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A UNIFIED RHEOLOGICAL MODEL FOR MODELLING STEEL BEHAVIOUR IN FIRE CONDITIONS

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Abstract:

This paper presents a newly developed rheological model capable of modelling the behaviour of carbon steel at high temperature under stress- and strain-rate controlled tests. By combining two serial Kelvin elements with the appropriate spring-and-damper constitutive behaviour models it is possible to model creep strain development under stationary and transient heating conditions. Furthermore, the model is able to take into account the inherent increase of the yield strength if the strain rate is raised to moderately high levels usually expected in a fire-induced collapse of the structure. Constitutive behaviour models for each of the rheological elements are based on the test data from which the Eurocode 3 stress-strain law originated. The model was verified by using the test results of constant stress- and strain-rate tests from various sources. Overall comparison of results indicates the applicability of the proposed rheological model to structural fire engineering analysis for steel grades S275 and S355.

Keywords:

Steel, creep, fire, coupon, damper, spring

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1. INTRODUCTION

Constitutive material modelling within the finite element method is based on the provision of a suitable material model which closely matches the material behaviour exposed to external load during the service life-time of a structure. The selected material model should be able to take into account as many significant material effects as possible so that the engineer is provided with a reliable calculation on which to judge the structure's load bearing capacity. Fire action represents one of the possible events during the service life-time of a building. In case of fire acting on a structure, a reduction of the material's resistance capacity and change of loading conditions often occurs. The former refers to the reduction of all of the material's mechanical strength and stiffness properties, and the latter refers to the thermally-induced change of force level and strain rate. These effects are accompanied by temperature distributions which depend on the variation of heating rate which occurs in a natural fire event.

Most of the accepted constitutive stress-strain models for representing steel behaviour at high temperature are based on a set of temperature-dependent stress-strain curves. Generally, the mathematical representation of these curves is generated by fitting the test data obtained from stress- or strain-rate-controlled testing of coupons heated under quasi-static conditions. Test data obtained under transient heating conditions is also used to generate material models. Strain-rate-controlled tests are usually conducted by applying strain rates which are codified to a standard testing procedure for determining steel strength [1,2]. The strain rate range used for testing is usually restricted to approximately 0.00025-0.0025s^{-1}, and the codified stress rate range is between 6-60 $MPas^{-1}$ [2]. Tests under transient heating conditions are also conducted using constant heating rates in the range 5-20°C/min, with 10°C/min being more frequently used than others in generating stress-strain models which are deemed to include creep implicitly [3,4].

The shape of the stress-strain model for steel at high temperature is essentially nonlinear for all models. However, there are differences between them in certain parts of the stress-strain curve, especially in the transition zone before the yield plateau starts [5].

At present there are a few stress-strain models which are used frequently in performance-based fire engineering. The older steel models [3, 6] used in modelling structural behaviour are based on a Ramberg-Osgood form of relationship. This type of model is based on a monotonically increasing curve which utilizes temperature-dependent coefficients to fit experimental data.

In Europe, the most widely-used stress-strain law is incorporated in the steel and composite structure Eurocodes [7, 8] for low-carbon structural and reinforcing steels, and for prestressing steel; this model comprises linear and elliptical zones, followed by a yield plateau. The model originates from a comprehensive coupon-study based largely on the transient-heating test methodology at 10°C/min [4].

Codified design models in the USA are given in the ASCE manual [9], and are based on a bilinear elasto-plastic temperature-dependent stress-strain model. The origin of the ASCE model is uncertain [5], but the shape of the stress-strain model suggests that the test data is probably taken from constant-temperature coupon tests conducted at a fast strain rate. Various researchers have proposed other types of stress-strain model, mostly based on curve-fitting of constant-temperature and transient test data [10-12].

Looking at the origins of general and codified design models it can be deduced that a stress-strain law determined from any particular set of test parameters should be limited in application to conditions similar to those under which the test results were obtained. The important parameters to which this applies are the applied heating rate, strain rate and stress rate during the test. As has been mentioned earlier, apart from the reduction of mechanical properties, changes of load level, strain rate and heating rate occur in steel during a natural fire event. A more comprehensive stress-strain model, which is sensitive to the crucial thermo-mechanical parameters which can change during fire exposure, needs to be developed for use in thermo-structural analysis. In order to develop such a constitutive law, the most convenient way is to develop a stress-strain model based on general rheological principles. Most previous research which has utilized rheological principles for high temperature analysis, such as the Burgers [13] and "standard" [14] solid models, has been based on the creation of rheological models for creep in steel at high temperature. The motivation for developing a unified rheological model in this study is the desire to unify the analysis of steel structures across the whole range of fire scenarios, taking into account all of the complex and time-dependent strain components which occur in the steel. The unified rheological model postulated in this paper aims to allow analysis which is temperature-, stress-, heating-rate- and strain-rate-sensitive. These are the major thermo-mechanical variables in fire.

The theoretical background for the developed rheological model is given in this paper, including a verification for each of its constitutive components. Several available material test studies have been used to test the model's capabilities and to verify its performance in constant-temperature and transient heating conditions in order to show the applicability of the model.

2. THEORETICAL BACKGROUND OF THE UNIFIED RHEOLOGICAL MODEL

2.1 Description of the rheological sub-models and the unified model

Three strain components can be defined [11] for steel at any temperature:

$$\varepsilon_{tot} = \varepsilon_{th}(T) + \varepsilon_{\sigma}(\sigma, T) + \varepsilon_{cr}(\sigma, T, t) \tag{1}$$

in which: ε_{tot} is the total strain, $\varepsilon_{th}(T)$ is the temperature-dependent thermal strain, $\varepsilon_{\sigma}(\sigma, T)$ is the stress-related strain (dependent on the applied stress σ and the temperature T) and $\varepsilon_{cr}(\sigma, T, t)$ is the creep strain (dependent on stress, temperature and time). The second and third of these are the most complex to determine, due to their dependency on a large number of thermo-mechanical variables. Since thermal strain depends only on temperature it can be modelled with relative ease, and it will not be considered as a separate strain variable within the rheological models. The unified model presented in this section and illustrated in Figure 1, is capable of providing only stress-related and creep strains.

Four different rheological models (three sub-models and a unified model) are presented in this chapter to represent the strain evolution in steel during fire, including the creep and strain-rate effects. The sub-models are used to calibrate each of the constitutive rheological components which are later implemented in the unified rheological model.

Model R1 is applied at very low strain rates, R2 at moderate strain rates and R3 at very fast strain rates. A unified rheological model proposed by the authors, which contains the physical attributes of the first three rheological sub-models, is denoted R4. This rheological model is intended to be applied over a wide range of strain rates. Variation of yield strength and the evolution of creep strain are taken into account for different rheological models by combining sub-models. The Kelvin-Voight element in model R2 is used for modelling the evolution of creep strain, and in model R3 is for modelling change of yield strength with strain rate.

Model R1 comprises a series combination of two spring elements, with each strain component being represented by a single nonlinear spring. The first spring represents a mechanical (stress-related) strain component which is based on a nonlinear stress-strain law, and the second spring represents long-term creep strain at lower strain rates. In this arrangement of elements there are no dampers connected in parallel to the springs, since the model is used for analyzing the strain response at very low strain rates, taking into account the reduction of yield strength which has been experimentally observed at very low strain rates. Since a series combination of rheological elements is used, the total strain is the sum of individual strains:

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \quad ; \quad \sigma = \sigma_1 = \sigma_2 \quad ; \quad \sigma_1 = k_1(\sigma, T)\varepsilon_1 \quad ; \quad \sigma_2 = k_2(\sigma, T)\varepsilon_2 \qquad (2)$$

in which: ε_1 and ε_2 are the strain components of the first and second Kelvin-Voight elements, σ_1 and σ_2 represent the stress components of the spring elements for the first and second Kelvin-Voight elements, k_1 and k_2 are the temperature- and stress- dependent functions and T is temperature.

The second model R2 comprises a spring element and a Kelvin-Voight element. The spring element represents the mechanical strain without accounting for the influence of change of strain rate, and the Kelvin-Voight element represents the viscous creep strain component. This type of rheological model is assumed to be valid for moderate strain rate ranges in which the strain-rate effects on the strain output can be treated as negligible. The stress-strain relationship for the spring element in R2, the differential equation for the Kelvin element and the total strain rate equation can be expressed as:

$$\sigma = k_1(\sigma,T)\varepsilon_1 \quad ; \quad \frac{\sigma}{c_2\left(\dot{\varepsilon},T\right)} = \frac{k_2(\sigma,T)}{c_2\left(\dot{\varepsilon},T\right)}\varepsilon_2 + \dot{\varepsilon}_2 \quad ; \quad \dot{\varepsilon} = \text{const} = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 \qquad (3)$$

in which: $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ represent the strain rate components of the first and second Kelvin-Voight elements, c_2 is the temperature- and strain rate- dependent function.

