Proceedings Paper:
Nguyen, Y, McLernon, DC and Ghogho, M (2016) Simplified Chirp Dictionary for Time-Frequency Signature Sparse Reconstruction of Radar Returns. In: 4th International Workshop on Compressed Sensing Theory and its Applications to Radar, Sonar and Remote Sensing (CoSeRa 2016). CoSeRa 2016, 19-22 Sep 2016, Aachen, Germany. Institute of Electrical and Electronics Engineers . ISBN 978-1-5090-2920-4

https://doi.org/10.1109/CoSeRa.2016.7745724

Simplified Chirp Dictionary for Time-Frequency Signature Sparse Reconstruction of Radar Returns

Yen Nguyen
School of Electronic and Electrical Engineering,
University of Leeds,
Leeds, UK.
Email: elythn@leeds.ac.uk

Des McLernon
School of Electronic and Electrical Engineering,
University of Leeds,
Leeds, UK.
Email: D.C.McLernon@leeds.ac.uk

Mounir Ghogho*
School of Electronic and Electrical Engineering,
University of Leeds,
Leeds, UK.
Email: M.Ghogho@leeds.ac.uk
* Also with the International University of Rabat, Morocco.

***Abstract*—In sparse reconstruction of the Doppler frequency, the chirp atom approach has been shown to give a better performance than its sinusoidal counterpart. Nevertheless, the chirp atom has a relatively large dimension and so its computational load is much greater compared to the sinusoidal atom. In this paper, we propose a simplified chirp dictionary that obtains a satisfactory time-frequency signature approximation of the signals, but with a computational load comparable to the sinusoidal atom. We estimate the chirp rate through the DTFT of the bilinear product at a certain lag, and the initial frequency is solved in the time domain.**

I. INTRODUCTION

Non-stationary signals are typically deployed to characterize speech, biomedical signals, sonar, and radar returns [1]–[3]. These signals' frequencies can be a constant, or linear/non-linear functions of time. In radar applications, they are commonly referred to as Doppler and micro-Doppler signals [4]–[7]. Being able to analyse these Doppler and micro-Doppler frequencies correctly is extremely critical in the radar field [8]–[12]. For example, we can measure the velocity and direction of a bulk motion or the vibration of targets' structures by examining respectively the frequency shifts or frequency modulations on the reflected signals.

There have been numerous methods of time-frequency distribution (TFD) analysis proposed. The TFD can be obtained by linear basis decomposition [13], [14] or quadratic time-frequency distribution, generally referred to as Cohen's class [15], [16]. Recently, compressive sensing (CS) has attracted much interest as a non-stationary signal reconstruction method. It is based on the fact that non-stationary signals are locally sparse in the TF domain [17]–[21]. Thus, data segments can be recovered even with compressed observations, under which previous methods fail to produce correct results. Incomplete samples, or random sampling in the field of radar can frequently happen due to range ambiguity, discarding noisy measurements, hardware simplification, sampling frequency limitations, or co-existence of various wireless services with active or passive sensing models [22]–[24]. Therefore, TFD approaches consistent with missing data are needed. A straightforward approach is to perform sparse reconstruction from windowed data in the time domain, deploying a partial Fourier basis [18]. This is similar to the spectrogram but compressive techniques are used and thus better results are attained. However, this method suffers from the trade-off between necessary measurements for accurate recovery and sparsity when considering the window size, and also the picket fence effect when there is a non-integer period in the analyzed data segments. Thus, another measurement basis is required to obtain more stable and reliable results.

In many situations, the frequency law of non-stationary signal segments can be represented as a weighted sum of piece-wise linear chirps, in which most of the parameter coefficients are zero. In this respect, they are sparse in the joint time-frequency domain. Thus, the segments' time-frequency signature can be recovered by sparse reconstruction with the measurement dictionary being chirp atoms [25], [26]. Greedy algorithms or convex optimization techniques are used to obtain the sparsest chirp combination that best describes the windowed signals. Compared with the sinusoidal dictionary method, better performance is obtained under both full and limited data because signals are more sparse in this representation, and longer windows can be employed without sensible reduction in sparsity. It is also not susceptible to the picket fence effect.

