts the macro-finance framework in a way that allows us to test the validity of the closed economy assumption. The econometric specification consists of two sub-systems. The first represents the world economy and is modelled using a reduced form specification with both OECD and US variables, allowing the two-way linkages between the US and the rest of the OECD to be studied. We identify a common non-stationary 'world' factor driving OECD inflation and US inflation interest rates. This is modelled using a latent variable, with another representing real interest rate movements. Although this was not the initial focus of our attention, this model reveals remarkably strong global effects on the US. We then develop a version of the model that also explains yield data for the US Treasury market using the standard arbitrage-free approach. The second sub-system is a model of the UK economy and Treasury bond market that allows for global influences and is estimated simultaneously with the first.

The rest of the paper is organized as follows. Section 2 specifies the structure of the macro-finance model. Section 3 describes the data and estimation method, reports the results of the specification tests. Section 3.3 discusses the empirical results for the preferred model (M1). Section 4 provides a summary of the key findings and offers some concluding remarks.

2 The model framework

This specification is based on the 'central bank model' which represents the behavior of the macroeconomy in terms of the output gap, inflation and the short term interest rate. This is often specified as a simple VAR, designed to reflect the broad reduced from empirical relationships between these variables, rather than structural linkages. This model has been modified by Kozicki and Tinsley (2001), Kozicki and Tinsley (2005) and Dewachter and Lyrio (2006) to add Kalman filters to allow for 'inflation asymptotes' or 'stochastic trends' that shift the equilibrium values of nominal variables like interest and inflation rates.[1] This 'KVAR' approach uses a closed economy model developed originally for the US (see Spencer (2008)), but we adapt it by allowing world factors, output, inflation and interest rates to affect the US as well as the UK economy.

2.1 The macro models

The world economy is represented by the OECD and the US, which still represents about a quarter of OECD GDP and effectively acts as the fulcrum for world real interest rates given the importance of its financial sector and the dollar[2]. Our macro system includes 8 observable macro variables: the aggregate OECD output gap g_t^{**} and inflation π_t^{**}; the US output gap g_t^*, inflation π_t^* and interest rate r_t^*; and the UK output gap g_t, inflation π_t and interest rate r_t. OECD variables represent the world economy and are denoted by (**)−superscripts, while US variables are denoted by (*)−superscripts and UK 'home country' variables are unsubscripted[3]. The estimation period of 1979-2007 was determined by the availability of discount bond equivalent data for the UK Treasury market (Section 3.1) as well as evidence of a structural break in the UK data when the Thatcher government came into power (Hendry and Mizon (1998), Clements and Hendry (1996)).

We started by estimating two separate closed economy macro-only KVARs, one for the 'world'(modeling g_t^{**} , π_t^{**}, g_t^*, π_t^* and r_t^*) and one for the UK (modeling g_t, π_t and r_t). Preliminary empirical analysis (see Table 2) suggested that inflation and interest rates were all non-stationary. Remarkably, we find (Table 3) that there is a nonstationary common trend driving OECD and US inflation rates with the cointegrating vector [1,-1], meaning that the

[1] These are also known as 'variable end-points,' and are non-stationary latent variables (i.e. integrated of order one: I(1)). They are modelled by the Kalman filter and designed to capture common trends that cause the associated nominal variables to be 'co-integrated'. This means that although these nominal variables are non-stationary, there is a linear relationship between them that is stationary.

[2] Although our OECD aggregate measures include the UK, the weight of the UK is small (5.55%).

[3] In the interests of simplicity we did not model the (average) OECD interest rate, relying instead upon the theory that in equilibrium this should be approximated by $(r_t^* + \pi_t^{**} - \pi_t^*)$.

OECD and US inflation asymptotes move on a one–for-one basis. This suggested the use of a single nominal stochastic trend to represent this common nominal trend in the world model: f_t^*. This was augmented by a stationary latent variable (z_t^*) to represent the effect of a factor that affects the real interest rate temporarily. Similarly, preliminary work on a stand alone macro-finance model for the UK suggested the use of two UK-specific factors: f_t is an I(1) stochastic trend representing the non-stationary trend in the nominal variables and z_t is a stationary I(0) variable representing real interest effects. Table 1 summarizes the order of integration of these variables.

Table 1: Stationarity of variables

	OECD-US variables	UK variables
Non-stationary: I(1)	$f_t^*, \pi_t^{**}, \pi_t^*, r_t^*$	f_t, π_t, r_t
Stationary: I(0)	g_t^*, g_t^{**}, z_t^*	g_t, z_t

2.2 The OECD-US macro KVAR

We use a KVAR(N^*) process to describe the joint OECD-US or 'world' macroeconomic dynamics under the real world or state density probability measure $\mathcal{P}$, where N^* is the order of the lag length. The BIC test indicated that a first order difference system was appropriate in this case ($N^* = 1$)[4]. This gives a relatively compact world macro-model:

$$\mathbf{x}_t^* = \boldsymbol{\kappa}^* + \boldsymbol{\phi}_z^* z_t^* + \boldsymbol{\phi}_f^* f_t^* + \boldsymbol{\Phi}_1^* \mathbf{x}_{t-1}^* + \mathbf{w}_t^* \tag{1}$$

$$\mathbf{w}_t^* = \mathbf{G}^* \mathbf{D}^{x*} \boldsymbol{\epsilon}_t^{x*}, \quad \boldsymbol{\epsilon}_t^{x*} \sim N(\mathbf{0}, \mathbf{I})$$

[4]BIC tests are carried out using OLS results.

BIC Tests			
Lags	1	2	3
The OECD US macro VAR	-61.1803	-60.9239	-60.4136
The UK macro VAR	-37.0716	-37.1694	-37.0849

where $\mathbf{x}_t^* = [g_t^{**}, g_t^*, \pi_t^{**}, \pi_t^*, r_t^*]'$ is the observed world macro vector and $\mathbf{w}_t^*$ is a 5×1 error vector; $\mathbf{D}^{x*}$ is a 5 by 5 diagonal matrix with positive diagonal elements; $\mathbf{G}^*$ is a 5×5 lower triangular matrix with unit diagonal. The real factor z_t^* and the nominal factor f_t^* processes are integrated of order zero (I(0)) and one (I(1)) respectively:

$$\begin{pmatrix} z_t^* \\ f_t^* \end{pmatrix} = \begin{pmatrix} \xi^* z_{t-1}^* \\ f_{t-1}^* \end{pmatrix} + \begin{pmatrix} v_t^* \\ u_t^* \end{pmatrix}; \quad where: \quad \begin{pmatrix} v_t^* \\ u_t^* \end{pmatrix} = \begin{pmatrix} \delta^{z*} & 0 \\ 0 & \delta^{f*} \end{pmatrix} \begin{pmatrix} \epsilon_t^{z*} \\ \epsilon_t^{f*} \end{pmatrix} \tag{2}$$

ϵ_t^{z*} and ϵ_t^{f*} are independent standard normal errors and ξ^* represents a mean reversion parameter that is less than unity in absolute value.

The macro-finance literature draws the important distinction between the 'asymptote' or asymptotic expectation $\bar{x}_t^* = E_t\left(\lim_{m\to\infty} x_{t+m}^*\right)$ of a vector like x_t^* conditional on the non-stationary nominal factors and its 'central tendency' $\tilde{x}_t^*$, the expectation conditional on all factors. E denotes the expectation under the state price density, measure $\mathcal{P}$. The macro model (1) has the central tendency $\tilde{x}_t^* = (\tilde{g}_t^{**}, \tilde{g}_t^*, \tilde{\pi}_t^{**}, \tilde{\pi}_t^*, \tilde{r}_t^*)'$ where[5]: $\tilde{x}_t^* = (\mathbf{I} - \mathbf{\Phi}_1^*)^{-1}\left(\boldsymbol{\kappa}^* + \boldsymbol{\phi}_z^* z_t^* + \boldsymbol{\phi}_f^* f_t^*\right) = \boldsymbol{\varphi}^* + \mathbf{R}_1^* z_t^* + \mathbf{R}_2^* f_t^*$. These parameters are restricted. $\mathbf{R}_2^* = (0\ 0\ 1\ 1\ 1)'$ is designed to incorporate the cointegration constraints identified in our preliminary regression results (Table 3), while $\mathbf{R}_1^* = (0\ 0\ 0\ 0\ 1)'$ allows us to interpret $z_t^* + \varphi_\rho^*$ as the central tendency of the real interest rate. $\boldsymbol{\varphi}^* = (0,\ 0,\ \varphi_\pi^{**},\ \varphi_\pi^*,\ (\varphi_\pi^* + \varphi_\rho^*))'$ (where these constants are to be estimated) makes the output gaps mean-reverting variables with zero asymptotes/central tendencies: $\bar{g}_t^{**} = \tilde{g}_t^{**} = \bar{g}_t^* = \tilde{g}_t^* = 0$. These long run tendencies are imposed by restricting the short run parameters: $\boldsymbol{\kappa}^* = (\mathbf{I} - \mathbf{\Phi}_1^*)\boldsymbol{\varphi}^*$; $\boldsymbol{\phi}_z^* = (\mathbf{I} - \mathbf{\Phi}_1^*)\mathbf{R}_1^*$; $\boldsymbol{\phi}_f^* = (\mathbf{I} - \mathbf{\Phi}_1^*)\mathbf{R}_2^*$.

[5] We use bold font $\mathbf{0}_{i,j}$ to denote an i by j zero matrix; bold font $\mathbf{0}$ to denote a zero matrix with appropriate dimension; bold font $\mathbf{I}_i$ to denote an i by i identity matrix; bold font $\mathbf{I}$ to denote an identity matrix with appropriate dimension.

2.3 The UK macro KVAR

Under $\mathcal{P}$, the general form of the UK macro model is given by a VAR(N) process. The BIC test (footnote 4) indicated that a second order difference system was necessary in this case ($N = 2$):

$$\mathbf{x}_t = \boldsymbol{\kappa} + \boldsymbol{\phi}_f f_t + \boldsymbol{\phi}_z z_t + \boldsymbol{\theta}\mathbf{X}_t^* + \boldsymbol{\Phi}_1 \mathbf{x}_{t-1} + \boldsymbol{\Phi}_2 \mathbf{x}_{t-2} + \mathbf{w}_t \quad (3)$$

$$\text{where : } \mathbf{w}_t = \mathbf{G}\mathbf{D}^x \boldsymbol{\epsilon}_t^x, \boldsymbol{\epsilon}_t^x \sim N(\mathbf{0}, \mathbf{I})$$

where $\mathbf{x}_t = [g_t, \pi_t, r_t]'$ is the observed UK macro vector, and $\mathbf{X}_t^* = (z_t^*, f_t^*, \mathbf{x}_t^{*\prime})'$ is the OECD-US state vector. The 3×7 matrix $\boldsymbol{\theta}$ parameterizes the various spillover effects from the state vector of the world sub-model to the UK economy. This is specified in the Section 2.4. The error term $\mathbf{w}_t$ is a 3×1 vector; $\mathbf{D}^x$ is a diagonal matrix with positive diagonal elements; $\mathbf{G}$ is a lower triangular matrix with unit diagonal. The model allows $\mathbf{x}_t$ to be affected by its lagged values, the real factor z_t, the nominal factor f_t, and the OECD-US state vector $\mathbf{X}_t^*$. The UK factor dynamics are similar to those for the world and are given by (2) after dropping subscripts.

In the closed economy version of the UK model (i.e. model M0, where $\boldsymbol{\theta} = \mathbf{0}$), the central tendency of $\mathbf{x}_t$ is driven by the factors z_t and f_t. In this case (3) has the central tendency $\tilde{x}_t = (\tilde{g}_t, \tilde{\pi}_t, \tilde{r}_t)' : \tilde{x}_t = (\mathbf{I} - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2)^{-1} \left(\boldsymbol{\kappa} + \boldsymbol{\phi}_z z_t + \boldsymbol{\phi}_f f_t\right) = \boldsymbol{\varphi} + \mathbf{R}_1 z_t + \mathbf{R}_2 f_t$,with the asymptote: $\bar{x}_t = \boldsymbol{\varphi} + \mathbf{R}_2 f_t$. We employ the cointegration restrictions: $\mathbf{R}_1 = (0\ 0\ 1)'$; $\mathbf{R}_2 = (0\ 1\ 1)'$ and $\boldsymbol{\varphi} = (0, \varphi_\pi, (\varphi_\pi + \varphi_\rho))'$ which are imposed as: $\boldsymbol{\phi}_z = (\mathbf{I} - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2)\mathbf{R}_1$, $\boldsymbol{\phi}_f = (\mathbf{I} - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2)\mathbf{R}_2$, $\boldsymbol{\kappa} = (\mathbf{I} - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2)\boldsymbol{\varphi}$. The M1 model (where $\boldsymbol{\theta} \neq \mathbf{0}$) allows for global influences and in this case the inflation and interest rate asymptotes are affected by f_t^*. The scale of this latent variable is however fixed by the $\mathbf{R}^*$ restrictions of the world model and its effect on UK inflation and interest rate effects are not restricted. As explained in Section 2.4, we apply restrictions (similar to those on $\mathbf{R}$) to ensure that it has the same asymptotic effect on interest and inflation rates with no asymptotic effect on the output gap.

2.4 Modeling global effects on the UK

One of the most important ways that macroeconomic impulses can be transmitted between different countries is through their bilateral exchange rates. However, it can be shown that if the terminal (or equilibrium) value of the real exchange rate is constant, the arbitrage-free complete-market assumptions of the macro-finance approach imply that exchange rates can be expressed as log-linear functions of the state variables of the model. So in the specific case of the UK, exchange rate variables are redundant because their effects should be picked up by domestic and overseas variables already included in the model, in the same way that the effects of long yields on the economy are picked up by model variables in a single country complete market setting. In this framework, the exchange rate should not exert any additional effect on a country like the UK. This is the basic approach used in this paper.

The main objective of this study is to investigate various global effects on the UK economy. However we started with a conventional macro-finance model in which the world economy did not affect the UK. This was estimated as two separate models of the world and the UK. In each case we started with a macro-only model, adding nominal and real factors, which gave the dynamics under $\mathcal{P}$. The yield data and associated prices of risk were then added to each model. This gave the baseline closed UK economy model M0 with $\boldsymbol{\theta} = \mathbf{0}_{3,7}$, to which we then added external effects. Recall that there are 7 variables in $\mathbf{X}_t^*$ ($z_t^*, f_t^*,\ g_t^{**}, g_t^*, \pi_t^{**}, \pi_t^*$ and r_t^*). Since the first two of these are latent variables, we call them 'common factors' in line with the literature cited in the introduction. Since the last five are observable variables we describe their influence as 'spillover' effects. In principle, all of these variables could affect the UK. This gives model M3 in which $\boldsymbol{\theta}$ in (3) is an unrestricted 3×7 coefficient matrix.

These external effects were investigated using the specification. $\boldsymbol{\theta} = (\mathbf{I} - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2)\,\boldsymbol{\Xi}$. This allows for 7 different types of spillover, which affect the UK dynamics through the parameters of the 3×7 matrix $\boldsymbol{\Xi}$. This matrix shows the long run effects, which can be restricted (as in $\mathbf{R}$). The 3×7 matrix $(\mathbf{I} - \mathbf{\Phi}_1 - \mathbf{\Phi}_2)\,\mathbf{\Xi}$ shows the short run impacts. In model M3 these are all unrestricted. However in model M2 the long run effects of the factors (z_t^* and f_t^*) are consistent with our interpretation that they represent central tendencies of world real interest and inflation rates. Thus, the long run effects of z_t^* on UK output and inflation are set to zero ($\chi_{11} = \chi_{21} = 0$). Then the nominal variables (f_t^*, π_t^{**}, π_t^* and r_t^*) are constrained in a way that ensures that the long run effect of the world inflation factor (and the other nominal variables which are anchored to this) only affects the UK's inflation rate, leaving its output gap and real interest rate unchanged. First, the UK output gap is isolated from nominal external variables using the restrictions: $\chi_{12} = \chi_{15} = \chi_{16} = \chi_{17} = 0$, where χ_{ij} is the element in the i-th row and j-th column of the matrix $\mathbf{\Xi}$. Second, a rise in the real world interest rate (a rise in r_t^*, with other world variables held constant) only affects UK nominal (and real) interest rates ($\chi_{27} = 0$). Third, a rise in all nominal world variables has the same long-run effect on UK inflation and nominal interest rates, leaving real rates unchanged. This combined effect is given by the parameter ν_{22} and the restriction is enforced by setting $\chi_{22} = \nu_{22} - \chi_{25} - \chi_{26}$ and $\chi_{23} = \nu_{22} - \chi_{35} - \chi_{36} - \chi_{37}$. This gives model M2, with the parameter matrix:

$$\mathbf{\Xi} = \begin{pmatrix} 0 & 0 & \chi_{13} & \chi_{14} & 0 & 0 & 0 \\ 0 & (\nu_{22} - \chi_{25} - \chi_{26}) & \chi_{23} & \chi_{24} & \chi_{25} & \chi_{26} & 0 \\ \chi_{31} & (\nu_{22} - \chi_{35} - \chi_{36} - \chi_{37}) & \chi_{33} & \chi_{34} & \chi_{35} & \chi_{36} & \chi_{37} \end{pmatrix}. \tag{4}$$

These constraints on $\mathbf{\Xi}$ ensure that the effects of the I(1) variables (f_t^*, π_t^{**}, π_t^* and r_t^*) preserve the zero tendency for g_t and have identical effects on π_t and r_t. Thus model M2 has 13 more parameters than M0, but 8 fewer then M3.

