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# **Spatial Representation for Navigation in Animals<sup>1</sup>**

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## **Abstract**

This article considers the problem of spatial representation for animat navigation systems. It is proposed that the global navigation task, or wayfinding, is best supported by multiple interacting sub-systems, each of which builds its own partial representation of relevant world knowledge. Evidence from the study of animal navigation is reviewed to demonstrate that similar principles underlie the wayfinding behaviour of animals including man. A simulated wayfinding system is described that embodies and illustrates several of the themes identified with animat navigation. This system constructs a network of partial models of the quantitative spatial relations between groups of salient landmarks. Navigation tasks are solved by propagating egocentric view information through this network using a simple but effective heuristic to arbitrate between multiple solutions.

### **Keywords:**

Animat AI; spatial representation; navigation; multiple sub-systems; quantitative models.

Shortened Title: Animat navigation.

## 1 Introduction

The approach of ‘Behaviour-based’ or ‘animat’ artificial intelligence (AAI) (Brooks, 1995; Maes, 1992; Meyer & Guillot, 1990; Wilson, 1990) emphasises the role of the agent’s continuous interaction with its environment in driving the selection and performance of behaviour. Further important themes include decentralized control; targeting perception to the acquisition of task-relevant information (rather than the delivery of a general description of the world); and incremental layering of competences to provide robustness to error or failure. A typical AAI system is constructed from a heterarchy of sub-systems each linking perception to action and fulfilling a specific behavioural role.

AAI has something of an ‘anti-representation’ bias. This is based in part on its success in excising representational elements from control systems. The classic example (see e.g. Maes, 1992) is the ‘wall-following’ robot that tracks the contour of a wall, negotiating bends and corners, by exploiting some simple, pre-wired reflexes and biases. This robot has little or no internal state, and certainly has no internal representation of the wall or of its own relationship to it. AAI has had considerable success in applying similar techniques to a range of local navigation skills, enabling mobile robots to move around in the immediate environment quickly and safely (Brooks, 1989; Connell, 1990; Soldo, 1990). For any mobile agent, however, an immensely valuable skill is that of finding and following paths to target locations in large-scale space (i.e. where targets may lie outside the visible scene). This skill will be referred to as global navigation or, to use a term common in the psychological literature, wayfinding. Unlike local navigation, which can be achieved with minimal internal state, wayfinding requires internal structures that encode at least some aspects of the agent’s past experience of its environment. The second section of this article therefore seeks to identify some characteristics that might distinguish an AAI approach to representation for the wayfinding problem.

AAI looks to biology to provide examples of robust solutions to difficult problems, and also seeks to inform biology by providing models of the mechanisms underlying animal behaviour. The literature on animal and human wayfinding has been an important source of inspiration for several navigational systems used in real and simulated autonomous robots (Kortenkamp, Weymouth, Chown & Kaplan, 1992; Kuipers & Byun, 1987; Mataric, 1990; Touretzky, 1994). Section 3 of the article therefore gives a brief review of evidence from the study of animal and human navigation, while section 4 attempts to find some convergent principles for navigation in both animals and animats.

A central objective of the article, and the topic of section 5, is to describe a simulated wayfinding system that embodies and illustrates several of the themes identified with animat navigation. This system, which has been partially described in (Prescott, 1993a; Prescott, 1993b), supports global navigation behaviour through the construction and use of multiple partial models of the quantitative spatial relations between groups of salient landmarks. The description given here introduces and proves a novel heuristic that substantially improves the effectiveness of the system in the presence of noise.

## **2 Representation for wayfinding in AAI**

The notion of representation is sufficiently controversial to justify a brief definition. Following Beer (1995) and Clark (to appear) identifiable elements of the agent's internal state will be described as 'representations' where assigning a content-bearing role is helpful in understanding the overall causal process (the agent-environment interaction). This definition puts the distinction between representation and not-representation firmly in the mind of the observer, where its value is in determining concise and predictive explanations<sup>i</sup>. It also leaves open the debate about the nature of representations, and in particular, does not equate representation with the notion of a central world model.

In fact, there is a clear lead for an AAI view of representation presented in the literature (Brooks, 1995; Maes, 1992; Mataric, 1995; Roitblat, 1994) that converges with many of the themes identified above. Some of the key elements of this view are as

follows. Each of the multiple sub-systems, that comprise the overall control architecture, maintains representations of those aspects of the world that it finds relevant. Accordingly, instead of a central and unified model of the world, knowledge is distributed between many partial representations focused on specific tasks. Where appropriate such representations may encode the agent's interaction with the world rather than modelling the structure of the world directly. There is no strong distinction between memory and process—knowledge may be intricately bound up with processing mechanisms. Finally, representations may be implicit and distributed in the Connectionist sense (e.g. Hinton, McClelland, & Rumelhart, 1986), in that an entity may be represented by a pattern of activity across many processing elements that emerges in the context of appropriate perceptual and internal states.

The following sub-sections consider the implications of this view for AAI approaches to the wayfinding problem.

## **2.1 Multiple, partial representations of the world**

The focus of much of the research in global navigation for mobile robots has been the task of planning optimal paths using an appropriately detailed and accurate metric map of the spatial layout of the environment (Chatila, 1986; Crowley, 1985; Iyengar, Jorgensen, Rao & Weisbin, 1985; Lozano-Perez, 1983; Rao, Stoltzfus & Iyengar, 1988; Turchan & Wong, 1985). The AAI approach suggests an alternative view in which the navigation problem is fragmented into a number of distinct, interacting sub-systems, where each subsystem builds its own partial representation of the world. Alternative sub-systems differ in the environmental characteristics, sensor modalities, and computational mechanisms they exploit, and the sorts of representations they construct. There are significant advantages to this fragmentation. First, the ability to exploit different wayfinding methods provides robustness by reducing the dependence on specific cues and sensor channels. Second, by exploiting multiple representations, alternative hypotheses about the structure of the environment can be encoded. Mutually consistent hypotheses confirm one another, whilst inconsistent hypotheses provide 'fall-back' when

navigation goes wrong (i.e. when the ‘first choice’ hypothesis fails there are other plausible candidates to be investigated). It will be appropriate to maintain separate models for these different sub-systems in as much as they exploit different spatial constraints, maintain useful alternative hypotheses, or suit representational forms that may not be easily or cheaply integrated into a unified model. A navigation system of this sort may exploit distributed representations of the environment (in the sense defined above)—that is, rather than coding explicit representations of estimates of spatial relationships, appropriate estimates may be generated dynamically through the interaction of multiple models, with each other and with available perceptual data, in the context of a specific navigation task.

### **2.3 Agent-centered representations**

One of the simplest forms of representation for wayfinding is a sequential record of the agent’s own actions in specific perceptual contexts. Memory for such sequences provides the basis for what is termed *route* knowledge (O’Keefe, 1990b; O’Keefe & Nadel, 1978). The ability to memorise and follow routes seems to account for many aspects of animal and human navigation behaviour (O’Keefe & Nadel, 1978). This principle has also been used to guide autonomous robots (Connell, 1988). The underlying representational form of route knowledge can be characterised as a chain of *stimulus x action -> next stimulus* associations. One of the attractions of such chains is that each link is built from the elementary sensorimotor experiences of the agent. There is no need to maintain an allocentric (world-centred) frame of reference, or to translate the (egocentric) experience of the agent into such a frame. The world itself is represented only in an indirect fashion. O’Keefe (1990c) identifies two categories for the elements of route knowledge—associations that involve approaching or avoiding specific cues are termed *guidances*, whilst those that involve performing a body-centred action (generally a rotation) in response to a cue are termed *orientations*. Such associations have a ‘deictic’ quality as defined by Agre (1991) in that objects are indexed according to the properties that make them relevant to the current situation and the ongoing task. For instance, the guidance

“move towards the red beacon”, or the orientation “turn left at the lights”, is each meaningful and unambiguous only in the context of appropriate circumstances and goals.

Although route knowledge can support effective wayfinding behaviour it is important to recognise that it has significant limitations. Since route models do not explicitly encode the underlying invariant spatial relations between locations in the environment, they are unsuited to the problem of determining novel paths between places. This lack of knowledge about how different places in the environment are related also means that it is difficult to find and re-establish position on a route if the agent becomes lost.

## **2.4 World-centered representations**

To support more flexible wayfinding behaviour than is possible on the basis of route knowledge, requires representations that encode the spatial layout of the environment. Further choices exist, however, as to the nature of the spatial relations that are learned. An important distinction, that has been a focus of interest in AAI research on wayfinding, is that between qualitative and quantitative spatial models. Several researchers associated with AAI (Kortenkamp et al., 1992; Kuipers & Byun, 1991; Shölkopf & Mallot, 1995) have advocated the use of qualitative, non-metric models of space, based principally on representations of topological spatial relations (connectedness, containment, and order), in preference to the quantitative, metric modelling approach that has been the norm in the robot navigation literature. A topological model of the layout of the environment can be assembled from sequential models of the type that underlie route knowledge. The development from route models to topological models is therefore of an incremental character fitting the ‘evolutionary’ bias of AAI that sees the development of complex or powerful systems as a process that exploits the building blocks provided by simpler and weaker systems (Brooks, 1995; Kuipers & Byun, 1991). Forming a layout representation requires the ability to recognise where route models overlap, implying the use of *place identification* skills that allow the agent to judge when experiences separated by arbitrary time intervals are generated by a single spatial location.

Though it has been suggested that topological modelling is less demanding in terms of sensor accuracy and computational resources than metric modelling (Kuipers & Byun, 1991; Kortenkamp, et al., 1992) such arguments are difficult to prove. Clearly topological modelling has a lesser need for accurate metric sense data, however, the problem of place identification is such that robustness to inaccurate or absent metric data must be achieved by improving the pick-up of non-spatial sensory characteristics—poor metric sensing must be compensated for by better pattern matching (an extended argument for view is given in Prescott, 1993b). Arguments about the relative computational expense of metric and non-metric systems will also be hard to make without direct comparisons between systems of similar competence. Costs will depend as much on the type of representation stored and the nature of the mechanisms used to handle inaccuracy and uncertainty, as on any inherent differences due to geometric character. Finally, the overheads associated with a given method will depend on its appropriateness to the navigational task, for instance, topological models may suit constrained environments of corridors and tunnels that are easy to segment into the vertices and edges of a graph representation but be less suited to open terrain (again see Prescott, 1993b).

Topological modelling is at one end of a range of alternatives to the use of metric models of spatial layout. One of the senses of qualitative navigation is the use of categorical or relative judgements of geometric relations implied by terms such as ‘near’, ‘far’, ‘closer’, ‘further’, ‘middle’. Some of these judgements can be based on uncalibrated sensor readings, for instance, a robot can follow the middle of a corridor by keeping left and right sensor readings in balance (e.g. Kuipers & Byun, 1991). Other qualitative judgements will require rough calibration or the use of fuzzy sets to classify sensor readings. Projective and affine geometries introduce additional, more quantitative, constraints and may be appropriate in some circumstances. For instance, the system described in Section 5 encodes affine spatial relations (although metric relations are recoverable given calibrated metric sensors), which allows co-ordinate transforms to be easily computed. Non-Euclidean geometries such as the ‘city-block’ metric may also be effective for some artificial environments such as mazes.

