Article:
Babiker, Mohamed Elhag and Lembessis, Vassilis (2016) Mechanical effects on atoms interacting with highly twisted Laguerre-Gaussian light. Physical Review A. 043854. pp. 1-5. ISSN: 1094-1622

https://doi.org/10.1103/PhysRevA.94.043854

Mechanical effects on atoms interacting with highly twisted Laguerre-Gaussian light

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(Received 21 July 2016; published 28 October 2016)

Laguerre-Gaussian (LG) light modes with large values of the radial index p and the azimuthal index l are considered. We give special emphasis on the mechanical effects on atoms arising from the Gouy phase and the phase due to the curvature of such light. It is shown that the additional phase terms for highly twisted light can lead to mechanical effects based on the substantial reduction of the axial wavenumber. As a consequence, axial recoil effects are diminished in the processes of stimulated emission and absorption of the light by the atoms. Both the scattering force and the momentum diffusion are affected by the inclusion of these normally ignored additional phase terms. The magnitudes of the effects are explored using order of magnitude analysis with typical parameters due to the atoms and the light.

DOI: 10.1103/PhysRevA.94.043854

I. INTRODUCTION

Soon after the discovery of Laguerre-Gaussian (LG) laser light it was recognized that such states of light could herald a new technique in the manipulation of the atomic gross motion since a LG photon carries not only a linear momentum vector but a quantized orbital angular momentum vector along the beam propagation axis. The realization of such beams in the laboratory was followed by a sizable amount of theoretical work on the mechanical effects of such beams on atoms [1–9]. A detailed review of work carried out on this subject is given in Ref. [10]. Another related new development has been the production of twisted matter (de-Broglie) waves. Such matter waves have been realized for electrons and neutrons and proposed for atoms [11–20]. It has been shown that the diffraction of an atom wave packet from a light mask with a forklike intensity profile results in the generation of atom vortex Bessel and LG beams endowed with the property of quantized orbital angular momentum [19].

By contrast, experimental investigations on gross atomic motion in twisted light are fewer and are still lagging behind theory. The common feature of most of the investigations is that they are limited to cases of the lowest values of the winding number l and, significantly, they also ignore modes with nonzero values of the radial mode index p. Advances in techniques for the generation of twisted light have recently enabled modes with very large radial index, p, and/or winding number l to be realized [21,22]. The role of the quantum numbers l,p characterizing the LG light fields have been investigated within the context of treating them as LG wave functions [23], and recently the "ignored quantum number" p within the context of quantum communications has been emphasized in Ref. [24] and a quantum mechanical theory featuring the effects of p has recently been presented [25]. The current experimental activity on the production of optical vortices with extremely large values of l and p [21,22] continues. It has been suggested that such beams can be exploited in the creation of concentric cylindrical lattices in which quantum Hall physics with cold atoms can be realized [26].

It turns out that in the study of the mechanical effects of LG light on atoms the consideration of large l and p values brings to the fore optical phase terms, which have so far been discarded because for small l and p values they are justifiably negligible. We show here that considerable modifications arise in the physics involving atomic gross motion primarily because the radiation forces exerted by the light on the atoms are modified and so are effects such as atom diffraction and the momentum diffusion coefficients. The modifications stem from phase gradients originating from the beam curvature and the Gouy phase most prominently near the focus plane of the LG light mode. Both the Gouy and the curvature phase terms have so far been ignored in the analysis, with the Gouy phase strongly dependent on the values of l and p.

The Gouy phase anomaly is a basic property of all focused beams. Although frequently discussed with reference to focused light beams, it is also known to arise in the cases of focused acoustic and electron beams. It was first discovered over 11 decades ago by Gouy who made direct measurements of the anomaly in optical beams [27,28]. Over the years the Gouy phase has been shown to play significant roles in a number of contexts as described in the interesting paper by Feng and Winful [29], who provided a physically transparent interpretation of the Gouy phase as originating from the in-plane spatial confinement of the focused beam. Hariharan and Robinson have given another explanation of the Gouy phase as a geometrical quantum effect, which arises as a result of the uncertainty principle whenever there is a modification of the volume of space in which the light beam is transmitted [30]. One of the most prominent manifestations of the Gouy phase is in the context of optical tweezers where it plays a role in the in-plane trapping of particles and leads to super-luminal phase velocities v_ϕ at focus. This suggests a subluminal group velocity v_g of the light in vacuum, which is in conformity with the product rule $v_\phi v_g = c^2$. Recent experiments suggest that light in vacuum travels at subluminal speeds for all beams, including Gaussian, Hermite-Gaussian, and Laguerre-Gaussian ones, which are endowed with lateral intensity spread. Of course, light only has its normal speed c in vacuum when propagating in the form of a plane wave [31].