The chirp approach, nevertheless, deploys a very large dimension measurement dictionary. Since there are two parameters to be estimated (i.e. the chirp rate and the initial frequency), the dictionary dimension can be equal to the square of the dimension when using the sinusoid atom. This very large atom set leads to a much heavier computation burden and a longer calculation time. Thus, in this paper, we propose the simplified chirp atom method. We will estimate the chirp rate (α, see in section II) through the DTFT of the bilinear product at a certain time lag. The initial frequency (β, see section II) is solved in time domain, with a lower dimensional dictionary than the computationally complex full chirp atom. The advantage of this approach is that we obtain acceptable estimation of the TF features of the signal but with computational complexity comparable to the sinusoidal atom.

The paper is organized as follows. Section II discusses the computational requirement of the full chirp dictionary approach. The simplified chirp atom method is then introduced in section III. Section IV includes simulation results. Finally, conclusions are given in section IV.

II. CALCULATION LOAD IN FULL CHIRP DICTIONARY APPROACH

In the full chirp dictionary approach, a discrete signal segment of length N_w is approximated as a sum of $K(K \geq 1)$ chirps. This means that we have to the estimate chirp rates (α) and the initial frequencies (β) of K chirps in each data segment. This task is carried out by CS techniques with a full chirp atom $\mathbf{\Psi}_F$. The parameter space of interest is [25], [26]:

$$\begin{aligned}&\mathbf{\Omega} = \{(\alpha, \beta) \quad \text{such that} \\ &|\alpha| \leq F_{\max} F_s / N_w \quad \text{and} \quad |\alpha N_w / F_s + \beta| \leq F_{\max}\},\end{aligned} \tag{1}$$

where $F_{\max}$ is the maximum frequency of the signal, $F_s(F_s = 2F_{\max})$ is the sampling frequency. The matrix $\mathbf{\Psi}_F$ is designed by uniformly sampling the 2D parameter space $\mathbf{\Omega}$. Let I denote the total number of chirp rate values in the discrete dictionary $\mathbf{\Psi}_F$. For the i^{th} chirp rate value in the dictionary, which we denote as $\tilde{\alpha_i}$, let $\tilde{\beta}_{i,j}$ denote the corresponding possible values for initial frequency, where $j = 1, 2, ..., J_i$. Note that the "~" refers to "dictionary values". The full chirp dictionary $\mathbf{\Psi}_F$ is defined as [25], [26]:

$$\begin{aligned}&\mathbf{\Psi}_F = [\mathbf{\Psi}_1, \mathbf{\Psi}_2, ..., \mathbf{\Psi}_I] \\ &\mathbf{\Psi}_i = [\boldsymbol{\psi}_{i,1}, \boldsymbol{\psi}_{i,2}, ..., \boldsymbol{\psi}_{i,J_i}] \\ &\boldsymbol{\psi}_{i,j}|_n = \exp\left(j2\pi(\tilde{\alpha}_i \frac{n^2}{2F_s^2} + \tilde{\beta}_{i,j} \frac{n}{F_s})\right) \\ &i = 1, ..., I, j = 1, ..., J_i, n = 0, 1, ..., N_w - 1.\end{aligned} \tag{2}$$

From (2), it can be seen that for each value of α_i, there are J_i columns in the dictionary corresponding to J_i values of initial frequency. Based on the parameter space $\mathbf{\Omega}$ in (1), we choose $I = F_s + 1$. Then, the dimension of the full chirp atom $\mathbf{\Psi}_F$ is:

$$Q_{\mathbf{\Psi}_F} = \lfloor \frac{3}{4}(F_s + 1)^2 + \frac{1}{4} \rfloor, \tag{3}$$

where $\lfloor . \rfloor$ is the "floor operator". So, for each window, $K\left(\frac{3}{4}(F_s + 1)^2 + \frac{1}{4}\right)$ calculations are implemented. Furthermore, as sliding windows are used, the chirp method faces a large computational burden. With the same manner of operation, the sinusoidal dictionary method only requires $O(KF_s)$ calculations, which gives it a big advantage over its chirp counterpart when large amounts of data are concerned. So, in this paper, we will propose a simplified chirp atom approach which is computationally efficient and still possesses all the strengths and advantages of the chirp dictionary method.

