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What is a large-scale dynamo?

G. Nigro[1,2], P. Pongkitiwanichakul[1,3], F. Cattaneo[1⋆] and S.M.Tobias[4]
[1] *Department of Astronomy and Astrophysics, University of Chicago, 5640 South Ellis Avenue, Chicago IL 60637, USA*
[2] *Universita della Calabria, Unita di Cosenza, I-87030, Arcavacata di Rende, Italy*
[3] *Department of Physics, Faculty of Science, Mahidol University, Bangkok 10400, Thailand*
[4] *Department of Applied Mathematics, University of Leeds, Leeds, LS2 9JT, UK*

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ABSTRACT
We consider kinematic dynamo action in a sheared helical flow at moderate to high values of the magnetic Reynolds number Rm. We find exponentially growing solutions which, for large enough shear, take the form of a coherent part embedded in incoherent fluctuations. We argue that at large Rm large-scale dynamo action should be identified by the presence of structures coherent in time, rather than those at large spatial scales. We further argue that although the growth-rate is determined by small-scale processes, the period of the coherent structures is set by mean-field considerations.

Key words: Dynamo – magnetic fields

1 INTRODUCTION

Astrophysical magnetic fields are generated by dynamo action. Even though they are generated in an exceptionally turbulent environment, they often display a remarkable degree of order in both space and time. A typical example is the twenty-two year solar cycle. The solar magnetic field has a large-scale dipole component (predominantly aligned with the rotation axis of the Sun); moreover temporal coherence is apparent in the location of emergence and number of active regions — a wave of magnetic activity moving from mid-latitudes to the equator on an eleven year timescale is clearly visible in the solar butterfly diagram (see e.g. Stix 2004). It is well-known that other late-type stars exhibit similar behaviour (Donahue et al 1996). On even larger scales, galactic magnetic fields are coherent on spatial scales, though in that case the long-term temporal evolution is unknown (Shukurov 2002, Kulsrud & Zweibel 2008).

This has led to the theoretical idea of a large-scale dynamo as a hydromagnetic mechanism to generate these large-scale fields. This approach is formalised in mean field electrodynamics (Moffatt 1978; Krause & Rädler 1980). This is an extremely idealised concept. The reality is that astrophysical fields do not just have large-scale components, they in fact have structure on multiple scales with indeed the strongest fields often being found at small scales. In reality the problem is not one of generating large-scale fields, rather how does one get a chaotic tubulent mess to have organisation on the large scales.

This issue is particularly acute at high magnetic Reynolds numbers (Rm). At low Rm, diffusion is so overwhelming that only large-scale magnetic structures are generated anyway and the problem of generation and organisation are one and the same. On the other hand at high Rm fluctuations can exist down to small scales. Furthermore, because in a turbulent environment it is the small scales that have the highest rate of strain, it is these and not the large scales that determine the rate at which magnetic structures are generated (Tobias & Cattaneo 2005).

These issues arise even at the most basic level of kinematic theory where the dynamo problem reduces to the solution of the induction equation for prescribed velocities. One would think that, because one is avoiding the complications of a full dynamic approach one could readily construct numerical models to study these problems. However, the combined requirements of high Rm and a scale separation between the small characteristic scale of the turbulent eddies and the assumed large scale magnetic structures (if they exist) makes the problem computationally challenging even in kinematic regime.

Progress can be made in special geometries that allow for dimensional reduction of the dynamo problem. This approach has been exploited by various authors (Roberts 1972; Galloway & Proctor 1992; Cattaneo & Tobias 2005) and most recently by Tobias & Cattaneo (2013), Cattaneo & Tobias (2014) who demonstrated numerically the existence of large-scale dynamo action in the form of propagating dynamo waves at high Rm. Here we take advantage of this breakthrough in order further to clarify the relationship between dynamo action and *large-scale* dynamo action.