2.5 The companion form

The companion or state space form of a model is obtained by writing it in the form of a first order difference equation. In the case of the world sub-system we stack (1) and (2), write the model as

$$\mathbf{X}_t^* = \mathbf{K}^* + \mathbf{\Phi}^*\mathbf{X}_{t-1}^* + \mathbf{W}_t^* \tag{5}$$

where $\mathbf{X}_t^* = (z_t^*, f_t^*, \mathbf{x}_t^{*\prime})'$; $\mathbf{W}_t^* = (v_t^*, u_t^*, \mathbf{w}_t^{*\prime})' = \mathbf{C}^*\mathbf{D}^*\boldsymbol{\epsilon}_t^*, \mathbf{W}_t^* \sim N(\mathbf{0}, \mathbf{\Omega}^*)$; $\mathbf{\Omega}^* = \mathbf{C}^*\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{C}^{*\prime}$; $\mathbf{K}^*$, $\mathbf{\Phi}^*$, $\boldsymbol{\epsilon}_t^*$, $\mathbf{C}^*$, $\mathbf{D}^*$ are defined in Appendix A.1. The macro models are estimated under the measure $\mathcal{P}$ showing the state density (i.e. the actual probability of any state). Financial models are usually developed under a risk-neutral probability measure such as $\mathcal{Q}^*$, which has the effect of adjusting these probabilities so that all assets have the same expected *dollar* rate of return. This adjustment is made by changing the parameters of (5) to get a congruent first-order process:

$$\mathbf{X}_t^* = \mathbf{K}^{*Q^*} + \mathbf{\Phi}^{*Q^*}\mathbf{X}_{t-1}^* + \mathbf{W}_t^{*Q^*} \tag{6}$$

$$where: \mathbf{K}^{*Q*} = \mathbf{K}^* - \mathbf{C}^*\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{\Lambda}_1^{**};\ \mathbf{\Phi}^{*Q*} = \mathbf{\Phi}^* - \mathbf{C}^*\mathbf{\Lambda}_2^*;\ \mathbf{W}_t^{*Q^*} \sim N(\mathbf{0}, \mathbf{\Omega}^*). \tag{7}$$

The parameters $\mathbf{\Lambda}_1^{**}$, and $\mathbf{\Lambda}_2^*$ reflect the 'prices of risk' associated with the world variables specified in Appendix A.2. Similarly the dynamics of $\mathbf{X}_t^*$ under the sterling risk-neutral measure $\mathcal{Q}$ (which equates sterling expected returns) are obtained by replacing $\mathbf{K}^{*Q^*}$ and $\mathbf{\Phi}^{*Q^*}$ in (6) by: $\mathbf{K}^{*Q}$ and $\mathbf{\Phi}^{*Q}$:

$$\mathbf{X}_t^* = \mathbf{K}^{*Q} + \mathbf{\Phi}^{*Q}\mathbf{X}_{t-1}^* + \mathbf{W}_t^{*Q} \tag{8}$$

$$where: \mathbf{K}^{*Q} = \mathbf{K}^* - \mathbf{C}^*\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{\Lambda}_1^*;\ \mathbf{\Phi}^{*Q} = \mathbf{\Phi}^* - \mathbf{C}^*\mathbf{\Lambda}_2^*; \mathbf{W}_t^{*Q} \sim N(\mathbf{0}, \mathbf{\Omega}^*) \tag{9}$$

The parameters $\mathbf{\Lambda}_1^*$ and $\mathbf{\Lambda}_2^*$ are specified in Appendix A.2.

The companion form of the UK macro model is:

$$\mathbf{X}_t = \mathbf{K} + \mathbf{\Theta}\mathbf{X}^*_{t-1} + \mathbf{\Phi}\mathbf{X}_{t-1} + \mathbf{W}_t \tag{10}$$

where $\mathbf{X}_t = \left(z_t, f_t, \mathbf{x}'_t, \mathbf{x}'_{t-1}\right)'$, $\mathbf{W}_t = \mathbf{A}\mathbf{D}^*\boldsymbol{\epsilon}^*_t + \mathbf{C}\mathbf{D}\boldsymbol{\epsilon}_t$, $\mathbf{W}_t \sim N(\mathbf{0}, \mathbf{\Omega})$; $\mathbf{\Omega} = \mathbf{A}\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{A}' + \mathbf{C}\mathbf{D}\mathbf{D}'\mathbf{C}'$ and the parameters $\mathbf{K}, \mathbf{\Theta}, \mathbf{\Phi}, \mathbf{A}, \mathbf{D}^*, \boldsymbol{\epsilon}^*_t, \mathbf{C}, \mathbf{D}, \boldsymbol{\epsilon}_t$ defined in Appendix A.1. Similarly the dynamics of $\mathbf{X}_t$ under the sterling risk-neutral measure $\mathcal{Q}$ (which equates sterling expected returns) are given by

$$\mathbf{X}_t = \mathbf{K}^Q + \mathbf{\Theta}^Q\mathbf{X}^*_{t-1} + \mathbf{\Phi}^Q\mathbf{X}_{t-1} + \mathbf{W}^{*Q}_t \tag{11}$$

$$where: \mathbf{K}^Q = \mathbf{K} - \mathbf{A}\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{\Lambda}^*_1 - \mathbf{C}\mathbf{D}\mathbf{D}'\mathbf{\Lambda}_1;\ \mathbf{\Phi}^Q = \mathbf{\Phi} - \mathbf{C}\mathbf{\Lambda}_2;\ \mathbf{\Theta}^Q = \mathbf{\Theta} - \mathbf{A}\mathbf{\Lambda}^*_2; \mathbf{W}^{*Q}_t \sim N(\mathbf{0}, \mathbf{\Omega}). \tag{12}$$

The parameters $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ are specified in Appendix A.2.

2.6 Yield models

Under the Gaussian assumptions, US dollar and UK sterling discount (or zero coupon) bond prices are log-linear in the state variables (See Appendix A.2 for more details). In the first case, the price $P^*_{\tau,t}$ at time t of a dollar payment at time $t+\tau$ can be represented as:

$$-\ln P^*_{\tau,t} = \eta^*_\tau + \mathbf{\Psi}^*_\tau\mathbf{X}^*_t \tag{13}$$

The coefficients η^*_τ, $\mathbf{\Psi}^*_\tau$ are defined by the recursion:

$$\begin{aligned} \eta^*_\tau &= \eta^*_{\tau-1} + \mathbf{\Psi}^*_{\tau-1}\mathbf{K}^{*Q*} - \frac{1}{2}\mathbf{\Psi}^*_{\tau-1}\mathbf{\Omega}^*\mathbf{\Psi}^{*\prime}_{\tau-1} \\ \mathbf{\Psi}^*_\tau &= \mathbf{J}^*_r + \mathbf{\Psi}^*_{\tau-1}\mathbf{\Phi}^{*Q*} = \mathbf{J}^*_r\left[\mathbf{I} - \left(\mathbf{\Phi}^{*Q*}\right)^\tau\right]\left(\mathbf{I} - \mathbf{\Phi}^{*Q*}\right)^{-1} \end{aligned} \tag{14}$$

with initial conditions: $\eta_1^* = 0, \mathbf{\Psi}_1^* = \mathbf{J}_r^*$, where $\mathbf{J}_r^*$ is a selection vector (with zero-one elements) that extracts r_t^* from $\mathbf{X}_t^* : r_t^* = \mathbf{J}_r^* \mathbf{X}_t^*$.

The external effects mean that the UK bond price is affected by the US state vector $\mathbf{X}_t^*$ as well as the UK state vector $\mathbf{X}_t$:

$$-\ln P_{\tau,t} = \eta_\tau + \mathbf{\Psi}_\tau \mathbf{X}_t^* + \mathbf{\Upsilon}_\tau \mathbf{X}_t \tag{15}$$

where $\eta_\tau, \mathbf{\Upsilon}_\tau, \mathbf{\Psi}_\tau$ are defined by the recursion relationships:

$$\eta_\tau = \eta_{\tau-1} + \mathbf{\Upsilon}_{\tau-1}\mathbf{K}^Q + \mathbf{\Psi}_{\tau-1}\mathbf{K}^{*Q} - \mathbf{\Upsilon}_{\tau-1}\mathbf{A}\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{C}\mathbf{\Psi}_{\tau-1}{}' - \frac{1}{2}\mathbf{\Psi}_{\tau-1}\mathbf{\Omega}^*\mathbf{\Psi}_{\tau-1}' - \frac{1}{2}\mathbf{\Upsilon}_{\tau-1}\mathbf{\Omega}\mathbf{\Upsilon}_{\tau-1}'$$

$$\mathbf{\Psi}_\tau = \mathbf{\Psi}_{\tau-1}\mathbf{\Phi}^{*Q} + \mathbf{\Upsilon}_{\tau-1}\mathbf{\Theta}^Q;\ \mathbf{\Upsilon}_\tau = \mathbf{J}_r + \mathbf{\Upsilon}_{\tau-1}\mathbf{\Phi}^Q \tag{16}$$

with: $\eta_1 = 0, \mathbf{\Psi}_1 = \mathbf{0}, \mathbf{\Upsilon}_1 = \mathbf{J}_r$, where $\mathbf{J}_r$ is a selection vector that extracts r_t from $\mathbf{X}_t$: $r_t = \mathbf{J}_r \mathbf{X}_t$.

The log bond models are obtained by substituting (7), (9) and (12) and the recursive solutions (14) and (16) into (13) & (15). Dividing by maturity and adding an error term gives the two empirical yield models:

$$y_{\tau,t}^* = -\ln P_{\tau,t}^*/\tau = \alpha_\tau^* + \mathbf{B}_\tau^* \mathbf{X}_t^* + e_{\tau,t}^* \tag{17}$$

$$y_{\tau,t} = -\ln P_{\tau,t}/\tau = \alpha_\tau + \mathbf{B}_\tau \mathbf{X}_t^* + \mathbf{\Gamma}_\tau \mathbf{X}_t + e_{\tau,t} \tag{18}$$

where $y_{\tau,t}^*$ and $y_{\tau,t}$ are the yields of the τ-period US and UK discount bonds respectively at time t; $e_{\tau,t}^*$ and $e_{\tau,t}$ are measurement errors; $\alpha_\tau^* = \eta_\tau^*/\tau$, $\mathbf{B}_\tau^* = \mathbf{\Psi}_\tau^*/\tau$, $\alpha_\tau = \eta_\tau/\tau$, $\mathbf{B}_\tau = \mathbf{\Psi}_\tau/\tau$, $\mathbf{\Gamma}_\tau = \mathbf{\Upsilon}_\tau/\tau$; $e_{\tau,t}^* \sim N(0, \sigma_\tau^{*2})$, $e_{\tau,t} \sim N(0, \sigma_\tau^2)$; $\sigma_\tau^* > 0, \sigma_\tau > 0$. The slope coefficients $\mathbf{B}_\tau^*$, $\mathbf{B}_\tau$ and $\mathbf{\Gamma}_\tau$ are known as the 'factor loadings' in this literature. Stacking the two yield models for

M different maturities:

$$\mathbf{y}_t^* = \boldsymbol{\alpha}^* + \mathbf{B}^*\mathbf{X}_t^* + \mathbf{e}_t^*, \mathbf{e}_t^* \sim N(\mathbf{0}, \boldsymbol{\Sigma}^*) \tag{19}$$

$$\mathbf{y}_t = \boldsymbol{\alpha} + \mathbf{B}\mathbf{X}_t^* + \boldsymbol{\Gamma}\mathbf{X}_t + \mathbf{e}_t, \mathbf{e}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}) \tag{20}$$

where $\mathbf{y}_t^* = [y_{m_1,t}^*, y_{m_2,t}^* ... y_{m_M,t}^*]'$; $\mathbf{y}_t = [y_{m_1,t}, y_{m_2,t} ... y_{m_M,t}]'$. We use 1, 2, 3, 5, 7, 10 and 15 year maturity rates for both bond markets, so that we have $\{m_1, m_2, m_3, ..., m_M\} = \{4, 8, 12, 20, ..., 60\}$ measured in calendar quarters. $\boldsymbol{\alpha}^*, \mathbf{B}^*, \mathbf{e}_t^*, \boldsymbol{\Sigma}^*, \boldsymbol{\alpha}, \mathbf{B}, \boldsymbol{\Gamma}, \mathbf{e}_t, \boldsymbol{\Sigma}$ are defined as: $\boldsymbol{\alpha}^* = [\alpha_4^*, \alpha_8^*, ...\alpha_{60}^*]'$, $\boldsymbol{\alpha} = [\alpha_4, \alpha_8, ...\alpha_{60}]'$, $\mathbf{B}^* = [\mathbf{B}_4^{*\prime}, \mathbf{B}_8^{*\prime}, ...\mathbf{B}_{60}^{*\prime}]'$, $\mathbf{B} = [\mathbf{B}_4', \mathbf{B}_8', ...\mathbf{B}_{60}']'$, $\boldsymbol{\Gamma} = [\boldsymbol{\Gamma}_4', \boldsymbol{\Gamma}_8', ...\boldsymbol{\Gamma}_{60}']'$, $\mathbf{e}_t^* = diag\left(e_{4,t}^*, e_{8,t}^*, ..., e_{60,t}^*\right)$, $\mathbf{e}_t = diag\left(e_{4,t}, e_{8,t}, ..., e_{60,t}\right)$, $\boldsymbol{\Sigma}^* = diag\left(\sigma_4^{*2}, \sigma_8^{*2}, ..., \sigma_{60}^{*2}\right)$ and $\boldsymbol{\Sigma} = diag\left(\sigma_4^2, \sigma_8^2, ..., \sigma_{60}^2\right)$.

2.7 The Kalman filter

The equations discussed so far are recursive. (1) and (3) show how the four real and nominal rate factors influence the macroeconomic variables, along with the lagged macro variables and the equation residuals[6]. Then (19) and (20) show how all these variables influence the bond yields, without these yields feeding back into the system at this stage. However the system is closed by a set of Kalman Filters (appendix A.3), which updates the four factors every quarter using the prediction errors in the observable macro and yield variables. Consequently, the closed model is highly simultaneous. In each period, the macro observations affect the yield curve via the factor revisions as well as their residuals. The bond yields affect the macro variables through the factor revisions. Indeed, it can be shown that the affine nature of the yield equations (19) and (20) means that the factors effectively play the same role in this kind of macro system as bond yields do when introduced directly into a macro VAR, as in Evans and Marshall (1998) for example[7].

[6] These residuals are conventionally labelled 'measurement errors', but also reflect model misspecification errors.

[7] The yield equations impose a linear relationship between each yield and the factors & macro variables. They add a measurement error, which is typically very small. This means yields can be closely approximated by linear

3 The empirical model

3.1 Data

These models were estimated and tested using quarterly time series of the macro variables and yields from 1979 Q1 to 2007 Q1. The three output gaps (g_t^{**}, g_t^*, g_t) are measured as the logarithmic difference between actual and estimated potential GDP and are obtained from the OECD website, (vintage December 2007). Inflation rates are measured as the annual logarithmic change in the consumer price index (CPI). The OECD inflation rate π_t^{**} is calculated using the aggregate OECD CPI (excluding high inflation countries)[8]. The US inflation rate π_t^* is calculated using the all items CPI (Source: US Bureau of Labor Statistics). The UK inflation rate π_t^* is calculated using the RPIX price index, which excludes mortgage interest payments (Source: Office for National Statistics). The US short interest rate r_t^* is the 3-month Treasury Bill rate and similarly r_t is the 3-month UK Treasury Bill rate. The US and UK macroeconomic data are shown in Figures 1a and 1b**.** The US yield data are continuous compounded zero coupon rates taken from McCulloch and Kwon (1991), updated by the New York Federal Reserve Bank. The UK yield data are continuous compounded zero coupon rates taken from the Bank of England website. These yield data are shown in Figures 2**.** Data summary statistics of both macro and financial data are reported in table 2.

3.2 Estimation

We estimate the world & UK macro KVARs (5, 10) and the US & UK yield models (19, 20) simultaneously using the Kalman Filter and Maximum Loglikelihood Estimation method (appen-

functions of the factors and macro variables. In this model the error variances are of the order of 10^{-10} to 10^{-14} for the 3 & 7 year maturities in both the US and UK markets. Since the macro equations are also linear, this means that we could get a model that was almost identical in likelihood if we replaced the lagged factors in say (1) by lagged 3 & 7 year US yields.

[8] OECD aggregate measures are GDP-weighted averages of individual OECD countries where the weight for the UK is 5.55%, and 35.94% for the US.

dix A.3). The loglikelihood is optimized numerically using the Nelder-Mead Simplex algorithm to solve the optimization problem (implemented in the MatLab *fminsearch* function).

The baseline closed UK economy model M0 has 146 parameters[9]. The M2 and M3 open economy models have 159 and 167 parameters respectively. Table 4 reports the optimized loglikelihood of each model. The LR tests in this table provide the basic result of this paper, showing that the closed economy model M0 is strongly rejected in favour of the open economy models. This table also shows the result of a LR test of M2 against M3, showing that the restrictions embodied in (4) are accepted at the conventional 95% level. Further work on this model showed that many of the price of risk parameters were poorly determined. Sequential elimination of risk parameters with a $t-$value of less than unity gave the model reported as M1.