III. SIMPLIFIED DICTIONARY APPROACH

A. Chirp rate estimation

Consider an arbitrary continuous signal $s_c(t)$, which consists of $K(K \geq 1)$ components:

$$s_c(t) = \sum_{k=1}^{K} A_k(t) \exp(j\phi_k(t)) + v_c(t), \tag{4}$$

where $0 \leq t \leq T$, $A_k(t), \phi_k(t)$ are the time-varying amplitude and phase of the k^{th} component, and $v_c(t)$ is white Gaussian noise. Sampling the continuous signal at the Nyquist rate $F_s(F_s = 2F_{max})$, where F_{max} is the maximum frequency of the signal $s_c(t)$, we have:

$$s(n) = \sum_{k=1}^{K} A_k(nT_s) \exp(j\phi_k(nT_s)) + v(nT_s), \tag{5}$$

where $n = 0, 1, ..., \lfloor T/T_s \rfloor$, and $T_s = 1/F_s$.

Similar to the full chirp atom approach, in this method, observations inside a short-time window are also approximated by the sum of piece-wise chirps. So breaking $s(n)$ into N_w-length blocks $\{s_m(n)\}_{n=0}^{N_w-1}$, the m^{th} block is calculated as:

$$s_m(n - u(m-1)) = s(n)h(n - u(m-1)), \tag{6}$$

where $n = u(m-1), u(m-1)+1, ..., u(m-1)+N_w-1$, $u(1 \leq u \leq N_w)$ is the shift between two consecutive windows, $m(m = 1, 2, ...)$ is the window index, and $h(n)$ is a rectangular window which is non-zero only for $0 \leq n \leq N_w - 1$.

Then the chirp-approximated m^{th} signal segment of $s(n)$ is written as:

$$\begin{aligned}s_m(n) &\approx \sum_{k=1}^{K} C_{k,m} \exp\left\{j2\pi\left[\alpha_{k,m}\frac{n^2}{2F_s^2} + \beta_{k,m}\frac{n}{F_s}\right]\right\} \\ &+ v_m(n) = \sum_{k=1}^{K} s_{k,m} + v_m(n),\end{aligned} \tag{7}$$

where $0 \leq n \leq N_w - 1$, $C_{k,m}, \alpha_{k,m}, \beta_{k,m}$ are respectively the complex amplitude, the chirp rate, and the initial frequency of the k^{th} chirp over the m^{th} window. Now $s_{k,m}$ is the chirp with parameters specified by $C_{k,m}, \alpha_{k,m}, \beta_{k,m}$. The instantaneous autocorrelation function (IAF) of $s_m(n)$ is expressed as:

$$\begin{aligned}C_{s_m s_m}(l, n) &= s_m(n+l)s_m^*(n-l) \\ &= \sum_{k=1}^{K} s_{k,m}(n+l)s_{k,m}^*(n-l) \\ &+ \sum_{\substack{i,j=1 \\ i \neq j}}^{K} s_{i,m}(n+l)s_{j,m}^*(n-l) \\ &= \sum_{k=1}^{K} AT_{k,m}(l, m) + \sum_{g=1}^{K(K-1)} CT_{g,m}(l, m),\end{aligned} \tag{8}$$

where l is time lag, $AT_{k,m}$ and $CT_{g,m}$ contain auto-terms and cross-terms, respectively, and are expressed as:

$$\begin{aligned}&AT_{k,m}(l, n) = \exp\left(j2\pi\frac{2\alpha_{k,m}l}{F_s}\frac{n}{F_s}\right)\exp\left(j2\pi\frac{2\beta_{k,m}l}{F_s}\right) \\ &CT_{g,m}(l, n) = \exp\left(j2\pi(\alpha_{i,m} - \alpha_{j,m})\frac{n^2}{2F_s^2}\right) \\ &\exp\left(j2\pi\frac{(\alpha_{i,m} + \alpha_{j,m})l}{F_s}\frac{n}{F_s}\right)\exp\left(j2\pi(\beta_{i,m} - \beta_{j,m})\frac{n}{F_s}\right) \\ &\exp\left\{j2\pi\left[\frac{(\alpha_{i,m} - \alpha_{j,m})l^2}{2F_s^2} + \frac{(\beta_{i,m} + \beta_{j,m})l}{2F_s^2}\right]\right\},\end{aligned} \tag{9}$$

where $i, j \in [1, K]$, $i \neq j$, $g \in [1, K(K-1)]$. Let $AT_{k,m}(\nu)|_{l=l_1}$, and $CT_{k,m}(\nu)_{l=l_1}$ be the DTFT of