⋆ E-mail: cattaneo@flash.uchicago.edu (FC)

2 GENERAL CONSIDERATIONS

The large-scale dynamo problem is usually discussed within framework of mean-field theory. Mean field theory in a broad sense is a theory of filtered equations, which is a time honoured activity that is applied in all branches of science. In dynamo theory, filtering introduces two advantages — one real and one perceived. The first advantage is the reason why filtering is attractive in many situations — filtering turns an equation with rapidly varying coefficients that may be extremely difficult to solve into one with smoothly varying coefficients that might be much easier to solve. In mean field electrodynamics the filtering eliminates the rapidly fluctuating part of the underlying turbulent velocity field. The second advantage is that the filtered equations are free of all the anti-dynamo theorems that plague the full equations; this is very peculiar to dynamo theory and why the theory is so popular. It makes sense then to look for solutions in very symmetric situations. For example Cowling's theorem (Cowling 1933) precludes the possibility of axisymmetric dynamo action; however it is still possible to have dynamo action that is axisymmetric on average. Thus frequently dynamo practitioners look for axisymmetric solutions of the filtered (mean-field) equations.

There are two problems with the mean-field approach, which may be related, that manifest themselves at high Rm. The first of these is that a given filtering may not be enough to control the fluctuations, with the result that the coefficients in the filtered equations are not as smooth as one would like (Cattaneo & Hughes 2009; Tobias & Cattaneo 2015). The second is that there is the underlying assumption, not often stated explicitly, that the solutions of the filtered equations should coincide with the filtered solutions of the full equations. It is not obvious that this is true. In fact, there are known examples where it isn't (Boldyrev, Cattaneo & Rosner 2005).

To fix ideas, we consider a simple example. In kinematic theory, once the (stationary) velocity is specified the dynamo problem essentially reduces to two things, determining the dynamo growth-rate and computing the structure of the dynamo eigenfunction. Now suppose that we define some spatial filter. It is reasonable to expect that application of the filter to the solutions of the full equations should correspond the eigenfunction of the filtered equations. That being the case, it would also be reasonable to assume that the growth-rate of the large-scale structure of the solutions should coincide with the growth rate predicted by the filtered equations. However at high Rm that is almost certainly never the case. The reason is that there is a unique growth-rate and all structures (small and large) grow at the same rate. As we mentioned in the introduction, in a turbulent environment, this rate is determined by the small scales that have been removed from the filtered equations. It is a common fallacy to assume that one may have a kinematic dynamo where the large and small scales grow at different rates.

Let us assume that the spatial structure obtained from the filtered equations is correct. A natural question then is to ask "What else does the filtered equation get right?". As argued above, it will not get the growth-rate correct. Consider a case which is more complicated, where the filtered equations have a growth rate and a frequency. For example, this could happen if there has been a breaking of isotropy of the filtered equations and a bifurcation to a travelling wave solution. In the full equations this would correspond to a breaking of the isotropy of the statistics of the velocity and therefore it is not unreasonable to expect that this will manifest itself as a breaking of symmetry of the statistics of the solutions. We believe that there are cases where the statistics of the symmetry breaking as measured say by the propagation speed of a wave pattern is the same both in the solutions of the filtered equations and those of the unfiltered equations in a statistical sense. This is because the effect of symmetry breaking is to move the frequency away from zero and is controlled by a change in the symmetry of the large scales. We stress again that this is not the case for the growth-rate[1] We note also that the small scales are too small to know that any of the statistical symmetries have been broken and therefore just do their own sweet thing.

A natural way to break large-scale isotropy is to introduce a large-scale shear to the velocity. This has two effects, it breaks the isotropy and as found by Tobias & Cattaneo (2015) has the effect of controlling the fluctuations, making the detection of the large-scale structure easier. In fact the second property has led these authors to formulate a general "suppression principle". This states that, at high Rm, large-scale organisation will only be detected if some mechanism is present to control the fluctuations at small-scales. It was further posited that this agent could be related to the shear, enhanced diffusion or nonlinear effects (in the dynamic regime). Here we shall illustrate some of these issues, by looking at a system related to that of Tobias & Cattaneo (2013).

