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Seto, H., Yagi, M., Aiba, N. et al. (3 more authors) (Accepted: 2016) Non-ideal Ballooning Mode Instability with Real Electron Inertia. Plasma and Fusion Research. ISSN: 1880-6821 (In Press)

Non-ideal Ballooning Mode Instability with Real Electron Inertia

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(Received 10 January 2009 / Accepted 24 January 2009)

Impacts of electron inertia with an electron skin depth (ESD) longer than the realistic value used in early numerical studies on non-ideal ballooning modes (NIBMs) are numerically investigated by a linearized 3-field reduced MHD model. In this paper, 4 different ESDs $d_e^* = 0, d_e, \sqrt{10}d_e, 10d_e$ are used for an resistivity dependence study of the growth rate of NIBMs, where $d_e = c_0\sqrt{\varepsilon_0 m_e/n_e e^2}$ is the real ESD and $d_e^* = 10d_e$ corresponds to an order of ESD used in a numerical study on collisionless ballooning mode (CBM) reported in [Kleva and Guzdar Phys. Plasmas **6**, 116 (1999)]. In the case with the real ESD $d_e^* = d_e$, a transition from resistive ballooning mode (RBM) to CBM occurs in the edge relevant resistivity regime, while the electron inertia effect is overestimated and the growth rate is almost independent of resistivity in the cases with $d_e^* = \sqrt{10}d_e$ and $10d_e$. These results indicate that the real ESD is one of key factors for the edge stability and turbulence analysis.

Keywords: non-ideal ballooning mode, electron inertia effect, reduced MHD model, linear analysis

DOI: 10.1585/pfr.4.000

It is one of the most important issues for predicting global confinement performance of fusion devices to understand plasma instability and transport phenomena in the edge region where collisionality drastically changes from collisionless to collisional. In the collisional regime, resistivity destabilizes a ballooning mode below the ideal stability limit (ISL), so called resistive ballooning mode (RBM) [1, 2]. On the other hand, in the collisionless regime, electron inertia also destabilizes collisionless ballooning mode (CBM) [3, 4, 5]. A CBM simulation with the real electron skin depth (ESD), however, has not been carried out so far due to limitation of computational resources, since its characteristic scale length is associated with the ESD, $d_e = c_0\sqrt{\varepsilon_0 m_e/n_e e^2} \sim 1.68 \times 10^{-3}$ [m] for the electron number density $n_e = 1.0 \times 10^{19}$ [m^{-3}]. A simulation of CBM with an ESD d_e^* longer than the real value by one order $d_e^* \sim 10d_e$ showed that the longer ESD makes the growth rate of CBM larger [6]. According to a local analysis, CBM with the real ESD can be destabilized only close to the ISL or for extremely high toroidal mode numbers for which fluid models are no longer valid [7]. In addition, electron inertia can be a key player in energy turbulent transport in the edge region as well as current diffusion [8] so that the simulation of CBM with the real ESD is also significant as a preliminary step for an edge turbulent simulation for L/H transition.

In this paper, impacts of the overestimated electron inertia on CBM/RBM instabilities in the resistivity regime relevant to the edge plasma are numerically investigated by BOUT++ code [9] which was applied to a linear in-

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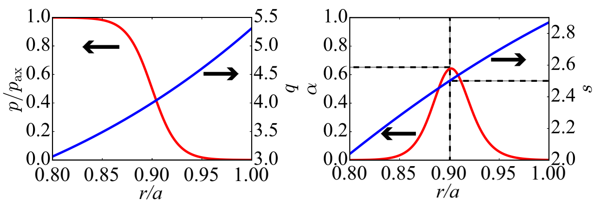

Fig. 1: The shifted circular equilibrium for CBM/RBM instability analysis: the radial profiles of p/p_{ax} and q (left) and the radial profiles of s and α (right), where the beta value at the magnetic axis is $p_{ax}/(B_{ax}^2/2\mu_0) = 5 \times 10^{-4}$.

