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On the Analysis of Device-to-Device Overlaid Cellular Networks in the Uplink under 3GPP Propagation Model

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***Abstract*—In this article we employ the third generation partnership project (3GPP) recommended path loss models for the analysis of cellular networks overlaid with D2D communication and channel inversion power control in the uplink. We characterize the coverage and average network throughput with the help of stochastic geometry. More specifically, we develop tractable expressions for the coverage in cellular and D2D modes. Our theoretical results differ significantly from previous work, which uses simple power law path loss models. The traditional methodology does not account for the presence of line-of-sight (LoS), non-line-of-sight (NLoS) and free space (FS) links. We demonstrate that such classification of links significantly impacts the inference which can be derived from the analysis for the design of overlaid D2D networks. In particular, we show that, contrary to the previous findings, the average throughput of the network does not saturate with the increase in the density of base stations (BS), but there exists an optimal mode selection threshold and BS density which maximizes the average throughput.**

I. Introduction

Direct device-to-device (D2D) communication is viewed as a key ingredient for the future generation wireless networks for improving the quality of experience (QoE) of the users [1]. By exploiting the close proximity of mobile devices, higher rates with lower power utilization can be achieved. Not only does D2D communication reduce the traffic overhead on the base stations (BS), but the single hop transmission (user-user) (as opposed to the conventional two-hop cellular communication (user-BS-user)) can also significantly reduce latency.

The ad hoc nature of D2D communication raises a new set of design challenges as to how to optimally integrate D2D communication within the current cellular infrastructure. Currently available literature on D2D communication in the uplink (UL) focuses on the analysis of spectrum sharing, interference mitigation, power control and mode selection techniques [1]–[3]. However, these works assume simplistic path loss models, which do not account for the line-of-sight (LoS) and non-line-of-sight (NLoS) links and also do not differentiate between the cellular and D2D links. It is well established that transmissions from the user equipment (UE) face a lot of obstructions as the distance to the intended receiver gets large because of the low antenna heights of the UEs. This effect is worsened in urban environments where D2D communication is most applicable. Recent studies on the analysis of LoS and NLoS communication focus only on single tier downlink cellular networks [4]–[7].

In this paper, we build upon the network model discussed in [2] for the overlaid D2D communication in the cellular UL with channel inversion power control by employing practical path loss models recommended by 3GPP for the transmissions from the UE to the BS [8] and the transmissions from the UE to UE [9]. The contributions of this paper are as follows:

- We borrow tools from stochastic geometry to fully characterize and obtain closed-form expressions for the average transmission power and coverage in cellular and D2D modes under the realistic 3GPP propagation model.
- We observe that for a given noise floor, the cellular coverage in the baseline model in [2] remains constant with the variation in BS density. Our enhanced model (based on 3GPP standards) indicates otherwise and shows that the cellular coverage decreases with an increase in the BS density. The normalized throughput of the network using the reference model saturates after a certain BS density threshold and increasing the BS density after that does not have any effect. On the contrary, our analysis with the 3GPP path loss model shows that there exists an optimal BS density, which maximizes the average throughput of the network.

The rest of the paper is organized as follows. Section II outlines the hybrid network setting. Section III discusses the preliminary analysis, which includes the derivation of the expected power in cellular and D2D modes. Section IV provides the main results of cellular and D2D coverage. Section V verifies the analysis of coverage with network simulations and discusses useful insights. Section VI concludes the paper.

II. System Model

We consider a UL scenario of a single tier cellular network overlaid with D2D communication and channel inversion power control. In this section, we briefly outline the important device, link and network level parameters which dictate the network performance.

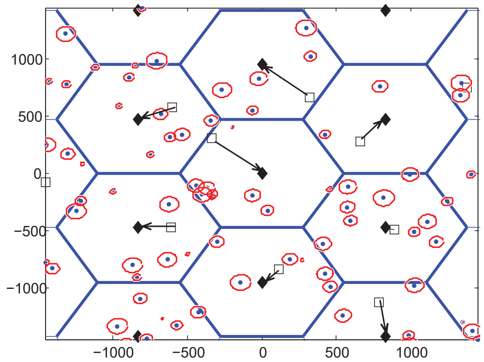

Figure 1: Network model. Diamonds represent BSs placed in the center of a regular hexagonal grid. The cellular UEs operating on a single channel are shown by squares. All UEs operating in D2D mode are shown by dots. The circle centered at each D2D UE represents its distance to the intended receiver. The D2D receiver lies anywhere on the perimeter of this circle.