This paper is organized as follows: In Sec. II we outline the main properties of a LG field for a mode with well defined l and

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p. Section III explores the modifications of the scattering force exerted by the light on a two-level atom when the full phase of the LG mode is taken into account. We present numerical estimates for the deviations from the conventional approaches where the Gouy and curvature terms have been omitted from the analysis. In Sec. IV we consider the diffusion coefficients associated with the motion of a two-level atom in a coherent LG laser beam with high values of l and p. Section V contains our conclusions and further comments.

II. LAGUERRE-GAUSSIAN LIGHT MODES: GOUY PHASE AND BEAM CURVATURE

In the paraxial approximation the electric field associated with a Laguerre-Gaussian mode, of wavelength $\lambda = 2\pi/k$ propagating in the z direction and polarized in the x direction is given by

$$\mathbf{E} = u_p^{|l|} e^{i\Theta_{lp}} \widehat{\mathbf{x}}, \tag{1}$$

where $u_p^{|l|}$ is the amplitude or mode distribution function,

$$u_p^{|l|} = E_{k00} \frac{C_{pl}}{\left(1 + z^2/z_R^2\right)^{1/2}} \left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right) \times e^{-r^2/w^2(z)} \tag{2}$$

and Θ_{lp} is the phase function,

$$\Theta_{lp} = kz + l\phi - (2p + |l| + 1)\tan^{-1}(z/z_R) + \frac{kr^2 z}{2\left(z^2 + z_R^2\right)}. \tag{3}$$

Here $L_p^{|l|}$ are the generalized Laguerre polynomials distinguished by the integers l and p. In a LG mode the integer l is referred to as the azimuthal or the winding number (also topological charge), which can be positive or negative. This determines the orbital angular momentum content as $l\hbar$ per photon carried by the mode, while integer $p > 0$ determines the number of the radial nodes in the mode as equal to $p + 1$. The factor C_{lp} is given by $C_{lp} = \sqrt{2p!/2\pi(|l|! + p!)}$; $w(z)$ is the width of the beam and it is equal to $w_0\sqrt{1 + z^2/z_R^2}$, where w_0 is the beam waist; E_{k00} represents the field amplitude of a Gaussian beam with the same power and beam waist, and $z_R = \pi w_0^2/\lambda$ is the Rayleigh range of the beam. In Eq. (3) the Gouy phase term is identified as the third term, namely,

$$\Theta_{\text{Gouy}} = -(2p + |l| + 1)\tan^{-1}(z/z_R), \tag{4}$$

and the curvature term is the last term, namely,

$$\Theta_{\text{curve}} = \frac{kr^2 z}{2\left(z^2 + z_R^2\right)}. \tag{5}$$

It is easy to see that the beam now has a total wave vector $\mathbf{K} = \nabla\Theta_{lp}$, where the gradient of the phase function is given by

$$\mathbf{K} = \nabla\Theta_{lp} = k\left[1 - (2p + |l| + 1)\frac{z_R}{k\left(z^2 + z_R^2\right)} + \frac{r^2\left(z_R^2 - z^2\right)}{2\left(z^2 + z_R^2\right)^2}\right]\widehat{\mathbf{z}} + \frac{l}{r}\widehat{\phi}. \tag{6}$$

where carets indicate unit vectors in cylindrical polar coordinates. This modified wave vector amounts to modifications in the photon recoil and the radiation pressure forces exerted on the atom as a result of its interaction with the beam.

