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# Study of improved confinement by a stepwise increase of the input heating power for tokamak plasmas

*Muhammad Asif<sup>1,2,\*</sup>, Mabruka Mohamed<sup>2</sup> and Eun-jin Kim<sup>2</sup>*

<sup>1</sup>*Department of Physics, COMSATS Institute of Information Technology,*

*Lahore 54000, Pakistan*

<sup>2</sup>*School of Mathematics and Statistics, University of Sheffield, Sheffield S3 7RH,  
United Kingdom*

E.mail\* : dr.muh.asif@gmail.com

## Abstract

The article is an extension of the brief study by [Sarah Douglas *et al.*, Phys. Plasmas 20, 114504 (2013)], where in the study a sinusoidal perturbation of the heating power has been studied. In this paper a stepwise increase of the heating power and its influence on the L-H transition are studied. Using a function,  $A \tanh(t/T)$  for the transition of input heating power for tokamak plasmas i.e. the addition of the perturbation,  $A \tanh(t/T)$ , to constant power  $q_0$  is shown to promote the confinement, leading to the L-H transition at a lower value of  $q_0$ , as compared to the case of constant  $q_0$  without the  $A \tanh(t/T)$  perturbation. It is seen that the input heating power  $Q$  that consists of constant part  $q_0$  in addition to a function  $A \tanh(t/T)$  provides

the L-H transition for relatively small  $A$  and much wider range values of  $1/T$  as compare to [Sarah Douglas *et al.*, Phys. Plasmas 20, 114504 (2013)].

## 1. Introduction

It was observed experimentally [1], that as the input power increases, the confinement degrades until it reaches a certain value at which it suddenly begins to improve into a high confinement state—this bifurcation is known as the L-H transition. The bifurcation occurs when zonal flow causes a transport barrier (steep temperature gradient) [2-6], to be produced at the edge of the plasma, improving plasma confinement. Zonal flow, excited by turbulence, is a poloidal flow with a small frequency and varies in the radial direction, thereby inhibiting radial transport [1–7]. Mean flows have smaller frequencies and move on a larger scale. Zonal flow is thought to take the lead in the suppression of turbulence before mean flow becomes large enough to influence it. Zonal and mean flows are amplified by different conditions, which permit them to take different roles in the transition. The study of the effect of zonal flows and its modeling has received a great attention in fusion research with on-going experimental tests (e.g., see Ref. 2 and references therein) to complement theoretical modeling.

Zero-dimensional models [7-22] are a phenomenological model and can be used for any size of tokamak by using appropriate parameter values, that is, the size of tokomaks can change values of parameters in the model. Such phenomenological model plays an important role in interpreting fusion plasma behavior. Specifically, in our case, by choosing variables to represent key macroscopic quantities such as the temperature gradient  $N$ , the strength of micro-turbulence  $E$ , and the magnitude of large-scale coherent nonlinear structures  $U$ , zero-dimensional models can model the L-H transition, and L-mode and H-mode confinement physics [10].

It was observed by H. Zhu, S. C. Chapman, and R. O. Dendy, called ZCD model [20], when a small oscillatory-in-time component is added to the steady heating rate, the ZCD model can exhibit a classic period-doubling path to chaos [23] i.e. the level of micro-turbulence  $E$  as the amplitude of oscillation is increased. In this case, the ZCD model may differ from the M. A. Malkov and P. H. Diamond [7], called MD model, which has one fewer variable for which oscillatory heating was studied in [21]. This distinctive phenomenology may offer a path to future experimental testing of the assumptions of zero-dimensional models, and perhaps distinguishing between them. Repeated on-off switching of electron cyclotron heating is now routine [24], so that quasi-oscillatory ECH scenarios are becoming realizable. In such scenarios, future experimental probe of the physical assumptions embodied in the model is required. However by the E.J. Kim and P. H. Diamond [10], called KD model, MD model and its ZCD extension also possess well defined scaling relations between energy confinement time and heating power, which can be calculated in near future. For such kind of studies the numerical solution of the time evolving system, as well as knowledge of its fixed points, are required, which is however not of our purpose in this paper.

This article is an extension of the also brief study Sarah Douglas [21], where in the study by Douglas [21] a sinusoidal perturbation of the heating power has been studied. In this paper a stepwise increase of the heating power and its influence on the L-H transition are studied. Both models are based on the two predator / one prey model of the L-H transition developed by [10]. These model has been very successful during the last years [20-22]. The aim of this paper is to

report on the interesting effect of a stepwise increase of the different heating powers on turbulence and confinement by extending [21]. The key question is if the modulation of the input power always helps the L-H transition, the transition occurring at a lower value of the constant power without modulation. If yes, what is the characteristics of the time varying input power that is best (or most efficient) in promoting the L-H transition. For a given constant input power  $q_0$ , what is the optimal choice of the time varying input power. To shed light on these issues, we consider the input power  $Q$  that consists of constant part  $q_0$  (as usually done) and in addition to a stepwise increase function  $A \tanh(t/T)$  and provides the L-H transition for relatively small  $A$  and much wider range values of  $1/T$  as compare to [21]. There is an interesting relation between  $A$  and  $w$  that leads to the L-H transition for different values of  $q_0$ .

