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Isospin symmetry at high spin studied via nucleon knockout from isomeric states

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One-neutron knockout reactions have been performed on a beam of radioactive ^{53}Co in a high-spin isomeric state. The analysis is shown to yield highly-selective population of high-spin states in an exotic nucleus with a significant cross section, and hence represents a technique that is applicable to the planned new generation of fragmentation-based radioactive beam facilities. Additionally, the relative cross sections among the excited states can be predicted to a high level of accuracy when reliable shell-model input is available. The work has resulted in a new level scheme, up to the 11^+ band-termination state, of the proton-rich nucleus ^{52}Co ($Z = 27$, $N = 25$). This has in turn enabled a study of mirror energy differences in the $A = 52$ odd-odd mirror nuclei, interpreted in terms of isospin-non-conserving (INC) forces in nuclei. The analysis demonstrates the importance of using a full set of J -dependent INC terms to explain the experimental observations.

Isospin symmetry arises from the near identical nature of the strong nuclear interaction regardless of which nucleons are involved (e.g. [1]). Under this assumption, and in the absence of electromagnetic effects, the proton and neutron can be considered as two states of the same particle, the nucleon. Heisenberg [2] assigned an isospin quantum number, $t = \frac{1}{2}$ for a nucleon, with projection $t_z = -\frac{1}{2}$ ($+\frac{1}{2}$) for the proton (neutron), respectively. For nuclei, therefore, we expect to find isobaric analogue states (IAS), of a given isospin T , in a set of nuclei with $T_z (= (N - Z)/2) = -T \rightarrow +T$ which. In the absence of isospin-breaking terms (such as the electromagnetic interaction), these IAS would be identical and degenerate. In reality, any isospin-breaking interactions will lift this degeneracy, and hence the differences in behaviour between IAS yields direct information on these interactions. Given that the Coulomb interaction is well understood, this has the potential to shed light on how isospin-breaking effects of nuclear origin manifest in nuclei, which is the long-term goal of this study. Mirror energy differences (MED), defined as $MED_\alpha = E_{\alpha,T,T_z=-1}^* - E_{\alpha,T,T_z=+1}^*$, where α denotes a state label and E^* is excitation energy), can yield important information on two-body interactions of the form $V_{pp} - V_{nn}$ that must be used in conjunction with the Coulomb interaction to provide a good theoret-

ical description – see for example [3–7]. These studies have raised fundamental questions about the influence of isovector interactions in nuclear structure. In this Letter, we present a new high-spin study of the odd-odd nucleus ^{52}Co ($Z = 27$), the proton-rich member of the $T = 1$ mirror pair $^{52}\text{Co}/^{52}\text{Mn}$, using a novel technique to access high-spin states in this exotic system.

A wider goal of contemporary nuclear physics is to evaluate, through spectroscopy, fundamental nuclear properties at the limits of nuclear existence through studies such as this. Rare-isotope facilities are now at the forefront, creating beams of radioactive nuclei through isotope-separation and post-acceleration techniques (ISOL–e.g. [8]) or using in-flight separation of exotic nuclei created following relativistic fragmentation reactions (e.g. [9]). For the most exotic nuclei, the information accessed tends to be restricted to the ground state, or excited states of relatively low spin, through mass measurements, decay spectroscopy and in-beam reactions such as knockout and Coulomb excitation. For higher-spin states, the traditional method is the heavy-ion fusion evaporation. Whilst some progress has been made in using fusion reactions with ISOL beams (e.g. [10]), high-spin studies far from stability remain exceptionally challenging.