A. Spatial Model and Mode Selection

The BSs (intensity λ_b) are placed inside a regular hexagonal grid. The transmitting UEs are distributed in space according to a homogeneous Poisson point process (HPPP) $\Phi_u \in \mathbb{R}^2$ with intensity λ_u. We assume that only a fraction ε of the UEs can participate in D2D communication. The intended receiver of each D2D enabled user is placed at distance L from the user, where L is a Rayleigh distributed RV with probability density function (PDF) $f_L(x) = 2\pi\zeta x \exp(-\zeta\pi x^2)$. It is further assumed that the D2D enabled UE communicates in D2D mode only if the distance L is below a certain threshold μ, otherwise cellular mode is selected. The probability of D2D mode selection is then given as $\mathbb{P}[L \le \mu] = 1 - \exp(-\zeta\pi\mu^2)$. Because of the independent thinning property of the HPPPs [10], the UEs operating in D2D mode constitute an HPPP $\Phi_d \in \mathbb{R}^2$ with intensity $\lambda_d = \varepsilon\lambda_u \left(1 - \exp(-\zeta\pi\mu^2)\right)$ and the cellular UEs constitute a HPPP $\Phi_c \in \mathbb{R}^2$ with intensity $\lambda_c = (1-\varepsilon)\lambda_u \exp(-\zeta\pi\mu^2)$.

It is assumed that the cellular UEs are associated with the nearest BS. Notice that the transmitting D2D UE and its intended receiver may not be present in the same cell due to the ad hoc nature of the D2D network. Without any loss of generality, the performance in cellular mode is measured at a typical BS in cellular mode and a typical D2D receiver in D2D mode. For the sake of analytical tractability, we exploit the stationarity property of HPPP. Therefore, in cellular mode, the typical node is assumed to be at the origin. A similar process can be repeated to position a typical D2D receiver at the origin by translating the PPP of the D2D receivers. Fig. 1 displays the network spatial model under consideration.

B. Propagation Model and Power Control

We consider that the radio signal experiences, small scale flat fading, which is complemented by the attenuation due to the large scale path loss. We assume a Rayleigh fading environment, where the channel power $h(\mathbf{x}_1, \mathbf{x}_2)$ between arbitrary locations $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^2$ is an i.i.d unit-mean exponential RV. Because of the i.i.d channel gains, we denote $h(\mathbf{x}_1, \mathbf{x}_2) = h$ in the rest of the analysis. We adopt the path loss model specified by the 3GPP standard in [9]. Consequently, the path loss model for the UE located at a distance x from its corresponding BS[1] is given as

$$l_c(x) = \begin{cases} A_{c,l}x^{-\alpha_{c,l}} & \text{with probability } Pr_c^{LOS}(x) \\ A_{c,n}x^{-\alpha_{c,n}} & \text{with probability } 1 - Pr_c^{LOS}(x), \\ \text{both for} & 0 \le x \le r_c, \\ & \\ A_{c,n}x^{-\alpha_{c,n}} & x > r_c, \end{cases} \quad (1)$$

where $A_{c,l}$ and $\alpha_{c,l}$ are the cellular LoS reference path loss and path loss exponents respectively, $A_{c,n}$ and $\alpha_{c,n}$ are the cellular NLoS reference path loss and path loss exponents respectively, r_c is a constant based on practical measurements and $Pr_c^{LOS}(x)$ is the probability of having a LoS link of the transmitting UE with the BS at distance x. It is given as [6]

$$Pr_c^{LOS}(x) = \begin{cases} 1 - \frac{x}{r_c} & 0 \le x \le r_c, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