## 2. Modeling confinement transitions

We consider the model [7] which reduces the number of parameters in the original model [10]. In this model [7], which is already adopted by [21], two predator ( $V$  and  $U$ ) one prey model ( $E$ ), the temperature gradient (which comes directly from the heat input) feeds the mean flow and dissipates as the turbulence and temperature gradient increase. Mean flow reduces turbulence and zonal flow while the zonal flow eats turbulence, creating the observed oscillatory behavior similar to [21]. The heat input generates turbulence, which generates zonal flows. This complicated relationship is modeled [7,21] by the following Eqs. (1)–(4), where  $\vartheta$ ,  $\xi$ ,  $\eta$ ,  $\rho$  and  $\sigma$  are constants:

$$\frac{dE}{dt} = (N - E - V^2 - U)E, \quad (1)$$

$$\frac{dU}{dt} = \vartheta \left( \frac{E}{1 + \xi V^2} - \eta \right) U, \quad (2)$$

$$\frac{dN}{dt} = Q(t) - (\rho + \sigma E) N, \quad (3)$$

$$V = N^2 \quad (4)$$

In the following, the effect on confinement of the system due to the application of various input powers is reported. The parameter values of [21] are fixed as  $\vartheta = 19$ ,  $\xi = 1.7$ ,  $\eta = 0.12$ ,  $\rho = 0.55$  and  $\sigma = 0.6$

Mode	Zonal flow	Turbulence	$q_0$ value
L	$U=0$	$E>0$	Up to 0.08
T	$U \neq 0$	$E \neq 0$	0.08- 0.564
Hysteresis: T/QH	$U \neq 0 / U=0$	$E \neq 0 / E = 0$	0.564-0.589
QH	$U=0$	$E=0$	0.59 onwards

**TABLE I.** Different modes depending on  $q_0$  values.

We start our research finding by identifying different modes when the input power is constant in time as  $Q=q_0$ . For a fixed value  $Q=q_0$ , we solve Eqs. (1)–(4) for a sufficiently long time by taking

the time average of  $E$ ,  $V$ , and  $N$  after removing the initial time transient and plot them in Figure 1, for different values of  $q_0$ . It can be seen that only turbulence  $E$  grows at first and then once it reaches a certain level, it excites the zonal flow  $V$ , which in turn reduces the turbulence.

At roughly  $Q=0.15$ , zonal flow exceeds turbulence for the first time and at roughly  $Q=0.38$  reaches a maximum where it is controlled by mean flow to be brought back below turbulence. At  $Q=0.57$ , they both die. For our chosen parameter values, different modes which are characterized by the values of zonal flow and turbulence are summarized in Table I. Here T mode, which is a result of modulation between the zonal flow and turbulence, turns out to contain two different regimes depending on the value of  $q_0$ : stationary state for  $0.08 < q_0 < 0.51$  and the oscillatory state for  $0.51 < q_0 < 0.564$ . For a small interval of  $0.564 < q_0 < 0.589$ , the mode becomes either QH or T depending on initial condition (i.e., bistability). It is also already reported in [21]. In order to see the effect of a function,  $A \tanh(t/T)$  for the transition of input heating power, we take the input power to have two parts as  $Q(t) = q_0 + A \tanh(t/T)$  and perform the following four steps:

- (i) For a chosen value of  $q_0$ , we solve Eqs. (1)–(4) and compute mean value of  $E$ ,  $U$ , and  $N$  by taking time-average after removing the initial transient.
- (ii) We repeat (i) by varying the values of  $A$ ,  $1/T$ .
- (iii) We plot results from (ii) as a function of  $A$  and  $1/T$  on a 3D plot.
- (iv) We repeat (i)–(iii) by choosing different values of  $q_0$ , which are representative of Figure 1.

In order to see the effect of the input heating power  $Q$ , 3D plot from (i)–(iv) are shown in Figures 2 and 3 for different values of  $q_0$ . For most  $q_0$  values, there is a tendency for the

turbulence to decrease for relatively small  $A$  and much wider range values of  $1/T$  as compare to [21]. The effect of the function  $A \tanh(t/T)$ , input power is rather modest up to  $q_0 < 0.37$ , at which point its effect becomes much stronger, leading to the QH mode for relatively small  $A$  and much wider range values of  $1/T$  as compare to [21]. For a very low  $q_0$  (0.075) as shown in Figure 2(a), a decrease in turbulence and rise in zonal flow is visible for large  $A$  and much wider range values of  $1/T$ , can be seen in Figure 2(b-c). This trend becomes more clear as  $q_0$  is increased. At  $q_0 = 0.37$ , the slight small  $A$  and much wider range values of  $1/T$  as compare to [21], are able to reduce turbulence completely to a zero value (i.e., leading to QH mode). Figure 2(d), shows another example of such case when turbulence reaches zero when  $q_0 = 0.4$ .