The first indication of the expected modifications of pressure forces can be seen by considering the linear momentum density carried by the beam. This is defined as the time-averaged Poynting vector, $\mathbf{S} = \epsilon_0 \mathbf{E} \times \mathbf{B}$, the real part of which is given by [1]

$$\mathbf{S} = i\frac{\epsilon_0}{2}\langle \mathbf{E} \times \mathbf{B}\rangle = i\frac{\epsilon_0}{2}\left(v_p^{|l|}\nabla v_p^{|l|*} - v_p^{|l|*}\nabla v_p^{|l|}\right), \tag{7}$$

where $v_p^{|l|} = u_p^{|l|} e^{i\Theta_{lp}}$. Substituting for $u_p^{|l|}$ from Eq. (2) we have after some algebra,

$$\mathbf{S} = \epsilon_0\left(u_p^{|l|}\right)^2\left\{\left[k - (2p + |l| + 1)\frac{z_R}{z^2 + z_R^2} + \frac{kr^2\left(z_R^2 - z^2\right)}{2\left(z^2 + z_R^2\right)}\right]\widehat{\mathbf{z}} + \frac{l}{r}\widehat{\phi} + \frac{2kzr}{z^2 + z_R^2}\widehat{\mathbf{r}}\right\}. \tag{8}$$

From the results in Eqs. (6) and (8) we see that both the wave vector and the Poynting vector expressions are modified compared with the expressions given so far in the literature provided that we work with parameters that can make the contributions from the Gouy phase Eq. (4) and the curvature phase Eq. (5) of appreciable sizes.

III. MODIFIED RADIATION PRESSURE FORCES

It is well known that the radiation pressure on a two-level atom undergoing an electric dipole transition at near resonance with the light is the vector sum of two distinct forces, namely the dissipative force and the dipole force, which depend on the detuning (defined as $\Delta_0 = \omega - \omega_0$, with ω_0 the atomic transition frequency and ω the frequency of the light), on the half-width Γ of the upper quantum state of the two-level atom, and on the Rabi frequency Ω_{lp}. In the saturation limit where $\Omega_{lp} \gg \Delta,\ \Gamma$, the dissipative force can be approximated to

$$\langle \mathbf{F}_{\text{diss}}\rangle_{lp} \approx \hbar\Gamma\nabla\Theta_{lp}. \tag{9}$$

With the forces acting on the atom due to the LG light defined as outlined above, the dynamics of the center of mass $\mathbf{R}$ can be evaluated. However, earlier treatments to date have invariably ignored the Gouy phase and the curvature effects and have discussed lowest order LG modes, most notably the $l = 1$, $p = 0$ doughnut mode [10]. As pointed out earlier, experimental techniques for mode generation have advanced considerably. Higher-order twisted light modes can now be readily created in the laboratory with large values of l and p. We now seek to explore the effects that can arise for large l and p in the dynamics of the atoms subject to this kind of light modes.

Consider the situation in which the atoms move near the focus plane of the LG mode such that $z \ll z_R$. When substituted in the dissipative force expression Eq. (9), the $\widehat{\phi}$ force component generates a light-induced torque, which is responsible for the azimuthal dynamics of the atom leading to their rotation. The $\widehat{\mathbf{z}}$ component, on the other hand, determines their axial dynamics. To investigate the dynamics near the focus we proceed in a series expansion of the involved quantities with respect to the variable (z/z_R). The field phase

Θ_{kpl} can be written as

$$\Theta_{pl} = \left[kz_R - (2p+|l|+1) + \frac{kr^2}{2z_R}\right]\frac{z}{z_R} + O\left(\frac{z}{z_R}\right)^3. \quad (10)$$

The beam waist $w(z) = w_0\sqrt{1+(\frac{z}{z_R})^2}$ can be approximated as

$$w(z) = w_0\left[1 + O\left(\left(\frac{z}{z_R}\right)^2\right)\right]. \quad (11)$$

The gradient terms originating from the Gouy phase and the curvature phase terms when taken together amount to an effective axial wave vector denoted by k_{eff}, so that the phase gradient in the vicinity of the focus plane can be written as

$$\nabla\Theta_{pl} \approx k_{\mathrm{eff}}\widehat{\mathbf{z}} + \frac{l}{r}\widehat{\phi}, \quad (12)$$

where k_{eff} is given by the equation

$$k_{\mathrm{eff}} \approx k\left(1 - \frac{(2p+|l|+1)}{kz_R} + \frac{r^2}{2z_R^2}\right). \quad (13)$$

The above relations show clearly that the axial wave vector is modified from k to k_{eff}. In the specific case $p = 0$ and making use of the relation $z_R = \pi w_0^2/\lambda$ we find that the winding number l should be close to $(kw_0)^2$ and since $kw_0 \gg 1$, only LG beams with large value of l could exhibit a nonnegligible effect, i.e., such that k_{eff} differs significantly from k.