The path loss model for the UE-UE link is slightly more involved and is given as

$$l_d(x) = \begin{cases} A_{fs}x^{-\alpha_{fs}}, & 0 \le x \le r_{fs}, \\ A_{d,l}x^{-\alpha_{d,l}}, & r_{fs} < x \le r_d, \\ & \\ A_{d,l}x^{-\alpha_{d,l}} & \text{with probability } Pr_d^{LOS}(x) \\ A_{d,n}x^{-\alpha_{d,n}} & \text{with probability } 1 - Pr_d^{LOS}(x), \\ \text{both for} & x > r_d. \end{cases} \quad (3)$$

where A_{fs}, $A_{d,l}$ and $A_{d,n}$ are the free space, D2D LoS and D2D NLoS reference path losses respectively; α_{fs}, $\alpha_{d,l}$ and $\alpha_{d,n}$ are the free space, D2D LoS and D2D NLoS path loss exponents respectively; r_d is a constant based on measurements; $r_{fs} = q f_c$ is the free space distance which depends on the carrier frequency f_c, where $q = 2.56/c$ m/Hz is a constant depending on UE's antenna heights and the speed of light c; $Pr_d^{LOS}(x)$ is the probability of having a LoS link between the transmitting UE and the UE at a distance x. It is given as

$$Pr_d^{LOS}(x) = \begin{cases} 1, & r_{fs} \le x \le r_d, \\ \frac{r_d}{x}\left(1 - \exp\left(-\frac{r_d}{x}\right)\right)\exp\left(-\frac{r_d}{x}\right), & x > r_d. \end{cases} \quad (4)$$

We use the following non-linear approximation to simplify the expression for $Pr_d^{LOS}(x)$ in (4).

$$Pr_d^{LOS}(x) \approx \begin{cases} 1, & r_{fs} \le x \le r'_d, \\ \frac{r'_d}{x}. & x > r'_d, \end{cases} \quad (5)$$

where $r'_d = r_d + v$ and v is a small displacement term.

[1] Here we assume $x = \|\mathbf{x}_1 - \mathbf{x}_2\|$, $\mathbf{x}_2 = \mathbf{o}$, $\mathbf{x}_1 \in \Phi_u$

1) Channel Inversion Power Control: The power received at a distance x in cellular or D2D mode (in the absence of noise) can be quantified as

$$P_r^{(i)} = P_i l_i(x) h,\ i = \{c, d\}, \tag{6}$$

where P_i is the UE transmit power in mode i. We adopt uplink channel inversion power control, where a transmitting UE inverts the path loss to serve the intended receiver. This implies

$$P_i = \rho_i l_i^{-1}(x)\ i = \{c, d\}. \tag{7}$$

Here, ρ_i is the sensitivity of the receiver in mode i. The small scale fading gain is not included in power control as it has little effect on the long term statistics and it removes the need to estimate h at every transmission slot. Furthermore, we assume that the links suffer from both co-channel interference and additive white Gaussian noise at receiver front end.

C. Spectrum Access Model

We assume that the available spectrum is divided between D2D and cellular networks. Thus, the D2D transmitters operate in an overlay mode in a disjoint spectrum partition. This enables network operator to suppress the inter-tier interference without sophisticated coordination mechanism. A fraction β of the bandwidth is allocated to the D2D UEs, while the remaining $1-\beta$ is allocated to the cellular UEs. For cellular communication, there is no intra cell interference, i.e. only one UE is transmitting on a given channel in a cell at the particular time.

III. Transmit Power Analysis

Quantification of the average transmit power of UEs in cellular and D2D modes is central for further performance analysis. More specifically, both coverage and attainable rates are coupled with the average transmit power, which shapes the received signal strength and co-channel interference. To this end, we first derive the expected transmit power of the UEs in cellular mode.

Lemma 1. *The average power of a UE in cellular mode with channel inversion power control and 3GPP path loss model for UE-BS link is given as*

$$\mathbb{E}[P_c] \approx \rho_c' \left\{ A_{c,l}^{-1} \left[\frac{y^{a_{2,l}}}{a_{2,l}} - \frac{y^{a_{3,l}}}{a_{3,l}} \right] + A_{c,n}^{-1} \left[\frac{y^{a_{3,n}}}{a_{3,n}} - \frac{y^{a_{2,n}}}{a_{2,n}} \right] \right\} + \frac{A_{c,n}^{-1}\rho_c}{(\pi\lambda_b)^{\alpha_{c,n}/2}\left(1+\frac{\alpha_{c,n}}{2}\right)} \tag{10}$$

where $\rho_c' = 2\pi\lambda_b\rho_c$, $y = \min(r_c, R)$ $a_{2,j} = (\alpha_{c,j}+2)$ *and* $a_{3,j} = (\alpha_{c,j}+3)$, $j = \{l, n\}$.