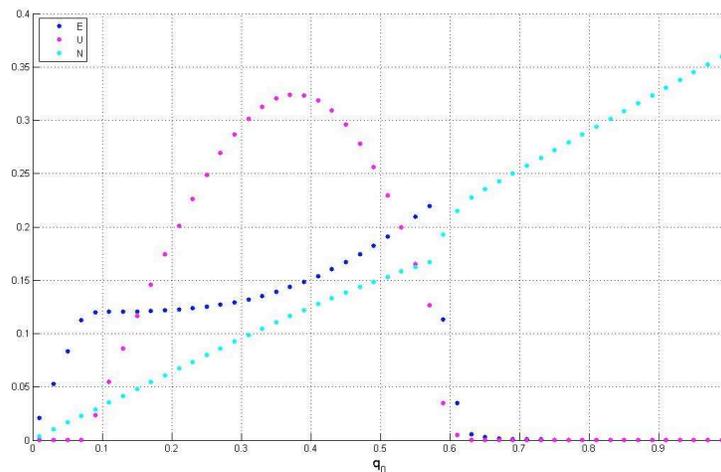
Finally all other values of  $q_0 = 0.516$ ,  $q_0 = 0.55$  and  $q_0 = 0.57$ , for relatively small  $A$  and much wider range values of  $1/T$  as compare to [21], are able to reduce turbulence completely to a zero value (i.e., leading to QH mode) as shown in Figures 3 (a-d). In comparison, above  $q_0 = 0.516$  where the system bifurcates from stationary to oscillatory state,  $1/T$  can take much wider range values while  $A$  can be relatively small. So with a larger  $q_0$ ,  $A$  and  $1/T$  can be more varied. The minimum  $A$  reducing with increasing  $q_0$  value is intuitive since higher  $q_0$  value guarantees its proximity to the bifurcation point  $q_0 = 0.59$  (see Table I).

As the stepwise increase of the input power leads to the QH mode with zero turbulence when  $q_0$  is equal or greater than 0.37, the area at which the turbulence is zero on these 3D plots is extracted and the edge of each area is presented in Figure 4. It can be seen that the input heating power  $Q$  that consists of constant part  $q_0$  in addition to a function  $A \tanh(t/T)$  provides the L-H transition for relatively small  $A$  and much wider range values of  $1/T$  as compare to [21]. The