However, radioactive nuclei can be created in high-

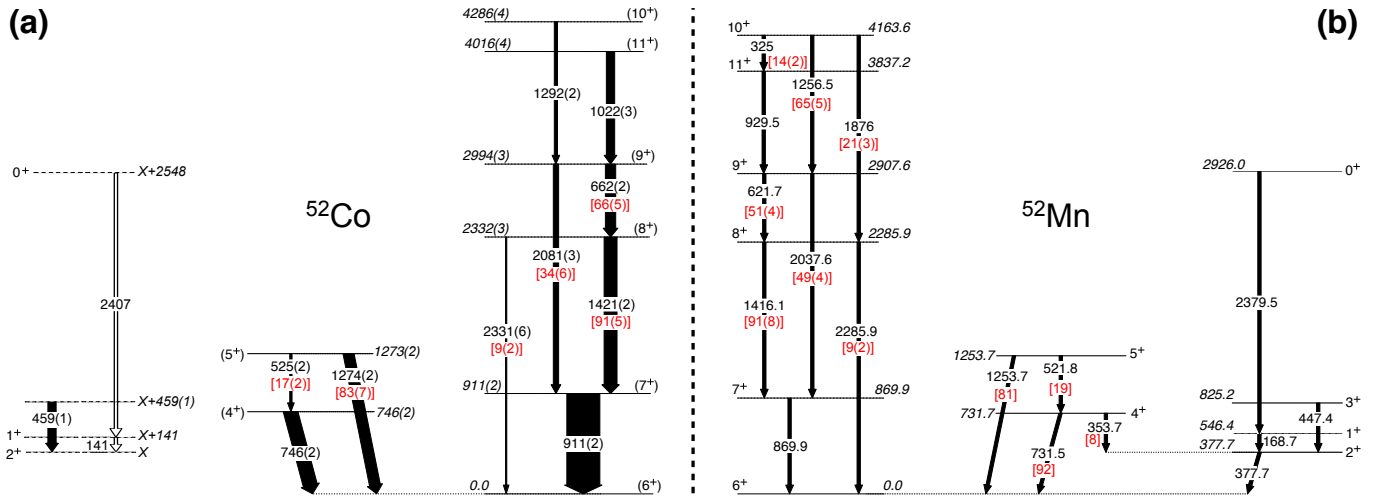


FIG. 1. (a) The scheme for ^{52}Co , deduced from this work (solid lines/solid arrows). The arrow widths are proportional to the relative intensities of γ rays observed. The dashed lines and hollow arrows are taken from previous work [25]. (b) The decay scheme [23] for ^{52}Mn , where the lowest-energy state of each spin is shown. The numbers in square parentheses are measured relative branching ratios (normalised to 100) where there is more than one γ decay from a state.

spin isomeric states in fragmentation reactions at relativistic energies (e.g. [11]). A highly effective method, recently extensively employed, is to identify exotic fragments in-flight, implant them post separation and perform γ -ray spectroscopy of decays below the isomeric states (e.g. [12, 13]). The possibility of using isomeric beams to perform in-beam reactions has long been considered as a potentially powerful method (see e.g. [14]) and there have been some pioneering experiments to perform, for example, Coulomb excitation [15] or fusion [16] reactions with radioactive beams in high-spin isomeric states. In this work, the high-spin study of ^{52}Co was performed using a new in-flight approach – namely a *knockout* reaction on an isomeric beam – a method that has the capability of creating nuclei further from stability, and at higher spins, than the isomer itself. In the current work, a one-neutron knockout reaction, from a high-spin 247 ms isomer in ^{53}Co , was shown to populate states up to $J^\pi = 11^+$ in ^{52}Co . The direct nature of the reaction results in selective population of high-spin states. We show that when coupled to a reliable reaction-model calculation, this yields a highly sensitive method for high-spin in-beam spectroscopy of exotic nuclei.

The experiment was performed at the National Superconducting Cyclotron Laboratory at Michigan State University, where a secondary beam of ^{53}Co ($T_z = -1/2$) was produced via the fragmentation of a 160 MeV/nucleon ^{58}Ni primary beam impinging upon a thick ^9Be primary target. The resulting fragments were then separated by the A1900 fragment separator [17, 18] and identified from their time of flight. The ~ 77 MeV/nucleon ^{53}Co secondary beam impinged on a 188-mg/cm 2 ^9Be target at the reaction target position of the S800 [19, 20]. In-flight γ rays from the knockout reaction residues were recorded by the Segmented Germanium Array (SeGA)

detectors [21], positioned in two rings at 37° and 90° , with respect to the beam axis. Unique particle identification was achieved through measuring the energy loss in the S800 ionization chamber and the time-of-flight through the spectrograph.