Proof: For tractability, we approximate the hexagonal cell with a circular cell of same area $1/\lambda_b$. The radius of the cells is then given as $R = (\pi\lambda_b)^{-\frac{1}{2}}$. Taking expectation of (7) over the distance gives $\mathbb{E}[P_c] = \int_0^R \rho_c l_c^{-1}(x)\, f_X(x)\, dx$, where $f_X(x)$ is the distribution of the distance of the UE from its BS. Since the tagged user is uniformly distributed in πR^2, $f_X(x) = \frac{2x}{R^2} = 2\pi\lambda_b x$. We get

$$\mathbb{E}[P_c] = 2\pi\lambda_b\rho_c \Bigg[A_{c,l}^{-1} \int_0^y \left(1 - \frac{x}{r_c}\right) x^{\alpha_{c,l}+1} dx + \frac{A_{c,n}^{-1}}{r_c} \int_0^y x^{\alpha_{c,n}+2} dx + A_{c,n}^{-1} \int_y^R x^{\alpha_{c,n}+1} dx \Bigg].$$

Solving the above integrals results in the expression in (10). ∎

The following Lemma gives the expected transmit power of the D2D UEs.

Lemma 2. *The average power of a UE in the D2D mode with channel inversion power control and 3GPP path loss model for the UE-UE link is given as*

$$\mathbb{E}[P_d] = K \Bigg[\frac{\omega\left(y_{fs}, 0, b_1(\alpha_{fs})\right)}{A_{fs} z^{b_1(\alpha_{fs})}} + \frac{\omega\left(y_d, y_{fs}, b_1(\alpha_{d,l})\right)}{A_{d,l} z^{b_1(\alpha_{d,l})}} + \frac{r_d'\, \omega\left(\mu, y_d, b_2(\alpha_{d,l})\right)}{A_{d,l} z^{b_2(\alpha_{d,l})}} - \frac{r_d'\, \omega\left(\mu, y_d, b_2(\alpha_{d,n})\right)}{A_{d,n} z^{b_2(\alpha_{d,n})}} + \frac{\omega\left(\mu, y_d, b_1(\alpha_{d,n})\right)}{A_{d,n} z^{b_1(\alpha_{d,n})}} \Bigg], \tag{11}$$

where $y_{fs} = \min(r_{fs}, \mu)$, $y_d = \min(r_d', \mu)$, $z = \pi\zeta$, $K = z\rho_d/1 - \exp(-z\mu^2)$, $b_1(a) = 1 + a/2$, $b_2(a) = (1+a)/2$, $\omega(x_1, x_2, b) = \Gamma(zx_1^2, b) - \Gamma(zx_2^2, b)$ *and* $\Gamma(x, a) = \int_0^x t^{a-1} \exp(-t)\, dt$, *is the lower incomplete Gamma function.*

Proof: The proof is along similar lines as that for Lemma 1. The expected D2D transmit power can be represented as

$$\mathbb{E}[P_d] = \int_0^\mu \rho_d l_d^{-1}(x)\, f_{L|L<\mu}(x)\, dx, \tag{12}$$

where $f_{L|L<\mu}(x) = f_L(x)/1 - \exp(-\zeta\pi\mu^2)$. Substituting (3) and (5) and into (12) and evaluating the piecewise integral, we obtain $\mathbb{E}[P_d]$ in (11). ∎

IV. Analysis of Coverage and Throughput

The SINR at the intended receiver is characterized as $SINR_i = \frac{\rho_i h}{I_i + \sigma^2}, i = \{c, d\}$, where σ^2 is the noise power and I_i is the interference power at the receiver. Due to the exponentially distributed channel power h, the probability that the SINR is greater than a certain modulation dependent threshold is expressed as

$$\Upsilon_i = \mathbb{P}\left[SINR_i \geq \theta_i\right] = \exp\left(-s_i\sigma^2\right) \mathcal{L}_{I_i}(s_i), \tag{13}$$

where $s_i = \frac{\theta_i}{\rho_i}$ with $i = \{c, d\}$. $\mathcal{L}_{I_i}(s_i)$ is the Laplace transform of the interference. It is evident from (13) that in order to fully characterize the cellular and D2D coverage probabilities, we need to obtain expressions for $\mathcal{L}_{I_c}(s_c)$, and $\mathcal{L}_{I_d}(s_d)$. The following theorem gives the Laplace transform of interference in the cellular mode.