3.3 Empirical results

In view of these findings we now focus on the estimates obtained from the M1 model[10]. Table 5 reports the parameter estimates and $t-$values. The fitted values of the macro and yield variables are shown in Figures 1 and 2. The estimated common and country specific factors are shown in Figure 3. This reveals a sharp downward movement in f^*as oil prices fell in 1986 followed by a gradual downtrend, which this model identifies as a common global trend. The factor z^* clearly coincides with movements in US monetary policy, increasing sharply during the Volker deflation of 1979-80 and falling sharply under the Greenspan stimuli of 1991-2 and 2001-2[11]. The decline in f exhibits strong downward movements around the time of the 1982 Falklands War and the 1983 election which gave Mrs.Thatcher a second term in office and again after the election of Mr. Blair in 1997 when the Bank of England gained its independence. Figure 4 shows the central

[9] These are: (ξ^*, ξ, δ^{z*}, δ^{f*}, δ^z, δ^f, $\boldsymbol{\varphi}(2)$, $\boldsymbol{\varphi}^*(3)$, $\mathbf{\Phi}_1^*\,(25)$, $\mathbf{\Phi}_1\,(9)$, $\mathbf{\Phi}_2\,(9)$, $\mathbf{\Xi}(0)$, $\mathbf{G}^*\,(10)$, $\mathbf{G}\,(3)$, $\mathbf{D}^{x*}\,(5)$, $\mathbf{D}^x\,(3)$, λ_1^{z**}, λ_1^{f**}, $\boldsymbol{\lambda}_1^{x**}\,(5)$, λ_1^{z*}, λ_1^{f*}, $\boldsymbol{\lambda}_1^{x*}\,(5)$, λ_1^z, λ_1^f, $\boldsymbol{\lambda}_1^x\,(3)$, λ_2^{z*}, λ_2^{f*}, λ_2^z, λ_2^f, $\mathbf{\Lambda}_2^{x*}\,(25)$, $\mathbf{\Lambda}_2^x\,(9)$, $\mathbf{\Sigma}^*\,(7)$, $\mathbf{\Sigma}\,(7)$).

[10] Estimates for the other models are available upon request from the authors.

[11] Subsequent regression analysis supports this interpretation, showing that it is negatively correlated with an indicator of credit availability taken from the Federal Reserve's Loan Officer Survey.

tendencies in US and UK inflation and interest rates implied by these factors.

3.3.1 Macroeconomic behavior

These macro–yield systems are first analyzed in terms of their impulse responses. These show the effect that a one period unit shock of each type would have on the system and are calculated using the Wold representation of the system as described in Hamilton (1994). Because these shocks are correlated empirically, we cannot interpret them as supply, demand, monetary or other shocks. Instead we follow convention and work with orthogonalized innovations (i.e. ϵ^*_{t+1} and ϵ_{t+1}) using the triangular factorization defined in (5) and (10). However, our factor models do distinguish the effects of non-stationary nominal shocks (which are picked up by f^*_t and f_t) from those of the stationary real shocks (represented by z^*_t and z_t) and the transition errors in the macro equations. Since they are cointegrated, nominal shocks have a permanent effect on the nominal variables of the system, while other effects are transient. As in the model of Kozicki and Tinsley (2005), the use of Kalman filters to pick up the effect of unobservable inflation shocks helps to solve the notorious price puzzle - the tendency for increases in policy interest rates to anticipate inflationary developments and apparently cause inflation.

The OECD-US macro responses are depicted in Figure 5. Although US monetary policy influences the rest of the OECD, other US macroeconomic variables apparently have little effect. However, OECD-wide shocks have a very significant effect on the US - note in particular the large short-run effect of g^{**} on g^* and π^{**} on π^*. Figure 6a shows how these external shocks influence the UK. z^* and r^* have a strong short-run impact on r, which has the effect of depressing output temporarily. Inflation and interest rates respond quickly to f^*, which has a permanent effect. Spillovers from OECD and US macro shocks are also significant, in particular π^{**} and π^* both seem to influence π positively in the short run. US output (g^*) has a remarkably strong impact on UK output, consistent with the old saying that ‘if the US sneezes the UK catches a cold’.

While these external responses are oscillatory, the responses to the UK shocks shown in Figure 6b are relatively fast and monotonic.

3.3.2 Yield curve behavior

The impulse responses for the yield models are depicted in Figure 8, 9a and 9b. They combine these macro responses with the yield factor loadings. These loadings depend upon the dynamics of the system under the risk neutral measures (14 and 16), which in turn depend upon the risk adjustments as well as the dynamic behavior under $\mathcal{P}$. Many of the risk adjustment coefficients were poorly determined and were eliminated in M1. The remaining coefficients are reported in Table 5c.

The orthogonality and admissibility assumptions mean that the factor risk premia [12] associated with each latent variable just depend upon that variable. For example, the dollar risk premium associated with exposure to z_t^* is: $\delta^{z*}\lambda_t^{z**} = -(\lambda_1^{z**}(\delta^{z*})^2 + \lambda_2^{z*}z_t^*)$. Empirically, λ_2^{z*} was insignificant in M2 and was set to zero in M1, meaning this premium is constant over time. The parameters $\delta^{f*}, \lambda_1^{f**}$ and λ_2^{f*} determining the premia associated with f_t^* are all highly significant. The factor risk premia for the macro variables (21) depend linearly upon these premia as well as those associated with the residual macro effects, which are determined by $\boldsymbol{\lambda}_1^{x**}$ and $\boldsymbol{\Lambda}_2^{x*}$ in (25) and (26). One notable feature of these results is that changes in the nominal US interest rate have a very significant effect on the prices of risk associated with the three nominal world variables ($\boldsymbol{\Lambda}_{2,i5}^{x*}, i = 3, 4, 5$). Parameters ($\lambda_1^z$, λ_2^z and λ_2^f) associated with exposure to z and f in the UK market are all significant.

The implied factor loadings for the US market are depicted in Figure 7a and are strongly influenced by z^* and f^*. Although the loadings on the macro residuals are relatively small, the

[12] Since this is a complete financial market model, it is possible to form portfolios that track each of the shocks and state variables, being uniquely exposed to the associated risk. The 'price of risk' associated with any variable is the factor risk premium or expected excess return on its tracker portfolio, divided by its standard deviation. This is a measure of the associated risk-reward ratio.

spot rate provides the link between the macroeconomic model and the term structure and by construction has a unit coefficient at a maturity of one quarter. The spot rate loadings decline rapidly over the next few maturities, where the yield curve is strongly influenced by the behavior of the real rate factor. In contrast, because f^* is persistent the loading on this moves up to unity quickly and stays relatively high, so that it acts as a 'level' factor. Figure 7b shows the UK loadings. As in the US, the UK spot rate influences the very short yields; while 1 - 5 year maturity yields are strongly influenced by the behavior of the real rate factors z and z^* and the long yield is largely determined by the nominal factors: f and f^*. The US spot rate also has some impact in the 1-2 year area, while the effect of the OECD and US output gap and inflation residuals seem to be more influential than those of the UK.

3.3.3 Analysis of Variance

The contribution of different shocks to the volatility of the system can be shown using Analysis of Variance (ANOVA) techniques. The results indicate the effect these shocks would have at different time horizons if the error process was suddenly started (having been dormant previously), as a percentage of the total variance, again using the Wold representation. This exercise takes into account both the relative volatility of these shocks and their effect on the variables of the system shown by the impulse responses. The triangular factorization (5) and (10) works like a principal component factorization, attributing the maximum influence to the latent variables and only a residual influence to the macro shocks, which have relatively little impact on the variance of the system.

The key distinction here is between the non-stationary nominal variables and the stationary real variables. The former follow a common world stochastic trend or random walk (with an additional trend influencing the UK variables) and thus have unbounded asymptotic variance. This means that the proportion of the variance in the nominal variables attributed to the nominal factors f_t^* and f_t approaches 100% as the time horizon increases, overshadowing the effect of the stationary shocks. However, the other shocks do influence stationary indicators like the output gap and the real interest rate. They also affect the slope & curvature of the yield curve which are mimicked by stationary yield differentials: respectively the 15 year less one year and the 5 year less average of the 15 year and one year yields[13]. The 'level' component is mimicked by the 15 year yield which is non-stationary.

This distinction is clearly reflected in Figure 10a, which reports the results for the US macro variables. The second panel shows that the volatility of the nominal variables f^*, π^{**},π^* and r^* is progressively dominated by the effect of shocks to f^*. The short run volatility of US inflation and interest rates appear to be affected by world rather than US inflation shocks. This effect is also evident in the UK results reported in Figure 10b. Tables 6a and 6b show the ANOVA results for the 1, 5 and 15 year maturity yields, decomposing their variances into the effect of the factors and macro residuals. These are progressively dominated by the shocks to the nominal factors, with which they cointegrate. Figures 11a and 11b show the results for the associated yield components. The level components, being non-stationary, are again dominated by f_t^* and f_t. However, the real rate factors strongly influence the behavior of the 'slope' components, with additional effects coming from the output gaps and spot rates. Consistent with the view that the business cycle affects curvature, the output gaps have a marked effect on the curvature components in both markets.

4 Conclusion

We develop a multi-country macro-finance model to study international economic and financial linkages. This allows for cross-country effects between the US and other OECD countries by introducing aggregate OECD output and inflation variables into a standard closed economy US

[13] We are very grateful to a referee for this suggestion.

KVAR. We find that although US monetary policy influences the rest of the OECD, other US macroeconomic variables have little effect. OECD output and inflation variables have a significant impact on the US, suggesting that it is inappropriate to model the US using the standard closed economy macro-finance specification. Consistent with earlier work on international economic linkages, we find strong evidence of a common inflation trend and real business cycle, as well as real interest rate spillovers. More of a surprise, the OECD and US inflation trends are cointegrated, moving together on a one-for-one basis. The residual effects of OECD output and inflation do not have much impact in the US Treasury market, but nor do US output and inflation, a finding consistent with previous closed-economy macro-finance studies.

We then look at the effects that these world economic variables have on the UK economy and Treasury bond market. The results confirm our prior view that it is inappropriate to model countries such as the UK using a closed economy specification. We find that the global inflation factor affects UK inflation, although its effect is less than one-for-one and is complemented by a UK-specific inflation trend. The global real interest rate factor is also influential. These linkages help explain the comovement of the US and UK bond yields. OECD output residuals have a particularly strong effect on UK inflation. Moreover, although the US business cycle does not appear to have much of an effect on the rest of the OECD, it does have a significant effect on the UK.

In contrast to the mainstream finance models of the bond market (Duffie and Kan (1996), Dai and Singleton (2000), and Duffee (2002)) which only employ yield data, the latent variables and parameters of this model reflect innovations in macroeconomic as well as yield data. The latent variables are aligned with inflation and real interest rate trends. They dominate the behavior of the 'slope' and ' level' components of the yield curve, while the residual errors in the macro equations help explain the 'curvature' component, consistent with the view that this is associated with the business cycle. The model is consistent with the traditional three-factor finance specification in this sense, but links these factors to the behavior of the macroeconomy. This research opens the way to new open-economy studies of monetary policy and a much richer bond market specification, incorporating the best features of the international interdependence and macro-finance models.

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A Appendix

A.1 The companion form of the macro models

Stacking (1) and (2) gives the companion form (5) where: $\mathbf{W}_t^* = \mathbf{C}^*\mathbf{D}^*\boldsymbol{\epsilon}_t^*$, $\mathbf{K}^* = (0, 0, \boldsymbol{\kappa}^{*\prime})'$ and $\boldsymbol{\Phi}^*$, $\boldsymbol{\Omega}^*$, $\mathbf{C}^*$,$\mathbf{D}^*$ are defined as:

$$\boldsymbol{\Phi}^* = \begin{pmatrix} \xi^* & 0 & \mathbf{0}_{1,5} \\ 0 & 1 & \mathbf{0}_{1,5} \\ \xi^*\boldsymbol{\phi}_z^* & \boldsymbol{\phi}_f^* & \boldsymbol{\Phi}_1^* \end{pmatrix}; \mathbf{C}^* = \begin{pmatrix} 1 & 0 & \mathbf{0}_{1,5} \\ 0 & 1 & \mathbf{0}_{1,5} \\ \boldsymbol{\phi}_z^* & \boldsymbol{\phi}_f^* & \mathbf{G}^* \end{pmatrix}; \boldsymbol{\epsilon}_t^* = \begin{pmatrix} \epsilon_t^{z*} \\ \epsilon_t^{f*} \\ \boldsymbol{\epsilon}_t^{x*} \end{pmatrix}, \mathbf{D}^* = \begin{pmatrix} \delta^{z*} & 0 & \mathbf{0}_{1,5} \\ 0 & \delta^{f*} & \mathbf{0}_{1,5} \\ \mathbf{0}_{5,1} & \mathbf{0}_{5,1} & \mathbf{D}^{x*} \end{pmatrix}$$

Similarly, stacking (3) and the UK analogue of (2) gives the companion form of the UK macro model (10), where $\mathbf{K}, \boldsymbol{\Theta}, \boldsymbol{\Phi}, A, \mathbf{C}, \mathbf{D}, \boldsymbol{\epsilon}_t$ are defined as:

$$\mathbf{K} = \begin{pmatrix} 0 \\ 0 \\ \boldsymbol{\kappa} + \boldsymbol{\theta}\mathbf{K}^* \\ \mathbf{0}_{1,3} \end{pmatrix}, \boldsymbol{\Theta} = \begin{pmatrix} \mathbf{0}_{1,7} \\ \mathbf{0}_{1,7} \\ \boldsymbol{\theta}\boldsymbol{\Phi}^* \\ \mathbf{0}_{1,3} \end{pmatrix}, \boldsymbol{\epsilon}_t = \begin{pmatrix} \epsilon_t^{z} \\ \epsilon_t^{f} \\ \boldsymbol{\epsilon}_t^{x} \\ \mathbf{0}_{1,3} \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \mathbf{0}_{1,7} \\ \mathbf{0}_{1,7} \\ \boldsymbol{\theta}\mathbf{C}^* \\ \mathbf{0}_{3,7} \end{pmatrix}$$

$$\boldsymbol{\Phi} = \begin{pmatrix} \xi & 0 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ 0 & 1 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ \xi\boldsymbol{\phi}_z & \boldsymbol{\phi}_f & \boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_2 \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{I}_3 & \mathbf{0}_{3,3} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ 0 & 1 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ \boldsymbol{\phi}_z & \boldsymbol{\phi}_f & \mathbf{G} & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \delta^{z} & 0 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ 0 & \delta^{f} & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{D}^{x} & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} \end{pmatrix}$$

A.2 Modeling the price of risk and bond prices

The shift from the state price density $\mathcal{P}$ to any risk neutral measure is implemented by multiplying the state probability density by an appropriate state-dependent non-negative utility weight (called the Radon-Nikodym derivative). For the US market this is M^*_{t+1} and for the UK it is M_{t+1}. The expected values of a variable like X^*_{t+1} or $P^*_{\tau-1,t+1}$ under $\mathcal{P}$ and $\mathcal{Q}^*$ are thus related by: $E^{Q^*}_t\left[X^*_{t+1}\right] = E_t\left[M^*_{t+1}X^*_{t+1}\right]$[14]. Similarly, dropping the asterisks for the UK: $E^Q_t\left[X_{t+1}\right] = E_t\left[M_{t+1}X_{t+1}\right]$. In the essentially affine specification of Duffee (2002) these adjustment factors are loglinear in the state variables: $-\ln M^*_{t+1} = \frac{1}{2}\boldsymbol{\Lambda}^{**\prime}_t\boldsymbol{\Lambda}^{**}_t + r^*_t + \boldsymbol{\Lambda}^{**\prime}_t\boldsymbol{\epsilon}^*_{t+1}$; $-\ln M_{t+1} = \frac{1}{2}\boldsymbol{\Lambda}^{*\prime}_t\boldsymbol{\Lambda}^*_t + \frac{1}{2}\boldsymbol{\Lambda}'_t\boldsymbol{\Lambda}_t + r_t + \boldsymbol{\Lambda}^{*\prime}_t\boldsymbol{\epsilon}^*_{t+1} + \boldsymbol{\Lambda}'_t\boldsymbol{\epsilon}_{t+1}$. $\boldsymbol{\Lambda}^{**}_t$ and $\boldsymbol{\Lambda}^*_t$ are price vectors of the risks which are associated with shocks to the US state vector X^*_{t+1} under $\mathcal{Q}^*$ and $\mathcal{Q}$ respectively; $\boldsymbol{\Lambda}_t$ is the price risk vector associated with shocks to X_{t+1} under $\mathcal{Q}$. It can be shown that the expectations under $\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{Q}^*$ are related by:

$$E^{Q*}_t\left(\mathbf{X}^*_{t+1}\right) = E_t\left(\mathbf{X}^*_{t+1}\right) - \mathbf{C}^*\mathbf{D}^*\boldsymbol{\Lambda}^{**}_t \tag{21}$$

and

$$E^Q_t\begin{pmatrix}\mathbf{X}^*_{t+1}\\ \mathbf{X}_{t+1}\end{pmatrix} = E_t\begin{pmatrix}\mathbf{X}^*_{t+1}\\ \mathbf{X}_{t+1}\end{pmatrix} - \begin{pmatrix}\mathbf{C}^*\mathbf{D}^* & \mathbf{0}\\ \mathbf{AD}^* & \mathbf{CD}\end{pmatrix}\begin{pmatrix}\boldsymbol{\Lambda}^*_t\\ \boldsymbol{\Lambda}_t\end{pmatrix} \tag{22}$$