Qualitative models of spatial layout have an important advantage over route knowledge, in that they allow the agent to generate novel routes (using known path segments). However, such models do not allow the determination of ‘straight-line’ routes or of short-cuts that cross unexplored terrain—such skills require knowledge of the quantitative spatial relations, direction and distance, between locations. In view of the importance of effective wayfinding to autonomous agents this is a very significant point in favour of the use of quantitative spatial knowledge.

As noted above, many quantitative navigation systems seek to construct maximally accurate metric world models. Such an aim can justify the construction of a unified model, into which all available observations are integrated, as the most efficient way to minimise errors in estimates of spatial position (Smith, Self & Cheeseman, 1990). However, from the point of view of a navigator, accuracy may not be the key criterion for determining the adequacy of its spatial knowledge. This is shown by recognizing that effective wayfinding rarely needs an accurate assessment of the position of a remote target—a navigator who begins with just a coarse estimate of the required heading can ‘home-in’ on the distant goal by making successive corrections based on incoming perceptual data (Jamon, 1990). Rather than seeking to maximise accuracy, the AAI approach emphasises strategies for coping with the inevitable error and uncertainty attached to acquired knowledge. This can involve the use of opportunistic planning techniques—of which the homing-in strategy is an example—to exploit sensor data whenever possible and minimise the use of stored knowledge, and the use of the redundancy—provided by multiple competences—to provide robustness to sensor error or failure. The position of this article is that quantitative navigation methods, developed along these lines, can support robust and effective wayfinding in autonomous agents.

### **3 Representation in natural navigation systems**

There are several good reasons why AAI looks for clues in the understanding of natural systems for the design of the artificial. One is to move beyond the hazardous

domain of folk psychology and introspection that has been a fertile ground for several questionable ideas, not least those concerned with the notion of central world model. A further reason is that natural systems embody robust solutions to difficult problems. Although on a species level these may be somewhat specialised, the existence of characteristic properties that extend across species suggest principles of general importance to the design of autonomous agents. The literature on navigation in natural systems, suggests that there is, indeed, a great diversity of solutions to the wayfinding problem, and that many natural systems are highly specialised to specific environmental niches. It is argued below, however, that there is a commonality across many species which relates to the use of multiple sub-systems for the control of navigation behaviour, and the exploitation of quantitative knowledge of environmental spatial relations.

## **2.1 Animal navigation**

The literature on animal spatial learning differentiates the wayfinding skills of most invertebrates and lower vertebrates, from those of higher vertebrates (birds and mammals). In particular, it suggests that wayfinding in many invertebrates uses path integration mechanisms (integrating changes in position from sensory signals), compass senses (determining orientation by using non-local cues such as the sun, or by sensing physical gradients such as geomagnetism), and piloting (orienting to specific remembered stimulus patterns)

(Cartwright & Collett, 1979; Cartwright & Collett, 1983; Cartwright & Collett, 1987; Wehner, 1983; Wehner & Menzel, 1990). Invertebrates do not appear to memorise the spatial layout of their environment, and as a consequence, their wayfinding behaviour may be restricted to homing and retracing familiar routes<sup>ii</sup>. In contrast, there is evidence that higher vertebrates do learn the spatial layout of their environments (see, for example, Gallistel, 1990; O'Keefe, 1990c; O'Keefe & Nadel, 1978; Olton, 1982) enabling them to generate and follow more efficient paths to distant targets. The literature on vertebrate navigation further suggests a discontinuity between route knowledge and the use of metric layout representations (Gallistel, 1990; O'Keefe, 1990b; O'Keefe & Nadel, 1978).

For instance, O'Keefe (O'Keefe, 1990b; O'Keefe & Nadel, 1978) has argued that there are two largely separate navigation systems used by mammals including man. The first of these, which he calls the *taxon* system, is supported by chains of guidance and orientation associations and underlies some of the route-following abilities seen in animals. O'Keefe's second system, called the *locale* system, constructs a layout model describing the metric spatial relations between locations in the environment. Evidence for the existence of this system and its independence from taxon strategies consists of both observational and laboratory studies of animal behaviour (e.g. Chapuis, Thinus-Blanc, & Poucet, 1983; Poucet, Thinus-Blanc, & Chapuis, 1983), and neurophysiological studies suggesting that different brain structures underlie the two systems (Morris, Garrud, Rawlins & O'Keefe, 1982). Gallistel (1990), who provides an extensive review of research on animal navigation, also concludes that many animals navigate on the basis of metric models of space.

O'Keefe's distinction between taxon and locale systems follows a long line of research into 'response' versus 'place' knowledge in animal navigation (see Olton, 1979; Olton, 1982; Gallistel, 1990) that shows the existence of complimentary navigation systems in animals. This literature also demonstrates, that there may be no simple hierarchical arrangement of control. For instance, in experiments on the maze learning behaviour of rats, response knowledge (chained motor responses) appeared to predominate in some experimental situations and place knowledge (encoded spatial relations) in others. The importance in animal navigation of multiple, redundant, mechanisms for achieving navigational goals, and the existence of mechanisms for arbitrating between available alternatives is demonstrated by a number of studies. The following briefly reviews some of this research.

Poucet and co-workers (Poucet, et al., 1983; Chapuis, et al., 1983) provide evidence from cats and dogs for the distinction between taxon and locale systems and demonstrate some of the characteristics of the arbitration process that chooses between them. In their experiments, animals were forced to choose between two paths from a starting point to a feeding bowl. The length of each path and its angular deviation from the straight line to

the goal was determined by an arrangement of screens that were either transparent or opaque. When the screens were transparent, the animals preferred the path with the smallest angular deviation even where this involved travelling a greater distance. When the screens were opaque animals preferred the shortest route regardless of the angular deviation. These results suggest that, when the goal is visible, a guidance system (taxon) is triggered that causes the animal to try to move along the direct line toward the goal, whereas, when the goal is not visible, the locale system is operated which selects the shortest available path. Interestingly the taxon system predominates over the locale system in the case of a visible goal, even though, in this case, it results in sub-optimal navigation behaviour.

Etienne (1992) reports experiments that show the existence, in the golden hamster, of independent wayfinding mechanisms derived from different sensory modalities, specifically, by dead-reckoning (proprioception) and with respect to (visual) landmarks. Hamsters have effective dead reckoning skills that are sufficient to relocate their nest in darkness. However, in lighted conditions, hamsters were found to orient primarily using visual information about local landmarks. In conflict situations, where a landmark (a single light spot) was rotated relative to the learned position, the hamsters homed using either the landmark information or their dead-reckoning sense. When the visual information and dead reckoning produced highly divergent paths dead reckoning was used, however, with smaller discrepancies visual information took priority over path integration. This last finding suggests a relatively sophisticated arbitration process for handling inconsistencies between different sources of direction information.

Experiments by Collett, Cartwright, and Smith (1986) with gerbils demonstrate the use of multiple redundant spatial representations derived from a common source. These animals appear to encode goal positions (buried sunflower seeds) in terms of individual visible landmarks by using some form of direction sense. In one experiment, gerbils were trained to locate a food cache at the centre of an array of two landmarks. When the distance between landmarks was doubled the gerbils searched at two sites each at the correct distance and orientation to one of the landmarks rather than at the centre of the two

locations (as some theories of a landmark ‘map’ might predict). In a further experiment the gerbils were trained to go to a goal-site at the centre of a triangle of three landmarks. During testing the distance of one landmark to the centre was doubled. Collett et al. report that the animals spent most of their search time around the place specified by the two landmarks, ignoring the one that broke the pattern. They interpreted this result in the following way:

“The gerbil is thus equipped with a useful procedure for deciding between discrepant solutions. When most of the landmarks agree in specifying the same goal, with just a few pointing to other sites, the chances are that the majority view is correct and that the additional possibilities result from mistakes in computation or from disturbances to the environment.” (Collett, et al., 1986).

Collett et al. are therefore suggesting that the multiple encoding of landmark-goal relations by gerbils occurs to provide the system with robustness. The gerbils appear to maintain distinct representations of the relative position of the goal to each landmark, rather than constructing a unitary representation of the landmark array and the goal. The ‘best guess’ as to where the target is located is decided by some form of ‘voting’ arbitration scheme at the moment when such a decision is needed.

Among researchers who regard the locale system in animal navigation as a metric cognitive map (Gallistel, 1990; McNaughton, Chen & Markus, 1991; O’Keefe, 1990b; O’Keefe & Nadel, 1978) there is some disagreement over the relative importance of dead reckoning and environmental fixes in constructing the map. This may suggest that the locale system is less of a unitary system than has been supposed. McNaughton, Chen, and Markus (1991), and Gallistel (1990) have argued that the principal sources of position information for map-building are dead-reckoning skills and compass senses. These skills are used to maintain an estimate of current position and heading relative to the origin (e.g. the nest) of a global allocentric frame. Places are coded in terms of the distance and direction from this origin or from single landmarks whose positions in the global frame are already stored. McNaughton et al. have termed such a mapping scheme a *vector*

*coding* system. An alternative view, proposed, for instance, by O'Keefe (1990c), is that a place is encoded in terms of the array of local landmarks visible from that position. In other words, that the cognitive spatial representation stores the locations of potential goals in allocentric coordinate frames determined by groups of salient cues. Such a system is here called a *relational coding*. Both vector and relational coding theories are backed up by substantial empirical evidence, hence, it seems unlikely that this debate will be easily resolved in favour of one side or the other. It seems plausible instead, that, in appropriate circumstances, animals may exploit both vector and relational coding mechanisms to generate complimentary models of space.

## 2.2 Human navigation

One of the motivations often cited (Kuipers & Byun, 1991; Yeap, 1988) for investigating qualitative navigation systems is research on human wayfinding. Much of this literature follows a theory originating with Piaget (Piaget, Inhelder & Szeminska, 1960) that human spatial knowledge has a hierarchical structure and is acquired through a stage-like process. Specifically, Piaget, and later Siegel and White (1975), argued that a fundamental stage in the acquisition of spatial knowledge is the construction of qualitative models of the environment from more elementary sensorimotor associations. This representation may be supplemented later by distance and direction information to form a more detailed quantitative map. Computational models, inspired by the (Piagetian) human wayfinding literature, have been described by Leiser (Leiser & Zilbershatz, 1989) and by Kuipers (Kuipers, 1982; Kuipers & Byun, 1991; Kuipers & Byun, 1987; Kuipers & Levitt, 1988). Kuipers, in particular, has developed a number of robot simulations based on the hypothesis of a 'semantic hierarchy' of spatial knowledge, which has been influential in other recent work on robot wayfinding (e.g. Levitt & Lawton, 1990). The 'semantic hierarchy' hypothesis sees the acquisition of spatial knowledge as assembling spatial knowledge on four levels termed *sensorimotor*, *procedural*, *topological*, and *metric*, it thus proposes, as in this article, that navigation behaviour should be supported by multiple representations<sup>iii</sup>.