Model R3 represents a single Kelvin-Voight element, comprised of a nonlinear spring element and a nonlinear damper element in parallel. This element represents a mechanical strain component with the ability to increase its yield strength because of the strain-rate effect, which generally increases the yield strength of steel. The damping coefficient in this rheological model is assumed to have a very high value so that it has its main effect at higher strain rates. At such high strain rates the creep strain component is expected to be very low compared to the mechanical strain. A parallel connection between the elements divides the total stress into the two components:

$$\sigma = \sigma_1 + \sigma_2 \quad ; \quad \sigma_1 = k_1(\sigma,T)\varepsilon \quad ; \quad \sigma_2 = c_1\left(\dot{\varepsilon},T\right)\dot{\varepsilon} \qquad (4)$$

The fourth model, R4 comprises a series combination of two Kelvin-Voight elements. The first of these represents the mechanical strain component in the same way as model R3; the second represents viscous creep strain. The damping coefficient of the former element is assumed to have a very low value compared to the damping coefficient of the latter. Both damping coefficients are assumed to have temperature- and strain-rate-dependency. The differential equation for each of the Kelvin elements of model R4, and the total strain rate, can be expressed as:

$$\frac{\sigma}{c_i} = \frac{k_i}{c_i}\varepsilon_i + \dot{\varepsilon}_i \ \text{ for } i = 1,2 \quad ; \quad \dot{\varepsilon} = \text{const} = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 \qquad (5)$$

In this case two differential equations have to be integrated with respect to time, and the appropriate strain rate solution at a current time interval is the sum of the individual rates for the two elements. Equation (5) can be solved using Euler integration, which is utilized in the solver presented in Figure 3 by using small time increments for integration.

Two different types of solution are developed for the presented rheological models. These are strain-rate- and stress-controlled, and are used for modelling of these respective types of test.

2.2 Test studies for calibration and verification

A wide range of test studies has been used to calibrate and verify the constitutive components of the unified rheological model. These studies are based on both transient and strain-rate-controlled coupon tests of steel grades currently available in Europe.

The central study selected for verification of the unified model is that by Kirby et al. [4,15]. This study was used as a basis for creating the original Eurocode 3 stress-strain law, and consisted of a series of transient coupon tests. The heating rates were varied in the range 2.5-20°C/min, with stress levels ranging between 25 and 350MPa. British standard steel grades designated 43A and 50B were tested in the study; these correspond to the current Eurocode 3 steel grades S275 and S355, respectively.

A study by Latham and Kirby [16] was selected for determination of the damper constant c_1. This study was based on a series of constant-strain-rate tests on steel of grade Fe430A (S275). Temperatures ranging from 20-800°C and strain rates from 0.002-0.1/min were used in these tests.

A study by Boko et al. [17] was used for verification of the unified rheological model. The study was conducted on a small series of transient coupon tests of a more recent alloy, of steel grade S355, at various stress levels between 50 and 400MPa. A single heating rate of 10°C/min was used in these transient tests. In addition, coupon tests at a fixed strain rate of 0.0002/s in the temperature range 20-750°C were conducted to determine the stress-strain relationship of the tested steel grade.

Tests conducted by Renner [18] studied the influence of different strain rates on the stress-strain material law of steel grade S275 in the range 400-700°C. In total 25 coupons were tested at three different displacement speeds (0.7-6.0 mm/min) at each temperature level. This study was selected for verification of the unified rheological model.

A study conducted by Harris [19] was based on the analysis of the effects of short-term creep on the stress-strain material law of steel grade S275. Three strain rates (0.00047-0.00165 s^{-1}) within the temperature range 400-650°C were used in this study, which was used for verification of the damper constant c_1.

A study conducted by Bull et al. [20] involved high-temperature constant-strain-rate tests on M20 grade 8.8 bolts. Strain rates ranging from 0.002-0.02 min^{-1} and temperatures from 550-700°C were analysed within the study, which was used for verification of the unified model in order to test its capabilities outside the range of contemporary steel grades.

2.3 Constitutive rheological components

The constitutive relationships of all four rheological models are functions of their spring and damper components. As mentioned earlier, the spring component represents the basic mechanical strain model. The constitutive model for spring k_1 in this study was chosen to be the recently-developed creep-free Eurocode 3 stress-strain model [21]. This decision was made since the Eurocode 3 stress-strain law is based on experimental data using standardized testing regimes, and is appropriate for modelling mechanical strain in the medium-strain-rate test range. An additional reason for using this particular model was that the Eurocode 3 form of stress-strain curve is widely accepted in performance-based structural fire design across Europe and in the scientific community.

This type of modified (creep-free) model follows the original Eurocode 3 model, except that the yield strain $\varepsilon_{y,\theta}$ is reduced to 1% in order to exclude the implicit creep content. The details of the procedure for extracting implicit creep can be found in [22]. The shape of the modified Eurocode 3 model and its comparison with the original are presented in Figure 2.

The constitutive model for spring k_2 is identical to that for spring k_1. The only difference is that the temperature-dependent yield strength $f_{y,\theta}$ of k_2 has been reduced to 80% of the original yield strength of k_1. This reduction has been chosen to match the slow-strain-rate tests by Latham and Kirby [16], since a reduction of yield strength of up to about

20% has been experimentally observed at very low strain rates. This study was chosen to calibrate the constitutive model of spring k_2. Figure 4 presents a comparison between modelling results using model R1 and the test results from [18-19], for the k_2 spring model.

The equations which describe the k_1 and k_2 constitutive models plotted in Figure 2 (modified EC3) are defined by the following expressions [7]:

$$\sigma = \varepsilon E_{a,\theta} \quad (\text{ for } \varepsilon \le \varepsilon_{p,\theta}) \tag{6}$$

$$\sigma = f_{p,\theta} - c + (b/a)\left[a^2 - (\varepsilon_{y,\theta} - \varepsilon)^2\right]^{0.5} \; (\text{for } \varepsilon_{p,\theta} < \varepsilon < \varepsilon_{y,\theta}) \tag{7}$$

$$\sigma = f_{y,\theta} \;\; (\text{for } \varepsilon_{y,\theta} < \varepsilon < 0.04) \tag{8}$$

Parameters a^2, b^2 i c can be obtained from the following expressions:

$$\begin{aligned} a^2 &= (\varepsilon_{y,\theta} - \varepsilon_{p,\theta})(\varepsilon_{y,\theta} - \varepsilon_{p,\theta} + c/E_{a,\theta}) \\ b^2 &= c(\varepsilon_{y,\theta} - \varepsilon_{p,\theta})E_{a,\theta} + c^2 \\ c &= \frac{(f_{y,\theta} - f_{p,\theta})^2}{(\varepsilon_{y,\theta} - \varepsilon_{p,\theta})E_{a,\theta} - 2(f_{y,\theta} - f_{p,\theta})} \end{aligned} \tag{9}$$

Where: $\varepsilon_{p,\theta} = f_{p,\theta}/E_{a,\theta}$ and $\varepsilon_{y,\theta} = 0.01$. Parameters $f_{p,\theta}$, $f_{y,\theta}$, $E_{a,\theta}$ are respectively the proportional limit, yield strength and modulus of elasticity at temperature θ which are defined in [7]. For strain values higher than 0.04 within the temperature range of 400°C, strain hardening of steel is taken into account according to the Annex A of EN1993-1-2.

Both damper parameters c_1 and c_2 are dependent on the damper stress level and strain rate. Damper c_1 represents the increase of yield strength as strain rates increase. Values for c_1 are determined from the fast-strain-rate tests of Latham and Kirby [16], with the help of rheological model R3 which does not have a creep strain component. The damper values for c_1 and c_2 are determined using the following nonlinear relationship:

$$c_i = \frac{\sigma_{d_i}}{\dot{\varepsilon}_i} \; ; i = 1, 2 \tag{10}$$

The relationship between stress and strain rate for damper c_1 was obtained using the test data from Latham and Kirby's study, which is presented in Figure 5(a). Table 1 presents the values of damper stress which is plotted in Figure 5(a). As mentioned earlier, a comparison with other fast-strain-rate test results [18-19] and the rheological model R3 is given in Figure 5(b). The damping coefficient c_1 always has a smaller value than that for c_2 when utilizing equation (10). The damping coefficient c_2 is determined using a logarithmic relation between strain rate and temperature. This type of relationship is presented in Figure 6(a) and can be obtained by using any creep model or stationary creep test data. The relationship presented in Figure 6(a) is obtained by creating a strain rate against temperature relationship from a series of curves of creep strain against temperature, obtained with the help of a prescribed creep model at constant stress levels and pre-defined heating rates.