These equations explain the factor risk premia in terms of the prices of risk. For example (21) takes $\boldsymbol{\Lambda}^{**}_t$, the vector showing the excess return to standard deviation ratios associated with $\boldsymbol{\epsilon}^*_{t+1}$; multiplies these by the standard deviations $\mathbf{D}^*$ to get their differential drift and then maps these into the factor risk premia (the differential drift in $\mathbf{X}^*_{t+1}$) using $\mathbf{C}^*$. Following Duffee (2002), these prices of risk are affine in the state vector. For the US dollar investors, they price market

[14] The operator E^M_t denotes the conditional expectation at time t under measure $\mathcal{M}$. V_t is the variance operator, which is the same under all measures.

risks as below:

$$\mathbf{\Lambda}_t^{**} = \mathbf{D}^* \mathbf{\Lambda}_1^{**} + \mathbf{D}^{*-1} \mathbf{\Lambda}_2^{**} \mathbf{X}_t^* \tag{23}$$

where $\mathbf{D}^*$ is defined in (5). For the UK investors, the prices of risk are:

$$\begin{pmatrix} \mathbf{\Lambda}_t^* \\ \mathbf{\Lambda}_t \end{pmatrix} = \begin{pmatrix} \mathbf{D}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{\Lambda}_1^* \\ \mathbf{\Lambda}_1 \end{pmatrix} + \begin{pmatrix} \mathbf{D}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{\Lambda}_2^* & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_2 \end{pmatrix} \begin{pmatrix} \mathbf{X}_t^* \\ \mathbf{X}_t \end{pmatrix} \tag{24}$$

$\mathbf{D}$ is defined in (10). We assume that the price of risk vectors $\mathbf{\Lambda}_t^{**}$ and $\mathbf{\Lambda}_t^*$ only differ in the constant terms, so we have $\mathbf{\Lambda}_2^{**} = \mathbf{\Lambda}_2^*$. The parameters $\mathbf{\Lambda}_1^{**}$, $\mathbf{\Lambda}_2^{**}$, $\mathbf{\Lambda}_1^*$, $\mathbf{\Lambda}_1$, $\mathbf{\Lambda}_2^*$ and $\mathbf{\Lambda}_2$ are specified as below:

$$\mathbf{\Lambda}_1^{**} = \begin{pmatrix} \lambda_1^{z**} \\ \lambda_1^{f**} \\ \boldsymbol{\lambda}_1^{x**} \end{pmatrix}, \mathbf{\Lambda}_1^* = \begin{pmatrix} \lambda_1^{z*} \\ \lambda_1^{f*} \\ \boldsymbol{\lambda}_1^{x*} \end{pmatrix}, \mathbf{\Lambda}_2^{**} = \mathbf{\Lambda}_2^* = \begin{pmatrix} \lambda_2^{z*} & 0 & \mathbf{0}_{1,5} \\ 0 & \lambda_2^{f*} & \mathbf{0}_{1,5} \\ \mathbf{0}_{5,1} & \mathbf{0}_{5,1} & \mathbf{\Lambda}_2^{x*} \end{pmatrix}, \tag{25}$$

$$\mathbf{\Lambda}_1 = \begin{pmatrix} \lambda_1^{z} \\ \lambda_1^{f} \\ \boldsymbol{\lambda}_1^{x} \\ \mathbf{0}_{3,1} \end{pmatrix}, \mathbf{\Lambda}_2 = \begin{pmatrix} \lambda_2^{z} & 0 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ 0 & \lambda_2^{f} & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{\Lambda}_2^{x} & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} \end{pmatrix}, \tag{26}$$

where $\boldsymbol{\lambda}_1^{x**}, \boldsymbol{\lambda}_1^{x*}$ are 5 by 1 column vectors; $\boldsymbol{\lambda}_1^{x}$ is a 3 by 1 column vector; $\mathbf{\Lambda}_2^{x*}$ is a 5 by 5 matrix and $\mathbf{\Lambda}_2^{x}$ is a 3 by 3 matrix and the rest of the parameters are scalars. Substituting (5), (10), (23), and (24) into (21) and (22) gives (7), (9) and (12).

Absent arbitrage, the prices of the $\tau-$period US discount bonds at time t are given by the time-discounted values of the risk-neutral expectations of their prices in the next period:

$$P_{\tau,t}^* = \exp\left(-r_t^*\right) E_t^{Q*}\left[P_{\tau-1,t+1}^*\right] \tag{27}$$

where r_t^* is the one-period risk-free US interest rate generated by (6). Since the state variables are normally distributed, these prices are lognormally distributed, with expected values given by the well known formula for the expectation of a lognormal variable: $\ln P_{\tau,t}^* = \exp[-r_t^* + E_t^{Q*}\left(\ln P_{\tau-1,t+1}^*\right) + \frac{1}{2}V_t\left(\ln P_{\tau-1,t+1}^*\right)]$. Substituting (6) and (13) into it and equating coefficients on the state variables with those in (13) gives the recursion restrictions (14). Similarly, dropping asterisks from the above log equation gives the UK bond relationship: $\ln P_{\tau,t} = \exp[-r_t + E_t^Q\left(\ln P_{\tau-1,t+1}\right) + \frac{1}{2}V_t\left(\ln P_{\tau-1,t+1}\right)]$. Substituting (8), (11), and (15) into it and equating coefficients with those in (15) give (16).

A.3 Kalman Filter Estimation

The model is estimated using the Kalman filter. We rewrite the system in the state-space form (Harvey (1989)):

$$\mathbf{Z}_t = \mathbf{M} + \mathbf{N}\mathbf{Z}_{t-1} + \mathbf{V}_t, \mathbf{V}_t \sim N(\mathbf{0}, \mathbf{\Pi}) \tag{28}$$

$$\mathbf{Y}_t = \mathbf{L} + \mathbf{H}\mathbf{Z}_t + \mathbf{E}_t, \mathbf{E}_t \sim N(\mathbf{0}, \mathbf{Q}) \tag{29}$$

The 15×1 state vector is $\mathbf{Z}_t = [\mathbf{X}_t^{*\prime}, \mathbf{X}_t^{\prime}]^{\prime}$, the 22×1 observation vector is $\mathbf{Y}_t = [\mathbf{y}_t^{*\prime}, \mathbf{x}_t^{*\prime}, \mathbf{y}_t^{\prime}, \mathbf{x}_t^{\prime}]^{\prime}$ and:

$$\mathbf{V}_t = \begin{pmatrix} \mathbf{C}^* & \mathbf{0} \\ \mathbf{A} & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{D}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \boldsymbol{\epsilon}_t^* \\ \boldsymbol{\epsilon}_t \end{pmatrix}.$$

The transition equation (28) is the joint OECD-US-UK macro model that consists of (5) and (10), where the parameters are $\mathbf{M}^{\prime} = [\mathbf{K}^{*\prime}, \mathbf{K}^{\prime}]$ and:

$$\mathbf{N} = \begin{pmatrix} \mathbf{\Phi}^* & \mathbf{0} \\ \mathbf{\Theta} & \mathbf{\Phi} \end{pmatrix}, \mathbf{\Pi} = \begin{pmatrix} \mathbf{C}^* & \mathbf{0} \\ \mathbf{A} & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{D}^*\mathbf{D}^{*\prime} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}\mathbf{D}^{\prime} \end{pmatrix} \begin{pmatrix} \mathbf{C}^* & \mathbf{0} \\ \mathbf{A} & \mathbf{C} \end{pmatrix}^{\prime}$$

The measurement equation (29) consists of the yield models (19) and (20), and has the parameters:

$$\mathbf{L} = \begin{pmatrix} \boldsymbol{\alpha}^* \\ \mathbf{0} \\ \boldsymbol{\alpha} \\ \mathbf{0} \end{pmatrix}, \mathbf{H} = \begin{pmatrix} \mathbf{B}^* & \mathbf{0} \\ [\mathbf{0}, \mathbf{I}_3, \mathbf{0}] & \mathbf{0} \\ \mathbf{B} & \boldsymbol{\Gamma} \\ \mathbf{0} & [\mathbf{0}, \mathbf{I}_3, \mathbf{0}] \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} \boldsymbol{\Sigma}^* \, \mathbf{0} \, \mathbf{0} \, \mathbf{0} \\ \mathbf{0} \;\; \mathbf{0} \, \mathbf{0} \, \mathbf{0} \\ \mathbf{0} \;\; \mathbf{0} \, \boldsymbol{\Sigma} \, \mathbf{0} \\ \mathbf{0} \;\; \mathbf{0} \, \mathbf{0} \, \mathbf{0} \end{pmatrix}$$

The Kalman filter (Harvey (1989)) uses the predicting equations:

$$\hat{\mathbf{Z}}_{t|t-1} = \mathbf{M} + \mathbf{N}\hat{\mathbf{Z}}_{t-1} \tag{30a}$$

$$\mathbf{P}_{t|t-1} = \mathbf{N}\mathbf{P}_{t-1}\mathbf{N}' + \boldsymbol{\Pi} \tag{30b}$$

and updating equations:

$$\hat{\mathbf{Z}}_t = \hat{\mathbf{Z}}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{H}'\mathbf{F}_t^I \left(\mathbf{Y}_t - \mathbf{L} - \mathbf{H}\hat{\mathbf{Z}}_{t|t-1}\right) \tag{31a}$$

$$\mathbf{P}_t = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{H}'\mathbf{F}_t^I\mathbf{H}\mathbf{P}_{t|t-1} \tag{31b}$$

$$\mathbf{F}_t = \mathbf{H}\mathbf{P}_{t|t-1}\mathbf{H}' + \mathbf{Q};\ \mathbf{F}_t^I = \mathbf{F}_t^{-1};\ \mathbf{F}_t^D = |\mathbf{F}_t| \tag{31c}$$

The conditional mean for $\mathbf{Y}_t$ at time $(t-1)$ is: $\hat{\mathbf{Y}}_{t|t-1} = E_{t-1}(\mathbf{Y}_t) = \mathbf{L} + \mathbf{H}\hat{\mathbf{Z}}_{t|t-1}$ and the prediction error of $\mathbf{Y}_t$ at one period ahead is,

$$\mathbf{u}_t = \mathbf{Y}_t - \hat{\mathbf{Y}}_{t|t-1} = \mathbf{Y}_t - \left(\mathbf{L} + \mathbf{H}\hat{\mathbf{Z}}_{t|t-1}\right) \tag{32}$$

Given initial conditions for the state vector $\mathbf{Z}_t$ and the covariance matrix of the estimation error $\mathbf{P}_t$, $\hat{\mathbf{Z}}_t, \hat{\mathbf{Z}}_{t|t-1}, \hat{\mathbf{Y}}_{t|t-1}$ and $\mathbf{u}_t$ follow by using these equations recursively for each period. They are substituted into the log-likelihood function:

$$\ln L\left(\mathbf{P}|\mathbf{Y}_{t=1,\ldots,T}\right)=\sum_{t=1}^{T}\ln f\left(\mathbf{Y}_t|\mathbf{Y}_{t-1},\mathbf{P}\right)=c-\frac{1}{2}\sum_{t=1}^{T}\ln\mathbf{F}_t^D-\frac{1}{2}\sum_{t=1}^{T}\mathbf{u}_t'\mathbf{F}_t^I\mathbf{u}_t \tag{33}$$

where $f\left(\mathbf{Y}_t|\mathbf{Y}_{t-1}\right)$ is the probability distribution function of $\mathbf{Y}_t$ conditional on $\mathbf{Y}_{t-1}$; $\mathbf{F}_t^D,\mathbf{F}_t^I$ are defined in (31); $\mathbf{u}_t$ is defined in (32); c is the constant term. The parameter set $\mathbf{P}$ is obtained by optimizing (33).

B Tables

Table 2: Data summary statistics: 1979Q1-2007Q1

Macro Variables								
	g^{**}	π^{**}	g^{*}	π^{*}	r^{*}	g	π	r
Mean	-0.553	3.872	-0.763	3.998	6.089	-1.188	4.817	8.142
Std.	1.510	2.614	1.951	2.728	3.273	2.505	3.605	3.484
Skew.	-0.259	1.732	-0.896	2.050	0.819	-0.305	2.027	0.499
Excess Kurt.	0.493	2.288	1.675	3.718	0.571	-0.155	4.038	-0.977
ADF	-2.967*	-1.749	-3.367*	-2.248	-1.804	-2.999*	-1.867	-1.624
The US Yield (subscript is the number of month to maturity)								
	y_4^*	y_8^*	y_{12}^*	y_{20}^*	y_{28}^*	y_{40}^*	y_{60}^*	
Mean	6.535	6.813	7.005	7.278	7.469	7.635	7.955	
Std.	3.300	3.214	3.101	2.965	2.861	2.731	2.565	
Skew.	0.699	0.658	0.658	0.701	0.702	0.701	0.682	
Excess Kurt.	0.316	0.121	0.012	-0.081	-0.226	-0.290	-0.302	
ADF	-1.528	-1.389	-1.538	-1.344	-1.198	-1.131	-1.117	
The UK Yield (subscript is the number of month to maturity)								
	y_4	y_8	y_{12}	y_{20}	y_{28}	y_{40}	y_{60}	
Mean	8.140	8.197	8.249	8.334	8.391	8.384	8.169	
Std.	3.213	3.081	3.027	3.013	3.042	3.054	2.925	
Skew.	0.319	0.263	0.247	0.217	0.195	0.195	0.221	
Excess Kurt.	-1.131	-1.066	-0.986	-0.972	-1.014	-1.001	-0.864	
ADF	-1.454	-1.454	-1.474	-1.42	-1.299	-1.146	-1.078	

Note:
1. The data are annualized percentage rate.
2. 5% significance level for ADF test is -2.89; the lags of ADF test is determined by AIC.
3. Output gaps are from OECD website; CPI/RPIX inflation and 3 month Treasury bill rates are from Datastream. Yields are discount bond equivalent data compiled by the New York Fed and Bank of England. Mean denotes sample arithmetic mean expressed as percentage p.a.; Std. standard deviation and Skew.& Excess Kurt are standard measures of skewness (third moment) and excess kurtosis (fourth moment). ADF is the Adjusted Dickey-Fuller statistic for the null of non-stationarity. The 5% significance level is (-)2.877.

Table 3: Cointegration tests

Dependent Variable	π_t^*
Regressor	π_t^{**}
Coefficient (β)	$\beta = 0.998$
Residual (u_t)	$u_t = \pi_t^* - \beta * \pi_t^{**}$
ADF test	
H0: u_t is I(1)	-4.385(*)
LR test	
H0: $\beta = 1$	0.004

Note:
(*) This hypothesis is rejected at 5% significant level, indicating that π_t^* and π_t^{**} are cointegrated. The LR test accepts the hypothesis $\beta = 1$, implying that the cointegrating vector for π_t^* and π_t^{**} is [1,-1].

Table 4: Estimated Model Log Likelihood and LR tests

Model	Parameters			Loglikelihood		
Specification	k(M)	k(3)-k(M)	k(2)-k(M)	L(M)	2(L(3)-L(M))	2(L(2)-L(M))
M0	146	21	13	274.0	279.8(*)	271.4(*)
Closed economy					(0.00)	(0.00)
				H0:	M0	M0
				H1:	M3	M2
M1:	133	34	26	406.6	14.6	6.2
Preferred					(0.9985)	(0.9999)
				H0:	M1	M1
				H1:	M3	M2
M2:	159	8		409.7	8.4	
Restricted					(0.3954)	
open economy				H0:	M2	
				H1:	M3	
M3:						
Unrestricted	167			413.9		
open economy						

(*) The hypothesis is rejected at the 5% significance level.
The number in brackets is the p-value: the probability that the null hypothesis is true.