One of the difficulties with some of the early studies on human navigation was the assumption that a metric ‘cognitive map’ would be something like an image of a cartographic map. Experiments that found errors in spatial knowledge—distortion, gaps, holes, fault-lines, and asymmetries—were therefore taken as evidence against the use metric representations and in favour of more qualitative systems. However, although there is little support for this notion of a metric “picture in the head” (see, for instance, Kuipers, 1982), there is good evidence that metric spatial representations, of a quite different kind, form an important element of human wayfinding competence (see Presson and Hazelrigg, 1984; Scholl, 1992; and McNamara, 1992, for reviews). Some of the distinctive properties of these representations are demonstrated in an experiment by Scholl (1987), which contrasted the spatial knowledge acquired through direct experience of the environment with that acquired indirectly by memorising a cartographic map. In the case of knowledge acquired from a map, a task of ‘pointing to unseen targets’ showed that the stored representation had a preferred orientation—pointing was easier when the subject was aligned with the North-South axis of the map. Further, with this type of knowledge all coded locations could be accessed with equal ease (as though one was ‘looking down’ on an aerial map of the environment). In contrast knowledge acquired from direct experience showed no preferred (absolute) orientation although, importantly, targets in front of the body were located more easily than ones behind. Scholl (1992) argues, on the basis of a number of studies of this nature, that human cognitive spatial representations encode allocentric spatial relations abstracted from the stream of local egocentric views experienced during movement. Furthermore, this store of knowledge has no inherent orientation. Whenever it is accessed, the representation of the world appears to be automatically aligned with the body, anticipating objects that are about to become visible. From this view, the internal representation is not a static and explicit model within which the navigator tracks his or her own position and orientation. Instead, spatial knowledge seems to be encoded in an implicit and distributed store, from which perceptual experience evokes a set of expectations that are always centred and oriented to the navigator’s current perspective.

#### 4 A Multiple schemata view of animat navigation

Arbib (1989, 1990) has proposed the term *schemata* to describe active representational systems or “perceptual structures and programs for distributed motor control” (Arbib, 1989). A similar definition is given by Neisser (1976) for whom a schema is an “active information seeking structure” defined by “information pickup and action”. Neisser’s conception of human cognitive maps as ‘orienting schemata’ has been adopted by Scholl (1987, 1992) as a framework for understanding the dynamic nature of the spatial representations underlying human wayfinding. Here, and in (Prescott, 1993b; Prescott, 1994), I argue for a ‘multiple schemata’ view of the use of spatial representations for navigation in animats.

The emphasis of a multiple schemata approach, as defined here, is not on constructing unified representations such as global topological or metric maps but rather on establishing multiple, partial models. Each schema will encode spatial constraints derived from the interaction with the environment, which in the context of appropriate sense data, can determine navigation behaviour. For instance, as in Etienne’s experiments with the golden hamster, a ‘dead reckoning’ schema can maintain an estimate of the homing vector based on proprioception, whilst a second ‘landmark’ schema represents the relative position of the home with respect to a salient landmark. Both schemata can then independently direct homing behaviour, or can combine for a more robust result. Different schemata are defined, not so much by the sort of geometric relations they preserve, but by the channels and mechanisms through which the encoded relations are derived and applied. For instance, separate metric schemata can derive from different sensory modalities as is the case with the ‘dead reckoning’ and ‘landmark’ schemata. They can also derive, however, from within a single sensory modality by reference to different environmental cues. For example, the observed position of a single distant landmark (e.g. the sun) can be employed to encode a direction constraint distinct from spatial localisations obtained with respect to local landmarks. Indeed, different individual landmarks or landmark groups can supply complimentary models as is demonstrated by

Collett et al.'s experiments with gerbils, and in the system described below. Distinct schemata may also be derived from within a given modality and by reference to the same external cues but by employing different computational mechanisms. For instance, the visual characteristics of landmarks might be used to model topological relations while other mechanisms extract distance and direction from the visual scene. Only in this latter sense will geometric content provide an important criterion for distinguishing between schemata.

For a given navigational task, arbitration schemes will be needed to determine the relative contributions of available schemata to navigation behaviour. This arbitration could be carried out through some fixed subsumption mechanism whereby, for instance, knowledge derived from a metric representation of spatial layout could override route-following strategies. Alternatively a more sophisticated system would seek to discriminate between multiple schemata by judging each according to its expected accuracy or reliability. In this way, reliable identification of a distinctive landmark might override estimates of spatial position or orientation determined by some metric reckoning process. Although some tasks may require the temporary creation of a unified model (drawing a graphical map of the environment might constitute such a task), in general the underlying representations will remain distinct so that (possibly conflicting) alternatives are not lost.

A multiple schemata view can help in understanding the evolution of multiple wayfinding competences in animals (as reviewed above). It also provides support for the essentially pragmatic approach in AAI to robot navigation. AAI has rightly begun by developing competences similar to O'Keefe's taxon systems that exploit the good odometry information available to robots and use orienting and guidance strategies to implement route-following behaviours (e.g. Connell, 1988). To achieve wayfinding skills of the more powerful and flexible variety seen in vertebrates, taxon mechanisms must be complimented by navigation systems that encode the spatial layout of the environment. Such systems may, however, consist of multiple, partial models of the kind proposed here. The final section of this article describes a system of this nature.

## 5 A distributed animat navigation system

This section describes in detail a distributed mapping system that constructs multiple partial models of quantitative spatial relations. The system is built using local co-ordinate frameworks defined by groups of salient landmarks. Specific locations are redundantly coded by their relations to a number of local landmark frames. Large-scale spatial relations are implicitly encoded in a network of nodes encoding local landmark/frame relations. The computations required to construct these models from noisy, egocentric metric data require only simple mathematics (indeed a connectionist learning rule can suffice) and have memory requirements roughly proportional to the number of goal-sites and landmarks stored. The task of determining direction or path information to a distant goal is performed by a parallel, spreading activation<sup>iv</sup> search in which estimates of landmark positions are propagated through the network. As landmark positions are redundantly encoded, the system generates multiple solutions for any task and uses an arbitration procedure to select between alternatives. While moving towards a goal the system exploits run-time error-correction by incorporating egocentric fixes on sighted landmarks to increase its robustness to noise in the representation or perceptual system. There is no permanent, explicit representation of the environment layout in this system, instead, the changing local view of the agent triggers a changing pattern of activity in the network that can be interpreted as a dynamic ‘map’ centred on and oriented to the agent.

It will be proposed that this method can provide robust support for wayfinding without being expensive in terms of computational power or memory, and without requiring complex coding mechanisms. These properties encourage the view that similar mechanisms could support wayfinding for an autonomous mobile robot. Although the system is not proposed as a model of any natural navigation system, it will also be suggested that it provides some useful insights into the character of such systems.

## 5.1 Relational models of space

A quantitative model of environmental layout is achieved by integrating observations from different egocentric view-points into representations with respect to environment-centred co-ordinate frames. There are two principle methods for representing spatial relations in allocentric co-ordinate frameworks. The first is that all objects are referenced with respect to a single global framework. As has already been noted, this method has certain advantages if the goal is to maintain the most accurate overall model. The second method is to form a network of local frameworks, where each frame is defined with respect to an individual object or group of objects. The relation between any two distant objects is then found by a sequence of transformations between adjoining reference frames (Brooks, 1985; Levitt & Lawton, 1990). This second, *relational coding* method, which exploits multiple partial representations, is preferred here for the reasons given in the previous section. The system to be described makes no use of many important sources of location information such as compass senses or dead reckoning. Clearly a robust navigation system will seek to use spatial knowledge derived through such channels. The aim here, however, is limited to the investigation of a distributed architecture that generates multiple redundant codings using information from a single modality.

The task of navigating a large-scale environment using relational methods divides into three problems: identification and re-identification of salient landmarks; encoding, and later remembering, goal locations in terms of sets of visible local cues; and finally, calculating paths between positions that share no common view. The first task, landmark identification, has been considered (though not entirely solved!) elsewhere, both from the point of view of animal and robot navigation systems (Levitt & Lawton, 1990; O'Keefe, 1990b; Zipser, 1983a; Zipser, 1986). In this article, landmarks are taken to be objects (or parts of objects) with *locally* distinctive secondary characteristics that can be identified with (roughly) zero-dimension locations in egocentric space. The agent is assumed to be able to detect suitable landmarks and determine, at least approximately, their positions relative to itself. This simplification is made in order to allow the investigation of the logically distinct problems of encoding and remembering relational models.

If position estimates can be accurately determined from the stored relational models then the place identification problem, which is critical to the task of constructing any layout model, is easily resolved—places (or landmarks) are identical that have the same coordinates with respect to the allocentric frame. Generally, however, we wish to assume error, possibly of a large or cumulative nature, in the estimates of spatial position. In this case, the identification problem still exists though the estimates of spatial position now provide powerful constraints to any matching process that exploits non-spatial sensory characteristics.

### 5.1.1 Proposed relational coding methods

A proposal for a relational coding system, presented as a theory of the hippocampal mapping system in the rat, has been provided by O'Keefe (1990a, 1990c) who suggests that the rat brain computes the origin and orientation of a polar coordinate frame from the vectors giving the egocentric locations of the set of salient visible cues. Specifically, he proposes that these location vectors are averaged to compute the origin (or *centroid*) of the polar frame, and that the gradients of vectors between each pair of cues are averaged to compute its orientation (or *slope*). Goal positions can then be recorded in the coordinate system of this allocentric frame in a form that will be invariant regardless of the position and orientation of the animal. This idea is illustrated in Figure 1.

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Figure 1 about here.

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This proposal has, however, a number of problematic characteristics. First, the computation of the slope is such that the resulting angle will differ if the cues are taken in different orders. Since any ordering is essentially arbitrary, a specific sequence will have to be remembered in order to generate the same allocentric frame from all positions within sight of the landmark set. Second, as landmarks move out of sight, are occluded by each other, or new ones come into view, the values of the slope and centroid will change.

Rather than changing the global frame each time a landmark appears or disappears it seems more judicious to maintain multiple local frames based on subsets of the available cues. These would supply several mutually-consistent encodings making the mapping system robust to changes in individual landmarks.

The use of multiple local frames has been proposed by Levitt and Lawton (1990). They observe that the minimum number of landmarks required to generate a coordinate frame is two (in two-dimensions, three in three-dimensions). They also provide a useful analysis of how the constraints generated by multiple local frames can be combined, even in the presence of very poor distance information, to provide robust location estimates. To calculate, from a novel position, a goal location that has been encoded in a two-landmark frame requires non-linear computations (trigonometric functions and square roots). It also requires that an arbitrary ordering of the two landmarks is remembered in order to specify a unique coordinate system. Among the concerns that has motivated the recent interest in qualitative modelling has been the resource demands of this need for continuous, computationally expensive co-ordinate transforms (Kortenkamp et al., 1992).