$AT_{k,m}(l,n)$ and $CT_{g,m}(l,n)$ at $l = l_1$, then:

$$|AT_{k,m}(\nu)|_{l=l_1}| = \delta\left(\nu - 2\alpha_{k,m}\frac{l_1}{F_s}\right)$$
$$|CT_{g,m}(\nu)|_{l=l_1}| = W(\nu) * \delta\left(\nu - \frac{l_1(\alpha_{i,m}+\alpha_{j,m})}{F_s}\right) *$$
$$\delta\left(\nu - (\beta_{i,m} - \beta_{j,m})\right), \tag{10}$$

where $W(\nu)$ is the DTFT of $\exp\left(j2\pi(\alpha_{i,m} - \alpha_{j,m})\frac{n^2}{(2F_s^2)}\right)$. Now (10) shows that the spectral representation of the auto-terms are delta functions whose locations are determined by only the chirp rates. So if $\nu_{AT_{k,m}}$ corresponds to the frequency location of the spectrum of the auto-terms then the chirp rates are approximated by:

$$\hat{\alpha}_{k,m} = \frac{\nu_{AT_{k,m}} F_s}{2l_1}. \tag{11}$$

In addition, (10) shows that while the cross-terms are mostly located away from the origin $\nu = 0$ Hz, the auto-terms are clustered around $\nu = 0$ Hz. Thus, most of the cross-terms can be removed by a LPF without significantly altering the auto-terms.

B. Simplified chirp dictionary algorithm

In the vector form, the signal over the m^{th} window in (7) can be expressed as:

$$\mathbf{S}_m = \mathbf{\Psi}_c \mathbf{X}_m + \mathbf{V}_m \tag{12}$$

where $\mathbf{S}_m = [s_m(0), ..., s_m(N_w - 1)]^T$, $\mathbf{V}_m = [v_m(0), ..., v_m(N_w - 1)]^T$ and $\mathbf{X}_m$ has K non-zero components. The compact dictionary matrix, $\mathbf{\Psi}_c$, is designed for each signal component inside the windowed data. The chirp rate value in $\mathbf{\Psi}_c$ is estimated by algorithm in III-A, and denoted as $\hat{\alpha}$. Let $\tilde{\beta}_j$ denote the corresponding possible values for initial frequency, where $j = 1, 2, ..., J$. The compact chirp dictionary $\mathbf{\Psi}_c$ is defined as:

$$\mathbf{\Psi}_c = [\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, ..., \boldsymbol{\psi}_J]$$
$$\boldsymbol{\psi}_j|_n = \exp\left(j2\pi(\hat{\alpha}\frac{n^2}{2F_s^2} + \tilde{\beta}_j\frac{n}{F_s})\right) \tag{13}$$
$$j = 1, ..., J, n = 0, ..., N_w - 1.$$

As $\hat{\alpha}$ has only one value, the compact chirp dictionary only has J columns, where from $\mathbf{\Omega}$ in (1), $J = \lfloor F_s - |\hat{\alpha}|T_w + 1\rfloor$. We choose $J = F_s + 1$, and thus the dimension of the compact chirp atom now becomes:

$$Q_{\mathbf{\Psi}_c} = J = F_s + 1. \tag{14}$$

Thus, when using the simplified chirp dictionary, the number of calculations is about $O(K(F_s + 1))$. Since $K < N_w \ll J$, $\mathbf{X}_m$ is highly sparse and solving for $\mathbf{X}_m$ in equation (12) becomes a sparse recovery (or CS) problem. The algorithm of the simplified chirp dictionary used in this paper is based on Orthogonal Matching Pursuit and has following steps:

INPUT:

- Signal $s(n)$ of length L.
- Signal vector $\mathbf{S} = [s(0), ..., s(L-1)]^T$.
- Windowed signal vector $\mathbf{S}_m = \mathbf{S}((m-1)u + n)$, $0 \le n \le N_w - 1$. Initialize $m = 1$.
- Lag value $l = l_1$

OUTPUT:

- Matrix of selected chirp $\mathbf{\Phi}$.