Table 5a: Estimated macro dynamic parameters for Model M1

Parameters	Estimates	t-value	Parameters	Estimates	t-value
$\Phi^*_{1,11}$	0.8242	15.45	$\Phi_{1,11}$	0.6828	7.38
$\Phi^*_{1,12}$	0.1234	2.98	$\Phi_{1,12}$	-0.1843	-2.19
$\Phi^*_{1,13}$	-0.1857	-3.16	$\Phi_{1,13}$	-0.0977	-1.12
$\Phi^*_{1,14}$	0.1055	2.02	$\Phi_{1,21}$	0.5278	8.32
$\Phi^*_{1,15}$	-0.0310	-1.21	$\Phi_{1,22}$	1.0974	13.79
$\Phi^*_{1,21}$	-0.1363	-1.45	$\Phi_{1,23}$	0.0595	0.71
$\Phi^*_{1,22}$	1.0149	13.68	$\Phi_{1,31}$	0.0604	3.37
$\Phi^*_{1,23}$	-0.3201	-2.67	$\Phi_{1,32}$	0.0693	1.68
$\Phi^*_{1,24}$	0.1735	1.73	$\Phi_{1,33}$	0.5781	11.39
$\Phi^*_{1,25}$	-0.0233	-0.49	$\Phi_{2,11}$	0.2865	2.99
$\Phi^*_{1,31}$	0.0047	0.09	$\Phi_{2,12}$	-0.0575	-0.77
$\Phi^*_{1,32}$	0.0329	0.79	$\Phi_{2,13}$	0.0762	0.99
$\Phi^*_{1,33}$	0.6701	10.48	$\Phi_{2,21}$	-0.4055	-5.74
$\Phi^*_{1,34}$	0.2520	4.73	$\Phi_{2,22}$	-0.2392	-3.51
$\Phi^*_{1,35}$	0.0220	0.79	$\Phi_{2,23}$	-0.1659	-2.23
$\Phi^*_{1,41}$	-0.0774	-0.77	$\Phi_{2,31}$	0.1403	6.50
$\Phi^*_{1,42}$	0.2278	2.86	$\Phi_{2,32}$	-0.0385	-1.25
$\Phi^*_{1,43}$	0.0871	0.79	$\Phi_{2,33}$	0.0905	2.15
$\Phi^*_{1,44}$	0.8478	8.94	ξ^*	0.8582	67.28
$\Phi^*_{1,45}$	-0.0229	-0.42	ξ	0.7348	9.48
$\Phi^*_{1,51}$	-0.0007	-0.01	φ^{**}_π	0.0118	6.71
$\Phi^*_{1,52}$	0.2374	2.59	φ^*_π	0.0125	6.74
$\Phi^*_{1,53}$	0.3511	2.18	φ^*_ρ	0.0060	4.98
$\Phi^*_{1,54}$	-0.0048	-0.03	φ_π	0.0109	6.31
$\Phi^*_{1,55}$	0.2457	4.67	φ_ρ	0.0112	4.51

Table 5b: Estimated volatility parameters for Model M1

Parameters	Estimates	t-value	Parameters	Estimates	t-value
δ^{z*}	0.0018	14.1292	G^*_{53}	0.5317	2.6448
δ^{f*}	0.0014	11.3162	G^*_{54}	0.2398	1.3346
Δ^{x*}_{11}	0.0009	23.4850	G_{21}	-0.3603	-6.5879
Δ^{x*}_{22}	0.0008	24.6268	G_{31}	0.1836	6.4429
Δ^{x*}_{33}	0.0009	23.7912	G_{32}	0.4221	5.1873
Δ^{x*}_{44}	0.0010	22.6541	σ^*_4	0.0003	36.6620
Δ^{x*}_{55}	0.0017	18.3075	σ^*_8	0.0001	22.9169
δ^z	0.0020	12.3732	σ^*_{12}	1.0E−5	2.5469
δ^f	0.0010	11.7009	σ^*_{20}	0.0001	63.2917
Δ^x_{11}	0.0015	19.9312	σ^*_{28}	1.0E−5	2.0416
Δ^x_{22}	0.0016	19.4822	σ^*_{40}	0.0002	19.0227
Δ^x_{33}	0.0014	19.7143	σ^*_{60}	0.0004	30.8444
G^*_{21}	1.4566	17.5776	σ_4	0.0004	1.9664
G^*_{31}	0.1375	1.5553	σ_8	0.0002	45.0365
G^*_{32}	-0.0002	-0.0019	σ_{12}	1.0E−7	0.0317
G^*_{41}	0.5395	3.0397	σ_{20}	0.0001	39.1631
G^*_{42}	0.0384	0.2144	σ_{28}	1.0E−7	0.1263
G^*_{43}	1.6634	15.6134	σ_{40}	0.0003	33.3596
G^*_{51}	1.0666	5.5906	σ_{60}	0.0007	2.6044
G^*_{52}	0.0405	0.2551			

Table 5c: Estimated price of risk parameters for Model M1

Parameters	Estimates	t-value	Parameters	Estimates	t-value
λ_1^{z**}	758.7318	1.5306	$\lambda_{2,51}^{x*}$	0	-
λ_1^{f**}	-106.0564	-4.8775	$\lambda_{2,52}^{x*}$	-0.2681	-1.1415
$\lambda_{1,1}^{x**}$	0	-	$\lambda_{2,53}^{x*}$	0.4161	2.5814
$\lambda_{1,2}^{x**}$	0	-	$\lambda_{2,54}^{x*}$	-0.1783	-1.2249
$\lambda_{1,3}^{x**}$	22456.6296	1.2411	$\lambda_{2,55}^{x*}$	-0.3368	-3.6649
$\lambda_{1,4}^{x**}$	-23815.5112	-1.2395	λ_1^{z*}	0	-
$\lambda_{1,5}^{x**}$	-1380.8322	-0.6531	λ_1^{f*}	0	-
λ_2^{z*}	0	-	$\lambda_{1,1}^{x*}$	-2355.5854	-1.0160
λ_2^{f*}	0.0065	5.7526	$\lambda_{1,2}^{x*}$	0	-
$\lambda_{2,11}^{x*}$	0	-	$\lambda_{1,3}^{x*}$	-2414.3796	-1.9568
$\lambda_{2,12}^{x*}$	0.4194	2.5656	$\lambda_{1,4}^{x*}$	0	-
$\lambda_{2,13}^{x*}$	0	-	$\lambda_{1,5}^{x*}$	0	-
$\lambda_{2,14}^{x*}$	0	-	λ_1^{z}	-201.6264	-3.0430
$\lambda_{2,15}^{x*}$	0	-	λ_1^{f}	0	-
$\lambda_{2,21}^{x*}$	0.6248	1.8357	$\lambda_{1,1}^{x}$	0	-
$\lambda_{2,22}^{x*}$	0.4424	1.2427	$\lambda_{1,2}^{x}$	501.0141	1.1957
$\lambda_{2,23}^{x*}$	0	-	$\lambda_{1,3}^{x}$	0	-
$\lambda_{2,24}^{x*}$	-0.4484	-2.1009	λ_2^{z}	-0.1969	-2.5363
$\lambda_{2,25}^{x*}$	0	-	λ_2^{f}	-0.0192	-3.1862
$\lambda_{2,31}^{x*}$	-0.1982	-1.8486	$\lambda_{2,11}^{x}$	-0.4495	-6.2731
$\lambda_{2,32}^{x*}$	0	-	$\lambda_{2,12}^{x}$	0.9840	5.0833
$\lambda_{2,33}^{x*}$	0.1312	2.0654	$\lambda_{2,13}^{x}$	-0.2398	-3.0614
$\lambda_{2,34}^{x*}$	0	-	$\lambda_{2,21}^{x}$	0	-
$\lambda_{2,35}^{x*}$	0.2665	4.3331	$\lambda_{2,22}^{x}$	0.0926	1.7946
$\lambda_{2,41}^{x*}$	0	-	$\lambda_{2,23}^{x}$	-0.0517	-1.5931
$\lambda_{2,42}^{x*}$	0	-	$\lambda_{2,31}^{x}$	0.2457	55.9467
$\lambda_{2,43}^{x*}$	0	-	$\lambda_{2,32}^{x}$	-0.0839	-2.1866
$\lambda_{2,44}^{x*}$	0	-	$\lambda_{2,33}^{x}$	0	-
$\lambda_{2,45}^{x*}$	-0.3684	-3.7289			

Table 5d: Estimated spillover matrix parameters for Model M1

Parameters	Estimates	t-value	Parameters	Estimates	t-value
ν_{22}	0.6847	6.80	χ_{24}	0.4788	1.26
χ_{31}	0.1795	0.92	χ_{34}	1.6464	1.68
χ_{13}	-3.3870	-1.40	χ_{25}	0.4139	1.62
χ_{23}	-0.4078	-0.69	χ_{35}	0.7525	3.82
χ_{33}	-1.7629	-1.17	χ_{26}	0.2282	1.02
χ_{14}	2.6129	1.67	χ_{36}	-0.8067	-3.60
			χ_{37}	0.4198	2.76

Table 6a: Proportion of US yield variance explained by macro and latent factors

% Variance explained by:	at:	Forecast horizon (quarters)				
		1	4	20	60	∞
x^*	short	23.44	12.74	7.33	4.47	0
	medium	2.79	1.30	0.74	0.41	0
	long	0.51	0.21	0.10	0.05	0
		1	4	20	60	∞
z^*	short	60.48	59.17	23.39	10.75	0
	medium	41.81	33.10	11.46	4.46	0
	long	9.85	7.07	2.07	0.72	0
		1	4	20	60	∞
f^*	short	16.08	28.09	69.28	84.79	100
	medium	55.39	65.59	87.80	95.12	100
	long	89.64	92.72	97.82	99.83	100

This table shows the proportion of the total variance explained by shocks to the the nominal and real factors and those of the macro residuals. Elapsed time is measured in quarters. The non-stationarity of these yields means that the variance attributed to f_t^* (and f_t in table (b)) approaches 100% as the time horizon increases, overshadowing the effect of the stationary shocks. (Short yield = 1 year yield; medium = 5 year yield; long = 15 year yield.) Note: short - 1 year yield; medium - 5 year yield; long - 15 year yield.

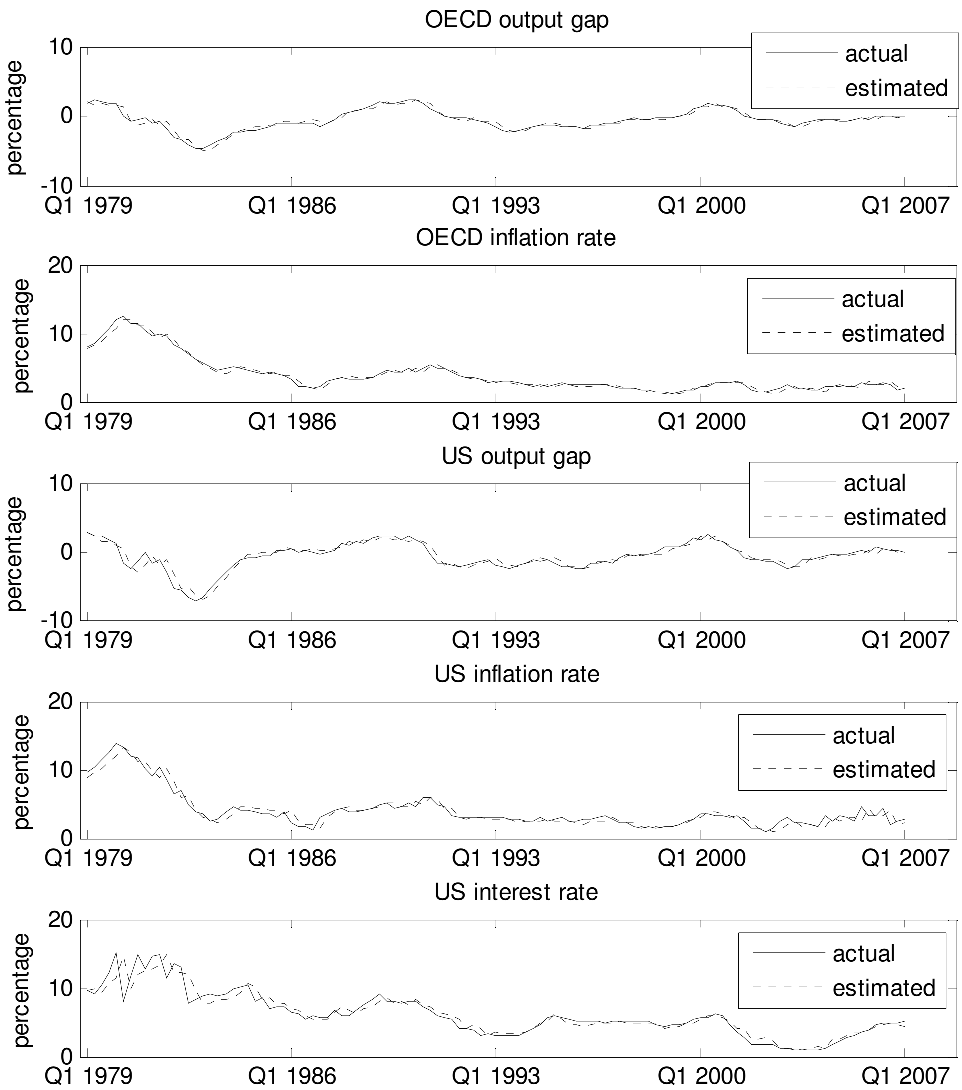
Table 6b: Proportion of UK yield variance explained by macro and latent factors

% Variance explained by:	at	Forecast horizon (quarters)				
		1	4	20	60	∞
x^*	short	5.21	4.11	3.68	3.38	0
	medium	2.96	2.31	1.04	0.55	0
	long	0.17	0.19	0.16	0.08	0
		1	4	20	60	∞
z^*	short	7.85	11.67	5.59	2.49	0
	medium	4.64	4.94	2.09	0.81	0
	long	0.48	0.53	0.26	0.09	0
		1	4	20	60	∞
f^*	short	6.25	13.10	35.88	38.06	46
	medium	16.46	21.96	30.20	28.79	29
	long	14.16	15.48	17.20	18.77	21
		1	4	20	60	∞
x	short	31.77	17.31	9.14	4.48	0
	medium	2.35	1.44	0.83	0.35	0
	long	1.31	0.85	0.44	0.17	0
		1	4	20	60	∞
z	short	40.92	36.95	11.04	4.85	0
	medium	43.88	29.85	8.74	3.35	0
	long	16.12	9.82	2.41	0.81	0
		1	4	20	60	∞
f	short	8.01	16.87	34.68	46.74	54
	medium	29.70	39.50	57.10	66.15	71
	long	67.75	73.13	79.53	80.09	79

Please see notes to Table 6a.