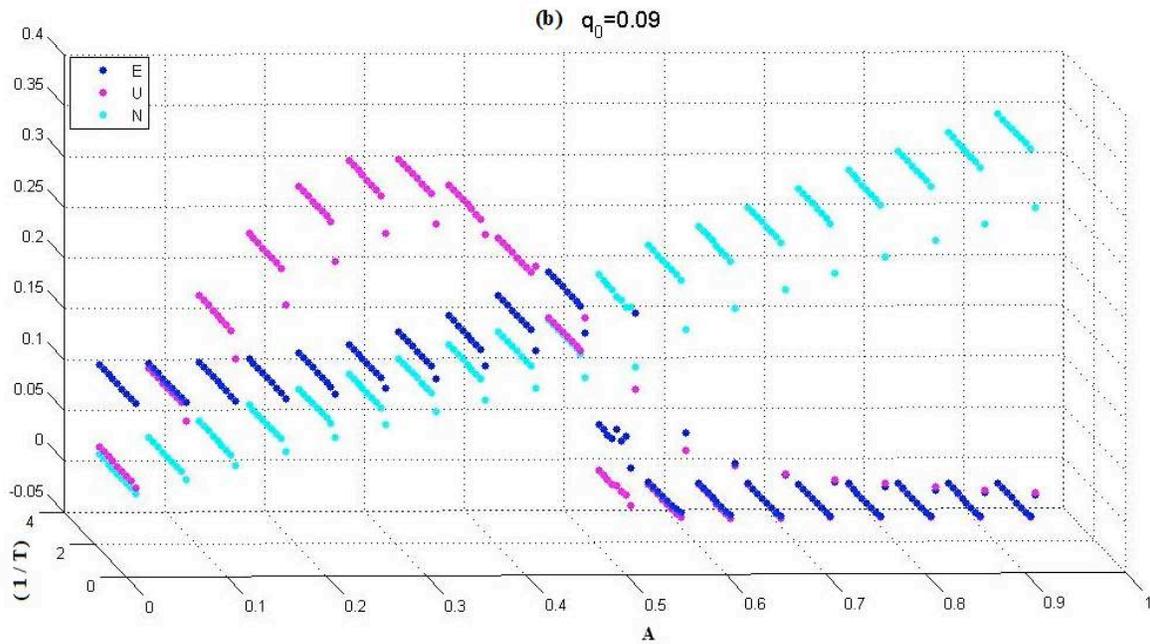
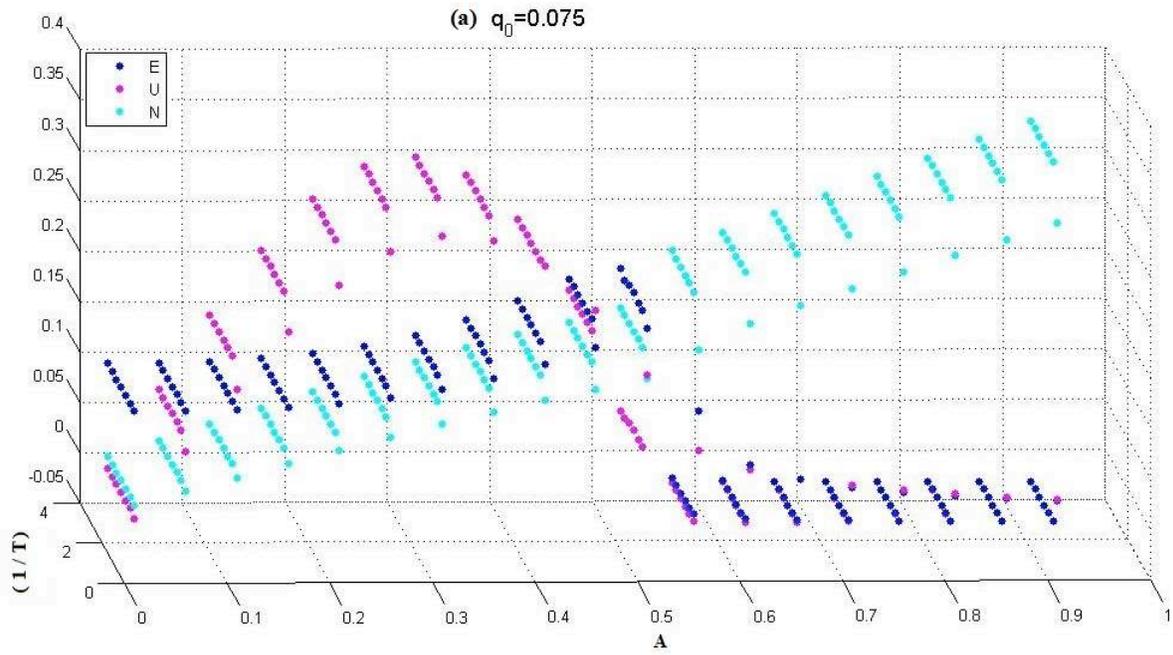
experimental test on our results as well as the extension to other models, would be of great interest in future.

### 3. Conclusion

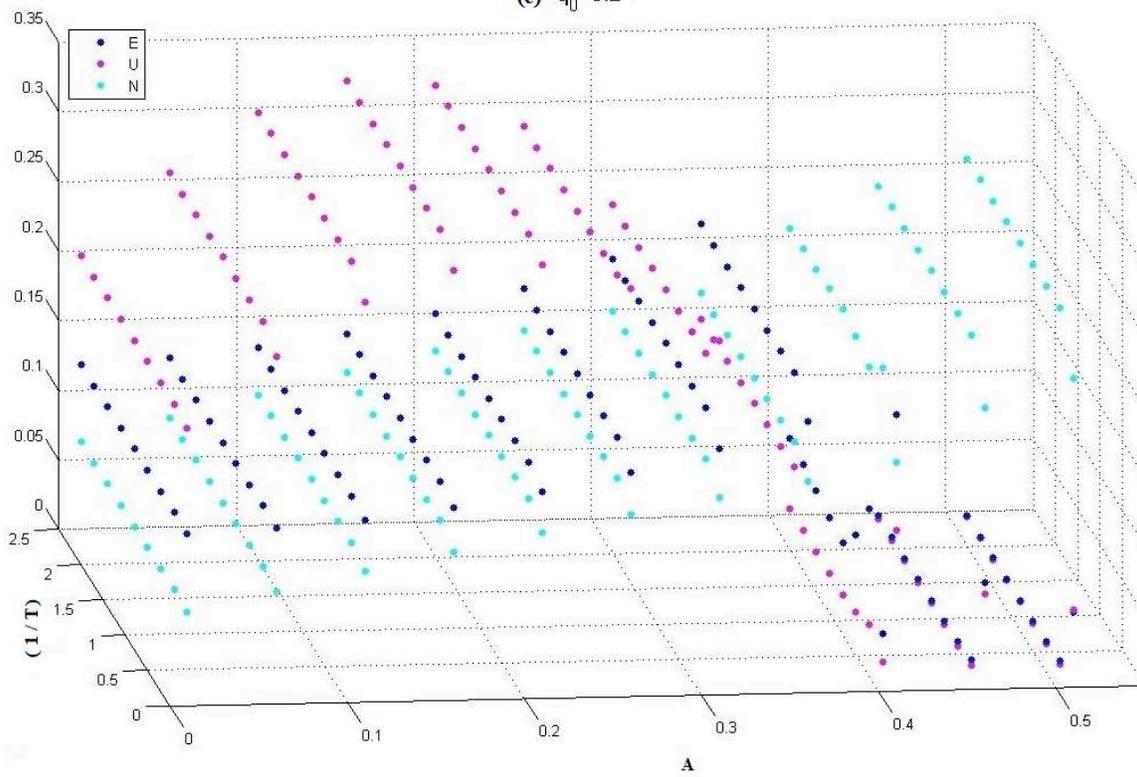
The main purpose of our paper is to repeat the analysis [21] using a function,  $A \tanh(t/T)$  for the transition of input heating power. We have obtained the different results than [21], as reported here, for a broad range of  $T$  values. We have reported an interesting results on plasma confinement using a function,  $A \tanh(t/T)$  for the transition of input heating power for tokamak plasmas. Specifically, the addition of the perturbation,  $A \tanh(t/T)$ , to constant power  $q_0$  is shown to promote the confinement, leading to the L-H transition at a lower value of  $q_0$ , as compared to the case of constant  $q_0$  without the  $A \tanh(t/T)$  perturbation. It is seen that the input heating power  $Q$  that consists of constant part  $q_0$  in addition to a function  $A \tanh(t/T)$  provides the L-H transition for relatively small  $A$  and much wider range values of  $1/T$  than [21].