Within the study, a single creep model was used as a background for determining this kind of relationship. Harmathy's creep model [24] has proved sufficiently accurate in previous studies [21,22, 23] modelling creep of European steel grades in fire. This model is based on a time-hardening rule. The creep strain is expressed as:

$$\varepsilon_{\mathrm{cr}} = \frac{\varepsilon_{\mathrm{cr},0}}{0.693} \cdot \cosh^{-1}\left(2^{\frac{Z\theta}{\varepsilon_{\mathrm{cr},0}}}\right) \qquad (\theta < \theta_0) \tag{11}$$

$$\varepsilon_{\mathrm{cr}} = \varepsilon_{\mathrm{cr},0} + \mathrm{Z}\theta \qquad (\theta \geq \theta_0) \tag{12}$$

$$\theta_0 = \varepsilon_{\mathrm{cr},0} / \mathrm{Z} \tag{13}$$

$$\theta = \int_0^t \exp^{\frac{-\Delta H}{RT_R}} dt \tag{14}$$

In which T_R is the temperature (°K), R is the universal gas constant (J/mol°K), *ΔH* is the creep activation energy (J/mol), Z is the Zener-Hollomon parameter (h^{-1}), $\varepsilon_{cr,0}$ is a dimensionless creep parameter, t represents time and θ represents temperature-compensated time. In order to utilize the creep model, the material parameters *Z, ΔH/R* and $\varepsilon_{cr,0}$ are borrowed from a research study conducted by Harmathy and Stanzak [25] for American steel Grade A36, whose yield strength is similar to Eurocode steel Grade S275. The results of calibration for the second Kelvin element in rheological model R2, which uses the constitutive model from Figure 6(a) and the creep model Cr_1 is given in Figure 6(b).

3. VERIFICATION OF THE UNIFIED RHEOLOGICAL MODEL R4

3.1 Stationary coupon tests

In this section the performance of the unified rheological model R4 is shown, and a comparison with different strain-rate-controlled test studies is presented. This comparison is presented in order to illustrate the applicability of the proposed rheological model for modelling high-temperature strain development in various steel grades.

Figure 7(a) presents a comparison of results from the study by Boko et al. [17] and the model R4. Boko's study is based on high-temperature strain-rate tests of steel coupons at $0.0002s^{-1}$. Figure 7(b) compares the model predictions with the study by Bull et al. [20], which reports medium-speed strain-rate tests on bolts of Grade 8.8. Figure 7(c) shows a further comparison against the study by Renner [18] for medium-strain-rate coupon tests of steel of Grade S275.

3.2 Transient coupon tests

This section shows how the unified rheological model R4 performs in modelling the strain development of steel coupons under transient heating conditions. Figure 8(a) presents a comparison of results between the model R4 and the study of Boko et al. [17] in modelling the strain development of steel coupons of Grade S355 at a heating rate of 10°C/min. Figures 8(b)-8(e) present comparisons of results between the model R4 and the tests by Kirby and Preston [15], which were based on transient heating of coupons of Grade S355 steel at heating rates of 2.5, 5, 10 and 20°C/min. Figures 9(a)-9(b) present similar comparisons between the model and test results of S275 coupons from the same study at heating rates of 10-20°C/min. A summary of the input parameters for the rheological model R4 used in modelling the test results from Figures 8(a)-8(e) is given in Table 2. Table 3 gives the analytical equations of the temperature- and strain-rate-dependent model of damper c_2. The functionality of model R4 is illustrated in Figure 10, which shows the reduction of the damping constant c_2, and its comparison with the value of creep strain which represents the output strain of the second Kelvin element for a simulation of S275 coupon response exposed to a stress of 100 MPa at a heating rate of 5°C/min.

4. DISCUSSION OF RESULTS

4.1 Spring component verification

A comparison between the maximum stress level obtained by the constitutive slow strain rate model R1 and the test results of Renner [18] and Harris [19] from Figure 4 indicates that the material models for the first and second springs of the rheological model R1 provide good predictions of the maximum stress in both test studies within the temperature range 500-700°C. The slow-strain rates in Renner's study varied between 0.0001-0.0002s^{-1} and in Harris's study between 0.0002-0.0005s^{-1}. This comparison was conducted in order to verify the material constitutive model for both springs of the unified rheological model, reflecting the steel response under slow strain rates at which a small reduction of yield strength occurs.

4.2 Damper component verification

Verification of the material model for damper c_1, which is used for modelling strain-rate effects in steel is presented in Figure 5(b). A comparison between the results from rheological model R3 (which is considered as a representative model for fast-strain-rate analysis) and the fast-strain-rate test results of Renner [18] and Harris [19] shows particularly good correlation when compared with Harris's results, giving support to the stress-strain rate dependency for damper c_1 shown in Figure 5(a). The fast-strain rates in Renner's study varied between 0.0007-0.0012s^{-1}, and in Harris's study between 0.0008-0.0015s^{-1}.

The verification of the damper model c_2 for modelling the development of creep strain is presented in Figure 6(b), and comparing this with the predictions of the analytical creep model Cr_1 indicates that the proposed temperature-strain rate dependency shown in Figure 6(a) is applicable. The rheological model used for calibration of the damper c_2 with creep model Cr_1 was R2, which is applicable for cases where creep-strain values are relatively high compared with the expected values of mechanical strain. The comparisons presented in Figures 4-6 are for verification of the material model for each of the rheological components which are an integral part of the unified rheological model R4.

4.3 Verification of stationary response

Further verification has been conducted by comparing the strain output of model R4 with selected test studies of steel coupons exposed to stationary heating conditions. This attempts to test the performance of model R4, using its previously calibrated constitutive components, in re-creating the strain output over temperatures between 500-700°C. The comparison with the results of stationary coupon tests in Figure 7(a, b) shows very good prediction of the stress-strain outputs of the tests from studies [17] and [20], indicating the applicability of the constitutive material components of model R4 for different steel grades; in particular, the damper c_1, which models strain-rate effects, seems applicable to steel grades S275, S355 and the bolt steel grade 8.8. Some discrepancy in predicting the yield strengths at 550 and 600°C is observed in Figure 7(c) from Renner's study.

4.4 Verification of transient response

Additional verification of the unified rheological model has been done by modelling the comprehensive transient-heating tests by Kirby and Preston [15]. These include transient tests at heating rates of 2.5, 5, 10 and 20°C/min for steel grade S355, plus 10 and 20°C/min for steel grade S275, as shown in Figures 8(a-e) and Figures 9(a-b). Slight discrepancies are observed from comparison with the tests at 20°C/min, for which the strain output from model R4 indicates earlier failure of the coupon than the experiment showed. This can potentially be explained through an additional strength gain of the steel due to the very high strain rate which occurred at this particular heating rate, and probably falls outside the range of strain rates for which damper c_1 was calibrated. This discrepancy is particularly observed in the transient tests [15] at 20°C/min when a very high stress level is applied; these stresses were 200 MPa for S275 and 300 MPa for S355.

The overall accuracy of the rheological model R4 can be appreciated from Table 3 by comparing the test results from this study and the model output for transient tests of

S355 at 150 MPa, at heating rates of 5 and 10°C/min. Verification of the model with the selected transient tests indicates that model R4 can reproduce the strain output very accurately at heating rates of 20°C/min and lower, where creep strain value is comparable to, or even greater than, stress-related strain.

It can also be seen from Figures 9 and 10 that the damper c_2 shows very high sensitivity to heating rates 2.5-5-10°C/min, indicating the substantial increase in creep strain evolution within this heating rate region.

4.5 Applicability of the proposed model

Comparison of results at heating rates between 2.5-20°C/min shows a very good match with the predictions of the unified model R4, indicating that it is applicable in transient heating conditions and at varying strain rates, which would generally describe the heating scenario for structural steel during a building fire event. The general applicability of the model R4 was demonstrated using a wide range of experimental data from different sources in order to test the reliability of the proposed model and its constitutive components.

Additionally, the results suggest that R4 is applicable for modelling the strain development in common low-carbon steel grades such as S275 and S355. This fact, combined with the good predictions for selected stationary and transient coupon test studies, indicates that the unified rheological model R4 is valid over a wide range of expected strain and heating rates.

5. CONCLUSION

A rheological model for modelling the strain development in structural steel during a fire scenario has been presented, together with verification studies which make use of previous test studies which include both stationary and transient heating regimes. The constitutive components of the rheological model have proved sufficiently accurate in predicting strain development in common steel grades, such as S275 and S355, used in Europe. The material models of each of the constitutive components can easily be adapted to different steel types and grades; this will of course demand some testing for calibration of the creep and strain rate enhancement elements. Therefore, in principle it can be applied to any type of steel, which makes its potential application universal. Further research regarding the model development will include the treatment of strain reversal in steel at high temperatures in order to fully capture the strain changes which occur in fire-affected steel-framed structures. Considering the comparisons which have been presented so far the following conclusions can be postulated:

- The unified rheological model has been shown to be applicable for modelling strain development in fire-affected steel for heating rates in the range 2.5-20°C/min;
- The constitutive components of the rheological model are sufficiently accurate to represent the strain development for low-carbon steel grades S275 and S355, on the basis of the verification of the unified model over a broad range of test studies;
- The model is applicable to any type and grade of steel, provided that the constitutive components are calibrated with respect to the material strain-rate enhancement effects and creep strain development.