3 MODEL AND RESULTS

In the spirit of the discussion above, the idea here is to solve the *full* induction equation at high Rm and examine the behaviour at large scales through some appropriate filtering procedure. Thus we solve

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Rm}\nabla^2 \mathbf{B}, \tag{1}$$

for the magnetic field $\mathbf{B}$ and a prescribed velocity $\mathbf{u}$, which we take to be a spatially periodic, random, quasi-two dimensional flow. These flows have all three components of the velocity, but only depend on two co-ordinates, x and y, say. Because of the invariance in the z-direction, the induction equation is separable with periodic solutions of the form $\mathbf{B}(x, y, z, t) = \mathbf{b}(x, y, t)\exp(ik_z z)$. This formally reduces the dynamo problem to a two-dimensional one for any value of k_z. These types of flows have been the workhorse of dynamo studies at high Rm (Roberts 1972; Galloway & Proctor 1992; Cattaneo & Tobias 2005).

The velocities here are made up of two pieces; the first is a random, helical flow, whose helicity we can control, with wavenumbers in the band $14/\pi L$ to $20/\pi L$, where L is is the periodicity in x and y (Pongkitiwanichakul et al 2016). The second is a large-scale shear flow of the form

$$\mathbf{u_s} = u_s \cos\left(\frac{2\pi y}{L}\right)\hat{\mathbf{x}}. \tag{2}$$

[1] The only way to make the growth-rate zero is to make Rm small.

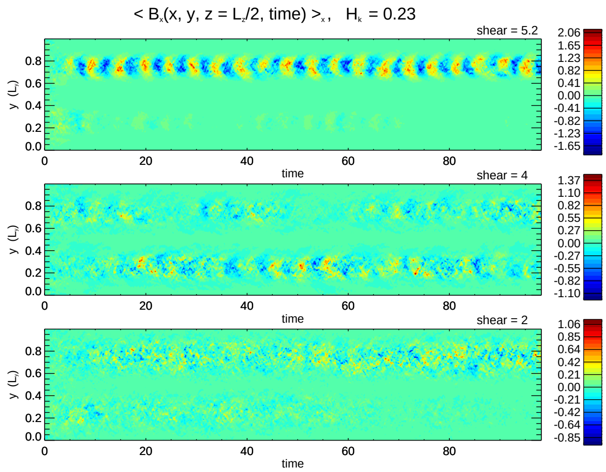

Figure 1. Activity versus time diagrams. Colour density plots of the B_x averaged in x as a function of time and y. Top panel: $u_s = 5.2$, Middle panel: $u_s = 4$, Lower panel: $u_s = 2$. Note that the relative amplitude of the patterns in the two bands of maximum shear depends on the (random) initial conditions.

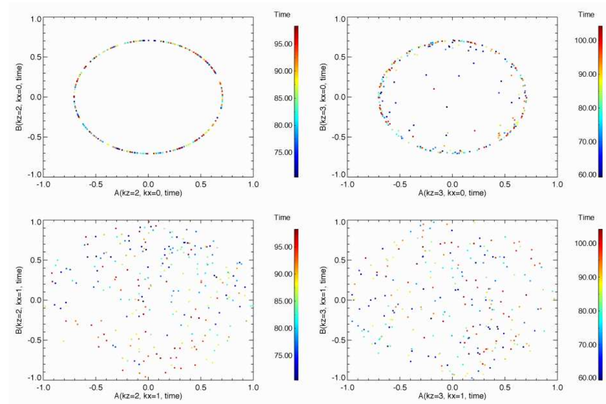

Figure 2. Phase diagrams. A_k versus B_k as defined in equation 3. The top panels are for $k = 0$ and the bottom panels are for $k = 1$. The left panels are for $u_s = 5.2$ and the right panels are for $u_s = 2$. The time ordering is given by the colour-bar to the right. For the $k = 0$ $u_s = 2$ case some of the off-circle points are at later times, hence corresponding to loss of phase-coherence rather than an initial transient.