stability analysis of ideal ballooning mode with the real electron mass [10]. The resistivity regime relevant to the edge plasma lies roughly from $\eta \sim 10^{-9}$ for $T_e \sim 10^3$ [eV] to $\eta \sim 10^{-7}$ with $T_e \sim 10^2$ [eV], where T_e is the electron temperature and the Spitzer resistivity $\eta \propto T_e^{-3/2}$ has been employed. Note that quantities without unit are normalized with the major radius R_{ax}, the magnetic field intensity B_{ax} and the poloidal Alfvén time t_A with the ion number density $n_i = 1.0 \times 10^{19}$ [m^{-3}] and the deuterium mass, where the subscript ax indicates values at the magnetic axis.

Fig.1 shows a shifted circular equilibrium with the pressure shear factor $\alpha \simeq 0.6$ and the magnetic shear factor $s \simeq 2.5$ at $r/a \simeq 0.9$ used for CBM/RBM instability analyses. This equilibrium is based on a linearized Grad-Shafranov equilibrium [11] and is far from the ISL $\alpha_{\mathrm{critical}} \sim 0.6s \sim 1.5$ for $s \simeq 2.5$ [12]. In CBM instability analysis, grid widths in the directions perpendicular to the field line must be shorter than the ESD. BOUT++ code employs a field-aligned coordinates (x, y, z) [13] so that the grid width in the flux surface label x is restricted by $\Delta x/RB_p \ll d_e \rightarrow \Delta x \ll d_e RB_p \sim 4\times10^{-5}$ and the grid width in the field line label z is restricted by $\Delta z RB_p/B \ll d_e \rightarrow \Delta z \ll d_e B/RB_p \sim 2\times10^{-2}$ respectively, where y is the parallel label. We employ an equally spaced grid for a 1/40 sector of torus with $\Delta x \sim 3\times10^{-6}$ for the radial domain corresponding to $0.8 \leq r/a \leq 1.0$, $\Delta y \sim 5\times10^{-2}$ for $0 \leq y < 2\pi$ and $\Delta z \sim 6\times10^{-4}$ for $0 \leq z < 2\pi/40$, which is fine enough for the real ESD.

For linear instability analyses of CBM/RBM, a 3-field reduced MHD (RMHD) model with resistivity and electron inertia is employed,

$$\frac{\partial}{\partial t}\frac{\nabla_\perp^2\tilde{\phi}}{B} = -B^2\nabla_\parallel\frac{\nabla_\perp^2\tilde{A}_\parallel}{B} + \mathbf{b}\times\boldsymbol{\kappa}\cdot\nabla\tilde{p}, \tag{1}$$

$$\frac{\partial\tilde{p}}{\partial t} = -\frac{\mathbf{b}\cdot\nabla_\perp\tilde{\phi}\times\nabla_\perp p}{B}, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\tilde{A}_\parallel - d_e^{*2}\nabla_\perp^2\tilde{A}_\parallel\right) = -\nabla_\parallel\tilde{\phi} + \eta\nabla_\perp^2\tilde{A}_\parallel, \tag{3}$$

where ϕ is the electrostatic potential, B is the magnetic field intensity, $A_\parallel$ is the magnetic potential, $\mathbf{b}$ is the unit vector along the equilibrium magnetic field, $\boldsymbol{\kappa}$ is the magnetic curvature and $\tilde{}$ indicates perturbed quantities respectively. In this model, the other kinetic effects are neglected to make the impact of the computationally large d_e^* clear.