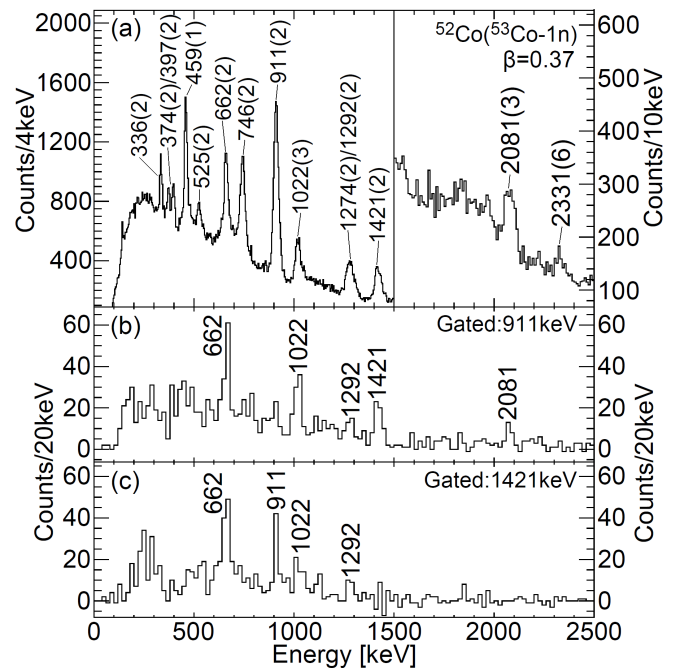


FIG. 2. (a) The Doppler corrected γ -ray spectrum for ^{52}Co fragments, identified following one-neutron knockout from ^{53}Co . An average β_{c} value of 0.37 was used. (b) and (c) Spectra from a γ - γ coincidence analysis, on the condition of coincidence with the (b) 911-keV or (c) 1421-keV transition.

Existing information on the structure of ^{52}Co comes

from the β - and β -delayed-proton decay of ^{52}Ni [22–25]. A number of high-lying (presumed 1^+) proton decaying states have been established [25] as well as three states of $J^\pi = 0^+, 1^+, 2^+$ connected by gamma decays [22, 25] – left side of Fig. 1(a). The 0^+ state is the IAS of the $T = 2$ ground state of ^{52}Ni . The excitation energies are unknown, even though the absolute binding energies have been measured through the proton decay of the 0^+ state [25]. The 2^+ state is expected to be isomeric, like its analogue in ^{52}Mn ($T_{1/2} = 21.1$ minutes [26]) which decays predominately via β -decay [27]. Recently, the beta-decay of the 2^+ isomer in ^{52}Co has been observed [24], with a half life of 102(6) ms.

The γ -ray spectrum for ^{52}Co from the current work is presented in Fig. 2(a), where a significant number of new transitions can be observed. The level scheme, resulting from the following analysis, is shown in Fig. 1(a).

The high-spin cascade in ^{52}Co and placement of the corresponding γ -ray transitions (from $10^+, 11^+ \rightarrow 6^+$) – Fig. 1(a) – was established experimentally using a γ - γ coincidence analysis, γ -ray intensity arguments and energy sums. Fig. 2(b) shows the background subtracted γ - γ coincidence spectrum, gated on the 911(2)-keV transition, which shows all of the γ -rays in this high-spin cascade. The spectrum in Fig. 2(c) is gated on the 1421(2)-keV transition, and here the same transitions, apart from the 2081(3)-keV transition, are observed. The use of this and further γ - γ analysis confirmed this cascade. A comparison with the main yrast sequence of ^{52}Mn – Fig. 1(b) – yields a clear state-by-state correspondence (energies and branching ratios) and so the spins and parities of the corresponding analogue states in ^{52}Mn are assigned. As these are not directly measured in ^{52}Co , they are placed in parentheses in Fig. 1(a).

The three remaining strong transitions, 459(1), 746(2) and 1274(2) keV in Fig. 2(a), do not have *any* strong transitions in coincidence with them. Hence, it is extremely likely that these are decays from states which are directly populated and feed the ground state or the 2^+ isomer. A comparison with ^{52}Mn suggests that the 1274(2)-keV and 746(2)-keV transitions are the the analogues of the 731.5 and 1253.7-keV transitions from the 4^+ and 5^+ respectively. The weak 525(2)-keV transition has the correct energy to complete this sum and, if this indeed also decays from the 5^+ state, the branching ratio of the two γ rays is consistent with the analogue transitions in ^{52}Mn . The remaining strong transition, the 459(1)-keV transition has a number of possible analogues in ^{52}Mn , all feeding the 2^+ isomer, from states with $J^\pi = 1^+, 2^+, 3^+, 4^+$. We cannot distinguish between these possibilities here.

The conservation of angular momentum dictates that only states up to $J^\pi = 7^+$ can be populated through one-neutron knockout from ^{53}Co , given the ground state of $J^\pi = 7/2^-$. However, states with angular momentum up to $J^\pi = 11^+$ are apparently observed with sizeable relative cross sections. This implies a strong population of the well-known $J^\pi = 19/2^-$ 247(12)ms isomer [28] in the ^{53}Co beam. Knockout of an $f_{7/2}$ neutron from this isomer

could, in principle, populate states between 6^+ and 13^+ . Indeed, states with $J > 7$ can *only* be populated from the isomer, and not from the ground state. It should be noted that $J^\pi = 11^+$ is the highest spin state available in the $f_{7/2}$ space without requiring excitations across the ^{56}Ni shell gap, and higher-spin states lie several MeV higher in energy and the transitions are not expected to be observable.