Flows of this type are known to be extremely effective dynamos, amplifying the magnetic field at a well-defined growth-rate (σ) comparable with a characteristic turnover time of the eddies. This growth-rate depends on k_z and, for the typical values of Rm that we consider here, there is a wavenumber of maximum growth such that the vertical size of the fastest growing magnetic structure is roughly twice its horizontal extent, which is itself comparable with the size of the eddies. For these values of Rm σ is largely independent of Rm (Tobias & Cattaneo 2013)

Because the characteristic scale of the magnetic structure is comparable with that of the eddies, and its lifetime is comparable with their turnover time, it is natural to interpret this as a small-scale dynamo. In the presence of shear the spatial structure of the magnetic fields remain essentially the same; however now a well-defined frequency of oscillation emerges that is revealed only when some appropriate form of filtering is introduced. This is illustrated in Figure 1, which shows the value of the x-average of the x-component of magnetic field for the plane $z = 0$ as a function of y and time, for three different values of the shear parameter u_s. Here the filter is spatial averaging in the x-direction. For the strongest shear a clear pattern is visible. Alternating regions of positive and negative mean B_x appear in a cyclic pattern. We note that the period of the oscillation is long compared with the characteristic turnover time of the underlying flow (approximately fifty times longer in this particular case). Even more importantly the pattern remains phase-coherent for many, many oscillations. For weaker shears the pattern becomes less evident and although the period of oscillations remains long compared with the turnover time, the pattern appears to lose phase coherence more rapidly.

Figure 1 was obtained by averaging in x, which corresponds to examing the $k_x = 0$ component of the magnetic field. One could ask if there are other magnetic patterns that have bigger wavenumbers and display some coherence. If so, what would be their lifetime? We know that if we go down

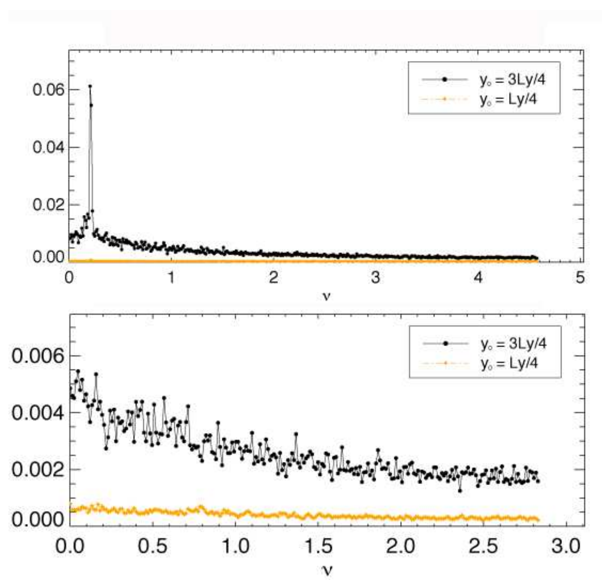

Figure 3. Frequency power spectra for the $k_x = 0$ component of B_x for the same two cases as in Figure 2; upper panel is for the higher shear. The horizontal axis is the actual frequency not the angular frequency.

to the scale of the eddies coherence is lost. If we believe that there is a pattern at scale $1/k$ it must take the form

$$A_k(y,t) \sin z + B_k(y,t) \cos z; \qquad (3)$$

then for a propagating wave there must be some functional relationship between A_k and B_k. In particular if one plots A_k as a function of B_k for a wavelike object then points corresponding to the values of A_k and B_k at different times should lie on a closed curve. Figure 2 shows the results of this procedure for $k_x = 0$ and $k_x = 1$ for two different values of the shear parameter that are the ones that correspond to the top and bottom panels of Figure 1. The y-values are selected to be the locations where the pattern is most ap-

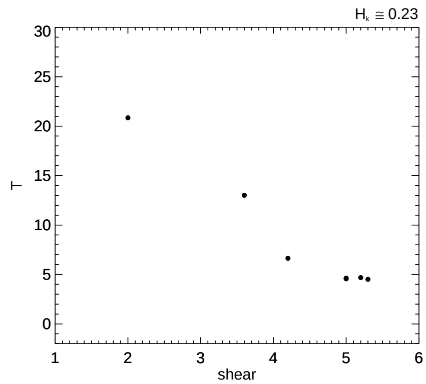

Figure 4. Dependence of the period of the dynamo waves, when these can be identified, as a function of the shear strength for fixed helicity.