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REFERENCES

[1] Annual Book of ASTM standards: metal test methods and analytical procedures, Vol. 03.01. Baltimore: ASTM International; 2005.

[2] EN 10002-1, *“Metallic materials* – Tensile testing – Part 1: Method of test at ambient temperature, European Commit*tee for Standardization“*, Brussels, 2001.

[3] Mäkeläinen, P., Outinen, J. and Kesti, J., “Fire design model for structural steel S420M based upon transient-state tensile test results”, Journal of Constructional Steel Research, 1998, 48; 47–57.

[4] Wainman D. E. and Kirby B. R., "Compendium of UK Standard Fire Test Data: Unprotected Structural Steel – 1 & 2," British Steel Corporation, 1988.

[5] Kodur, V. K. R., Dwaikat, M. M. S. And Fike, R., "High-Temperature Properties of Steel for Fire Resistance Modeling of Structures", ASCE Journal of Materials in Civil Engineering, 2010; 22(5):423-434.

[6] Burgess, I.W., El-Rimawi, J.A. and Plank , R.J., "A secant stiffness approach to the fire analysis of steel beams", Journal of Constructional Steel Research, 1988; 11:105-120.

[7] EN 1993-1-2:2005, *"Eurocode 3* – Design of steel structures – Part 1-2: General Rules – *Structural fire design"*, European Committee for Standardization, Brussels, 2005.

[8] EN1992-1-2:2004, *"Eurocode 2* – Design of Concrete Structures – Part 1-2: General Rules – *Structural Fire Design"*, European Committee for Standardization, Brussels, 2004.

[9] ASCE, "Structural fire protection" ASCE committee on fire protection, Manual No. 78, ASCE, Reston, Va., 1992.

[10] Poh, K. W., "Stress-Strain-Temperature Relationship for Structural Steel", Journal of Materials in Civil Engineering, 2001; 13(5):371-379.

[11] Y. Anderberg, „Modelling Steel Behaviour", Fire Safety Journal, 1988; 13(1):17-26

[12] Lie, T. T., ed. (1992). Structural fire protection, ASCE, New York

[13] Cowan M., Khandelwal K., "Modelling of high temperature creep in ASTM A992 structural steels", Engineering Structures, 2014; 80:426-434.

[14] Brnić J., Turkalj G., Lanc D., Čanadija M., Brčić M., Vukelić G., "Comparison of material properties: Steel 20MnCr5 and similar steels, "Modelling of high temperature creep in ASTM A992", Journal of Constructional Steel Research, 2014, 95; 81–89.

[15] Kirby, B.R. and Preston, R.R., "High Temperature Properties of Hot-Rolled, Structural Steels for Use in Fire Engineering Design Studies", Fire Safety Journal, 1988, 13:27-37.

[16] Latham D. and Kirby, B.R., "Elevated temperature behaviour of welded joints in structural steels", European commission, Technical steel research-final report 1998, 58-60.

[17] Boko, I., Torić, N. And Peroš, B., "Structural fire design parameters and procedures – analysis of the potential of Eurocode 3", Materialwissenschaft und Werkstofftechnik, 2012; 43(12):1036-1052.

[18] Renner, A., The Effect of Strain-Rate on the Elevated-Temperature Behaviour of Structural Steel, Research dissertation, University of Sheffield, Department of Civil and Structural Engineering, 2005.

[19] Harris, A., Modelling Short-Term Creep of Structural Steel at High Temperatures, Research dissertation, University of Sheffield, Department of Civil and Structural Engineering, 2009.

[20] Bull, L., Palmiere, E., Thackray, R., Burgess, I.W. and Davison, J.B., "Tensile Behaviour of Galvanised Grade 8.8 Bolt Assemblies in Fire", Journal of Structural Fire Engineering, 2015; 6(3):197-212

[21] Torić, N., Sun R.-R., Burgess, I.W., "Creep-free fire analysis of steel structures with Eurocode 3 material model", Journal of Structural Fire Engineering, 2015; Accepted for publication.

[22] Torić, N., Sun R.-R., Burgess, I.W., "Development of a creep-free stress-strain law for fire analysis of steel structures", Fire and Materials, 2015; Accepted for publication, DOI: 10.1002/fam.2347

[23] Torić, N., Harapin, A. and Boko, I., "Experimental Verification of a Newly Developed Implicit Creep Model for Steel Structures Exposed to Fire", Engineering Structures, 2013; 57:116-124.

[24] Harmathy, T. Z., "A Comprehensive Creep Model", Journal of Basic Engineering, 1967, 89(3): 496-502.

[25] Harmathy, T. Z. and Stanzak, W.W., *"Elevated*-Temperature Tensile and Creep *Properties of Some Structural and Prestressing Steel"*, National Research Council of Canada, Division of Building Research, Ottawa, 1970.

Figure Captions

Figure 1: Proposed rheological models

Figure 2: Constitutive stress-strain model for spring 1

Figure 3: Integration scheme for strain rate solver

Figure 4: Comparison of results between rheological model R1 and slow strain rate test results

Figure 5: Comparison of results between rheological model R3 and fast strain rate test results

Figure 6: Comparison of results between rheological model R2 and medium strain rate test results

Figure 7: Comparison with different stationary coupon tests

Figure 8: Comparison with different transient coupon tests for steel S355

Figure 9: Comparison with different transient coupon tests for steel S275

Figure 10: Comparison of damper c_2 value and the increase of creep strain value for simulation of the transient test of grade S275, at 100 MPa with heating rate of 5°C/min

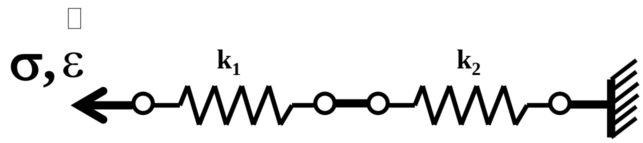
Table Captions

Table 1: Factors for determining damper parameter c_1

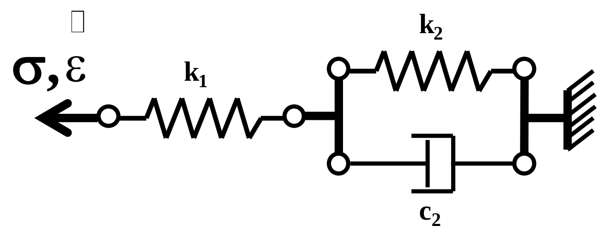
Table 2: Input parameters for the rheological model R4 from selected studies

Table 3: Constitutive equations for the damper coefficient c_2 at various stress levels

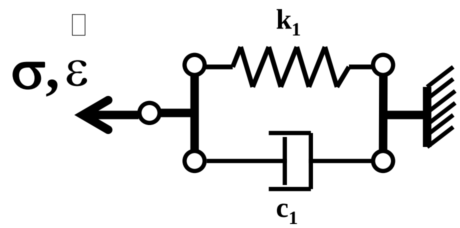
Table 4: Accuracy of the rheological model R4 for steel S355 at 150 MPa from study [15] at heating rates of 5-10°C/min