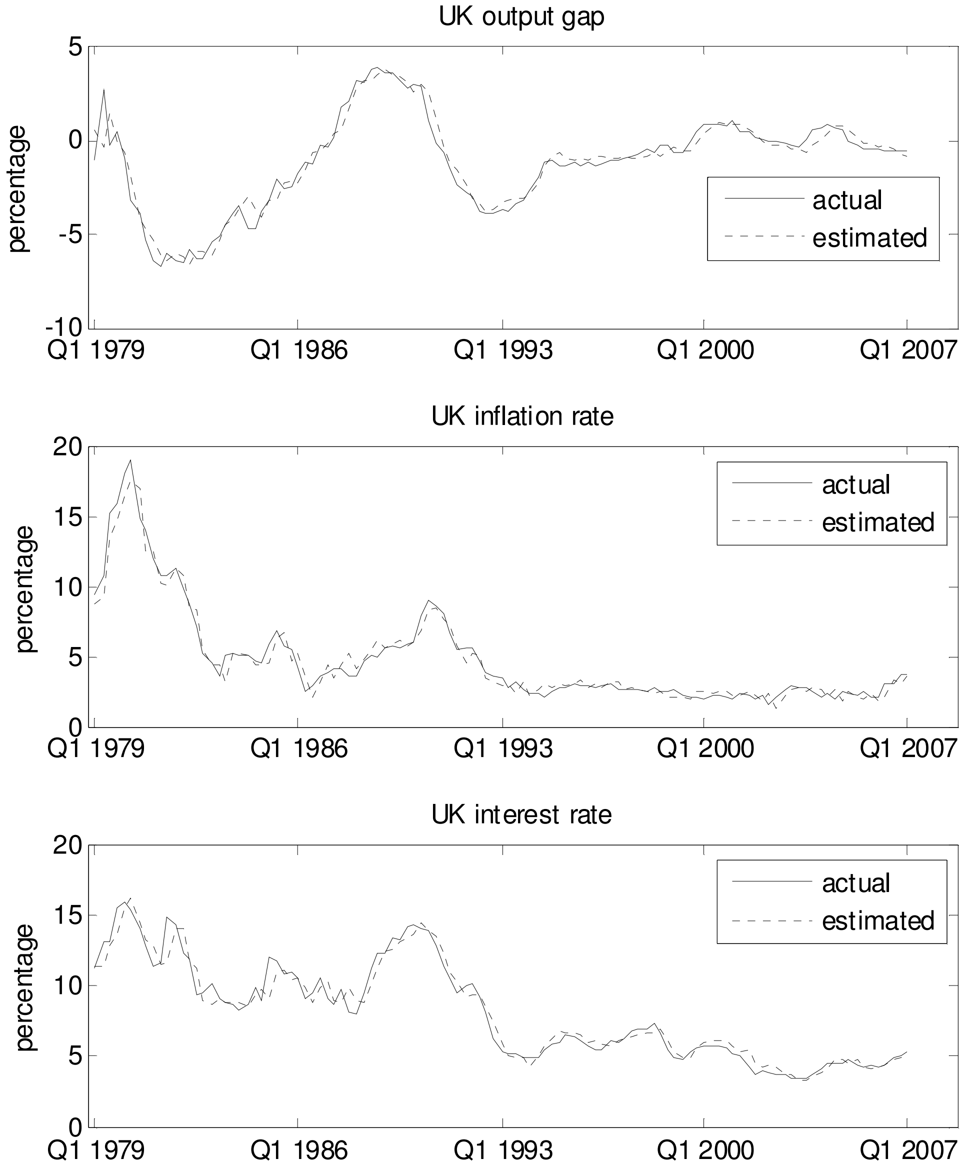
C Figures

Figure 1a: OECD US macro data



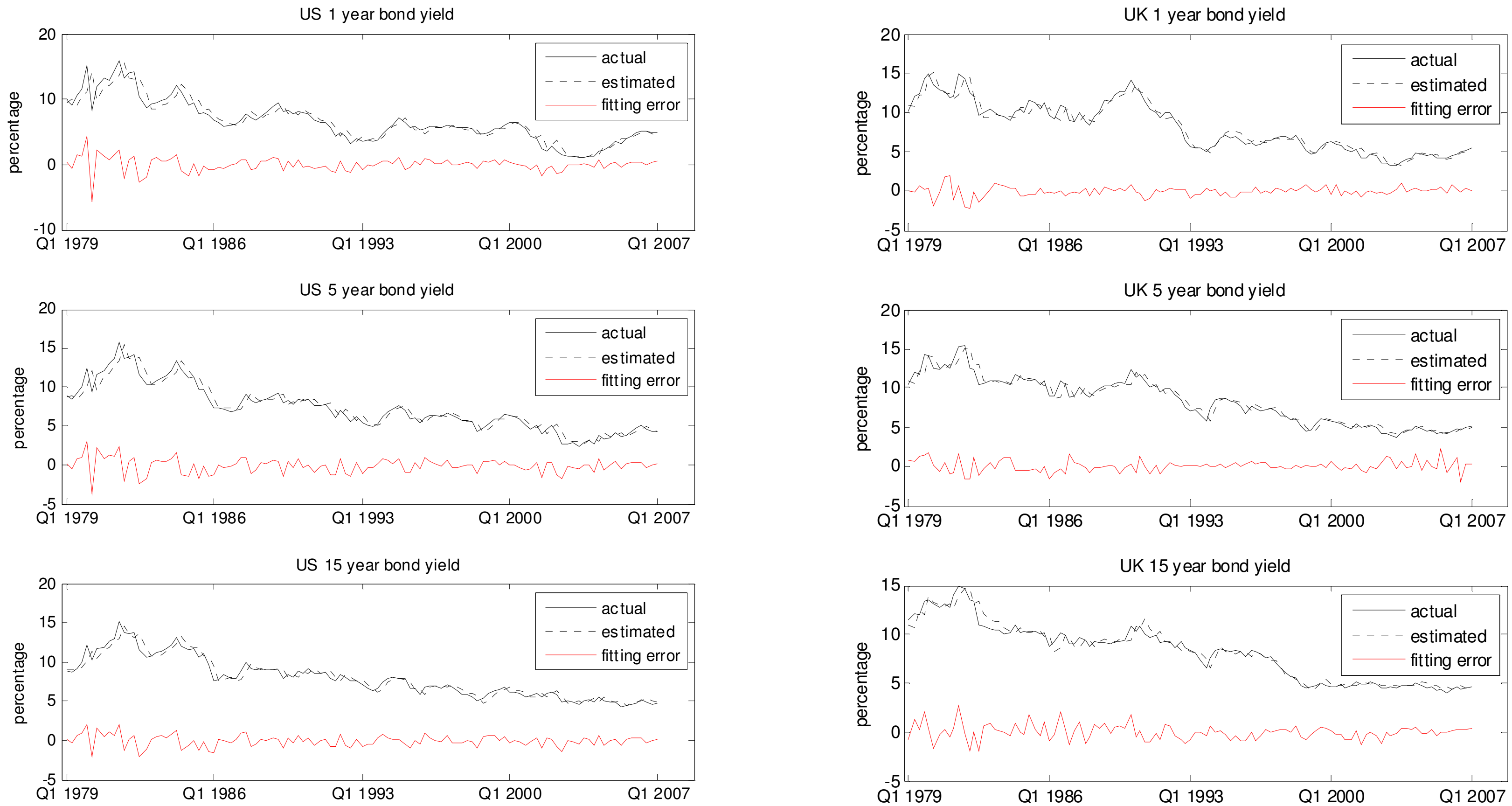
See notes to Table 2.

Figure 1b: UK macro data



See notes to Table 2

Figure 2: US and UK yield data



See notes to Table 2

Figure 3: The estimated latent factors

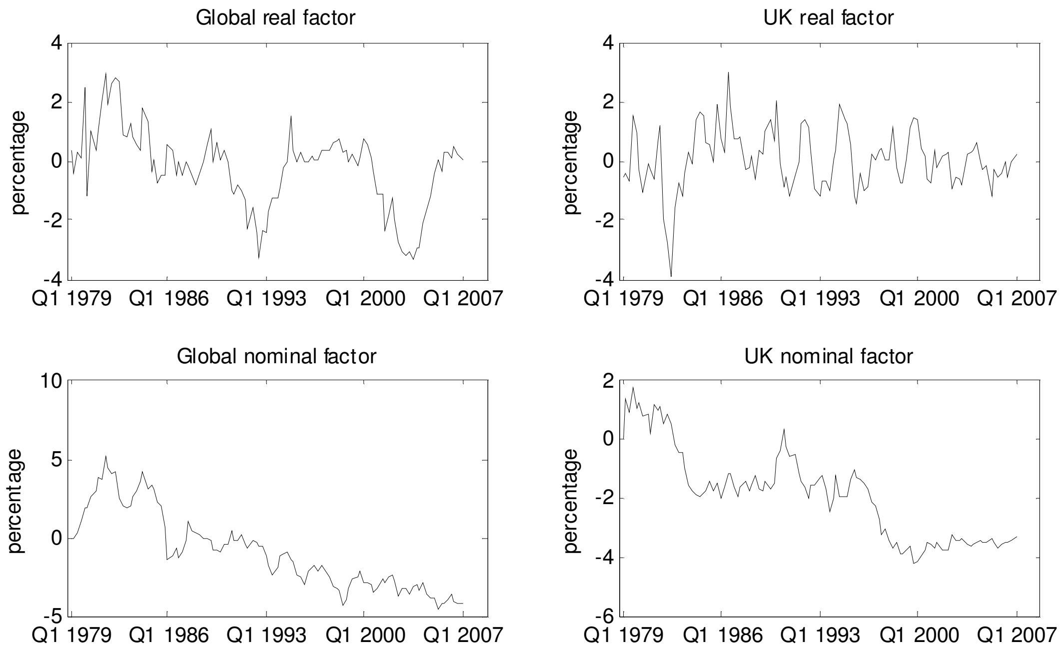
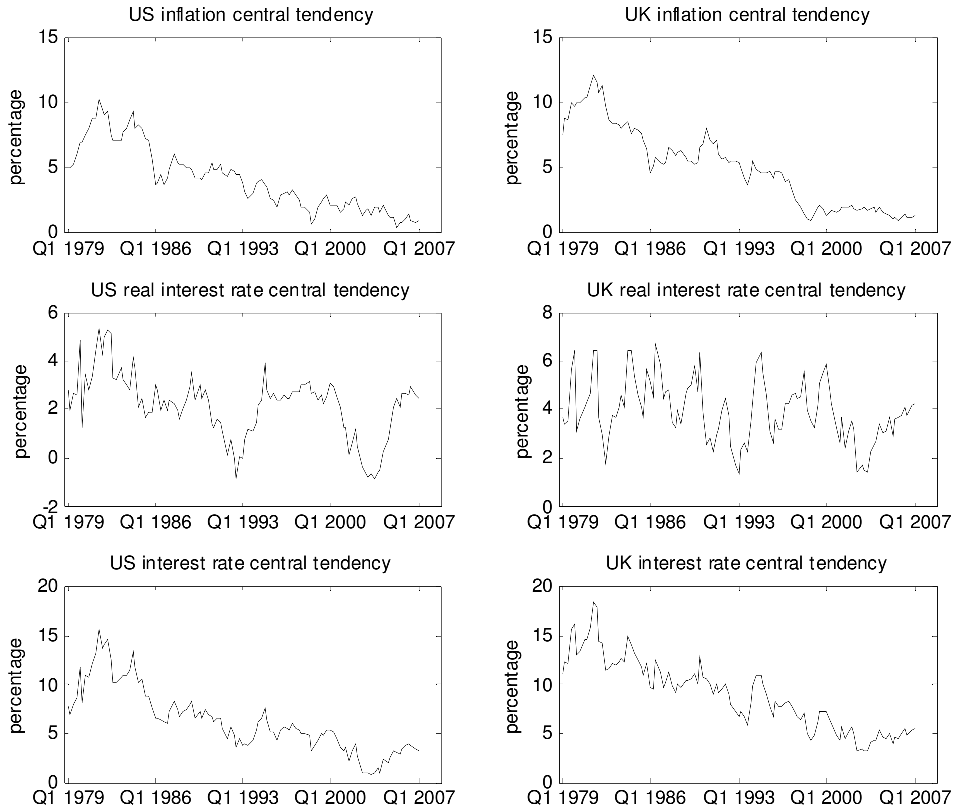


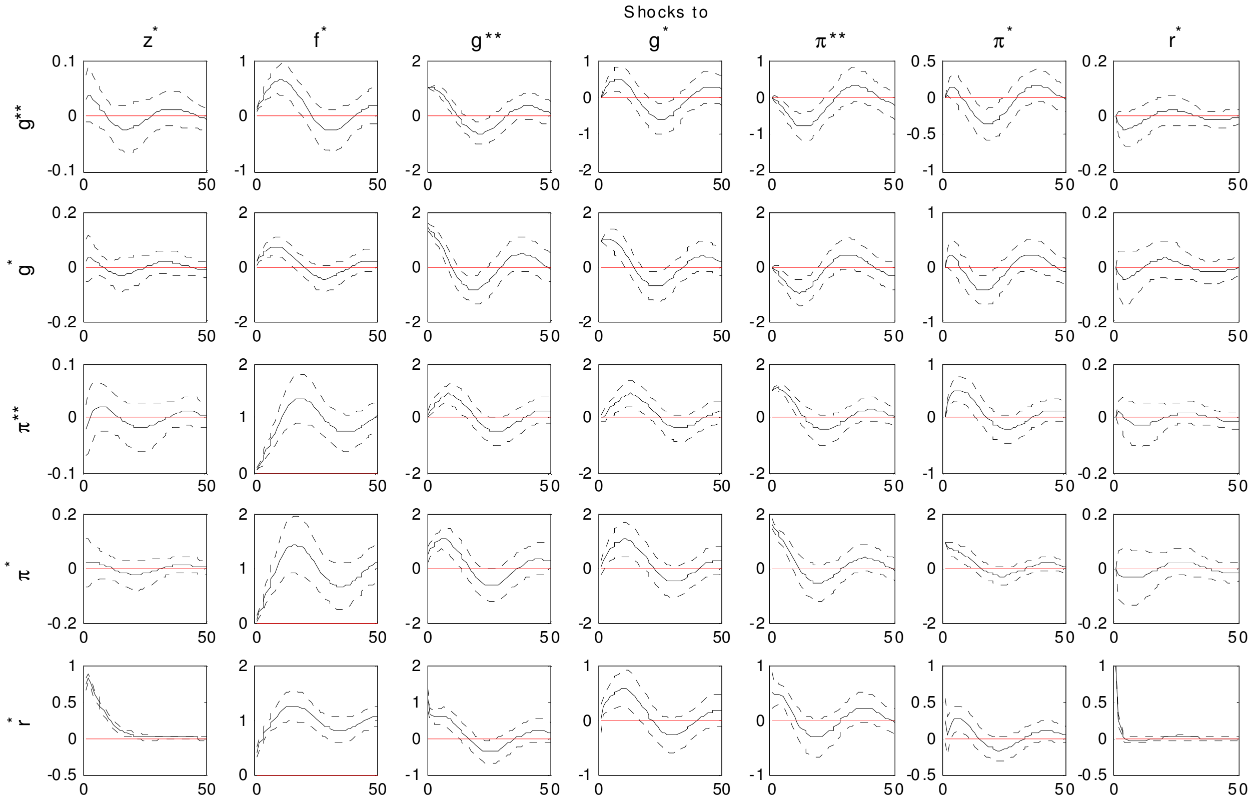
Figure 4: The central tendencies (or underlying values) of inflation and interest rates



The central tendencies of US inflation and interest rates are $\varphi_{\pi}^{*} + f^{*} = 0.0125 + f^{*}$ and $\varphi_{\pi}^{*} + f^{*} + \varphi_{\rho}^{*} + z^{*} = 0.0185 + z^{*} + f^{*}$ respectively. Since the OECD and US inflation tendencies both depend upon the same common trend f, they move together, differing by a small constant $\varphi_{\pi}^{**} - \varphi_{\pi}^{*} = -0.0007$. The central tendencies of UK inflation and interest rates are: $\varphi_{\pi} + c_{\pi} + f + \nu_{22} f^{*} = 0.0186 + f + 0.6847 f^{*}$ and $\varphi_{\pi} + \varphi_{\rho} + c_{r} + z + \chi_{31} z^{*} + f + \nu_{22} f^{*} = 0.0287 + z + 0.1795 z^{*} + f + 0.6847 f^{*}$ respectively where c_{π} and c_{r} are the last two elements of the 3 by 1 vector $\mathbf{\Xi} \cdot [\mathbf{0}, \boldsymbol{\varphi}^{*\prime}]$. These values are scaled up by 4 to be consistent with annual rates.

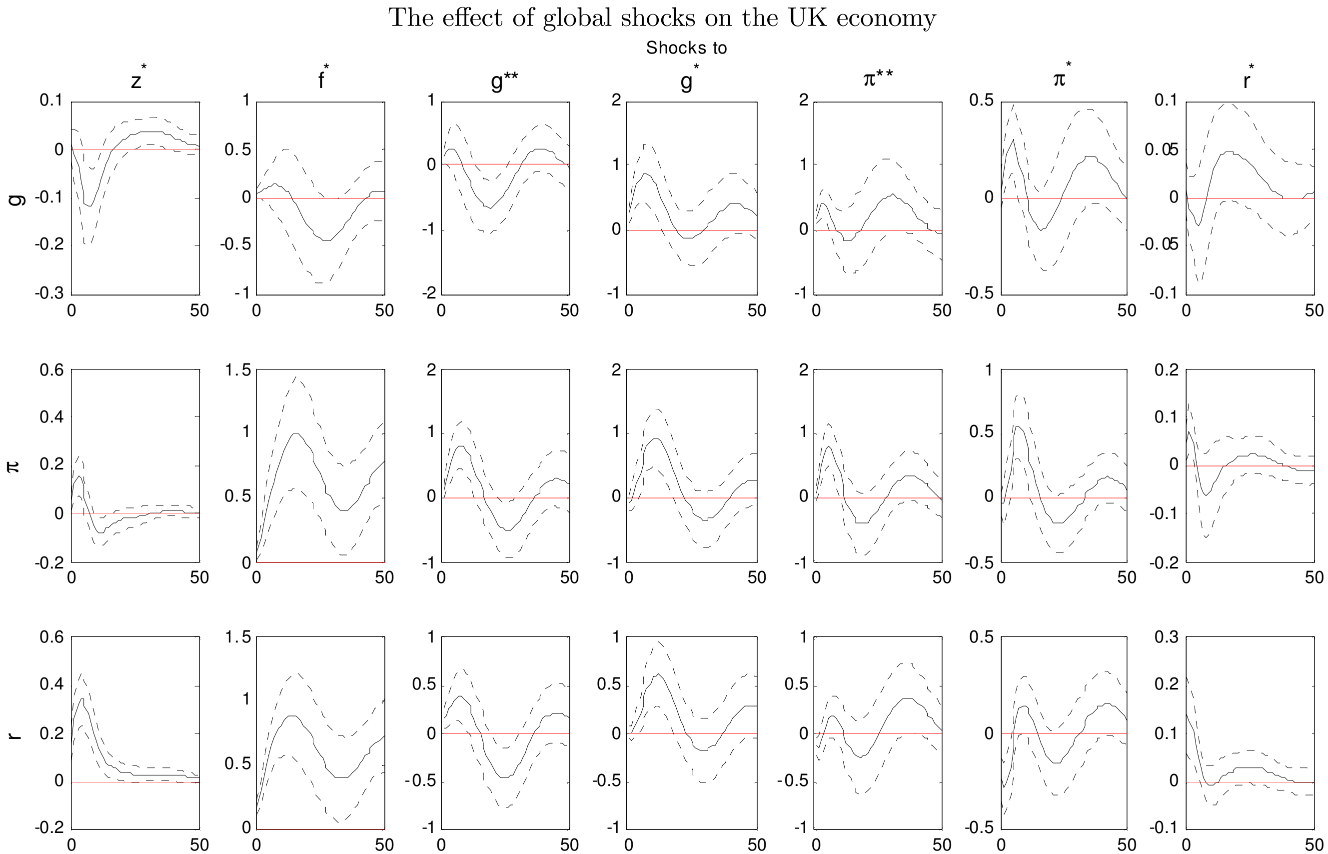
Figure 5: OECD-US macro impulse responses - with 95 percent confidence intervals

The effect of global/US shocks on the US economy



Each panel shows the effect of a shock to one the seven orthogonal innovations (ϵ^*) shown in (1) and (2). These shocks increase each of the driving variables in turn by one percentage point compared to its historical value for just one period. Since f^* is a martingale, the second shock (ϵ^{f^*}) has a permanent effect on inflation and interest rates, while other shocks are transient. Elapsed time is measured in calendar quarters.