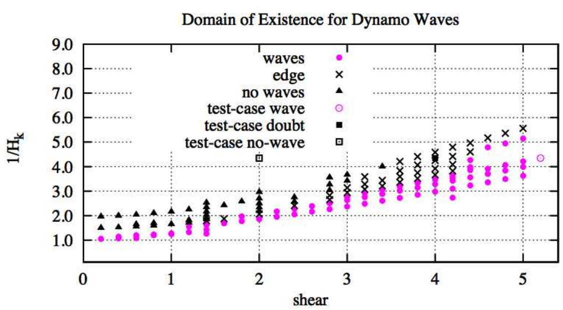

Figure 5. Domain of existence in the 1/Helicity-shear plane. For simple dynamo waves, the boundary should be a horizontal line.

parent. Clearly, for both cases, the $k_x = 0$ is a wave and its phase coherence is very long, whereas for $k_x = 1$ the points are all over the place, so there is no coherent pattern with a characteristic scale half the size of the box. In fact, one can repeat this procedure for any other value of k_x greater than unity and the result is always the same, so $k_x = 0$ is the only long-lived coherent pattern. Moreover as the shear is decreased some of the points move to the interior indicating that the oscillation occasionally loses coherence and is reset.

Now that we know that there is a pattern that is coherent over long times, we can identify its frequency by performing timeseries analysis. The spatiotemporal solution is analysed at two locations in the y-direction corresponding to the locations of maximum shear ($y = \pi/4$, $y = 3\pi/4$) in the $z = 0$ plane. The remaining signal is then a function of x and *time*. As we have shown that it is only the largest scale $k_x = 0$ that retains phase-coherence throughout the evolution, we perform the Fourier transform in time of this component and plot the resulting periodograms in Figure 3 for two values of the shear. What is clear from these is that for the case of the strongest shear the period of the oscillation is unambiguously identified and is given by $P = 1/0.206 \approx 5$. The signal for the weaker shear is much noisier with many peaks in the periodogram. Nevertheless a preferred frequency is detectable and yields a period of $P = 1/0.048 \approx 20$. The dependence of these frequencies *does* yield periods that are compatible with those that emerge from a theory based on the solution of the filtered equations (in this case mean-field electrodynamics), and is not determined by the small scale dynamo. This is illustrated in Figure 4 which shows the period of the dynamo waves - when they exist - as a function of the shear rate for “fixed” helicity. The period of the dynamo waves decreases with increasing shear in agreement with the predictions of mean-field theory (Parker 1955). From the previous discussion it should be clear that this is because the small-scale dynamo shows no phase coherence and so can not be responsible for setting the period for the oscillation of the large scales.

Clearly, higher shear produces more coherent wave-patterns, and we know that in the absence of shear one should not expect to detect wave patterns at all. Moreover in the absence of helicity the wave patterns should also be absent. Hence one should expect a boundary distinguishing regions where waves are detectable or not in the Helicity-Shear plane. A simple-minded application of mean-field theory gives that the critical dynamo number for the onset of large-scale dynamo action is made up of the product of the shear and the α-effect (Parker 1955), which in turn should be related to the helicity. That argument being correct, the boundary in the $S - \frac{1}{H}$-plane should be a horizontal line. Figure 5 shows what the boundary actually looks like for this case. For weak shear, that is indeed the case, breaking down when the shear is strong; for a fixed shear less helicity than expected is required to detect dynamo waves. There are a number of reasons why this may be the case. First, as argued above, shear suppresses the fluctuations making the detection of the large-scale pattern more straight-forward. Furthermore, there are inductive effects, other than the α-effect that contribute at high shear amplitude to the electromotive force (Krause & Rädler 1980).

4 DISCUSSION

In this paper we have considered dynamos driven by sheared helical random noise. These systems according to classical mean-field theory should have large-scale behaviour, but these assertions are based on low magnetic Reynolds number arguments where no other dynamo action can occur. This is not the case at moderate and high Rm, which is what we consider here. We find that indeed the dynamo operates to generate a magnetic structure whose amplitude grows exponentially in time, with a well-defined growth-rate. This growth-rate is entirely determined by small-scale processes, such as chaotic stretching and cancellation exponents (Du & Ott 1993) and *not* in any way related to mean-field effects.