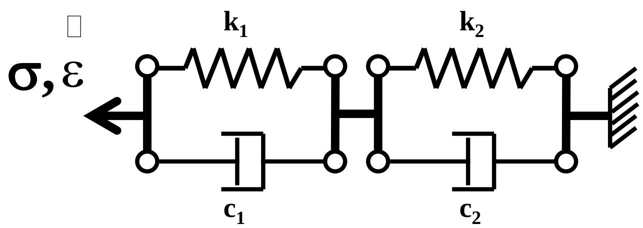
(a) R1



(b) R2

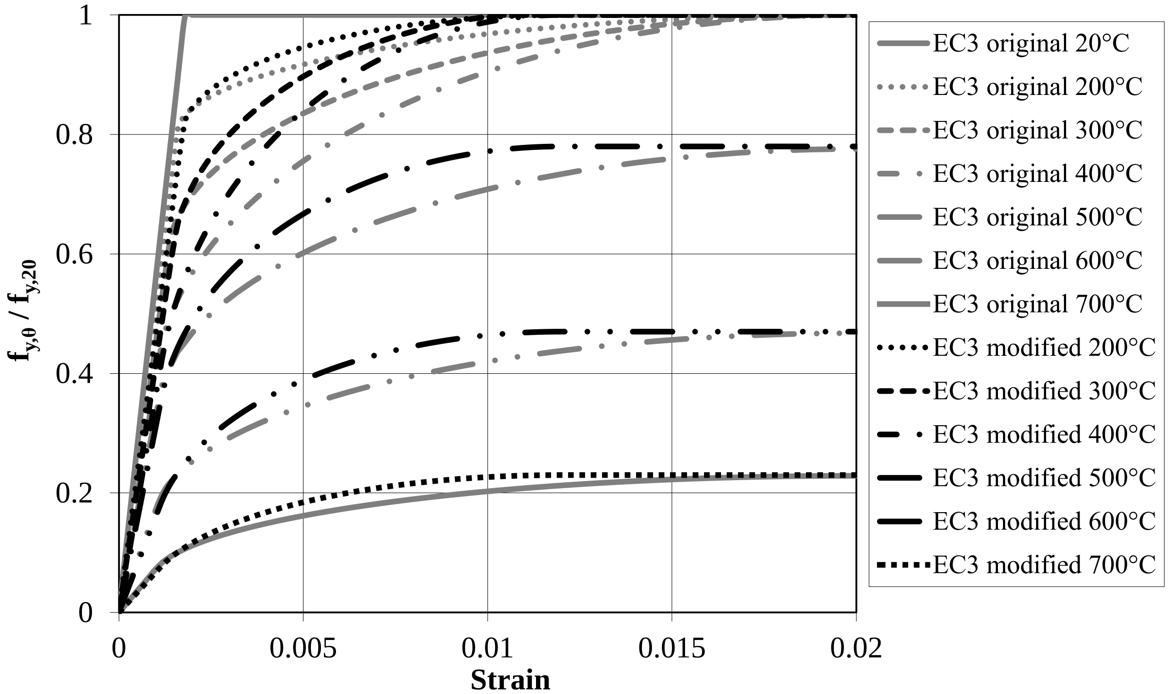


(c) R3



(d) R4

Figure 1



1
0.8
0.6
0.4
0.2
0
$f_{y,\theta}$ / $f_{y,20}$
0
0.005
0.01
0.015
0.02
Strain
EC3 original 20°C
EC3 original 200°C
EC3 original 300°C
EC3 original 400°C
EC3 original 500°C
EC3 original 600°C
EC3 original 700°C
EC3 modified 200°C
EC3 modified 300°C
EC3 modified 400°C
EC3 modified 500°C
EC3 modified 600°C
EC3 modified 700°C

Figure 2

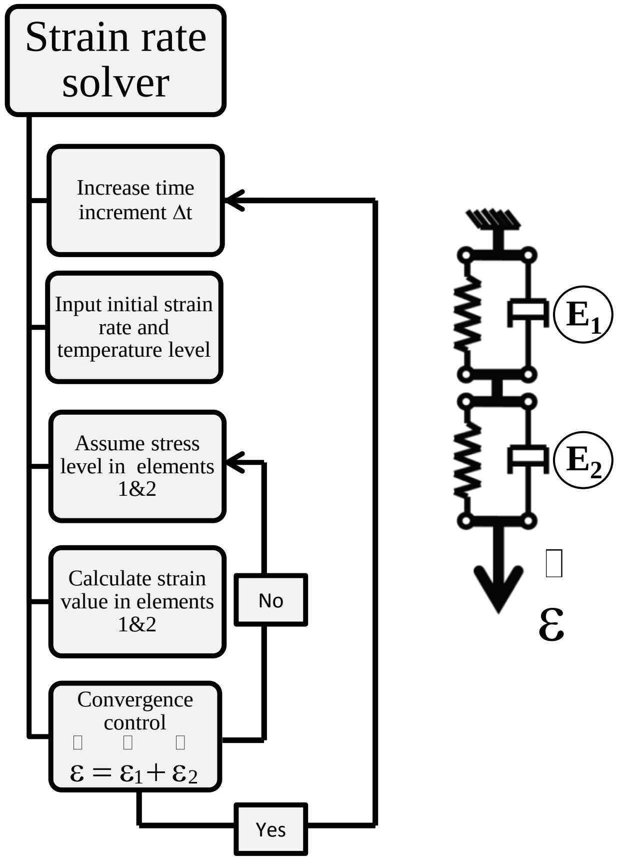
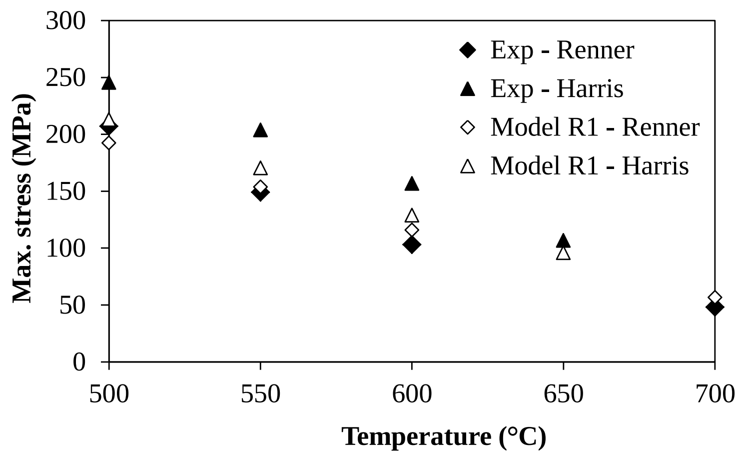
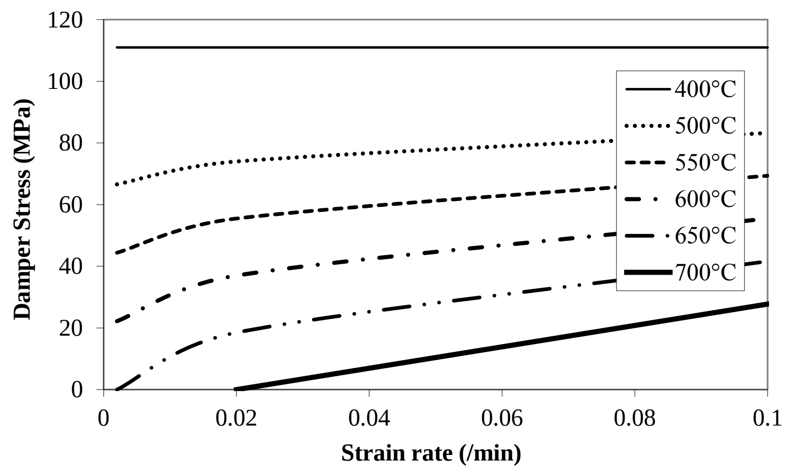

Strain rate solver
Increase time increment Δt
Input initial strain rate and temperature level
Assume stress level in elements 1&2
Calculate strain value in elements 1&2
No
Convergence control
ε = ε1 + ε2
Yes
E1
E2
ε

Figure 3

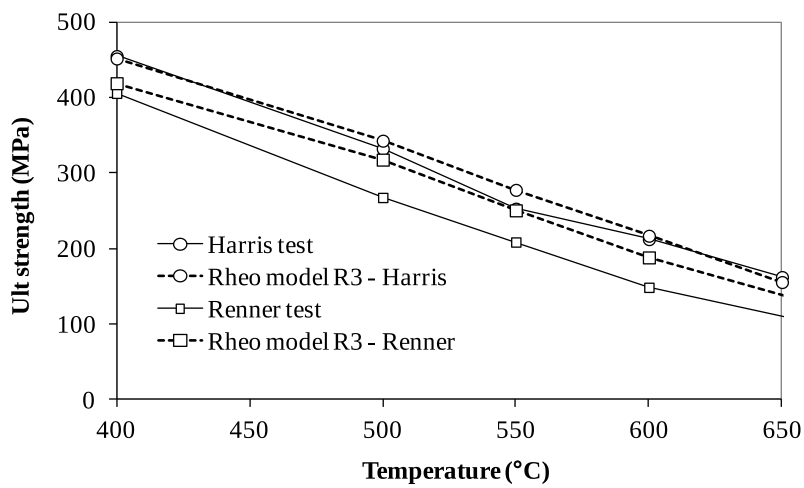


300
250
200
150
100
50
0
Max. stress (MPa)
500
550
600
650
700
Temperature (°C)
Exp - Renner
Exp - Harris
Model R1 - Renner
Model R1 - Harris

Figure 4

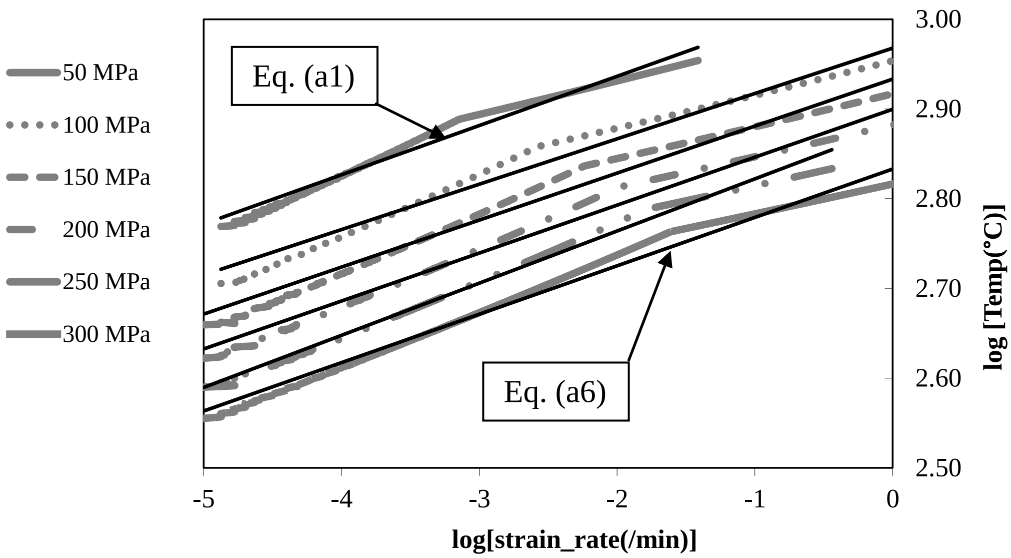


(a) Temperature and strain rate dependency for damper parameter c_1 (Adapted from study of Latham and Kirby [16])