PROCEDURE:

1) Initialize the residual $\mathbf{r_0} = \mathbf{S}_m$, matrix of selected chirps $\mathbf{\Phi}_i = \emptyset$, and the iteration counter $i = 1$.
2) Calculate IAF at $l = l_1$ ($C_{r_i r_i}(l,n)|_{l=l_1}$).
3) Calculate DTFT of $C_{r_i r_i}$. Estimate the chirp rate and build the compact chirp dictionary $\mathbf{\Psi}_c$ by (11) and (13).
4) Find the index λ_i, $\lambda_i = \arg\max_{j=1,...,J} | < \boldsymbol{r}_{i-1}, \boldsymbol{\psi}_j > |$.
5) Store the selected chirp $\boldsymbol{\psi}_{\lambda_i}$, $\mathbf{\Phi}_i = [\mathbf{\Phi}_{i-1}\,\boldsymbol{\psi}_{\lambda_i}]$.
6) Solve a least square problem to find the residue after subtracting the chirp
$$\mathbf{x}_i = \arg\min_{\mathbf{x}} \|\mathbf{\Phi}_i \mathbf{x} - \mathbf{S}_m\|_2$$
$$\mathbf{r}_i = \mathbf{S}_m - \mathbf{\Phi}_i \mathbf{x}_i.$$
7) Increment i, and return to step 2 if $i < K$ or $\|\mathbf{r}_i\|_2 > 0.05\|\mathbf{S}_m\|_2$. The magnitude of the selected chirps is stored in $\mathbf{x}_i$. If $i = K$ or$\|\mathbf{r}_i\|_2 \le 0.05\|\mathbf{S}_m\|_2$, move to the next windowed signal, increment m, and return to step 1.

There is another way to estimate the initial frequencies after the chirp rates are verified. According to (9), the initial frequencies can be approximated by the magnitude of the auto-terms' frequencies. However, this magnitude can be easily affected by noise, and thus its results are unreliable. The drawback of the simplified chirp dictionary method is that it does not perform well if too much data (over 50% of observations) is absent. This is because the missing samples in the IAF at any time lag can be double the number of missing samples for $s(n)$ in the time domain.

IV. SIMULATION RESULTS

For illustration purposes, we use three examples with different TFD methods including the Wigner-Ville distribution (WVD), sinusoid, and two chirp dictionary approaches. The WVD represents the quadratic TFD, which is vulnerable to missing samples and cross-terms and so it is unable to deliver an accurate TF estimation under compressed data. The WVD is simulated in order to compare it with the CS-related methods. The signals in each of the three cases are firstly sampled at the Nyquist rate, and then some samples are randomly removed. The sampling frequency is $F_s = 256$Hz, the total signal length is $L = 256$, 60% of the data is used to obtain the time frequency signature of the signal, and $SNR = 20$dB. The chirp dictionary methods deploy a rectangular window, and the sinusoidal atom method uses a Hanning window.

In the first example, the signal consists of two closelyalligned chirps, which is expressed as:

$$s(n) = \exp\left\{j2\pi[(0.1F_s)\frac{n}{F_s} + (0.3F_s)\frac{n^2}{2F_s^2}]\right\} + \exp\left\{j2\pi[(0.13F_s)\frac{n}{F_s} + (0.33F_s)\frac{n^2}{2F_s^2}]\right\} + v(n), \tag{15}$$

where $n = 0, 1, ..., L-1$. To capture enough data to resolve the two close frequency-spaced chirps, the window size is set to a large value, $N_w = 100$. The WVD gives a noisy TF distribution due to the missing signal entries and it is obviously plagued by cross-terms. These issues are mitigated when CS related methods are used. However, Fig. 1(b) shows the failure of local reconstruction of the sinusoidal method due to lack of sparsity when employing a long window. The sparsity, on the other hand, when chirp methods are in use, only depends on the number of piece-wise chirps inside the considered segment. Thus the two chirp dictionary methods are less sensitive to this issue, and the signal is clearly resolved as shown in Fig. 1(c) and Fig. 1(d).