However, within this object there is a large-scale pattern in the form of a travelling helical (dynamo) wave, that remains coherent for long periods of time and whose frequency *is* determined by mean-field effects. It is important to understand that this wave does not have a separate growth-rate from the rest of the magnetic structure. It can only be unambiguously identified from the rest of the structure by this persistent phase coherent signal; all other parts of the solution are incoherent in time. This raises an interesting point, related to how would one identify large-scale dynamo action for a non-oscillatory pattern. It is possible that this may be identified by a slowly-evolving pattern in the exponentially growing solution, should one of these exist. Here by slowly evolving we mean something that is changing on a timescale much longer than any characteristic timescale of the turbulent flow. For example in a spherical shell this could correspond to the axis of the dipole component of the magnetic field which may wander on a timescale much longer than the turbulent eddies that are driving the dynamo. Hence we believe that it is better to consider a definition of large-scale dynamo action that considers the timescale of evolution of the pattern, rather than one that relies on spatial scales alone.

Thus we find that at high Rm, large-scale dynamo action consists of a long-lived coherent pattern embedded in a sea of incoherent fluctuations. It is natural to speculate as to the relationship between the coherent and incoherent parts. If we think of them as signal and noise then one could conceive several different scenarios; an additive noise case in which the overall signal is the superposition of a periodic part and random noise, a multiplicative case in which the signal is periodic but with a randomly varying amplitude, and a random phase case in which the signal is periodic but where the phase of the oscillation varies as a random process. In principle, given a long enough signal, it should be possible to differentiate between these different possibilities using the techniques of timeseries analysis. This exercise may provide valuable insight into the physical processes that give rise to the large-scale organisation, and we plan to undertake this exercise in a subsequent investigation.

Of course the previous discussion, as is mean-field theory, is all framed within the kinematic approximation where the magnetic field does not act back on the turbulent flow. Thereby it gives no indication of what the relative amplitudes of the coherent part to the incoherent part of the magnetic structure will be once this process saturates. This is a subtle issue, for the kinematic evolution the relative amplitude of the coherent to the incoherent part is fixed in the growing solution. Once nonlinearity becomes important different scales may continue to grow at different rates, and when they saturate they may saturate at relative amplitudes completely different from those of the kinematic phase. Nonetheless the filtering technique used to identify coherent large-scale fields in the kinematic regime may continue to be utilised in the nonlinear regime.

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References cited

Boldyrev, S., Cattaneo, F., & Rosner, R. 2005, Phys. Rev. Lett., 95, 255001

Cattaneo, F., Emonet, T., & Weiss, N. O. 2003, Astrophys. J., 588, 1183

Cattaneo, F., & Hughes, D.W., 2009, Mon. Not. Roy. Ast. Soc., 395, L48

Cattaneo, F., & Tobias, S. M. 2005, Physics of Fluids, 17, 127105

—. 2014, Astrophys. J, 789, 70

Cowling, T.G.. 1933, Mon. Not. Roy. Ast. Soc., 94, 39

Donahue, R.A., Saar, S.H. & Baliunas, S.L., 1997, Astrophys. J, 466, 384

Du, Y. & Ott, E., 1993, J. Fluid Mech., 257, 265

Galloway, D. J., & Proctor, M. R. E. 1992, Nature, 356, 691

Krause, F. & Rädler, K.-H., 1980, Mean-field magnetohydrodynamics and dynamo theory, Pergamon Press, Oxford.

Kulsrud, R.M. & Zweibel, E.G., 2008, Rep. Prog. Phys., 71, 046901

Moffatt, H. K. 1978, Magnetic Field Generation in Electrically Conducting Fuids, Cambridge University Press.

Parker, E. N. 1955, Astrophys. J., 122, 293

Pongkitiwanichakul, P., Nigro, G, Cattaneo, F. & Tobias, S.M., 2016, Astrophys. J., submitted

Roberts, G. O. 1972, Royal Society of London Philosophical Transactions Series A, 271, 411

Shukurov, A., 2002, Ast. and Space Science, 281, 285

Stix. M., 2004, The sun: An introduction, Springer

Tobias, S. M., & Cattaneo, F. 2008, Journal of Fluid Mechanics, 601, 101

Tobias, S. M., & Cattaneo, F. 2013, Nature, 497, 463

Tobias, S. M., & Cattaneo, F. 2015, Journal of Plasma Physics, 81 (06), 395810601

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