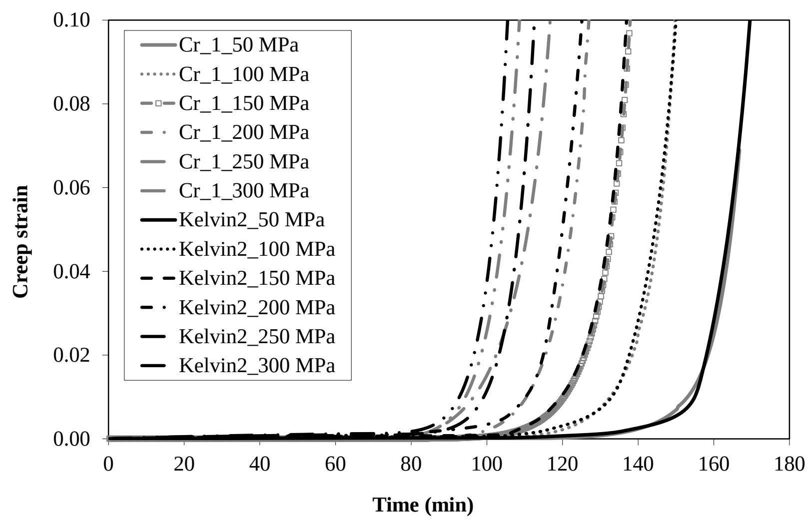


(b) Comparison with the fast strain rate results from studies of Renner [18] and Harris [19]

<u>Figure 5</u>

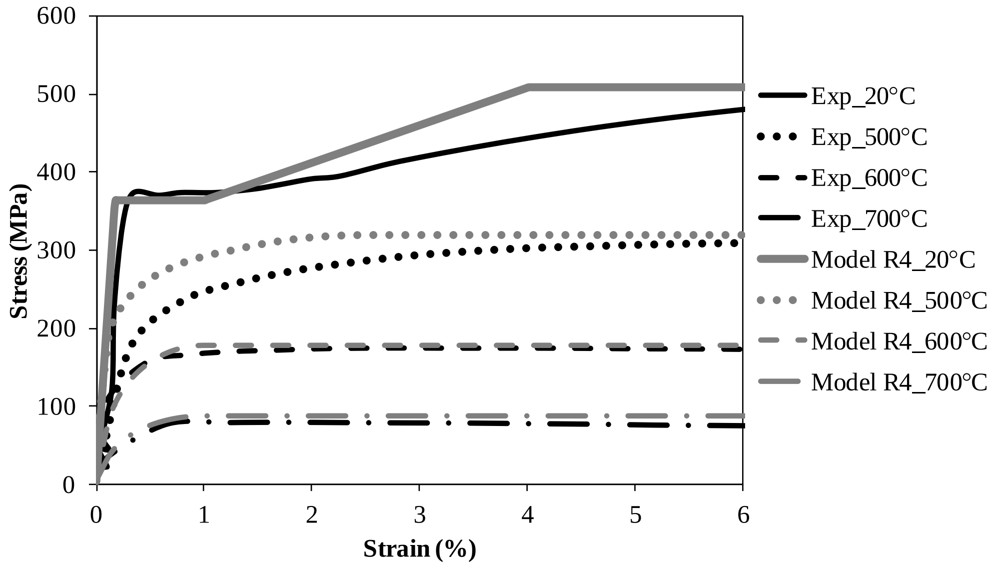


(a) Temperature and strain rate dependency of damper parameter c_2 (Adapted from creep model Cr_1 at 5°C/min)

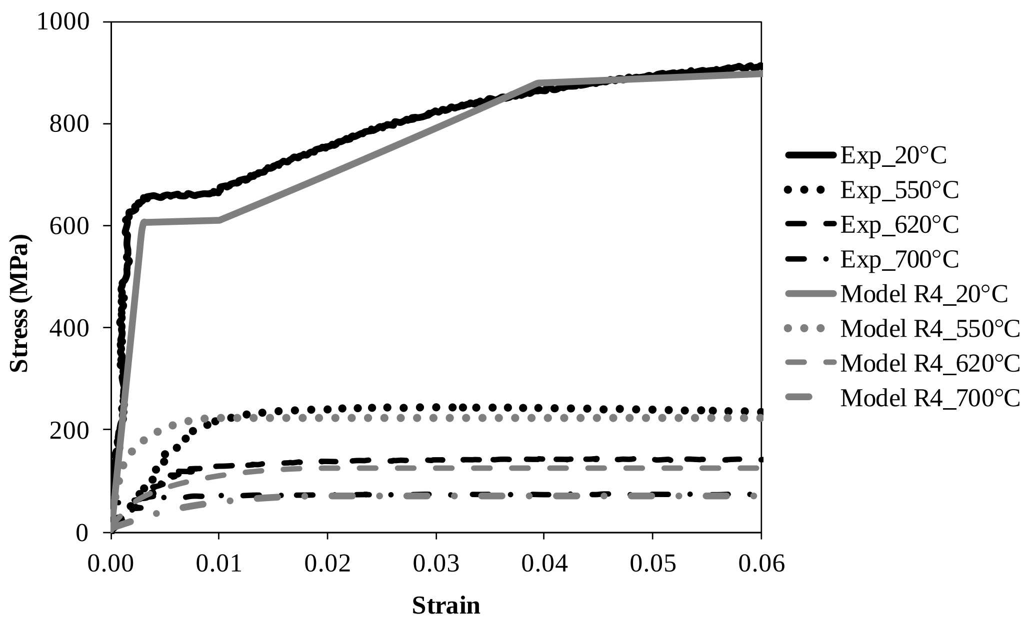


(b) Comparison of the strain output from Kelvin element 2 of rheological model R2 and the creep model Cr_1

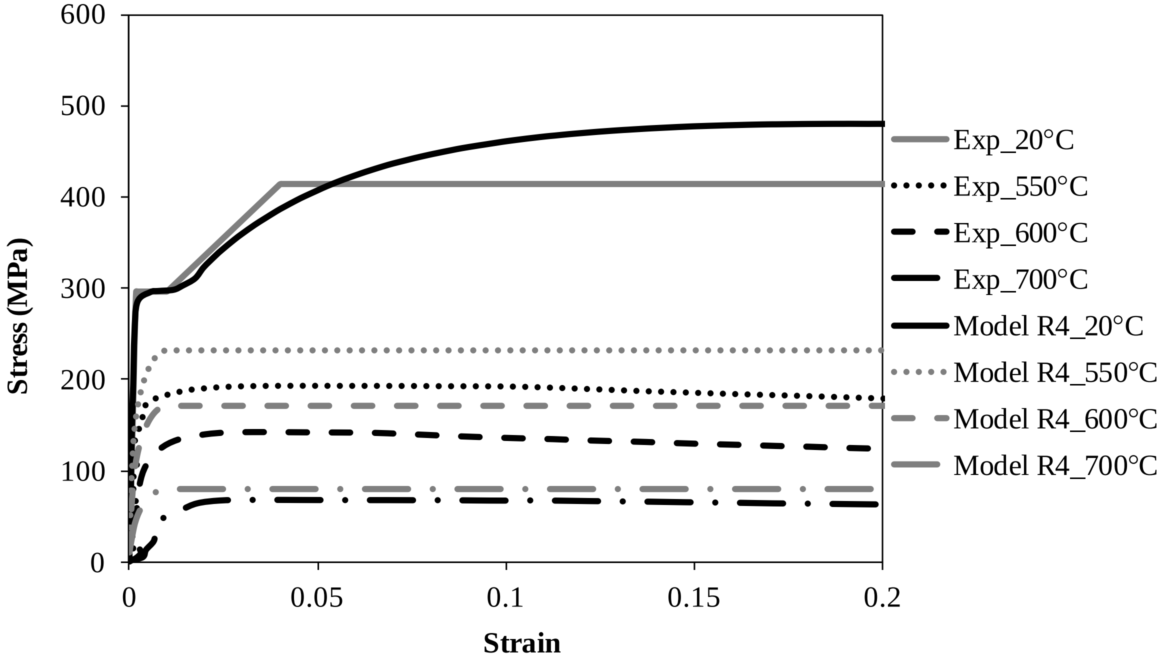
Figure 6



(a) Comparison with results from study [17] – strain rate 0.0002s^{-1}, steel grade S355