Resistivity dependences of growth rates of $n = 40$ CBM/RBM for 4 different ESDs $d_e^* = 0, d_e, \sqrt{10}d_e, 10d_e$ are summarized in Fig.2, where the growth rate and the resistivity are normalized as $\gamma t_A \rightarrow \gamma$ and $\eta/\mu_0 R_{\rm ax}^2 t_A^{-1} \rightarrow \eta$ respectively. We choose $n = 40$ as a typical toroidal mode number which is relevant to fluid models including RMHD models [2, 6, 10]. In the case with $d_e^* = 0$ where is only the RBM branch, the growth rate of RBM shows good agreement with a local theory $\gamma \propto \eta^{1/3}$ [1] in $10^{-8} \leq \eta \leq 10^{-7}$. The growth rate, however, becomes less sensible to the resistivity in $\eta > 10^{-7}$ since the eigen function of RBM is strongly localized in the parallel direction and its parallel derivatives become no longer negligible [2]. A transition from RBM to CBM occurs in the resistivity regime relevant to the edge plasma in the case with the real ESD $d_e^* = d_e$. This result indicates that CBM with realistic toroidal mode numbers for fluid models can be destabilized in the edge region even when equilibrium is not close to the ISL. Finally, in the cases with $d_e^* = \sqrt{10}d_e$ and $d_e^* = 10d_e$, CBM is overestimated and its growth rates are almost independent of resistivity in $\eta < 10^{-7}$. In addition, Fig.3 shows that the eigen function of CBM with $d_e^* = 10d_e$ goes to the strong ballooning limit rather than the weak ballooning limit in contrast to the case with the real ESD. These results indicate that the real ESD is one of key factors for the stability and turbulent analysis in the edge plasma where resistivity lies in the CBM/RBM transient regime. Further linear spectrum analyses on NIBM instability with other kinetic effect, especially ion diamagnetism which can stabilize middle-n modes, are future works.

This work is partly supported by MEXT KAKENHI Grant No. 16K18342. Computations were carried out on the Helios supercomputer at the Computational Simulation Centre of the International Fusion Energy Research Centre (IFERC-CSC).

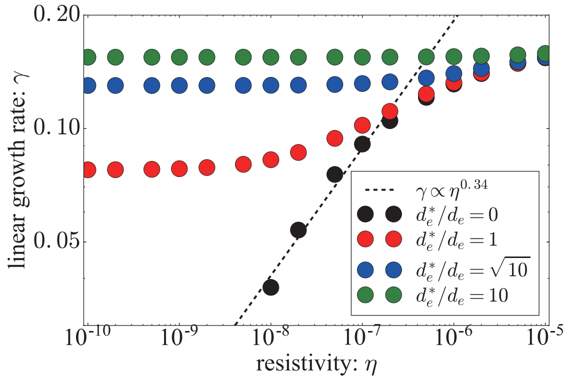

Fig. 2: Linear growth rates of $n = 40$ CBM/RBM as functions of resistivity for 4 different $d_e^* = 0, d_e, \sqrt{10}d_e, 10d_e$ which correspond to the computational electron masses $m_e^* = 0, m_e, 10m_e, 100m_e$ respectively for $d_e^* \propto \sqrt{m_e^*}$.

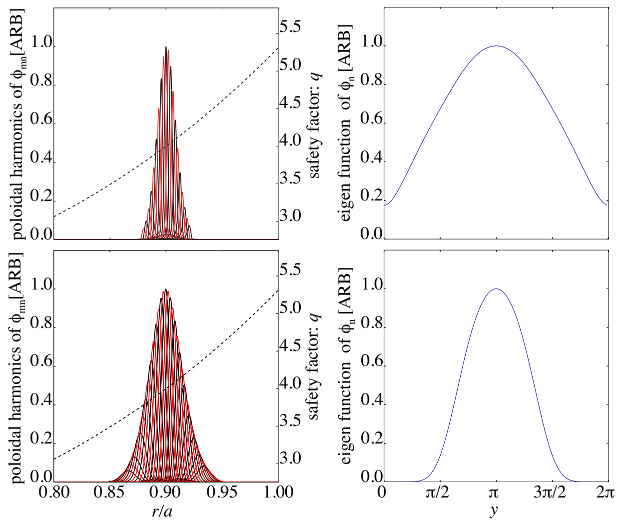

Fig. 3: Radial structure of poloidal harmonics (left) and eigen function in parallel direction (right) of $n = 40$ mode of $\tilde{\phi}$ in collisionless limit $\eta = 1.0\times10^{-10}$ with $d_e^* = d_e$ (upper column) and $d_e^* = 10d_e$ (lower column).

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