Zipser (1986), who had earlier considered a landmark pair method (Zipser, 1983b), points out that if one more landmark is used to compute the local frame then all the calculations are greatly simplified. In fact, all that is required to encode a goal location using three landmarks (in two dimensions, four in three dimensions) is that one constant is associated with each cue. Zipser called these constants *beta-coefficients*, they are, however, identical to the *barycentric* coordinates that have been known to mathematicians since Moebius (see for instance Farin, 1988). The system for large-scale navigation described below uses this three landmark method and it is therefore described in detail in the following section. In the remainder of the article the navigation problem will be considered as two-dimensional, however, the extension of these methods to three dimensions is straightforward.

### 5.1.2 The Beta-coefficient coding system

Figure 2 shows the relative locations of a group of three landmarks, hereafter termed an *L-trie*, labelled A, B, and C, seen from two different viewing positions  $V$  and  $V'$ . A goal site  $G$  is assumed to be visible only from the first viewpoint.

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 Figure 2 about here.  
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The column vectors  $\mathbf{x}_i = (x_i, y_i, 1)^T$  and  $\mathbf{x}'_i = (x'_i, y'_i, 1)^T$  give the location in homogeneous coordinates of object  $i$  in the egocentric frames centred at  $V$  and  $V'$  respectively. The two frames can therefore be described by the matrices

$$\mathbf{X} = [\mathbf{x}_A \quad \mathbf{x}_B \quad \mathbf{x}_C], \text{ and } \mathbf{X}' = [\mathbf{x}'_A \quad \mathbf{x}'_B \quad \mathbf{x}'_C] \quad (1)$$

If the three landmarks are distinct and not collinear then there exists a unique vector  $\beta = (\beta_A, \beta_B, \beta_C)^T$  such that

$$\mathbf{X}\beta = \mathbf{x}_G \text{ and } \mathbf{X}'\beta = \mathbf{x}'_G. \quad (2)$$

In others words, by remembering the invariant  $\beta$  the egocentric goal position from any new viewing position  $V'$  can be determined by the linear sums

$$\begin{aligned} x'_G &= \beta_A x'_A + \beta_B x'_B + \beta_C x'_C, \\ y'_G &= \beta_A y'_A + \beta_B y'_B + \beta_C y'_C, \\ (1 &= \beta_A + \beta_B + \beta_C). \end{aligned} \quad (3)$$

Note that since each constant is tied to a specific cue the ordering of the landmarks is irrelevant.

The beta vector can be determined directly by computing the inverse matrix  $\mathbf{X}^{-1}$  since

$$\begin{aligned} \beta &= \mathbf{X}^{-1}\mathbf{X}\beta \\ &= \mathbf{X}^{-1}\mathbf{x}_G \end{aligned} \quad (4)$$

Though this inverse calculation uses only simple arithmetic, the value of the beta-encoding as a possible biological model has been questioned on the grounds of its apparent mathematical complexity<sup>v</sup> (Worden, 1992). However, as Zipser (1986) points out, the same computation can be achieved by a gradient descent method, and so can be viewed as adapting the connection strengths of a simple learning unit using a supervised learning rule. A network architecture that instantiates such a mechanism is as follows. The network consists of two types of processing unit. The first are object-position units (*object-units* henceforth) whose (vector-valued) activations represent the locations in egocentric space of specific goal-sites and salient landmarks. The activations of these units can be set in two possible ways. First, for goals and landmarks that are visible, sensor systems identify the relevant objects and determine their positions in the agent's current egocentric reference frame. Second, for objects outside the current field of view, activations can be determined by propagating egocentric position estimates through the network as explained below. The second type of processor is termed a *beta-unit*. This unit receives input from three object-units and adapts its connection strengths (the beta coefficients) to match its output to the activation of a fourth.

An example of this architecture is illustrated in Figure 3, which shows a beta-unit G/ABC that receives the positions of the landmarks A, B, and C, and the goal, G, as its inputs. When all three landmarks and the goal are visible, the unit can adapt the weights  $(\beta_A, \beta_B, \beta_C)$  to reduce the discrepancy between its output  $(x, y, z)$  and the goal vector  $(x_G, y_G, 1)$ . When the three landmark nodes (A, B, and C) are active, and the goal G is inactive, an egocentric position estimate for G can be computed, according to equation 3, and propagated to the goal unit. To adapt the beta values by gradient descent, a suitable rule for the update at each iteration of the weight  $\beta_i$  from the  $i^{\text{th}}$  object is

$$\Delta\beta_i = \alpha[(x_G - x)x_i + (y_G - y)y_i + (1 - z)] \quad (5)$$

where the parameter  $\alpha$  is the learning rate<sup>vi</sup>.

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 Figure 3 about here.  
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To provide a more intuitive understanding of the beta-coding a geometrical interpretation can be given. Consider landmark  $A$  in L-trie  $ABC$  in Figure 4. The perpendicular  $h_A$  from  $A$  onto the line  $BC$ , defines one of the three axes of the *barycentric* coordinate frame generated by the spatial configuration of the three reference cues. For a given goal position  $G$ , the coefficient  $\beta_A$  with respect to landmark  $A$ , is the projection of  $G$  onto this axis. In other words,  $\beta_A$  is the ratio of the two perpendiculars  $h_G/h_A$  (or, equivalently, the ratio of the areas of the triangles  $GBC$  and  $ABC$ ).

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 Figure 4 about here.  
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This coding system in fact records *affine* rather than *metric* spatial relations, hence, another term for the coefficients is *affine coordinates*. However, assuming that the agent detects metric egocentric spatial relations according to a calibrated Euclidean measure, metric relations will be recoverable from the stored model.

## **5.2 Recognising poor cue configurations—a simple technique for improving the beta-coding**

An important problem that arises in using the beta-coding method with noisy data is that the spatial arrangement of the reference and target landmarks has a marked affect on the size of the error in the beta-coefficients and in any subsequent estimate of the target's position. For instance, if all the landmarks in the L-trie group lie exactly along a line, the matrix  $\mathbf{X}$  will be singular and the beta-coefficients for any target will be undefined. In general, landmark groups that are near collinear will also give large errors when computing cue positions in the presence of noise<sup>vii</sup>. The position of the target landmark relative to the L-trie is also significant—better estimates are obtained when the target lies

close to, or inside, the L-trie<sup>viii</sup>. Estimating the covariance of the errors in the beta-coefficients is difficult because of the non-linearities in their calculation, however, there is a simple heuristic that is effective at identifying poor cue configurations. Specifically the size of the beta-coefficient vector can be used to indicate the likely size of the error in the computed target position. This is possible because, as is shown below, the component of the error that changes with the configuration of landmarks is scaled by the value of the estimated beta vector—codings with larger vectors will, on average, have relatively large errors in the presence of perceptual noise. This technique makes few assumptions about the structure of the perceptual data and should therefore be applicable across a range of noise distributions. The following sub-sections show the derivation of this heuristic, and suggest how it can be applied in an estimator that improves the accuracy of a beta coding by combining repeated measures.

### 5.2.1 The size of the beta vector as an estimate of target error

In order to examine the relative stability of different cue configurations, the relationship between perceptual error, in observed cue positions, and the error in a computed target position is analysed. To account for noise in the perceptual system both of the expressions in equation (2) are expanded to show the (unknown) errors, hence,

$$\mathbf{x}_G + \delta\mathbf{x}_G = (\mathbf{X} + \delta\mathbf{X})(\boldsymbol{\beta} + \delta\boldsymbol{\beta}), \text{ and} \quad (6)$$

$$\mathbf{x}'_G + \delta\mathbf{x}'_G = (\mathbf{X}' + \delta\mathbf{X}')(\boldsymbol{\beta} + \delta\boldsymbol{\beta}) \quad (7)$$

We wish to see how the perceptual errors  $\delta\mathbf{X}$ ,  $\delta\mathbf{x}_G$ , and  $\delta\mathbf{X}'$  affect the computed error  $\delta\mathbf{x}'_G$  in the estimate of the target position. First, note that the matrix  $\mathbf{X}'\mathbf{X}^{-1}$  represents the co-ordinate transform between the reference frames  $\mathbf{X}$  and  $\mathbf{X}'$ . The two noise-free measures of the goal position are therefore related by

$$\mathbf{x}'_G = \mathbf{X}'\mathbf{X}^{-1}\mathbf{x}_G. \quad (8)$$

Multiplying both sides of (6) by this transform gives

$$\mathbf{x}'_G + \mathbf{X}'\mathbf{X}^{-1}\delta\mathbf{x}_G = \mathbf{X}'\mathbf{X}^{-1}(\mathbf{X} + \delta\mathbf{X})(\boldsymbol{\beta} + \delta\boldsymbol{\beta}). \quad (9)$$

Now subtract (9) from (7) to give the error in the target

$$\begin{aligned}\delta\mathbf{x}'_G &= (\mathbf{X}' + \delta\mathbf{X}')(\beta + \delta\beta) - \mathbf{X}'\mathbf{X}^{-1}(\mathbf{X} + \delta\mathbf{X})(\beta + \delta\beta) \\ &\quad + \mathbf{X}'\mathbf{X}^{-1}\delta\mathbf{x}_G \\ &= (\delta\mathbf{X}' - \mathbf{X}'\mathbf{X}^{-1}\delta\mathbf{X})(\beta + \delta\beta) + \mathbf{X}'\mathbf{X}^{-1}\delta\mathbf{x}_G.\end{aligned}\tag{10}$$

Since  $\mathbf{X}'\mathbf{X}^{-1}$  is a (noise-free) transformation that involves only rotation and translation it will be well-conditioned. Acting on  $\delta\mathbf{x}_G$ , the error in the original measure of the target position, this transformation therefore causes a rotation of the error but no change in its magnitude. From this we can conclude that the second term in equation 10 will be largely unaffected by the spatial configuration of the cues. The first term shows that the contribution to the target error  $\delta\mathbf{x}'_G$  due to noise in the measured reference cue positions is *scaled exactly* by the estimated beta vector. A suitable method of assessing the likely error generated by any beta coding is therefore to look at the size  $|\beta|$  of the vector estimate<sup>ix</sup>. In the following this will be referred to as the *beta-size* heuristic.

The above demonstrates analytically that the beta-size measure predicts a major component of the target error. The effectiveness of the measure has also been evaluated by Monte Carlo simulation using one thousand randomly generated L-trie/target configurations. For each configuration the beta vector was computed by matrix inversion under conditions of additive Gaussian noise<sup>x</sup> in all measured cue positions. An estimate of the target position for each configuration was then made using this estimated beta vector and new noisy measures of the L-trie. The error in the target estimate showed a Spearman's rank correlation of 0.8 with the size of the estimated beta vector<sup>xi</sup> confirming the latter as a good predictor of the relative error of alternative beta codings.