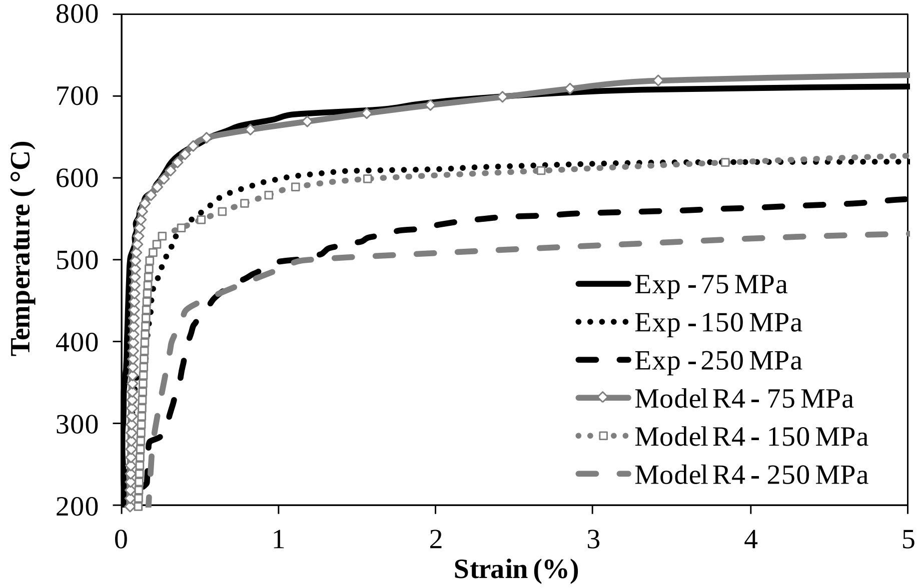


(b) Comparison with results from study [20] – medium strain rate 0.01min^{-1}, bolt grade 8.8

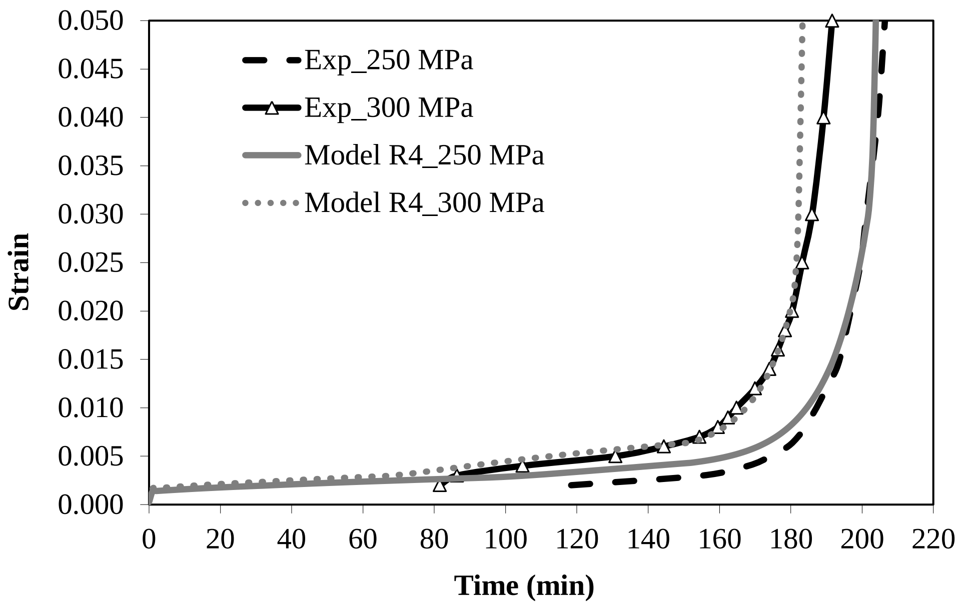


(c) Comparison with results from study [18] – medium strain rate (550°C – 0.0006152s^{-1}, 600°C – 0.0006750 s^{-1}, 700°C -0.0006033 s^{-1}), steel grade S275

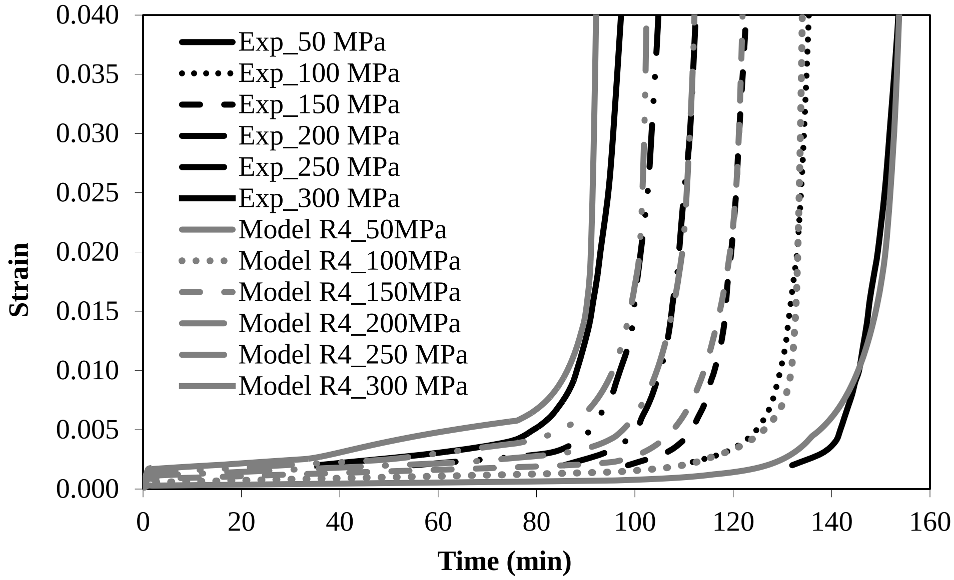
Figure 7



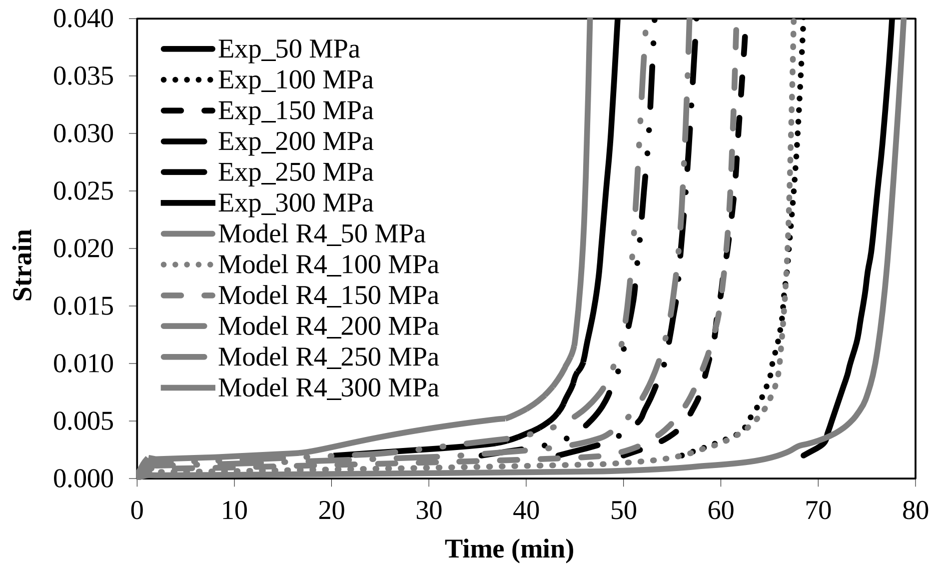
(a) Comparison with results from study [17] – heating rate 10°C/min , steel grade S355



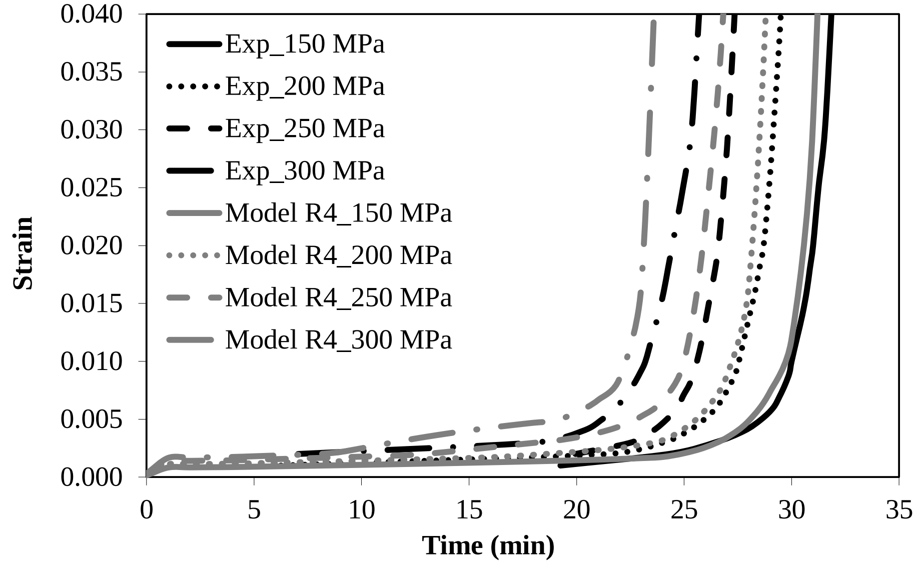
(b) Comparison with results from study [15] – heating rate 2.5°C/min , steel grade S355