To quantify further the size of the modifications we consider the following typical experimental scenario. Consider an LG mode of wavelength $\lambda = 2\pi/k = 852.35$ nm, with azimuthal and radial indices $l = 300$, $p = 3$, respectively. We focus on four different cases of beam waist with respective values $w_0 = 3\lambda$, 5λ, 10λ, and 20λ, and seek to explore how the effective wave vector k_{eff} changes with the radial position r near the focus plane, i.e., in the region at $z \approx 0$ of the beam. The plot of k_{eff} as a function of radius r, scaled in beam waist units, is shown in Fig. 1. There are four k_{eff} curves: a dotted one ($w_0 = 3\lambda$), a dashed one ($w_0 = 5\lambda$), a dot dashed one ($w_0 = 10\lambda$), and a long dashed one ($w_0 = 20\lambda$). The full spike-like curve is a scaled plot of the intensity of the specific LG beam being considered. It is in this region where the beam intensity and thus its mechanical effects on the atoms are considerable.

From Fig. 1 we see that as the beam waist becomes smaller the values of k_{eff} become considerably different from that of k. We also see that for $w_0 = 3\lambda$ the effective axial wave vector k_{eff} takes negative values at certain radial positions and this is to be interpreted by saying that locally the atom "sees" a beam traveling in the opposite direction. We must, however, be careful in interpreting this scenario since, as has recently been pointed out, when the focusing is very tight the generated LG beam is not a pure state as we have the production of modes with higher and lower winding numbers due to a small field component in the propagation direction so the above ideal picture does not apply [26,32].

A direct consequence of the modification of the axial wave vector is that the dissipative force on a two-level atom is also modified. In the saturation limit this force is now given by

$$\langle\mathbf{F}_{\mathrm{diss}}\rangle_{lp} = \hbar\Gamma k_{\mathrm{eff}}\widehat{\mathbf{z}} + \hbar\Gamma\frac{l}{r}\widehat{\phi}. \quad (14)$$

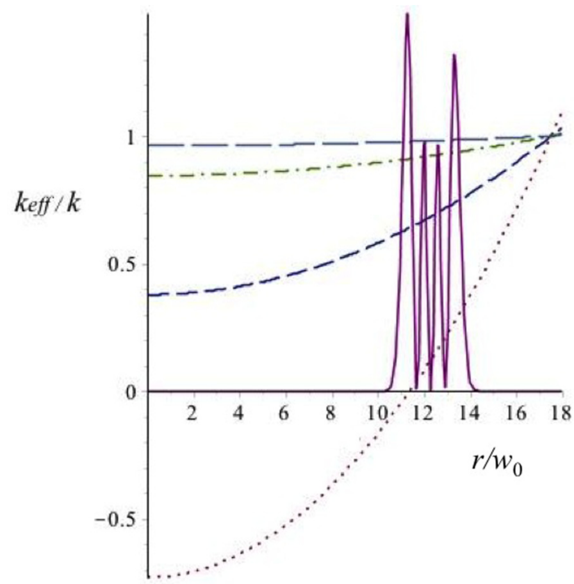

FIG. 1. The ratio k_{eff}/k as a function of the radial position, scaled in beam waist units, for a LG mode of wavelength $\lambda = 852.35$ nm and indices $l = 300$, $p = 3$. The ratio is given for four different beam waists: $w_0 = 3\lambda$ (dotted), $w_0 = 5\lambda$ (dashed), $w_0 = 10\lambda$ (dott-dashed), and $w_0 = 20\lambda$ (long dashed). The solid curve is a scaled plot of the intensity highlighting the regions where the beam intensity and thus its mechanical effects on atoms are considerable. The radial distance r is scaled in w_0 units.

Note that the axial ($\hat{\mathbf{z}}$) dissipative force, which in the absence of the anomalous Gouy and curvature phases is known to be given simply by $\langle\mathbf{F}_{\mathrm{diss}}\rangle_{lp} = \hbar\Gamma k\widehat{\mathbf{z}}$, is now modified by the inclusion of the additional phase terms. By contrast, these phase anomalies have no effect on the azimuthal $\hat{\phi}$ force component, which, in the saturation limit, gives rise to a light-induced torque of magnitude $\hbar\Gamma l$ acting on the atom about the beam axis. With reference to Fig. 1, which suggests that the effective wave vector is nullified on critical radial distances and changes its sign from negative to positive around them, we see that we could provide possible ways to handle atoms via LG light beams.