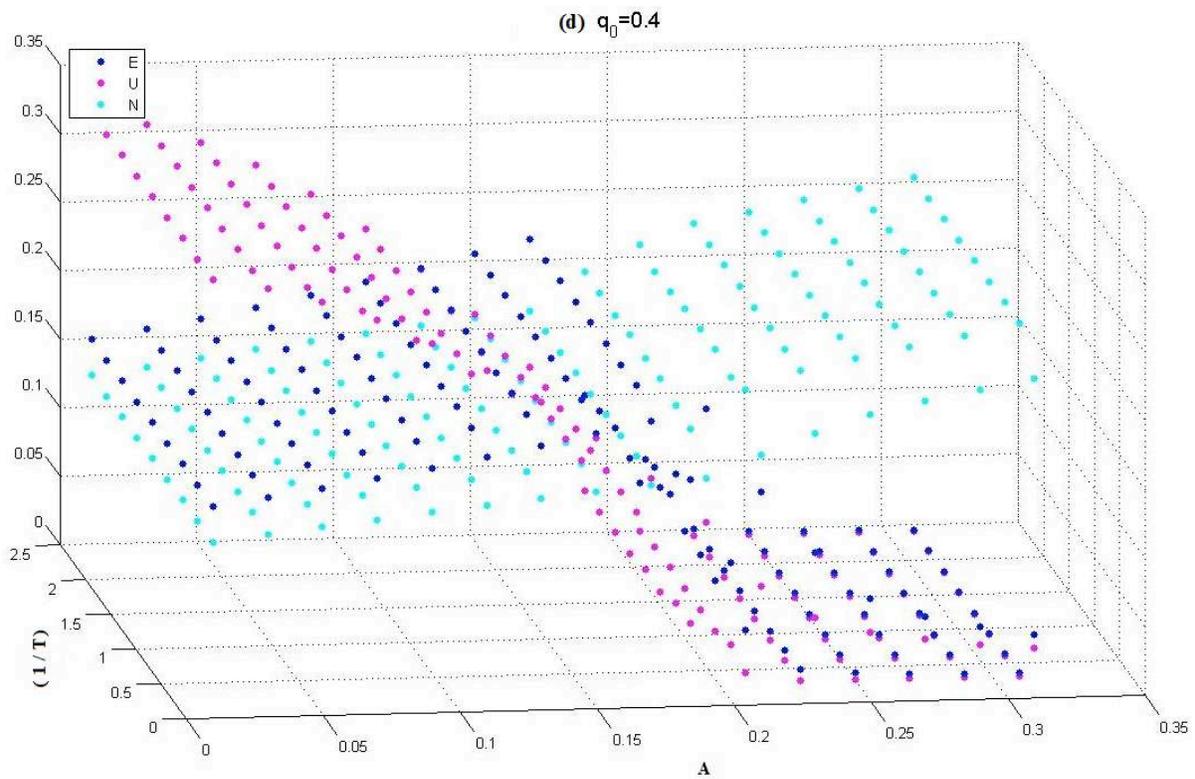


**Figure 1:** Mean values E (in blue) , U (in red) and N (in sky blue ) against constant input power  $Q=q_0$ .

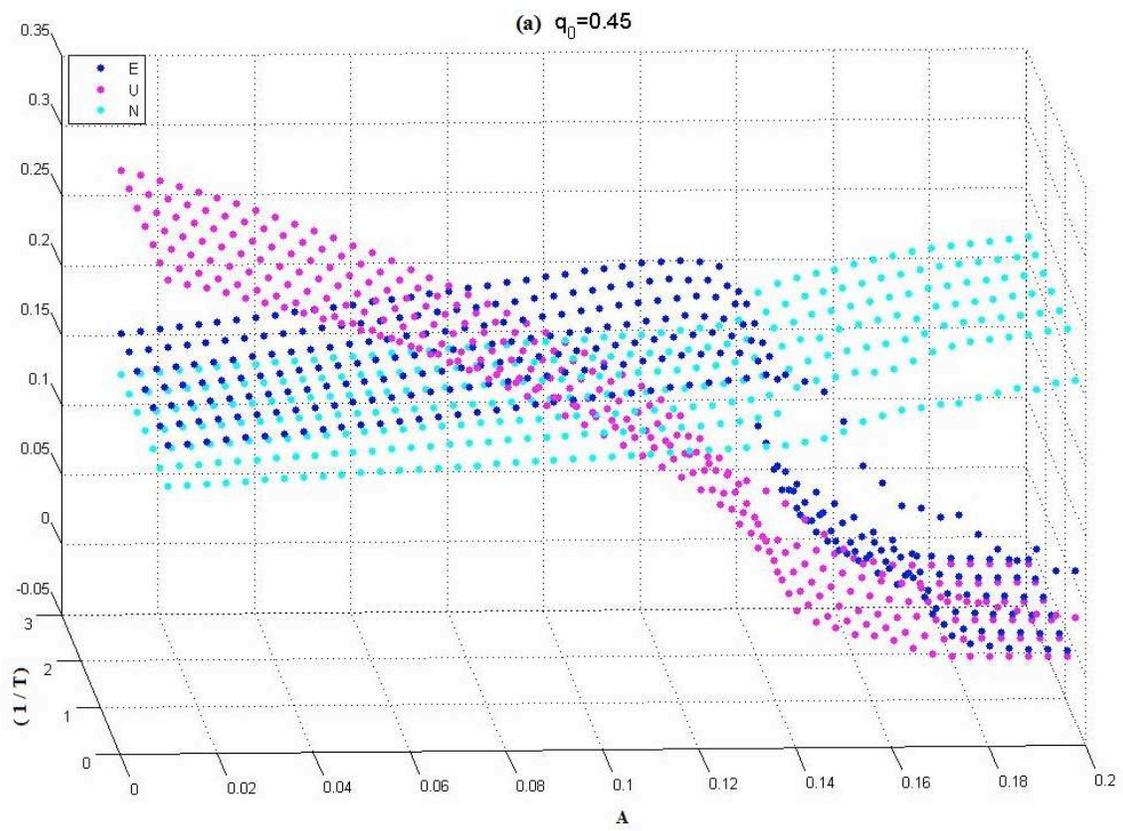


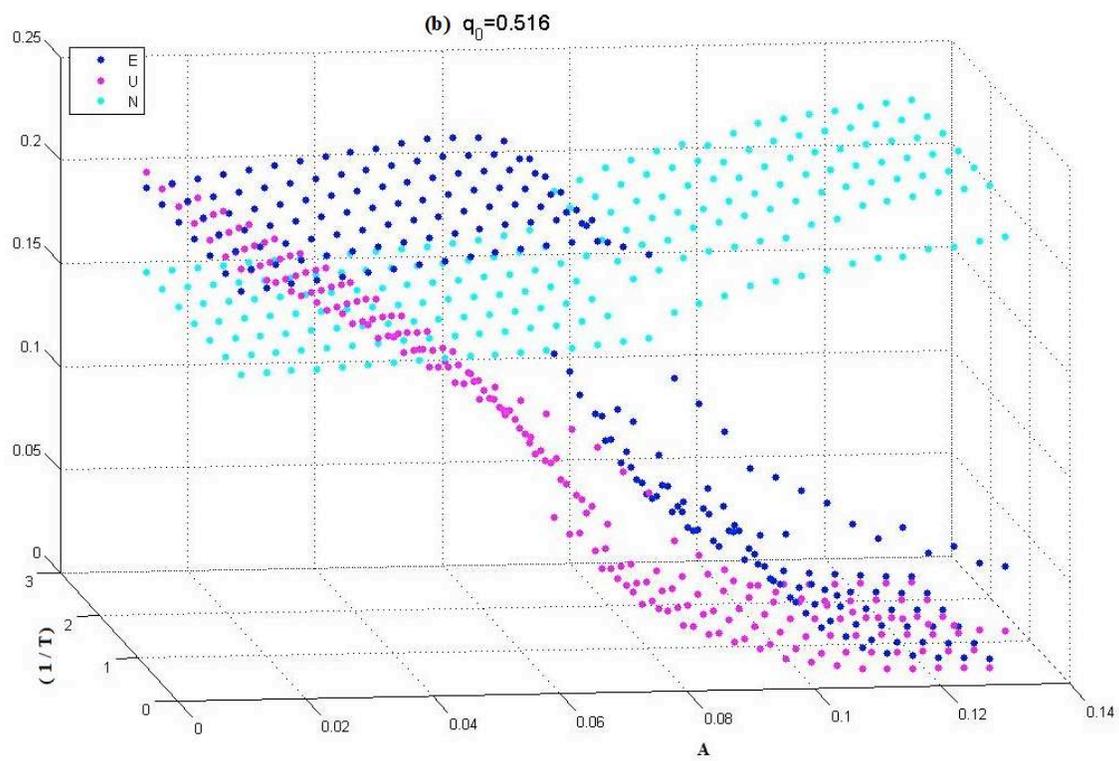
(e)  $q_0=0.2$



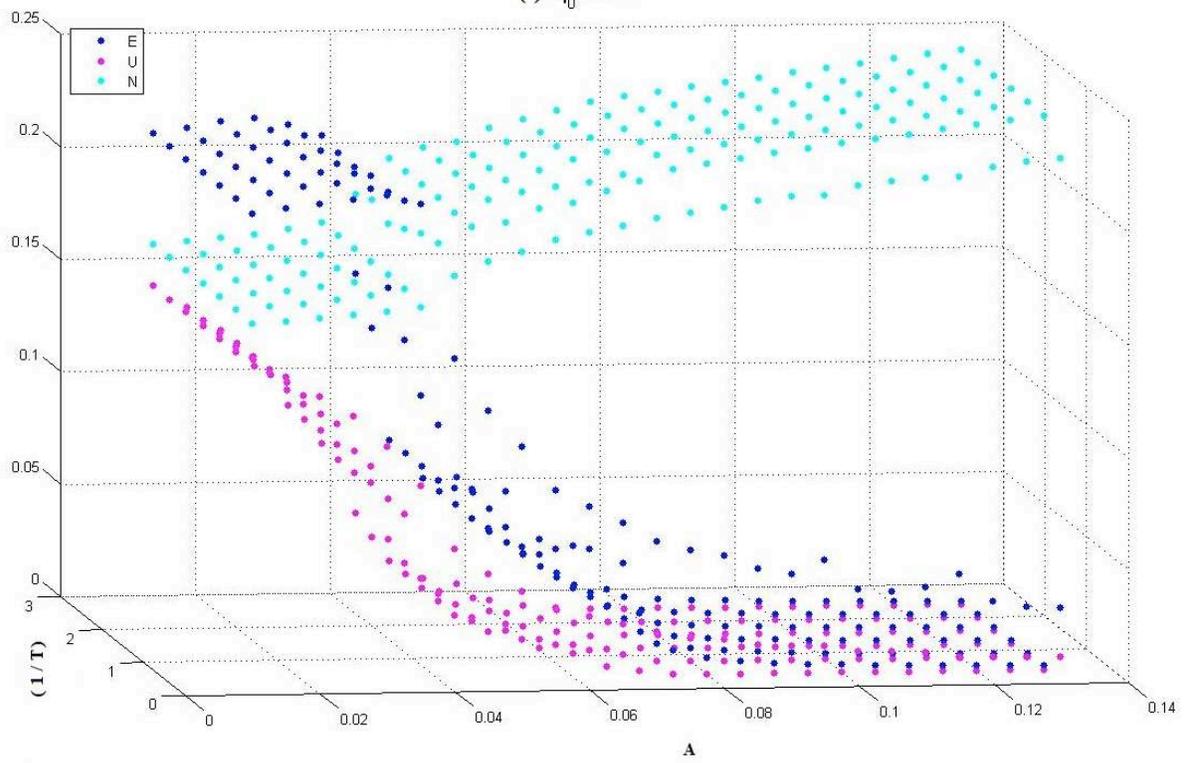


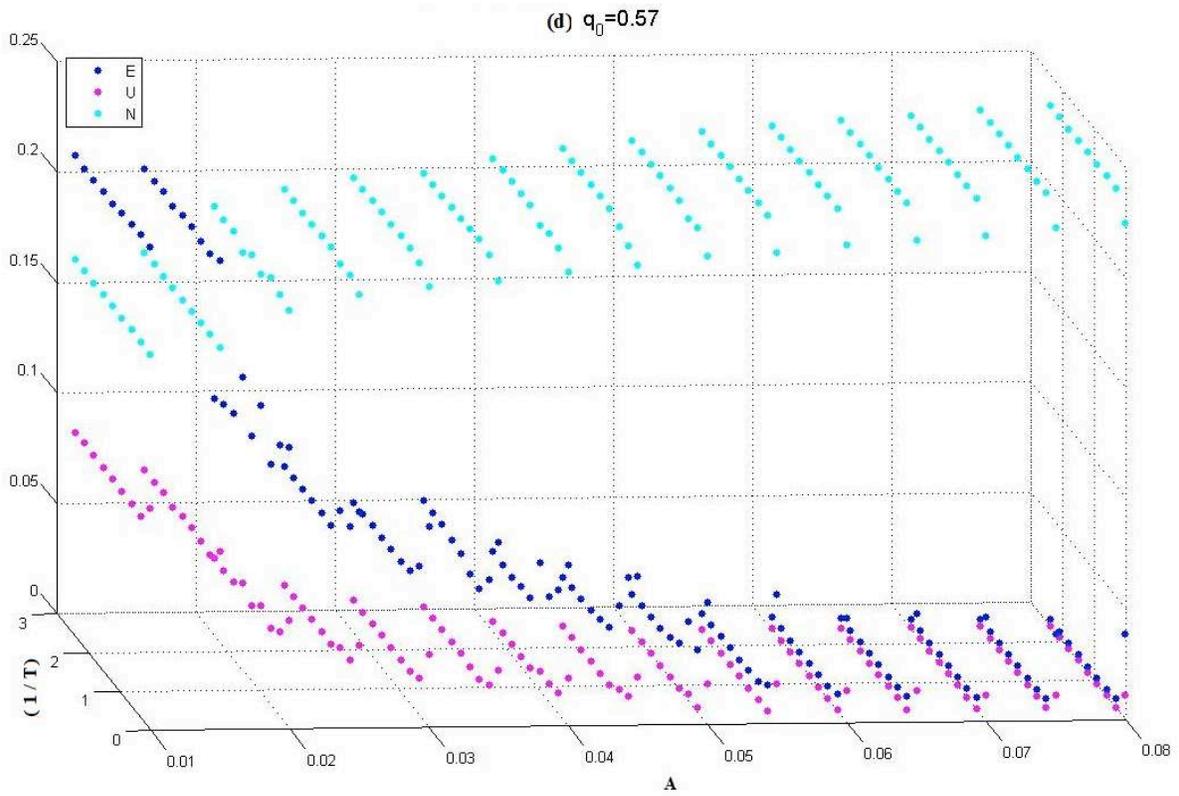