### 5.2.3 A robust estimator for the beta vector

When multiple estimates of the beta vector are obtained for the same landmark configuration, equation 10 indicates that accuracy will be better for estimates that are smaller than the mean observation. This suggests that a robust mechanism for combining multiple estimates would be a weighted average that gives more weight to smaller

estimates. This can be achieved by using a simple recursive estimator based on the standard Kalman filter (Maybeck, 1979). This estimator maintains two measures,  $\hat{\beta}$ , the current estimate of the vector, and a scalar  $c$  that substitutes for the variance estimate of a standard filter. A third quantity  $k$  is temporarily computed in order to perform each update but is not stored. Given a sequence of estimates  $\beta_1, \beta_2, \beta_3, \dots$ , appropriate updates are

$$\hat{\beta}_1 = \beta_1, \hat{\beta}_i = \hat{\beta}_{i-1} + k_i(\beta_i - \hat{\beta}_{i-1}), \quad (11)$$

$$c_1 = |\beta_1|, c_i = (1 + \alpha)(1 - k_i)c_{i-1}, \quad (12)$$

$$k_1 = 1, k_i = \frac{c_{i-1}}{(c_{i-1} + |\beta_i|)}, \quad (13)$$

(where  $\alpha$  is a constant which is either zero or takes a small positive value, see below).

This estimator is robust in that it prevents bad outliers (very large beta values) from skewing the estimate  $\hat{\beta}$ . The estimate itself will be biased to be below the mean, since, for the reasons outlined in the previous sub-section, it is better to under-estimate in this task than to over-estimate.

The beta-size estimator was also evaluated using a Monte Carlo simulation of one thousand L-trie/target pairs. Each cue configuration was sampled five hundred times with additive Gaussian noise in position estimates for each sample. The estimator was compared with a unweighted average computed using the same cue configurations and noise data. On average the beta-size estimator produced smaller errors at every stage of the estimation process than the unweighted estimator<sup>xii</sup>.

If there is considerable variation in the number of samples taken for different beta units this can significantly effect the relative accuracy of alternative codings making the *beta-size* measure a less effective means of selecting between them. However, if  $\alpha$  in equation 12 takes a small, non-zero value (e.g. 0.02) then  $\hat{\beta}$  will converge to a weighted running average of sampled beta estimates, and  $c$  will converge to be proportional to the expected size of the beta vector estimate<sup>xiii</sup>. In these circumstances, simulations have

shown that  $c$  can be used in place of  $|\hat{\beta}|$  as an indicator of the accuracy of alternative codings that is more robust to variation in sampling.

### 5.3 Modelling large-scale space

The beta coding method can be extended to determine the spatial relations between points over a wide environment that share no common landmarks. This is achieved by building a two-layer *relational network* of object and beta-units, which stores the positions of landmarks in the local frames defined by neighbouring L-trie groups. The resulting structure records the relationships between multiple local frames. Thereafter, the locations of distant landmarks (and goal sites) can be found by propagating local view information through this network. Zipser (1983b) and Levitt and Lawton (1990) have both discussed methods of this type for large-scale navigation using landmark-pair coordinate frames. The advantage of using the three-landmark method, however, is that following a sequence of transformations through the network is significantly simpler. Since all calculations are linear in the observed cue positions and independent of landmark order, the process can be carried out by spreading activation through the relational network (using the mechanism described in section 5.1.2 to accomplish each transition step). In contrast, a landmark-pair method would require networks of local processing units of considerably greater complexity in order to perform the necessary non-linear transformations between frames.

#### 5.3.1 Constructing a large-scale representation

The relational network that encodes the large-scale spatial model is constructed whilst exploring the environment. Specifically, each time the agent moves, new object-units are recruited to the network's lower layer to encode any visible landmarks that have not been encountered previously, and new beta-units are recruited to the upper layer for every novel L-trie/landmark combination in the set of visible cues. Each new beta unit is therefore connected to four object units—three representing the reference L-trie, and the fourth the landmark that is being encoded in this L-trie frame. Beta-coefficients are

calculated either by matrix inversion or by the iterative gradient descent rule as the agent moves within sight of the relevant cues.

Figure 5 shows an example of this learning process for a simple environment of five landmarks. From the current view-point, and assuming 360° perceptual capability four landmarks A, B, C, and D are visible for which the agent generates beta-units A/BCD, B/ACD, C/ABD, D/ABC. Following adequate exploration the network illustrated in Figure 6 will have been generated.

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 Figure 5 about here.  
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 Figure 6 about here.  
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Given this network the agent can determine the location of any target landmark when it is within sight of any group of three others. For instance if cues A, B, and C are visible and E is required, then the active object units will trigger D/ABC (activating object unit D) and hence E/BCD to give the egocentric location of the target. The method clearly generalises to allow the position of any goal site that is encoded with respect to an L-trie frame to be found. Note that the representation formed by the relational network encodes, in a direct manner, only the *local* spatial relations between neighbouring landmarks. Hence, although individual landmarks may be encoded in multiple local frames, the memory requirements for any environment is potentially  $O(n)$ —proportional to the number of landmarks, rather than  $O(n^2)$  as would be the case if all global relations between every pair landmarks were explicitly stored.

### 5.3.2 *The topology of the relational network*

The connectivity of the relational network implicitly defines an *adjacency graph* of the topological arrangement of local landmark frames. For instance, the network shown in figure 6 instantiates the graph shown in Figure 7. The links between nodes in this graph correspond to the beta-units, the nodes themselves are the L-trie co-ordinate frames (that are not explicitly represented in the relational network). The topological model of overlapping frames encoded in the adjacency graph constitutes a more abstract level of representation which is appropriate for some way-finding procedures (see below). -----

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 Figure 7 about here.  
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Although the graph shown here has entirely bilateral connections, there is nothing intrinsically symmetrical about the coding method. For instance, it would be quite possible to encode the relationship D/ABC and not the reverse A/BCD. This could happen if the agent, whilst moving through the environment, encodes the positions of landmarks in front with respect to those it is already passing, but not vice versa. This property of the mapping mechanism accords with observations of asymmetric spatial knowledge in humans (see Kuipers, 1982; McNamara, 1992).

## 5.4 **Wayfinding using the relational model**

### 5.4.1 *Orienting—determining large-scale spatial relations from a single view-point*

The ability to determine the direction and distance to distant locations is a useful competence that can support behaviours such as orienting toward a remote target, and moving in the direction of that target with the hope of finding a straight-line route. As has already been suggested, such estimates do not need to be very accurate in order to be of considerable use—it is often sufficient to start out in roughly the right direction and then make corrections en route.

As has been shown above, the relational network can compute large-scale spatial relations by propagating estimates of local cue positions. Rather than planning a specific sequence of frame transitions to determine the position of a distant target, the propagation of landmark positions can be seen as a parallel process which triggers all possible beta-units. In other words, activating the object-units of visible landmarks can trigger every beta-unit based on a sub-set of those landmarks. These new estimates then cause more beta-units to fire, and so on. This spreading activation process will result in egocentric position estimates for landmarks that are progressively more and more remote from the agent. For any specific target there will be multiple alternative estimates generated in this manner, each corresponding to a different sequence of local frame transitions. The system therefore provides a redundant coding of large-scale spatial relations.

Assuming noise in the perceptual mechanisms that determine the relative positions of visible cues (and hence noise in the stored beta-coefficients), there will be significant variation between the computed estimates that arrive via different transition sequences. Although the possibility exists for combining multiple estimates, this task is complicated by the overlap between the landmark sets that are used to generate different estimates. Overlapping sets produce estimates that are not independent and cannot be combined in a straightforward manner. The alternative to a combined estimate, is to use arbitration mechanisms that select one estimate out of the many. This raises the question of how the relative accuracy of different estimates can be judged. A model of the accumulation of noise would necessitate, at least, the computation and propagation of covariance estimates. Rather than pursue this course, cheaper methods have been sought that arbitrate on the basis of heuristic estimates of relative error. Three suitable heuristics are as follows.

The first and simplest approach to arbitrating between multiple estimates is to regard the first estimate that arrives at an object-unit as the best approximation to that landmark's position. This idea is motivated by the observation that each transition in a sequence of frames multiplies the noise in the position estimate. On average, therefore, the best estimate will be provided by the *shortest sequence* of frame transitions.

Assuming that the delay incurred for a frame transition is uniform throughout the network, such an estimate will be the first to arrive at the object-unit of a target (switching that unit into a state in which any further estimates can be discarded).

As has already been noted, however, the spatial arrangement of landmarks makes a significant contribution to the accuracy of different beta codings. Arbitration mechanisms based on the size of beta vectors have therefore been investigated with the aim of limiting some of the effects of poor cue configurations on propagated estimates. The rationale for both the heuristics described below is that a single poor transition in a sequence of frames can grossly magnify the error in the final estimate. The aim is therefore to try and identify sequences with particularly weak links and avoid using the estimates they generate.

First, in choosing between estimates provided by transition sequences of equal (shortest) length, the size of the largest beta vector in each generating sequence has been found to be a useful predictor of relative error. This *beta-max* heuristic is simple and economical to implement as it involves propagating just one additional value along with each cue estimate. The first position estimate to arrive at an object-unit is stored but replaced if a better estimate (according to the *beta-max* measure) comes along. Second, there may be circumstances where a longer sequence of transitions can provide a better estimate than the shortest possible sequence. This will arise most often where the latter exploits some particularly poor cue configurations. To reduce the likelihood of using very poor transitions, a *beta-threshold* heuristic has been adopted of excluding from the network beta units for which the size of the beta vector is above a fixed threshold. The effectiveness of both heuristics has been demonstrated empirically as described in section 5.4.3 below.

Determining the approximate direction and distance to a distant goal can support orienting and ‘straight-line’ behaviours as described. However, this mechanism will not always be an appropriate means of wayfinding—estimates for very distant targets may prove too inaccurate, or the direct line to a goal location may not prove to be a viable path. The following section therefore describes an indirect but more reliable wayfinding

method that exploits topological knowledge to determine, at the outset, a possible path to a distant goal, and then uses en route corrections to home-in on the target.

#### 5.4.2 Wayfinding—following a path to a goal location

A good strategy for reaching a goal is to move, not in the direction of the goal itself, but towards landmarks that are known to lie along a possible path to the goal. In the context of the relational network, this involves determining, before setting out, a sequence of adjacent landmark frames that links start and goal positions. The agent then finds its way by moving frame to frame through this topological sequence. Because perceptual information about expected landmarks should become available as each frame is crossed, the agent will be able to replace computed estimates of cue positions with ‘hard’ sensor data and so avoid the multiplicative build-up of noise. There is, however, an overhead incurred by this method through the need to calculate a suitable sequence of adjoining frames. Multiple sequences are possible so arbitration is again needed to decide between alternatives. The *shortest sequence* heuristic is appropriate here, although on the slightly different grounds that shorter sequences should (on average) give more direct routes. Other heuristics are possible, for instance, estimates of the actual distances covered by alternate routes could be calculated allowing a more informed judgement as to which is the shortest path.