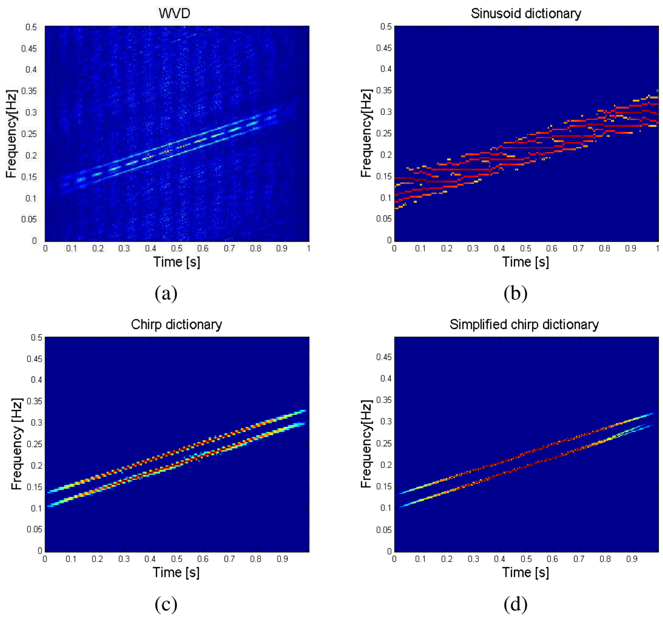

Fig. 1: TF (frequency normalized) signature for $s(n)$ in (15) with 40% data missing: (a) WVD; (b) Sinusoidal dictionary; (c) Chirp dictionary; (d) Simplified chirp dictionary.

In the second example, we use a three- component FM signal, which is expressed as:

$$s(n) = \exp\left\{j(0.1F_s)\cos(2\pi\frac{n}{F_s} + \pi) + j2\pi(0.2F_s)\frac{n}{F_s}\right\} + \exp\left\{j(0.1F_s)\cos(2\pi\frac{n}{F_s} + \pi) + j2\pi(0.3F_s)\frac{n}{F_s}\right\} + \exp\left\{j2\pi[(0.1F_s)\frac{n}{F_s} + (0.3F_s)\frac{n^2}{2F_s^2}]\right\} + v(n), \tag{16}$$

where $n = 0, 1, ..., L-1$. The window length is set to $N_w = 70$. The results are given in Fig. 2. It is evident in Fig. 2(a) that cross-terms and noise-like artifacts clutter the signal component and hide the pertinent signal structure when the WVD is employed. The sinusoidal dictionary approach reveals inaccuracy in the TF signature estimation since besides insufficient sparsity, it is vulnerable to the picket fence effect [26], resulting in frequency contents at false locations. The chirp dictionary approach can address this failure and the instantaneous frequency laws are resolved as seen in Fig. 2(c), and Fig. 2(d). The simplified chirp dictionary has some inaccurate approximation due to limited samples in the instantaneous autocorrelation domain, but the result is acceptable. Compared with its sinusoidal counterpart, it gives a better performance but with a similar calculation effort.

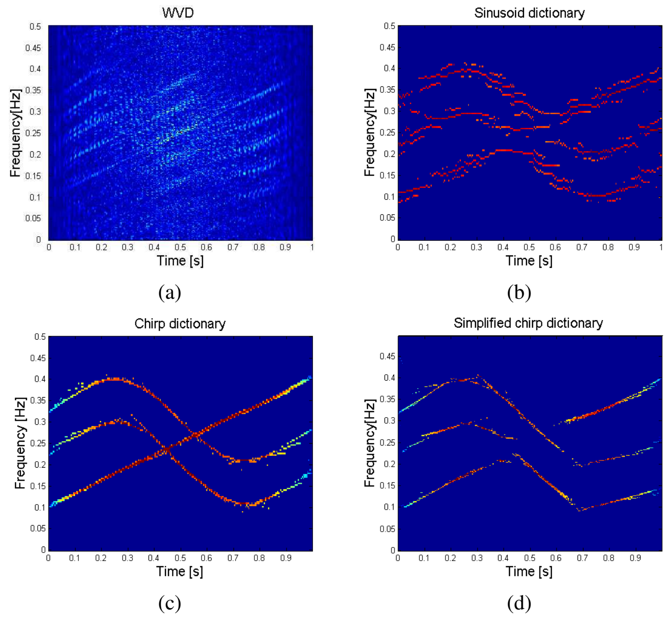

Fig. 2: TF signature (frequency normalized) for $s(n)$ in (16) when 40 % samples are missing: (a) WVD; (b) Sinusoidal dictionary; (c) Chirp dictionary; (d) Simplified chirp dictionary.