**Figure 2:** 3D plots of  $E$ ,  $U$ , and  $N/5$  as a function of  $A$  and  $1/T$  for (a)  $q_0=0.075$ , (b)  $q_0=0.09$ , (c)  $q_0=0.2$ , (d)  $q_0=0.4$ .



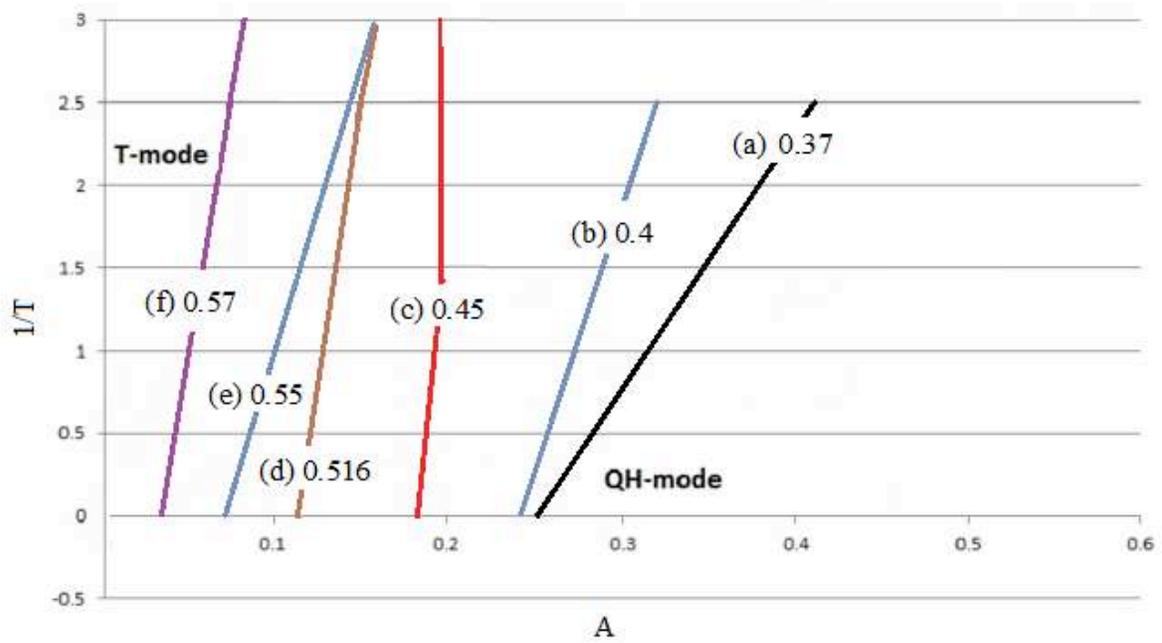


(c)  $q_0=0.55$





**Figure 3:** 3D plots of  $E$ ,  $U$ , and  $N/5$  as a function of  $A$  and  $1/T$  for (a)  $q_0=0.45$ , (b)  $q_0=0.516$ , (c)  $q_0=0.55$ ,  
(d)  $q_0=0.57$ .



**Figure 4:** The relation between  $A$  and  $1/T$  for a stepwise increase of the input power for the onset of the transition to QH-mode for different  $q_0$  values: the lower right and the upper left lines represent the QH and T-modes, respectively.

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