To explicitly determine the shortest frame sequence the process of propagating information through the relational network is reversed. In other words, a spreading activation search is performed from the goal back toward the start position. This is easiest to imagine in the context of the adjacency graph (Figure 7). The search process described here, and implemented in simulation, uses an explicit representation of the adjacency graph—containing a unit for each L-trie frame—which is constructed alongside the relational network. The spreading activation process through the graph is modelled as a series of synchronous updates that occur as follows. An L-trie node in the graph, for which there is a beta-unit encoding the goal, is activated and clamped on (i.e. its activity is fixed throughout the search). All other nodes are initialised with zero activity. The

signal at the goal is then allowed to diffuse through the network decaying by a constant amount for each graph link that is traversed<sup>xiv</sup>. Once the activation reaches a node local to the start position, the shortest sequence is easily determined—beginning with this start node, a path is traced which links each node to its most active neighbour until the goal is reached. An illustration of this spreading activation process has been given in (Prescott, 1993a; Prescott, 1993b). Since the adjacency graph is implicit in the relational network the possibility exists (though is yet to be explored) of implementing this search process directly in the relational network, without the need for an explicit graph model.

Having found an appropriate sequence of neighbouring frames, several methods can be used to implement path following behaviour. For instance, the agent could start by moving toward the average position of the landmarks vectors for the first L-trie in the path. Once that position is reached, the position of the next L-trie is generated and so on till the goal is reached. As landmarks along the path appear, propagated estimates of landmark positions are replaced with sensor data making the error in the final goal estimate independent of path-length. As each frame along the path is reached, the single unknown cue position for the subsequent frame can usually be computed by a single frame transition (if indeed it has not already come into view). The *beta-size* heuristic can therefore be applied to alternative estimates in a straightforward manner by preferring estimates with smaller beta vectors.

Both the orienting and path following procedures are illustrated in Figure 8. The figure shows an environment of fifty landmarks which was explored by a random walk for one hundred thousand learning steps<sup>xv</sup>. During exploration the perception of local landmark positions was perturbed by Gaussian additive noise with a standard deviation equal to 5% of the perceptual range of the agent. At each learning step the system encoded the relative locations of the four closest visible landmarks<sup>xvi</sup> using the matrix inverse method of beta computation together with the *beta-size* estimator. The network generated by this learning process was pruned according to a beta-threshold heuristic<sup>xvii</sup> resulting in 899 beta-units derived from 465 L-trie frames, where each landmark was redundantly encoded with respect to between 4 and 31 frames. The figure shows typical examples of orienting and

path-following in the presence of noise. Cumulative noise in the orienting method results in an estimated direction and distance (the dotted line) which is slightly off target. In contrast, the path-following system exploits sensor data as it becomes available and so is able to home-in on the target position.

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Figure 8 about here.  
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#### *5.4.3 The relational network as a dynamic map*

The spreading activation process that propagates local landmark positions through the relational net will eventually generate egocentric location estimates for all landmarks whose object-units have indirect connections with those encoding the visible scene. The resulting activity in the object-units of the network can be thought of as a dynamic ‘map’ of the environment that is arranged with the agent at the centre and oriented towards its current heading. This egocentric map was generated for the fifty-landmark environment for an agent at viewing position 2 shown in Figure 8. Figure 9 shows the positions of the landmarks in the agent’s egocentric frame, and the errors in these estimates compared to the exact (egocentric) positions.

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Figure 9 about here.  
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As a result of multiplicative error, the layout of landmarks is more accurately estimated close at hand than further away, however, the topological relations between landmarks are reproduced throughout. The nature of this ‘dynamic’ map demonstrates one of the major differences between the relational frame approach and methods that emphasise the construction of a permanent ‘map’ of environmental layout in which position errors are minimised or explicitly modelled. In the relational approach there is no

long-term, static representation of large-scale spatial relations. Instead, insofar as a large-scale map of any sort exists, it is described by the continuously changing activations of the units in the relational net that encode current estimates of landmark positions in the egocentric frame of the agent. There is a striking similarity between this property of the relational coding system and Scholl's (1987) observation that the human 'cognitive map' has no (absolute) preferred orientation but is always centred and oriented to the current viewing perspective.

The effectiveness of heuristics based on beta-size was investigated with respect to maps generated from different viewing positions in the fifty landmark environment (following the learning process described above). Maps computed using the *shortest sequence*, *beta-max* and *beta-threshold* heuristics were compared with maps generated using the *shortest sequence heuristic* alone (i.e. selection between sequences of identical length was random). Comparing position estimates generated from one hundred different viewing points, 76% of all estimates generated by the first method were as good or better than the estimates generated by the second method<sup>xviii</sup>. Arbitration based on the size of beta vector estimates therefore appears to be an effective means of improving the exploitation of spatial knowledge encoded in the relational network.

## 6 Future Work

One use which the dynamic map may serve is to aid in disambiguating perceptually similar landmarks. This can be achieved by treating the forward part of the dynamic map as a prediction of the upcoming scene. If the agent then sees a new landmark that is similar to one it already encodes, it can judge whether the two cues actually arise from the same object by the extent to which the place where the landmark appears agrees with the location predicted for it by the system. Preliminary simulation studies have shown this to be a useful constraint for the landmark identification task. Similar mechanisms might also be used to detect changes in the environment.

It has been suggested above that this mapping system generates a reasonably compact representation of large-scale space, however, for this to hold in arbitrary environments

some selectivity about the choice of reference landmarks may be required. As the number of landmarks within the agent's perceptual range increases there is a combinatorial increase in the number of potential L-trie/landmark pairings. In situations where there are five or more salient landmarks visible contemporaneously some pruning of the set of candidate L-tries frames may be desirable to prevent learning an excess number of redundant encodings for each landmark. In the work described above a simple heuristic has been used of encoding relations between just the four closest landmarks visible at each step. A better approach, however, would be to encode relationships only with respect to what are judged to be 'good' L-trie configurations (in the sense of producing beta-codings that are more stable with respect to perceptual errors). For example, in encoding a goal position, an L-trie configuration that is closer to an equilateral triangle should be greatly preferred to one that is near collinear. Similarly, four landmarks that form the corners of a square might be selected as a good configuration for mutually encoding one another. The issue of how this selection might be implemented efficiently is currently being explored, again, the ability to reliably predict the upcoming scene might prove a useful criteria for deciding whether or not additional encodings are needed. It is interesting to note that if an animal were to represent the world in this way we might predict that certain landmark configurations would be more salient than others. This hypothesis could even be tested by looking for blocking or overshadowing effects where memorising a goal location with respect to a good cue configuration inhibits or attenuates learning with respect to a poor configuration.

The system described here is specialised to exploit perceptual information about the relative locations of local landmarks, and makes no use of other sources of location information such as compass senses, dead reckoning, and route knowledge. It has been argued in this article, however, that such information should play a role in a robust navigation system. The problem of combining, or arbitrating between, directions derived from different wayfinding mechanisms is therefore a major research interest for the future.

## 7 Conclusions

This article has argued that the problem of navigation in animats may be best approached by discarding the goal of a unitary system with a global world model, in favour of the use of multiple systems—or *multiple schemata*—each supported by partial models. This approach, it has been argued, has a resonance with the general ethos of animat research and converges with current understanding of natural navigation systems.

This view constitutes a theoretical position some aspects of which have been explored through simulation. A navigation system has been developed which maintains multiple, partial and overlapping models of the environment based on the barycentric co-ordinate frames defined by groups of salient landmarks. The architecture of this system is envisaged as a network of simple computational units that can support parallel processing. It has been demonstrated that this system can generate multiple redundant estimates of both local and large-scale spatial relations. Techniques for arbitrating between the estimates have been developed which make few assumptions about the structure of the perceptual data and so should be applicable across a range of noise distributions. The system maintains no long-term, static representation of large-scale spatial relations, however, the continuously changing activations of the units in the network can be viewed as a dynamic map which is permanently oriented to the position and heading of the agent.

The limitations of simulation are such that the evaluation of the hypotheses presented here has barely begun. This task awaits more thorough investigation through the construction of genuine wayfinding robots.

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<sup>i</sup>As Beer (1995) points out, to justify the tag ‘representation’ requires more than just a correlation between the agents internal state and certain aspects of the external world. The additional requirement is that some useful ‘explanatory work’ is performed by making a distinction between the content-bearing and non-content-bearing elements of the system. With respect to understanding autonomous agents, Beer is sceptical that this requirement

can be met, whereas Clark (to appear) considers that the notion of representation is useful but needs to be reformulated along similar lines to those proposed here.

<sup>ii</sup>Gould

(1986)

has proposed a contrary view that insects do construct models of spatial layout, however, the balance of current evidence appears to be against this position (Wehner & Menzel, 1990).

<sup>iii</sup>There are significant differences between the Kuipers' hierarchical view and the view espoused here, in particular, with respect to the relative importance and priority assigned to the topological and metric levels of representation. Kuipers also advocates the construction of global models of space at different levels of the semantic hierarchy. These

differences are considered further in

(Prescott, 1993b; Prescott,

1994)

.

<sup>iv</sup>Spreading activation as a graph searching technique has a long history in cognitive

modelling

(Anderson,

1983)

and in the literature on graph search

(Dijkstra,

1959)

and path planning (e.g.

Crowley, 1985; Mataric,

1990 ).

^In defence of the biological plausibility of the beta coding method O'Keefe has proposed that matrix inversion may be one of the computations performed in the

hippocampus of the rat

(O'Keefe,

1990a)

.

<sup>vi</sup>In practice variable gain and momentum terms will speed convergence of the gradient descent version of the beta computation (as in many other connectionist learning

procedures

see e.g. Hertz, Krogh, & Palmer,

1991) . The rate of convergence also depends on the spatial configuration of the four cues, specifically, beta values will converge more slowly with poorer cue configurations (as defined below in section 4.2).

<sup>vii</sup>That near collinear L-trie configurations will give poor beta codings is evident from the observation that each beta-coefficient is inversely scaled by the perpendicular from one vertice of the triangle (see figure 4 and section 4.1.2). If the perpendicular is small, any amount of perceptual noise can severely distort the computed value of the coefficient. For any triangle that is near collinear, one of the perpendiculars will always be near zero in size.

<sup>viii</sup>That more reliable codings are obtained when the target lies near or inside the reference triangle follows from the proof given in section 4.2.1 that noise is scaled by the size of the beta vector. As is clear from figure 4, one or more components of the beta vector will increase in size as the position of the goal moves further away from the reference triangle.

<sup>ix</sup>Either the length of the beta vector (used throughout this paper) or the largest single component can be chosen as the *beta-size* measure—since the elements of the vector are constrained to sum to unity these two measures have a very high correlation.

<sup>x</sup>In the case of additive Gaussian noise, a cue positioned at  $(x,y)$ , relative to the viewing-point, is observed as  $(x + \delta x, y + \delta y)$  where  $\delta x$  and  $\delta y$  are drawn independently from a Gaussian noise distribution with a mean of zero and a fixed standard deviation.