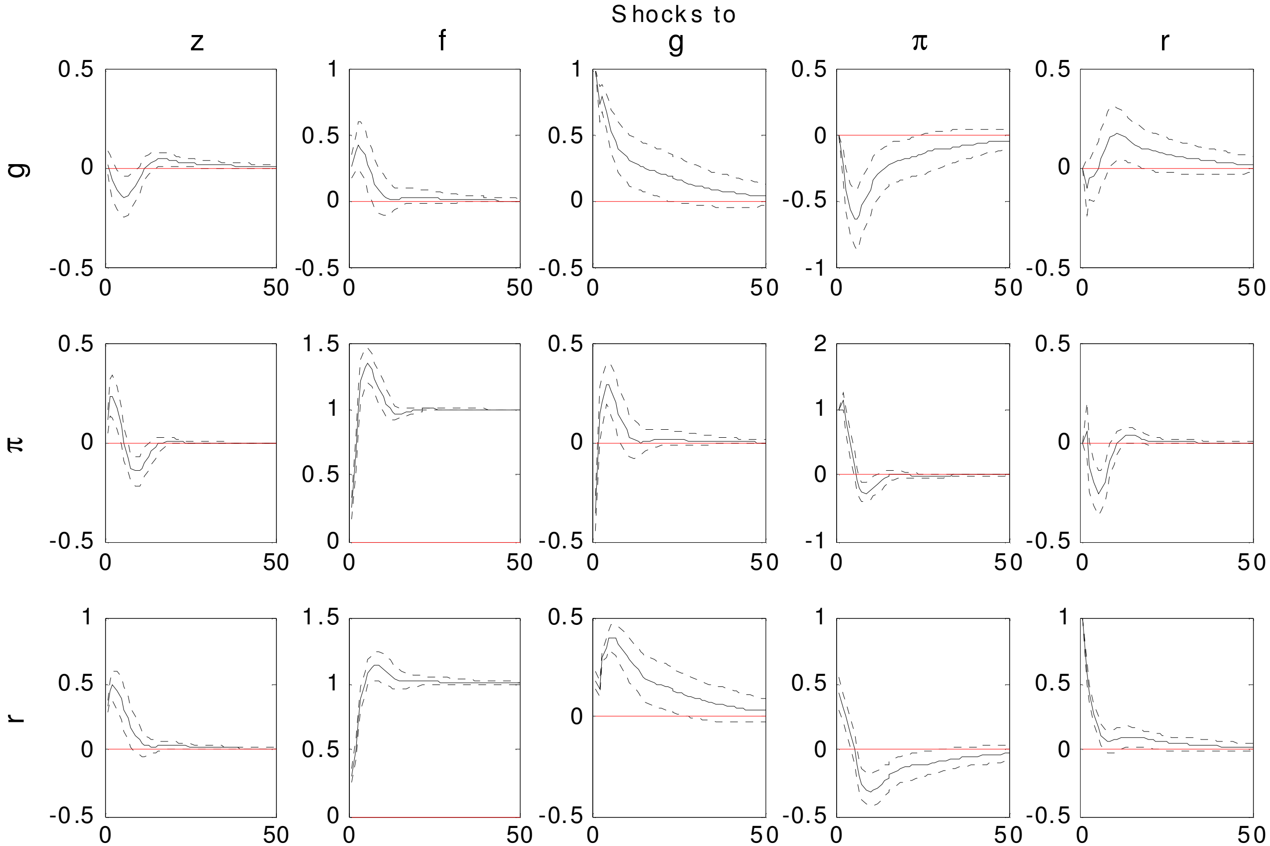
Figure 6a: UK macro impulse responses 1 - with 95 percent confidence intervals



Each panel shows the effect on the UK economy of a shock to one the five orthogonal innovations (ϵ) shown in (8) and (9). These shocks increase each of the driving variables in turn by one percentage point compared to its historical value for just one period. Since f is a martingale, the second shock (ϵ^f) has a permanent effect on inflation and interest rates, while other shocks are transient. Elapsed time is measured in calendar quarters.

Figure 6b: UK macro impulse responses 2: - with 95 percent confidence intervals

The effect of domestic shocks on the UK economy



Each panel shows the effect on the UK economy of a shock to one the seven orthogonal innovations (ϵ^*) shown in (1) and (2). These shocks increase each of the driving variables in turn by one percentage point compared to its historical value for just one period. Since f^* is a martingale, the second shock (ϵ^{f^*}) has a permanent effect on inflation and interest rates, while other shocks are transient. Elapsed time is measured in calendar quarters.

Figure 7a: Factor loadings of the US yield model

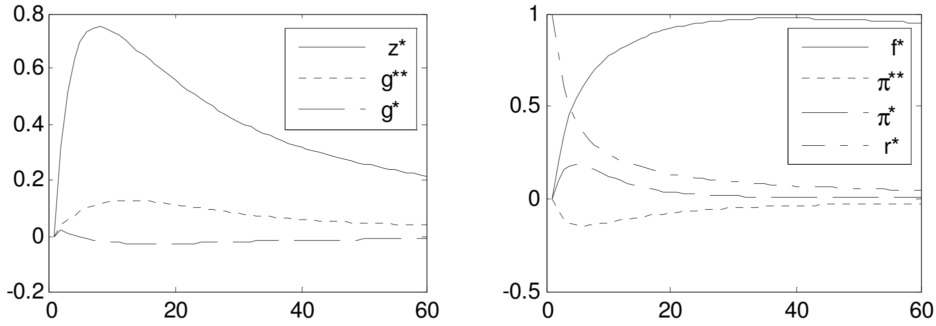


Figure 7b: Factor loadings of the UK yield model

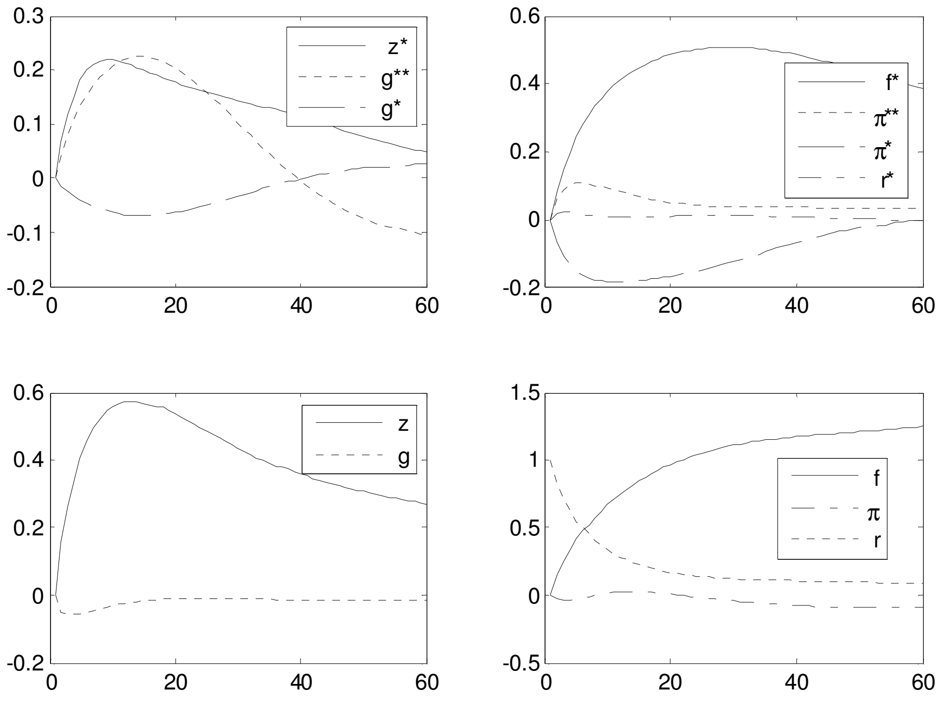
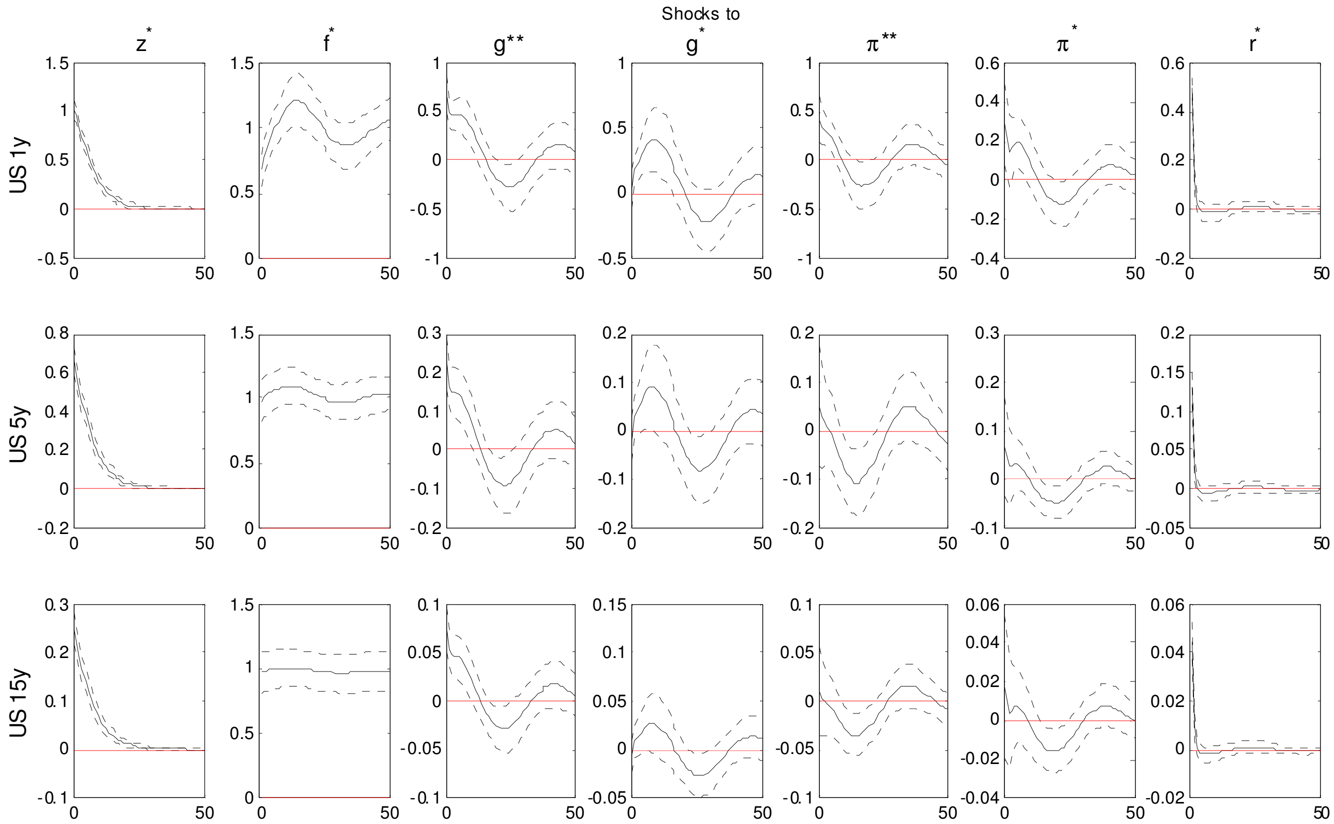


Figure 8: US yield impulse responses - with 95 percent confidence intervals

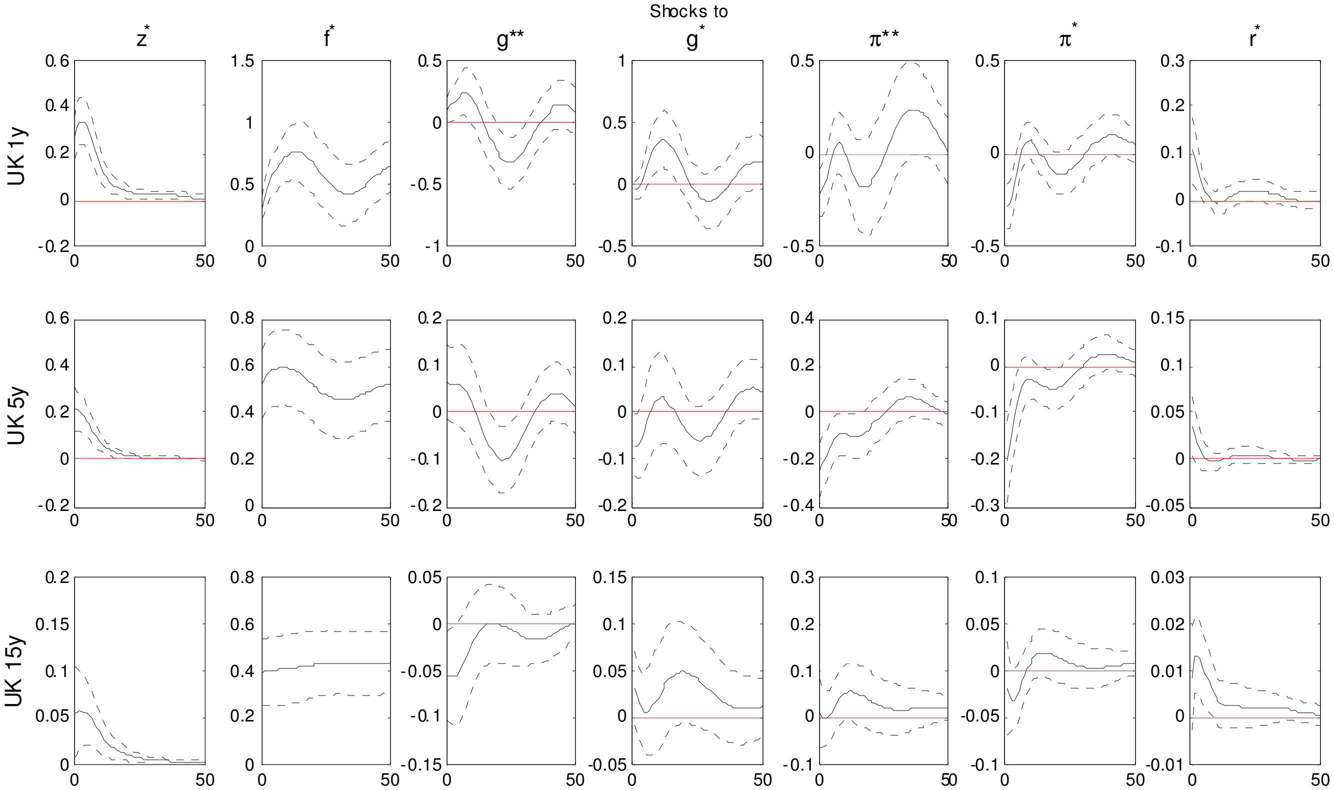
The effect of global/US shocks on the US yield curve



See notes to table 6. Since f^* and yields are cointegrated martingale processes, the second shock (ϵ^{f^*}) has a permanent effect, while other shocks are transient.