The mechanical effects of light on atom are very sensitive to the Doppler shift experienced by a moving atom. If the atom has a velocity $\mathbf{V}$ then the Doppler shift is given by $\delta_D = \{\nabla\Theta_{lp}\}\cdot\mathbf{V}$. This topic has been investigated analytically by Allen, Babiker, and Power [33], but it now seems clear that the Doppler shift too is subject to modifications due to Gouy and curvature phases for highly twisted light and there are also consequences in the context of the dynamics of the optical molasses in such LG beams [10].

IV. MOMENTUM DIFFUSION

It is well known that there are fluctuations of the radiation pressure forces due to some physical factors, which result in atomic momentum diffusion. Such fluctuations may arise from the fluctuations of the recoil momentum due to the fluorescence photons that are emitted in random directions, from the fluctuations in the number of fluorescence cycles in a specified time interval and from the fluctuations in the number of the absorbed photons [34]. These sources of diffusion are behind the heating of the translational degrees of freedom and so determine the limits of the mechanical action of the light on the atoms.

Whenever a photon is spontaneously emitted in a given direction, and since the direction of the emission is random, the atom recoils backwards and executes a random walk of step size $\hbar k$ in momentum space. The momentum diffusion coefficient D_{spont} associated with this process is given by

$$D_{\text{spont}} = \frac{1}{4}\hbar^2 k^2 \Gamma \frac{S}{1+S}, \tag{15}$$

with S the saturation parameter given by $S = \Omega_{lp}^2/2(\Delta^2 + \Gamma^2/4)$. Since spontaneous emission involves the vacuum modes D_{spont} does not depend on the details of the effective wave vector of the structured beam. This is not the case with the rest of the diffusion coefficients associated with the atomic motion in the light field.

The evaluation of the momentum diffusion coefficient D_{abs} due to the fluctuations of the number of absorbed photons in the case of an LG beam with high values of p and l indices leads to the following expression:

$$D_{\text{abs}} = \frac{1}{4}\hbar^2 k^2 \Gamma \frac{S}{1+S}(1+Q) \times \left[\left(1 - \frac{2p+|l|+1}{kz_R} + \frac{r^2}{2z_R^2}\right)^2 + \left(\frac{l}{kr}\right)^2\right], \tag{16}$$

where Q is the Mandel parameter [35] that represents a departure from Poissonian statistics and is defined as

$$Q = \frac{\langle(\Delta\hat{n})^2\rangle - \langle\hat{n}\rangle}{\langle\hat{n}\rangle}, \tag{17}$$

where $\hat{n}$ is the photon operator and, thus, $\langle\hat{n}\rangle$ is the mean photon count and $\langle(\Delta\hat{n})^2\rangle$ is the variance. The diffusion coefficient D_{dip} associated with the dipole force is given by

$$D_{\text{dip}} \approx \frac{1}{2\Gamma}(\hbar\nabla\Omega_{lp})^2. \tag{18}$$

We now proceed to estimate (in our case) the diffusion coefficient for a doughnut mode with $p = 0$, but large l and choose the saturation parameter S to be small, i.e., $S \ll 1$, and so $S/(1+S) \approx S$. We obtain from Eqs. (15) and (16) at the radius of maximal light intensity $r = w_0\sqrt{|l|/2}$, respectively, that

$$D_{\text{spont}} \approx \frac{1}{4}\hbar^2 k^{L^2} \Gamma \frac{\Omega_{l0}^2}{2\sqrt{|l|}(\Delta^2 + \Gamma^2/4)}, \tag{19}$$

and

$$D_{\text{abs}} \approx D_{\text{spont}}(1+Q) \times \left[\left(1 - \frac{\lambda^2(|l|+2)}{4\pi^2 w_0^2}\right)^2 + \frac{|l|\lambda^2}{2\pi^2 w_0^2}\right]. \tag{20}$$

The second and third terms in the square brackets represent modifications that would not be present in the corresponding relation for a Gaussian beam. These modification terms are of different physical origins. The first term is associated with fluctuations of the atomic momentum along the beam axis. As the azimuthal index l of the beam increases this contributes a larger negative value. The third term is proportional to the absolute value $|l|$ of the azimuthal index. This term is associated with fluctuations of the atomic angular momentum along the beam axis. Thus, because of the relative sign difference the two fluctuation terms compete with each other as the azimuthal index increases.