In the third example, we use data from human gait radar returns obtained at the Radar Imaging Lab of the Center for Advanced Communications at Villanova University, USA. The data is first uniformly sampled at the Nyquist rate with $F_s = 1000$ Hz, and then thinned by randomly removing 40% of samples. Sparsity level is assumed to be $K = 30$. The window length is $N_w = 128$, and we only use 128 frequency components to display the TF signature in order to zoom in on the instantaneous frequencies, and so partly mitigate drawbacks of the sinusoidal dictionary method. The results in Fig. 3 show that the simplified chirp dictionary approach can describe Micro-Doppler TF presentations of the torso and limbs under compressed observations.

V. CONCLUSION

The accurate piece-wise chirp approximations to the time-frequency signature of many Doppler and micro-Doppler signals motivate the use of the chirp dictionary for sparse reconstruction of the signal's local frequency structure under full

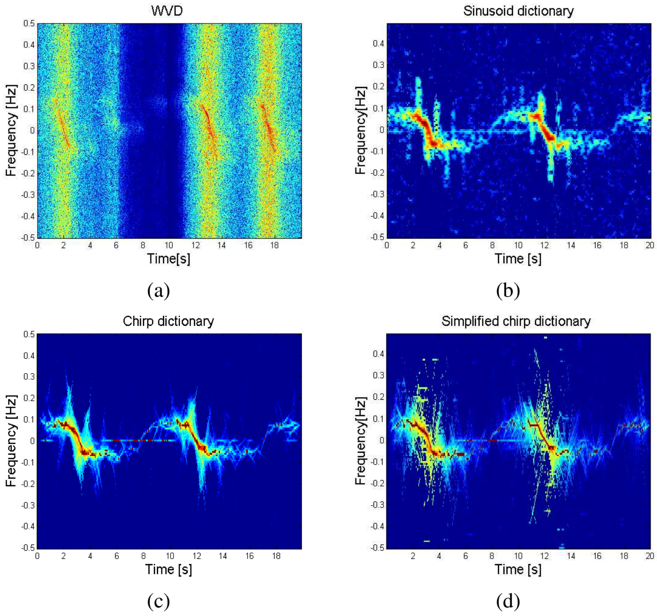

Fig. 3: TF (frequency normalized) signature of human gait radar return with 40% data missing: (a) WVD; (b)Sinusoidal dictionary; (b) Chirp dictionary; (c) Simplified chirp dictionary.

and incomplete data. Compared with the sinusoidal dictionary method or WVD, the chirp atom generally achieves better performance. The simplified chirp atom set helps reduce the amount of calculation, and thus saves time. Although it is quite vulnerable to incomplete samples, it provides satisfactory results and an extra choice when analysing non-stationary signals. When not many samples are absent, this method can enjoy a much faster implementation compared with the full-chirp dictionary method, and also a better performance in comparison with the sinusoidal dictionary method.

ACKNOWLEDGMENT

I would like to thank the Center for Advanced Communications at Villanova University, USA for both support and providing the simulation data.

REFERENCES

[1] W. C. Knight, R. G. Pridham, and S. M. Kay, "Digital signal processing for sonar," *Proceedings of the IEEE*, vol. 69, no. 11, pp. 1451–1506, 1981.

[2] T. Misaridis and J. A. Jensen, "Use of modulated excitation signals in medical ultrasound. part i: Basic concepts and expected benefits," *Ultrasonics, Ferroelectrics, and Frequency Control, IEEE Transactions on*, vol. 52, no. 2, pp. 177–191, 2005.

[3] M. G. Amin and W. Sun, "A novel interference suppression scheme for global navigation satellite systems using antenna array," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 5, pp. 999–1012, 2005.

[4] V. C. Chen, *The micro-Doppler effect in radar*. Artech House, 2011.

[5] V. C. Chen and H. Ling, *Time-frequency transforms for radar imaging and signal analysis*. Artech House, 2001.

[6] V. C. Chen, "Analysis of radar micro-doppler with time-frequency transform," in *Statistical Signal and Array Processing, 2000. Proceedings of the Tenth IEEE Workshop on*, pp. 463–466, IEEE, 2000.