<sup>xi</sup>The rank correlation is used to compare the *beta-size* measure with the target error because we wish to use the heuristic to *rank* alternative beta vector estimates. The size of the rank correlation does not change with any global increase or decrease in the level of perceptual noise, however, if the noise distribution is varied between cue configurations then this clearly will have some influence. Tests under conditions of *relative* Gaussian noise (where the standard deviation of the noise distribution increases linearly with distance) gave correlations between .75 and .72 for levels of upto 20% perceptual noise. This result confirms that the cue configuration is a major determinant of error and suggests that the size heuristic will be reasonably robust to other influences on the variance in the target error.

<sup>xii</sup>Errors in the estimate of the target position were compared after 10, 50, 100, and 500 samples—all differences between the beta-size estimator ( $\alpha = 0$  or  $\alpha = 0.02$ ) and the unweighted estimator were significant ( $p < 0.001$ , Wilcoxon signed rank test). The variance of the error distributions differed substantially between the two methods, the unweighted average having a higher variance in the computed target error throughout the estimation process.

<sup>xiii</sup>From equation 12 the change in  $c$  over one update step is given by  $c_i - c_{i-1} = (1 + \alpha)(1 - k_i)c_{i-1} - c_{i-1}$ . Setting this formula to zero and expanding  $k_i$  (using equation 13) shows that the possible stationary values for  $c$  are zero and  $\alpha|\beta_i|$ . Monte Carlo simulations indicate that the expected value of  $c$  converges to  $\alpha|\beta_i|$ , whilst  $\hat{\beta}$  converges to a weighted running average after approximately  $1/\alpha$  samples. The parameter  $\alpha$  acts as a ‘forgetting’ factor, that causes early measures to gradually drop out of the estimate, and is related to standard filtering techniques.

<sup>xiv</sup>The spread of activation from one L-trie node to the next is achieved by, at each time-step, updating the activity of each node to be equal to the maximum of its own activation and that of any of its immediate neighbours (multiplied by a decay factor) at the previous time-step.

<sup>xv</sup>This large number of learning steps was allowed to guarantee multiple exposures to all cue configurations, navigation between most points is actually possible after a much shorter training period. The time taken to learn the environment could also be reduced considerably by adopting a more intelligent exploration strategy than the random walk.

<sup>xvi</sup>This heuristic reduces the number of beta-units generated in positions where many landmarks are visible simultaneously, see section 4.5 for further discussion of this point.

<sup>xvii</sup> Beta-units were excluded from the relational network where the size of the beta vector was above a threshold value of 5.0.

<sup>xviii</sup>16% of estimates were tied (i.e. both methods generated exactly identical estimates). Similar results were obtained for networks generated under conditions of 1% (73%, 23% tied) and 10% (70%, 15% tied) additive Gaussian noise. The effect of sequence length was also examined. This analysis showed that the number of tied estimates falls significantly with sequence length but the overall proportion of good or better estimates changes very little (remaining between 65% and 75% for sequences of upto four transitions).

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## Figure Captions

Figure 1: O’Keefe’s proposal for an allocentric polar coordinate frame defined by ‘centroid’ and ‘slope’ measures determined from the positions of local landmarks in the agent’s egocentric coordinate frame. The arrows indicate two possible viewing positions. (Adapted from O’Keefe, 1990c)

Figure 2: Relative positions of three landmarks and a goal from two viewpoints V and V'. (Adapted from Zipser, 1986)

Figure 3: Figure 3: A beta-unit,  $G/ABC$ , for computing the position of a goal G with respect to three object-units for landmarks A, B, and C. See text for details of how the activity of the units and values of the  $\beta$  parameters are set.

Figure 4: Coding a goal position (G) relative to the barycentric coordinate frame determined by three landmarks A, B, and C. See text for further details.

Figure 5: An environment with five landmarks, the agent is oriented towards the right with field of view as indicated by the circle.

Figure 6: A relational network for the five-landmark environment. The network consists of an input/output layer of object-units and a hidden layer of beta-units that encode landmark transformations in specific local frames. Each beta-unit is connected to four object-units representing three reference landmarks and a target landmark.

Figure 7: The L-trie adjacency graph for the five-landmark environment. The edges in the graph correspond to the beta-units in the relational network. See text for further details.

Figure 8: Orienting (the dotted line) and following a path (the solid line) to a distant target. The positions of fifty distinct landmarks in the environment are indicated by letters. The target, in this example, is the landmark  $u$ . The circle shows the limit of the agent's perceptual system. The compass is intended to indicate the orientation of an external reference frame (the agent itself has no direction sense). The path determined for path-following used the sequence of adjacent landmark frames (Rsv HRs Hos Hoq goq Bgo BGg BGN GNh ANh Aah Lah Laf).

Figure 9: The 'dynamic' map in the agent's own egocentric coordinate frame generated from viewing position (2) in the previous figure. The upper picture shows the estimated position of landmarks in the agent's egocentric frame (note that the environment is now oriented to the agents current viewing position). The lower picture shows the error in the position estimate for each landmark. Note that the noise in the estimates for landmarks  $Q$ ,  $Y$ ,  $o$ , and  $q$  is generated by the perceptual system alone. For all other landmarks these errors are amplified by the errors in the beta-units of the network.

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**Figure 1**

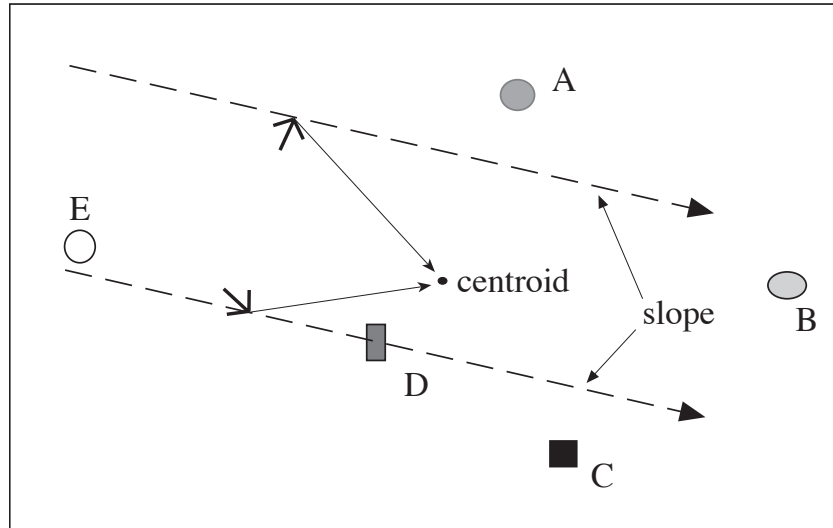


Figure 1: O'Keefe's proposal for an allocentric polar coordinate frame defined by 'centroid' and 'slope' measures determined from the positions of local landmarks in the agent's egocentric coordinate frame. The arrows indicate two possible viewing positions. (Adapted from O'Keefe, 1990c)

**Figure 2**

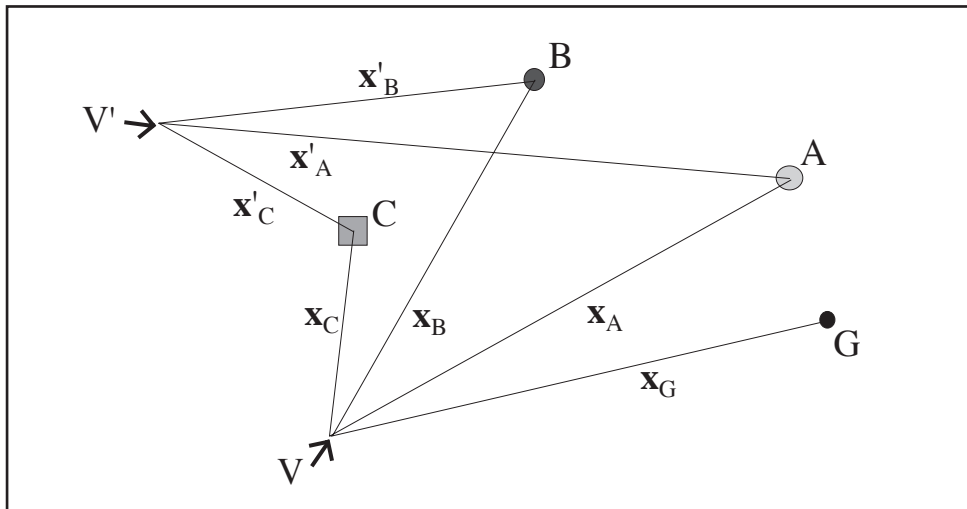


Figure 2: Relative positions of three landmarks and a goal from two viewpoints  $V$  and  $V'$ .  
(Adapted from Zipser, 1986)

Figure 3

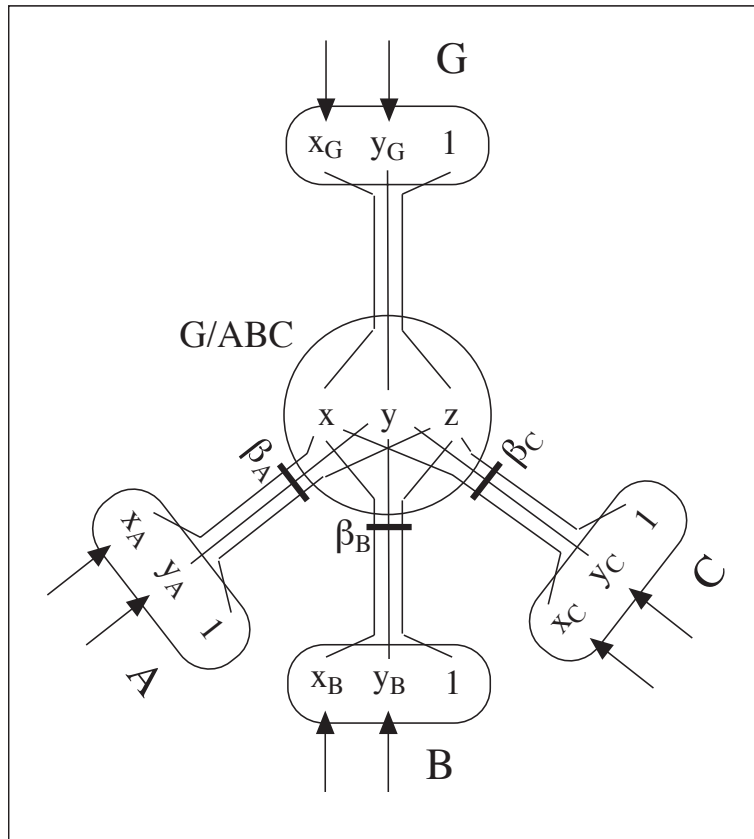


Figure 3: A beta-unit,  $G/ABC$ , for computing the position of a goal  $G$  with respect to three object-units for landmarks  $A$ ,  $B$ , and  $C$ . See text for details of how the activity of the units and values of the  $\beta$  parameters are set.