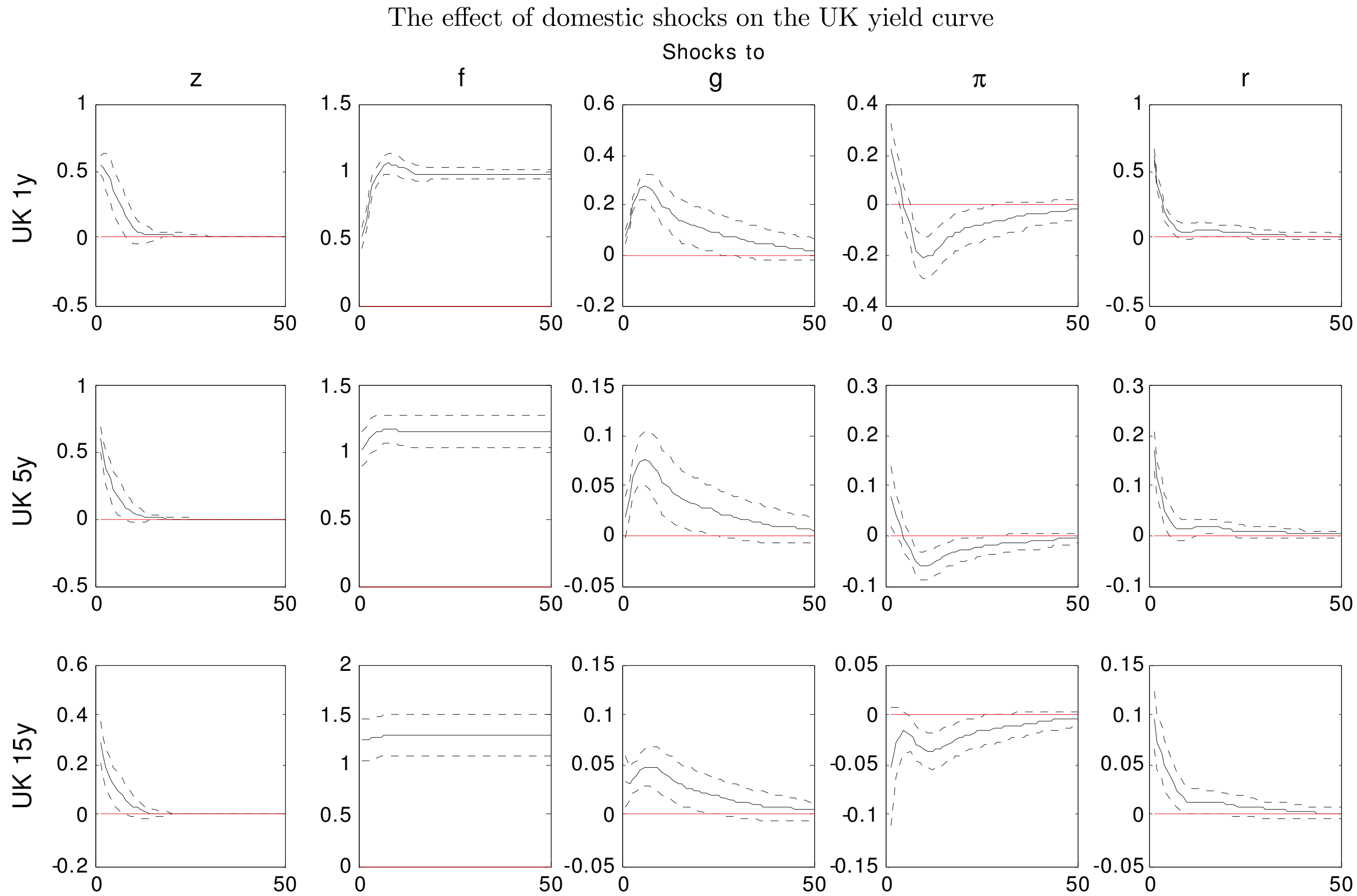
Figure 9a: UK yield impulse responses 1: - with 95 percent confidence intervals

The effect of global shocks on the UK yield curve



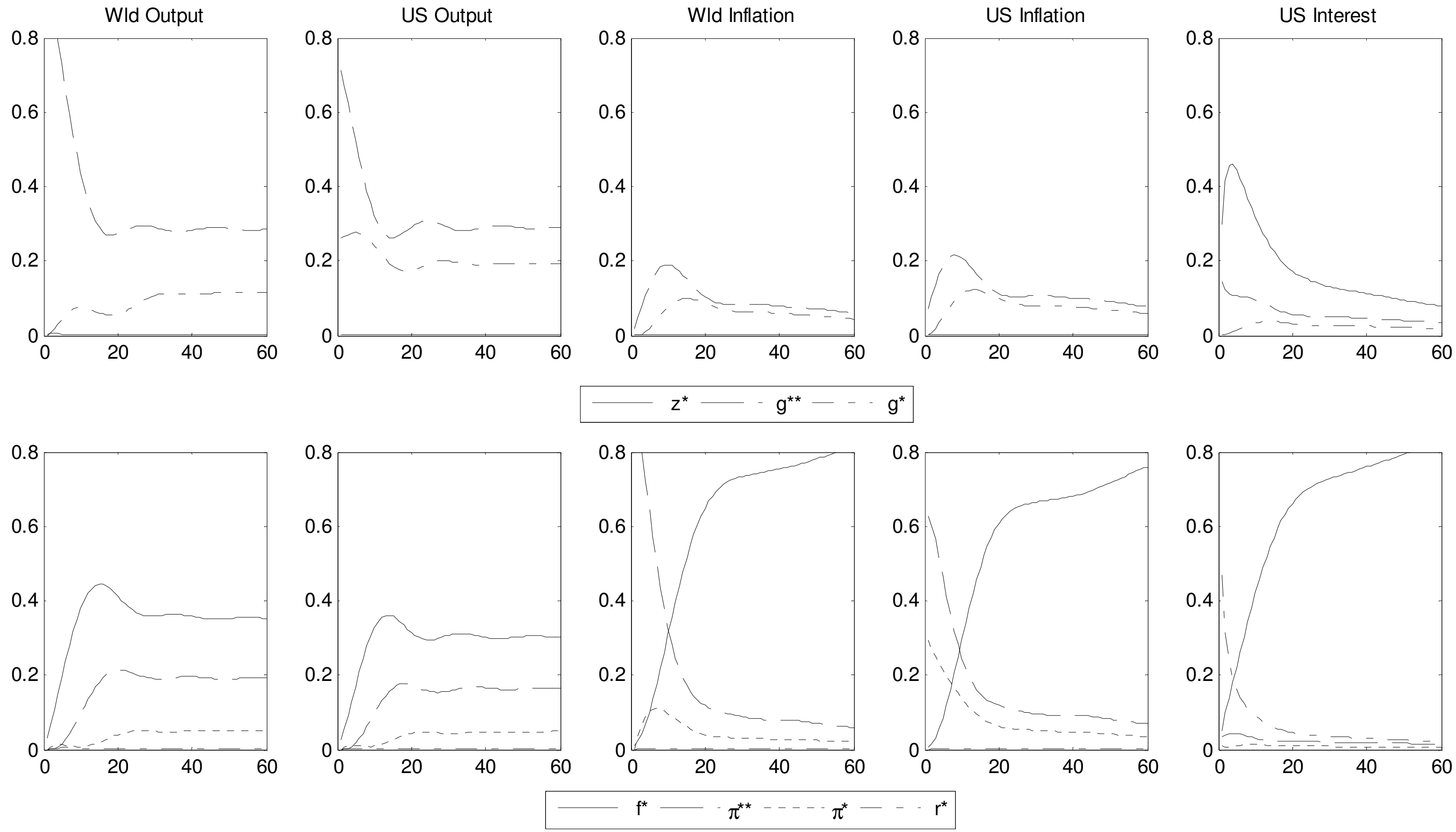
See notes to tables 6a and 8.

Figure 9b: UK yield impulse responses 2: - with 95 percent confidence intervals



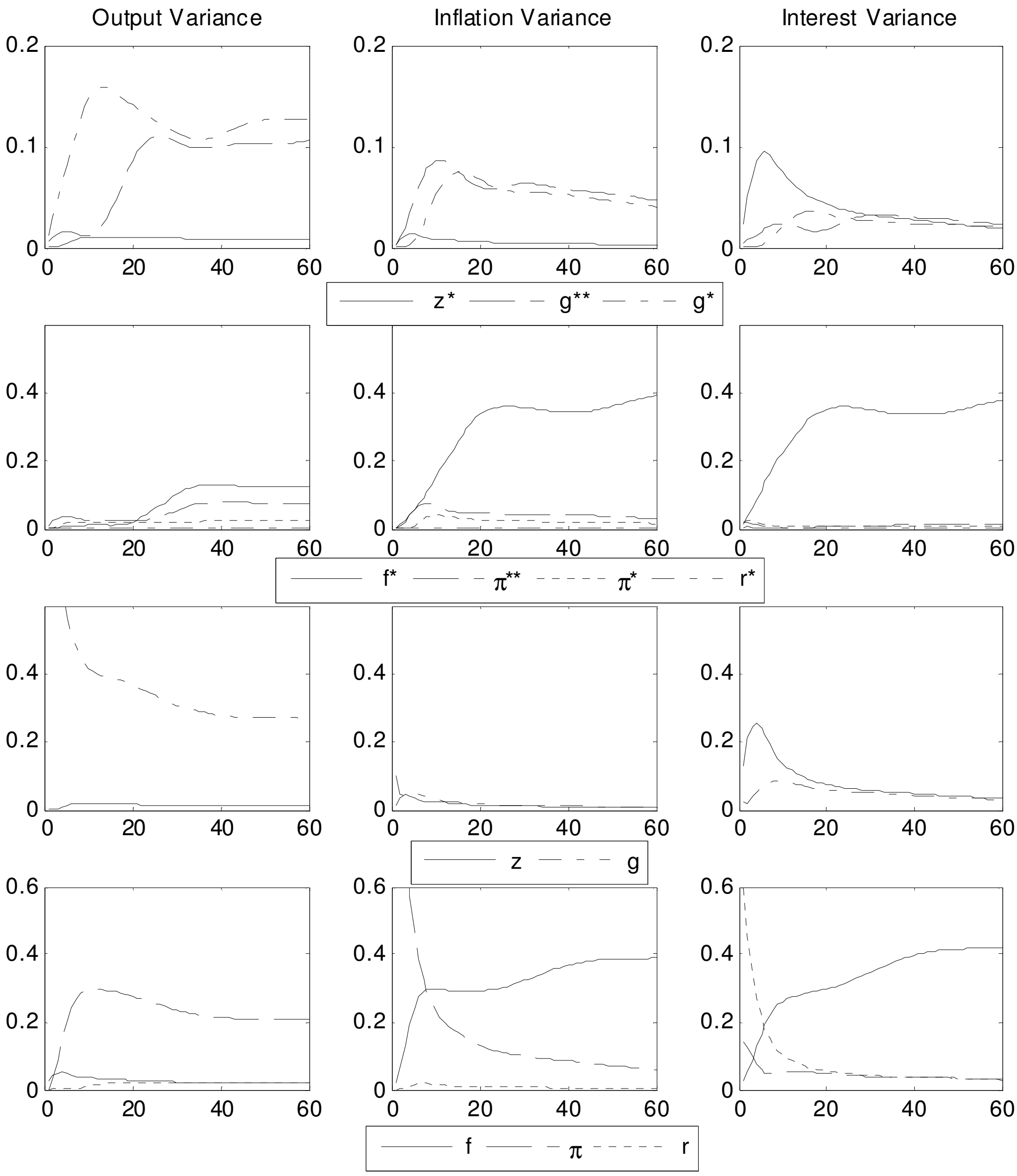
See notes to table 6b. Since f and yields are cointegrated martingale processes, the second shock (ϵ^f) has a permanent effect, while other shocks are transient.

Figure 10a: Variance Decomposition: OECD-US macro variables



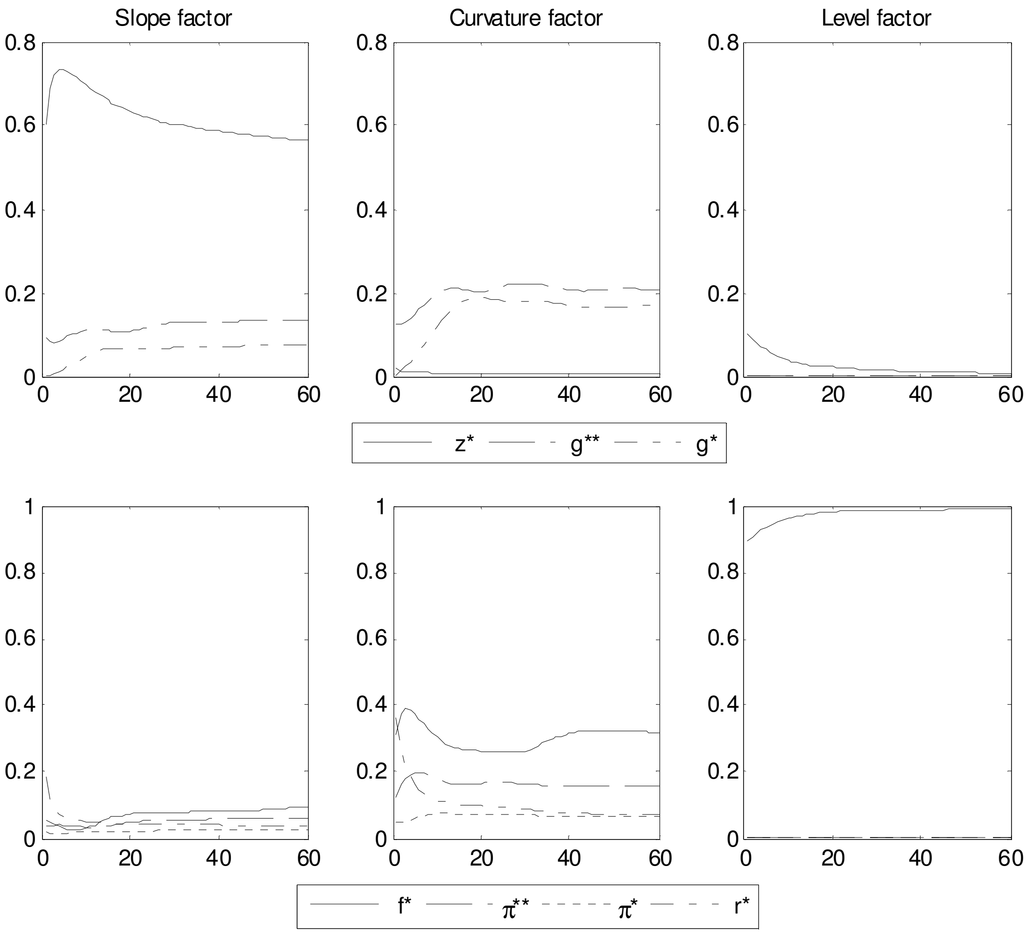
The first panel shows the proportion of the total variance of the OECD/US macro variables g**, g*, pi**, pi*, and r* explained by its real variables. The second panel shows the proportions explained by the nominal variables. Elapsed time is measured in quarters.

Figure 10b: Variance Decomposition: UK macro variables



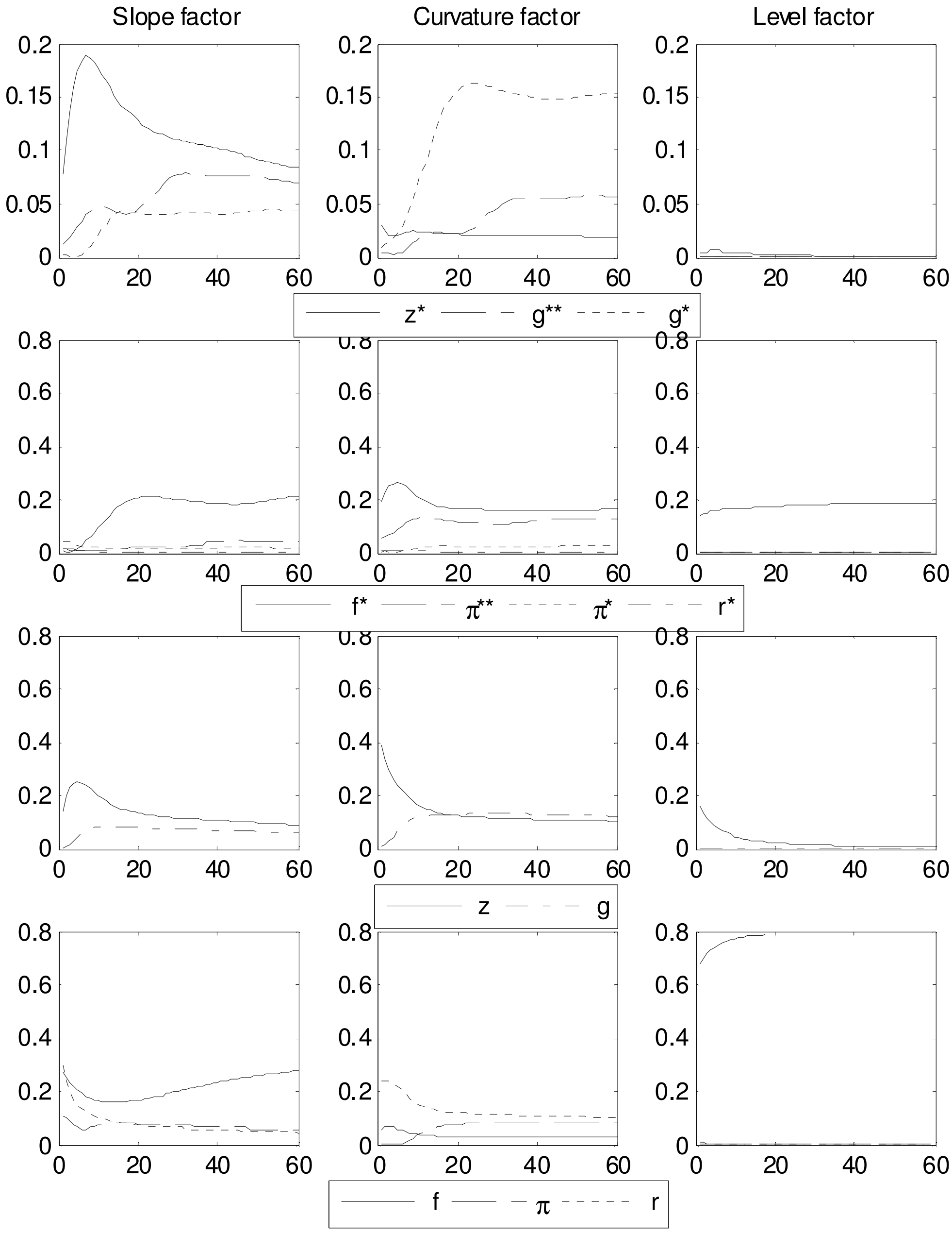
The first panel shows the proportion of the total variance of the UKmacro variables g, pi, and r explained by the US real variables. The second, third and last panel show the proportions explained by the US nominal, UK real and UK nominal variables respectively. Elapsed time is measured in quarters.

Figure 11a: Variance Decomposition: US yield factors



The plot shows the proportion of the total variance of the level, slope and curvature factors of the bond yields explained by shocks to the various driving variables. The 'level' factor is mimicked by the 15 year yield, the 'slope' factor is mimicked by the 15 year yield less one year yield, and the 'curvature' factor is mimicked by the 5 year yield less average of the 15 year and one year yields. Elapsed time is measured in quarters.

Figure 11b: Variance Decomposition: UK yield factors



Please see notes to Table 11a.