To check this hypothesis, we have performed calculations of the cross sections to these states, from both the ground state and isomer in ^{53}Co , and compared these with the experimental results. The single-nucleon removal cross sections were calculated under spectator-core approximation assuming eikonal reaction dynamics [29–31], with shell-model structure input. Valence nucleon radial wave functions were calculated in a Woods-Saxon plus spin-orbit potential, the geometry of which is constrained by Hartree-Fock calculations using a Skyme SkX interaction [32]. Full- pf shell-model calculations using the KB3G interaction [33] were used to compute the spectroscopic factors for the knockout process, utilizing the code NuShellX@MSU [34].

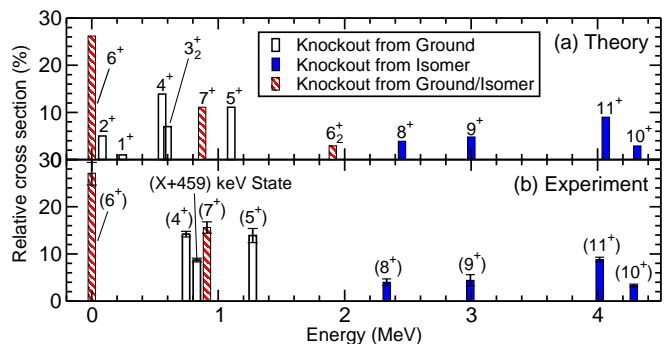


FIG. 3. (a) Calculated relative cross sections for states in ^{52}Co populated via one-neutron knockout from either the ^{53}Co ground state ($J^\pi = 7/2^-$), the high-spin isomeric state ($J^\pi = 19/2^-$) or both. A fractional population of the isomer of 27% has been assumed (see text). (b) The experimentally measured relative cross sections.

The calculations were performed, separately, for knockout from the ^{53}Co ground state and from the $J^\pi = 19/2^-$ isomer. The results, plotted as a percentage of the total cross section, are shown in Fig. 3(a). The lowest four states for all $J \leq 11$ were included in the calculation, but only those most-strongly populated are plotted. Although all states with J^π between 0^+ and 11^+ are predicted to be directly populated, the vast majority ($\sim 98\%$) of the predicted cross section is distributed among the 12 states shown in Fig. 3(a). All the remaining (not plotted) states have predicted individual population intensities of $< 0.25\%$. The cross sections are shown separately for states which can be accessed from (i) only the ground state, (ii) only the isomer and (iii) both the ground state and isomer. In making this plot, it is necessary to know the fraction of the beam that is in the isomeric state, which was not measurable. There-

fore, the isomeric fraction was allowed to vary until the relative population of the two groups of states (i) and (ii) above is similar to that observed. This yields a fraction of approximately 27% of the beam particles in the isomeric state. This has been used in Fig. 3(a). The decay is predicted to proceed principally to the lowest energy state for each spin. The exception is $J^\pi = 3^+$, where the model has two 3^+ states close in energy. The 3_1^+ state wave function is found to contain at least one proton excitation out of the $f_{7/2}$ shell, and hence has little overlap with the parent state in ^{53}Co , with the majority of this overlap present in the 3_2^+ state instead.

The experimentally measured relative cross sections, for all observed states, are shown in Fig. 3(b). Even though we are unable to deduce the state from which the 459-keV transition decays, it seems likely from this comparison that it corresponds to the $J^\pi = 3_2^+$ state in the model. The model suggests that the $J^\pi = 2^+$ and 1^+ states should be directly populated. However, the long lifetime of the 2^+ [24] and the low energy of the 1^+ state transition (141 keV – below the observational limit) prevent observation of the transitions from these states. The 6_2^+ state is also predicted to be populated which, in ^{52}Mn , decays by a 1956-keV transition. Hence the high energy and weak population again prevent clear identification of this transition in ^{52}Co .