(c) Comparison with results from study [15] – heating rate 5°C/min , steel grade S355

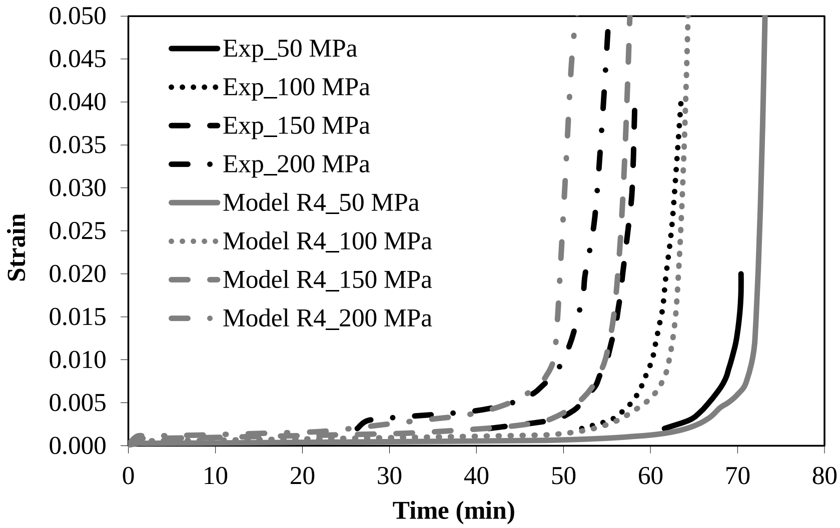


(d) Comparison with results from study [15] – heating rate 10°C/min , steel grade S355

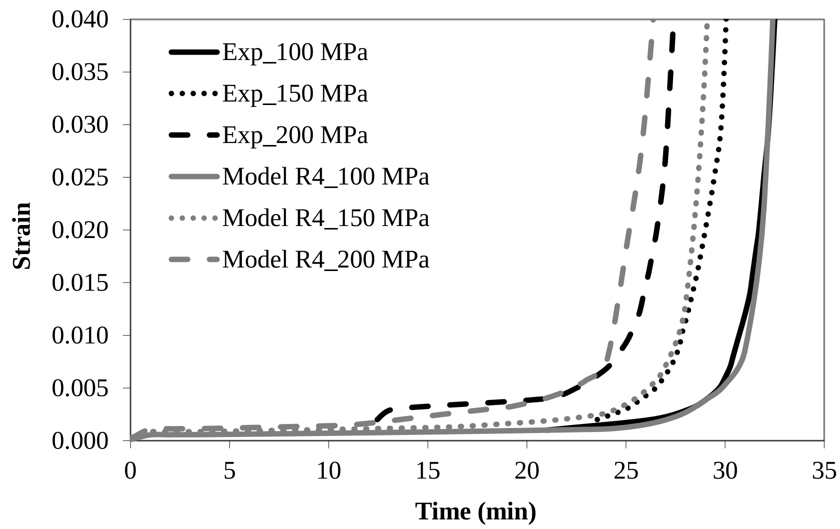


(e) Comparison with results from study [15] – heating rate 20°C/min , steel grade S355

Figure 8

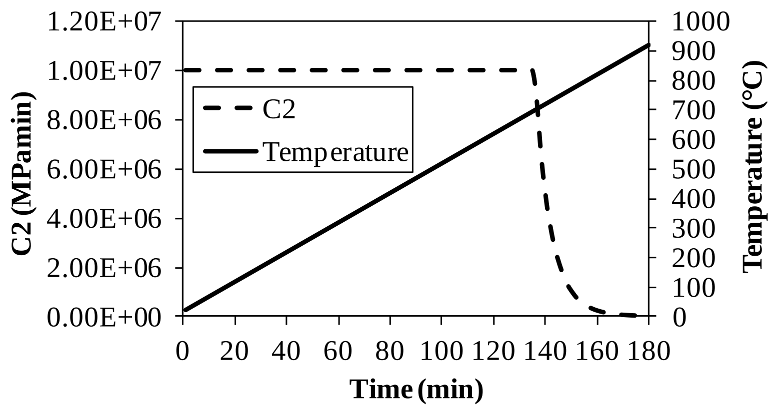


(a) Comparison with results from study [15] – heating rate 10°C/min , steel grade S275

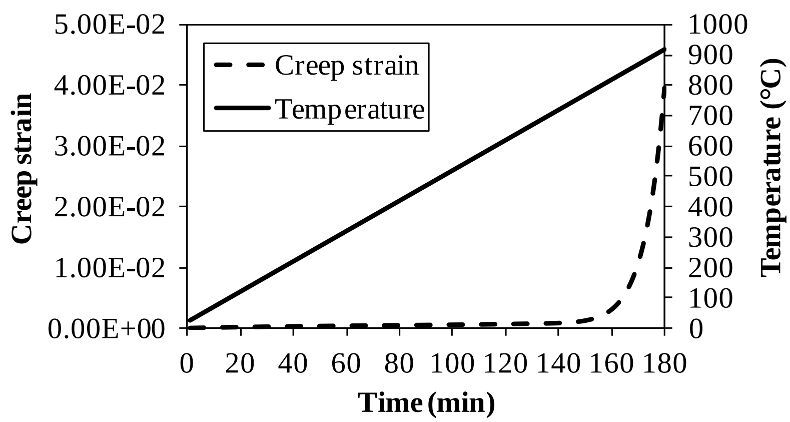


(b) Comparison with results from study [15] – heating rate 20°C/min , steel grade S275

Figure 9



(a) Reduction of the value of damper c_2 with respect to time



(b) Output strain values of the second Kelvin element

Figure 10

Table 1

Strain rate (min^{-1}) / Temperature (°C)	0.002	0.02	0.1
400	111.0	111.0	111.0
500	66.6	74.0	83.3
550	44.4	55.5	69.4
600	22.2	37.0	55.5
650	-	18.5	41.6
700	-	-	27.8

Table 2

Study/steel grade		Yield strength $f_{y,20}$ - 20°C (MPa)	Modulus of elasticity $E_{y,20}$ - 20°C (MPa)	Incremental time step **Δt** (min)	Reduction factors for yield strength and modulus of elasticity
Kirby & Preston [15]	S275	267.0	185000.0	1.0E-04	EN1993-1-2 (carbon steel)
	S355	357.0	185000.0		
Boko et al. [17]	S355	362.4	209000.0	1.0E-04	EN1993-1-2 (carbon steel)
Renner [18]	S275	308.0	193400.0	1.0E-04	EN1993-1-2 (carbon steel)
Harris [19]	S275	341.0	195833.0	1.0E-04	EN1993-1-2 (carbon steel)
Bull et al. [20]	Bolt grade. 8.8.	603.0	210000.0	1.0E-04	EN1993-1-2 (bolts)

Table 3

Background model Cr_1, 5°C/min Equations from Figure 6a	
$\log_{10}(T) = 0.05488208 \cdot \log_{10}(\dot{\varepsilon}) + 3.04615288 \quad \sigma = 50\,\text{MPa}$	(a1)
$\log_{10}(T) = 0.05055587 \cdot \log_{10}(\dot{\varepsilon}) + 2.96789280 \quad \sigma = 100\,\text{MPa}$	(a2)
$\log_{10}(T) = 0.05240341 \cdot \log_{10}(\dot{\varepsilon}) + 2.93324938 \quad \sigma = 150\,\text{MPa}$	(a3)
$\log_{10}(T) = 0.05343190 \cdot \log_{10}(\dot{\varepsilon}) + 2.89970035 \quad \sigma = 200\,\text{MPa}$	(a4)
$\log_{10}(T) = 0.05808014 \cdot \log_{10}(\dot{\varepsilon}) + 2.88001807 \quad \sigma = 250\,\text{MPa}$	(a5)
$\log_{10}(T) = 0.05393024 \cdot \log_{10}(\dot{\varepsilon}) + 2.83296461 \quad \sigma = 300\,\text{MPa}$	(a6)

Table 4

Temperature (°C)	Exp 5°C/min [15]	Exp 10°C/min [15]	R4 - 5°C/min	R4 - 10°C/min
601	1.00	0.80	1.278	0.924
607	1.20	1.00	1.548	1.105
611	1.40	1.13	1.649	1.167
615	1.80	1.33	1.895	1.358
618	2.00	1.50	2.06	1.454