**Figure 4**

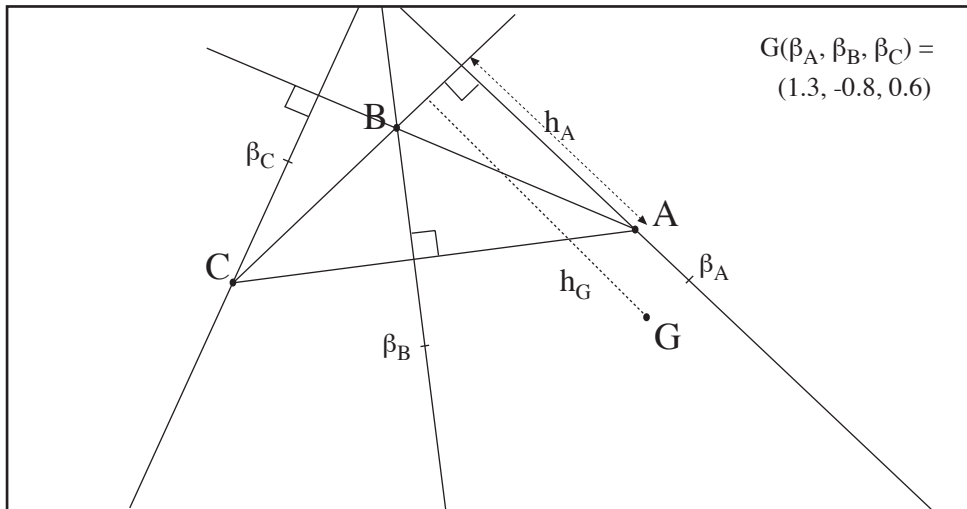


Figure 4: Coding a goal position (G) relative to the barycentric coordinate frame determined by three landmarks A, B, and C. See text for further details.

**Figure 5**

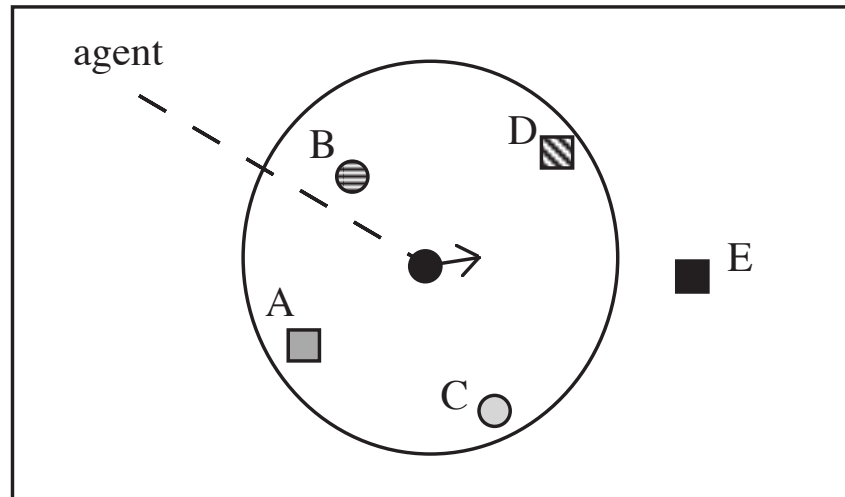


Figure 5: An environment with five landmarks, the agent is oriented towards the right with field of view as indicated by the circle.

**Figure 6**

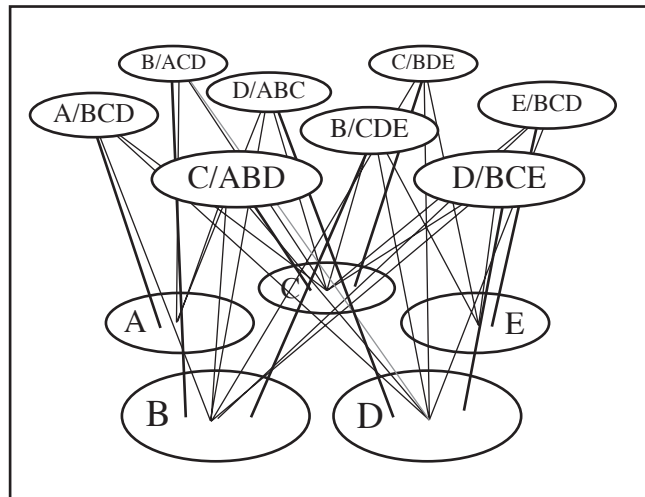


Figure 6: A relational network for the five-landmark environment. The network consists of an input/output layer of object-units and a hidden layer of beta-units that encode landmark transformations in specific local frames.

**Figure 7**

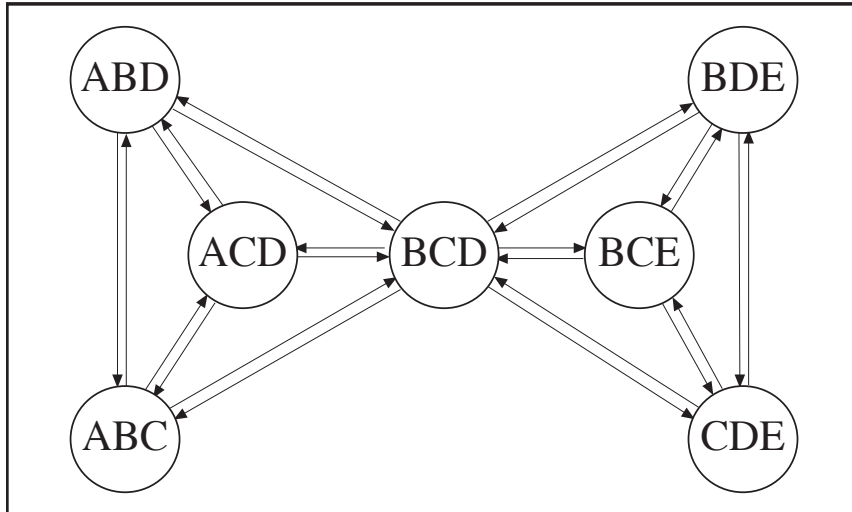


Figure 7: The L-trie adjacency graph for the five-landmark environment. The edges in the graph correspond to the beta-units in the relational network. See text for further details.

**Figure 8**

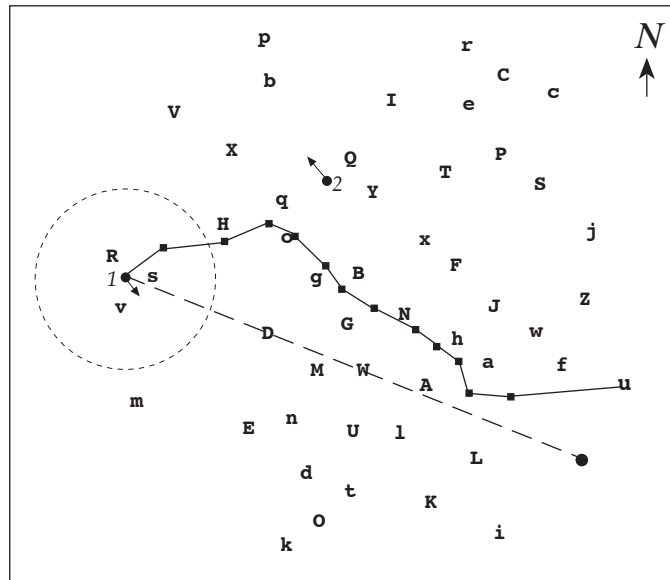


Figure 8: Orienting (the dotted line) and following a path (the solid line) to a distant target. The positions of fifty distinct landmarks in the environment are indicated by letters. The target, in this example, is the landmark *u*. The circle shows the limit of the agent's perceptual system. The compass is intended to indicate the orientation of an external reference frame (the agent itself has no direction sense). The path determined for path-following used the sequence of adjacent landmark frames (Rsv HRs Hos Hoq goq Bgo BGg BGN GNh ANh Aah Lah Laf).

**Figure 9**

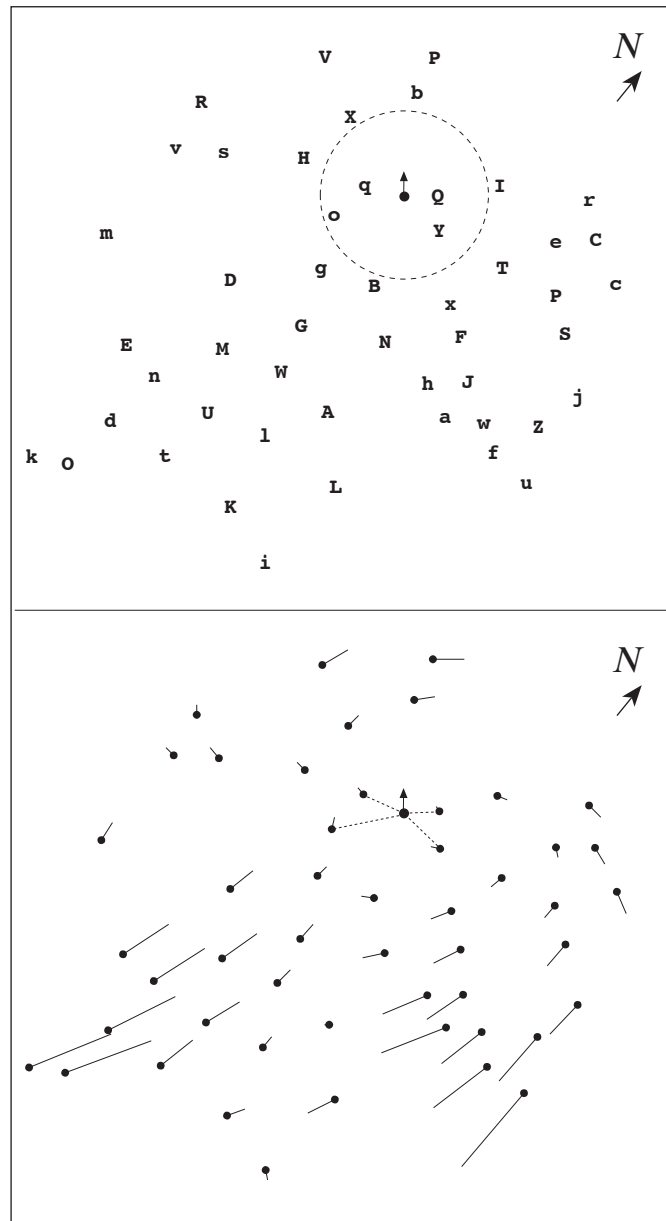


Figure 9: The ‘dynamic’ map in the agent’s own egocentric coordinate frame generated from viewing position (2) in the previous figure. The upper picture shows the estimated position of landmarks in the agent’s egocentric frame (note that the environment is now oriented to the agents current viewing position). The lower picture shows the error in the position estimate for each landmark. Note that the noise in the estimates for landmarks  $Q$ ,  $Y$ ,  $o$ , and  $q$  is generated by the perceptual system alone. For all other landmarks these errors are amplified by the errors in the beta-units of the network.