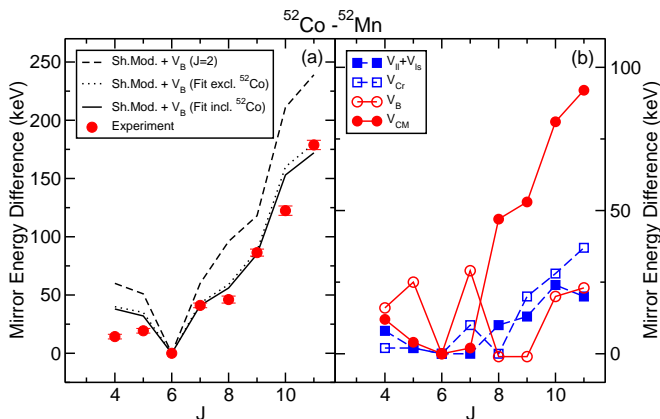


FIG. 4. (a) A comparison between experimental and shell-model MED. The three lines correspond to different methods for determining the INC term V_B . The dashed line uses a single +100 keV INC matrix element for $J = 2$ [4]. The solid line uses four INC matrix elements extracted from a fit across the $f_{7/2}$ -shell [5] – see text. The dotted line also uses four fitted matrix elements, but where the current data for ^{52}Co were excluded from the fit. (b) The four components of the shell-model MED (defined in the text), the sum of which yields the solid line in (a).

To summarise this analysis, even though the isomeric ratio has been favourably adjusted, there is an excellent agreement between Figs. 3(a) and (b), with a clear correspondence between experiment and theory on a state-by-state basis. This represents the first measurement and analysis of knockout solely from a high-spin isomer. In

terms of comparison of the relative cross-sections among the high-spin states, the agreement is excellent.

The mirror energy differences (MED) for the $A = 52, T = 1$ mirror pair, are shown in Fig. 4(a). The large rise in the MED from the ground state up to the $J^\pi = 11^+$ state is easily explained in an $f_{7/2}$ picture, in terms of the Coulomb effect of the angular momentum alignment of the three valence proton holes in ^{52}Mn , compared with the alignment of neutron holes in ^{52}Co . Analysis of these MED, in a large-scale shell-model calculation using the ANTOINE code [36], was performed using the full- pf valence space and the KB3G interaction [33]. The approach of Ref. [4] has been adopted, which has been shown to yield a reliable description of MED in the $f_{7/2}$ region. The contribution of four isospin-breaking effects to the MED are calculated. Three of the terms account for (a) the Coulomb two-body interaction (V_{C_M}); (b) the Coulomb effect of changes in radius (V_{C_r}) and (c) single-particle effects of Coulomb and magnetic origin ($V_U + V_{I_s}$). The final term (V_B) represents a further isospin-non-conserving interaction in addition to the usual two-body Coulomb term. In previous work, it was found that the inclusion of a single repulsive interaction of $V_B \approx +100$ keV for $f_{7/2}$ protons coupled to $J = 2$ proved highly effective in accounting for experimental MED data in this region [4]. The dashed line in Fig. 4(a) shows the prediction using this prescription. Here, it is clear that the agreement is, unusually for this mass region, quite poor.

In a recent systematic study of mirror nuclei in the $f_{7/2}$ shell [5], a full set of effective isovector ($V_{pp} - V_{nn}$) matrix elements has been extracted by fitting the shell model to all experimental MED. This has yielded matrix elements of $V_B = -72, +32, +8, -12$ keV for $J = 0, 2, 4, 6$ couplings of the $f_{7/2}$ orbital [5, 37] (as opposed to a single value of +100 keV for $J = 2$ alone). The results of a shell-model calculation, using these new values for V_B , are shown by the solid line in Fig. 4(a). The agreement is now excellent. This is the clearest evidence yet for the need to include a full set of isospin-breaking matrix elements for all J -couplings. The four terms in the MED calculation are shown in Fig. 4(b), where the fitted values of V_B have been used. In this mirror pair, the V_B contribution turns out to be unusually small, but only once all four matrix elements are included. It should be noted that the fit performed in reference [5] uses 93 MED data points which *include* the seven states reported here. We have performed the fit again excluding these, extracted a new set of $V_B(J)$ terms, and repeated the full MED calculation. This is shown by the dotted line in Fig. 4(a), and it is clear that the outcome is unchanged.

In conclusion, a new method has been used for accessing high-spin states in exotic nuclei – knockout from a high-spin isomer populated in a fragmentation reaction. It has been shown that a reaction model, coupled to spectroscopic factors determined in a full pf -shell model analysis, can predict the distribution of cross section among high-spin states with excellent accuracy.

The analysis has yielded a comprehensive level scheme of the proton rich nucleus ^{52}Co ($T_z = -1$). MEDs for the $T = 1$, $A = 52$ mirror pair were extracted and compared with shell-model calculations and interpreted in terms of isospin non-conserving interactions. The results show strong evidence for the need to include a full set of J -dependent INC terms in the analysis of mirror nuclei.

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