[7] M. G. Amin, *Through-the-wall radar imaging*. CRC press, 2011.

[8] C. Clemente, A. Balleri, K. Woodbridge, and J. J. Soraghan, "Developments in target micro-doppler signatures analysis: radar imaging, ultrasound and through-the-wall radar," *EURASIP Journal on Advances in Signal Processing*, vol. 2013, no. 1, pp. 1–18, 2013.

[9] Y. Kim and H. Ling, "Human activity classification based on micro-doppler signatures using a support vector machine," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 47, no. 5, pp. 1328–1337, 2009.

[10] M. G. Amin, "Time-frequency spectrum analysis and estimation for nonstationary random-processes," in *Time-Frequency Signal Analysis*, pp. 208–232, 1992.

[11] G. Matz and F. Hlawatsch, *Time-varying power spectra of nonstationary random processes*. na, 2003.

[12] F. Ahmad, M. G. Amin, and P. Setlur, "Through-the-wall target localization using dual-frequency cw radars," in *Defense and Security Symposium*, pp. 62010H–62010H, International Society for Optics and Photonics, 2006.

[13] S. G. Mallat and Z. Zhang, "Matching pursuits with time-frequency dictionaries," *Signal Processing, IEEE Transactions on*, vol. 41, no. 12, pp. 3397–3415, 1993.

[14] S. Ghofrani, M. G. Amin, and Y. D. Zhang, "High-resolution direction finding of non-stationary signals using matching pursuit," *Signal Processing*, vol. 93, no. 12, pp. 3466–3478, 2013.

[15] L. Cohen, *Time-frequency analysis*, vol. 299. Prentice hall, 1995.

[16] J. Jeong and W. J. Williams, "Kernel design for reduced interference distributions," *Signal Processing, IEEE Transactions on*, vol. 40, no. 2, pp. 402–412, 1992.

[17] P. Flandrin and P. Borgnat, "Time-frequency energy distributions meet compressed sensing," *Signal Processing, IEEE Transactions on*, vol. 58, no. 6, pp. 2974–2982, 2010.

[18] B. Jokanović, M. Amin, and S. Stanković, "Instantaneous frequency and time-frequency signature estimation using compressive sensing," in *SPIE Defense, Security, and Sensing*, pp. 871418–871418, International Society for Optics and Photonics, 2013.

[19] M. Amin, B. Jokanovic, and T. Dogaru, "Reconstruction of locally frequency sparse nonstationary signals from random samples," in *Signal Processing Conference (EUSIPCO), 2014 Proceedings of the 22nd European*, pp. 1771–1775, IEEE, 2014.

[20] Y. D. Zhang, M. G. Amin, and B. Himed, "Reduced interference time-frequency representations and sparse reconstruction of undersampled data," in *Signal Processing Conference (EUSIPCO), 2013 Proceedings of the 21st European*, pp. 1–5, IEEE, 2013.

[21] L. Stankovic, I. Orovic, S. Stankovic, and M. Amin, "Compressive sensing based separation of nonstationary and stationary signals overlapping in time-frequency," *Signal Processing, IEEE Transactions on*, vol. 61, no. 18, pp. 4562–4572, 2013.

[22] M. Amin, *Compressive sensing for urban radar*. CRC Press, 2014.

[23] C. Law, G. Jones, D. Backer, W. Barott, G. Bower, C. Gutierrez-Kraybill, P. Williams, and D. Werthimer, "Millisecond imaging of radio transients with the pocket correlator," *The Astrophysical Journal*, vol. 742, no. 1, p. 12, 2011.

[24] J. M. Peha, "Wireless communications and coexistence for smart environments," *Personal Communications, IEEE*, vol. 7, no. 5, pp. 66–68, 2000.

[25] Y. T. Nguyen, M. G. Amin, M. Ghogho, and D. McLernon, "Local sparse reconstructions of doppler frequency using chirp atoms," in *Radar Conference (RadarCon), 2015 IEEE*, pp. 1280–1284, IEEE, 2015.

[26] Y. T. Nguyen, M. G. Amin, M. Ghogho, and D. McLernon, "Time-frequency signature sparse reconstruction using chirp dictionary," in *SPIE Sensing Technology+ Applications*, pp. 94840H–94840H, International Society for Optics and Photonics, 2015.