In order to see the significance of the above diffusion coefficients, we first highlight their connection with the Rabi frequency of the LG beam. For a LG mode with $p = 0$ and the normalization factor $C_{lp} = \sqrt{(2^{|l|+1}/\pi|l|!)}$ the intensity is maximum at the radial coordinate $r = w_0\sqrt{|l|/2}$, so the corresponding Rabi frequency at this radial coordinate is given by $\Omega_{l0}^2 \approx \Omega_0^2/\sqrt{|l|}$, where Ω_0 is the Rabi frequency associated with a Gaussian beam of the same power and beam waist. The gradient of the Rabi frequency appearing in Eq. (18) involves a differentiation with respect to r and can be shown to be given approximately by $(\nabla\Omega_{l0})^2 \approx \sqrt{|l|}\Omega_0^2/w_0^2$. We see that as the azimuthal, or winding number, l, increases the Rabi frequency becomes smaller compared to the Gaussian wave case while the gradient of the Rabi frequency becomes larger. This means that the saturation parameter becomes smaller and the diffusion coefficient due to the fluctuations of the number of absorbed photons becomes smaller compared to the same coefficient for an atom irradiated by a Gaussian beam. By contrast the diffusion coefficient associated with the dipole force becomes stronger as the winding number l of the beam increases.

The above qualitative analysis suggests that the physics involves a random walk in momentum space where the corresponding "steps" are such that: (i) they do not have the same size since a spontaneous "kick" has a size different from a stimulated "kick" and (ii) these kicks have a spatial dependence since they depend on r. These "structured kicks" would have an impact in cases where cold atoms are diffracted through light masks created by LG beams with high values of p and l such as those arising in the production of atom vortex beams [19].

V. COMMENTS AND CONCLUSIONS

In this paper we have considered the Gouy phase and the curvature phase terms, which are naturally present in a Laguerre-Gaussian light mode but are often neglected in the context of the interaction of LG light with atoms. We have shown that highly twisted LG modes (i.e., when the values of radial index p and azimuthal index l are large) the anomalous phase terms become significant. Specifically we considered the semiclassical interaction of LG light with two-level atoms and the features arising on taking the two anomalous phases into consideration when compared with the case in which these phases are ignored. We have shown that the axial wave number of the highly twisted LG light is modified by the phase terms, with the effective wave vector diminished compared to its value in the absence of the anomalous phase terms. The immediate physical consequences for free atoms is that the radiation forces responsible for their translational motion in twisted light are significantly modified. All the associated effects that arise when the interaction of the beam with the atomic transition is at close resonance with the light (as in the case of optical molasses) would be modified accordingly [10]. Atoms trapped in doughnut beams can be used as a basis for atomtronics [36]. If ions are trapped they may create axial magnetic fields, which could have useful applications at the microscale. The modifications in question would have to be taken into account in those contexts.

Twisted light beams have been proposed for the creation of properly tailored light masks, which can diffract cold atoms and so generate atom vortex beams [19]. These atom beams have an amplitude function of the Bessel type but are endowed with a phase function as for an LG beam. If we employ highly twisted beams for the creation of such light masks then the generated atom vortex beams will have Gouy and curvature terms included in their phase function. The high diffraction orders of the generated atom vortices would then have higher angular momenta and consequently they would propagate with a smaller axial wave vector (i.e., travel more slowly) in the axial direction.

Highly twisted LG modes can now be produced routinely in the laboratory, so it is to be anticipated that the effects pointed out here would be revealed in future experiments, and they would then be taken into account in various physical applications. The significant reduction of the axial wave vector would feature in applications in recoil free spectroscopy experiments. The effects depend also on the extent of the beam focusing. It is the combination of tightly focused modes and large values of p and l that ensures significant changes in the atomic momentum. In the tight focusing regime the atomic motion can be loosely described as “blind” in the axial direction as it interacts with light. In this case the atom experiences primarily a torque which makes it rotate in the azimuthal direction moving in the plane transverse to the propagation direction of the beam.

ACKNOWLEDGMENTS

This project was funded by the National Plan for Science, Technology and Innovation (MAARIFAH), King Abdulaziz City for Science and Technology, Kingdom of Saudi Arabia, Award No. 11-MAT-1898-02. The authors thank Prof. D. Ellinas for useful and stimulating discussions.

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