Theorem 1. *The Laplace transform of interference on the BS from the cellular UEs outside the cell using the 3GPP path loss*

$$\mathcal{L}_{I_c}(s_c) = \exp\Bigg(-2\pi\lambda_b\Bigg\{\frac{\alpha_{c,n}^{-1}y^{2-\alpha_{c,n}}}{k_{c,n}(1-2/\alpha_{c,n})}\xi_1\left(\alpha_{c,n},k_{c,n},y\right)+\frac{1}{2}\left[y^2\xi_2\left(\alpha_{c,l},k_{c,l},y\right)-R^2\xi_2\left(\alpha_{c,l},k_{c,l},R\right)\right] + \frac{1}{3r_c}\left[y^3\xi_3\left(\alpha_{c,l},k_{c,l},y\right)-R^3\xi_3\left(\alpha_{c,l},k_{c,l},R\right)+y^3\xi_3\left(\alpha_{c,n},k_{c,n},y\right)-R^3\xi_3\left(\alpha_{c,n},k_{c,n},R\right)\right]\Bigg\}\Bigg), \quad (8)$$

$$\mathcal{L}_{I_d}\left(s_d\right) = \exp\Bigg(-\pi\lambda_d\Bigg\{r_{fs}^2\xi_2\left(\alpha_{fs},k_{fs},r_{fs}\right)+r_d^{'2}\xi_2\left(\alpha_{d,l},k_{d,l},r_d'\right)-r_{fs}^2\xi_2\left(\alpha_{d,l},k_{d,l},r_{fs}\right) + \frac{2r_d^{'2-\alpha_{d,l}}\xi_0\left(\alpha_{d,l},k_{d,l}r_d'\right)}{\alpha_{d,l}k_{d,l}\left(1-1/\alpha_{d,l}\right)}-\frac{2r_d^{'2-\alpha_{d,n}}\xi_0\left(\alpha_{d,n},k_{d,n},r_d'\right)}{\alpha_{d,n}k_{d,n}\left(1-1/\alpha_{d,n}\right)}+\frac{r_d^{'2-\alpha_{d,n}}\xi_1\left(\alpha_{d,n},k_{d,n},r_d'\right)}{\alpha_{d,n}k_{d,n}\left(1-2/\alpha_{d,n}\right)}\Bigg\}\Bigg) \quad (9)$$

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model for UE-BS link and channel inversion power control is given by (8),

where $y = \max\left(r_c, R\right),\ k_{c,j} = \left(s_c\mathbb{E}\left[P_c\right]A_{c,j}\rho_c\right)^{-1}, j = \{l,n\},\ \xi_1\left(a,k,x\right) = {}_2F_1\left(1,2/a;1-2/a;-\left(kx^a\right)^{-1}\right)$, $\xi_2\left(a,k,x\right) = {}_2F_1\left(1,2/a;1+2/a;-kx^a\right)$, $\xi_3\left(a,k,x\right) = {}_2F_1\left(1,3/a;1+3/a;-kx^a\right)$ *and* ${}_2F_1\left(a,b;c;x\right)$ *is the generalized hypergeometric function [11].*

Proof: The active interfering cellular users constitute a HPPP $\Phi_{c,a}$ with intensity λ_b as only one interfering user is present in a cell. The interference in this case is characterized as $I_c = \sum_{\mathbf{x}_m\in\Phi_{c,a}\backslash\mathbf{o}} P_{c_m}h_m l_c\left(\|\mathbf{x}_m\|\right)$.

The Laplace transform is then given as

$$\mathcal{L}_{I_C}\left(s_c\right) = \exp\left(-s_c\sum_{\mathbf{x}_m\in\Phi_{c,a}\backslash\mathbf{o}}P_{c_m}h_m l_c\left(\|\mathbf{x}_m\|\right)\right) \overset{(a)}{\approx} \exp\left(-2\pi\lambda_b\int_R^\infty\frac{x}{1+\left(s_c\mathbb{E}\left[P_c\right]l_c\left(x\right)\right)^{-1}}dx\right),$$

where (a) follows from the probability generating functional (PGFL) of PPP [10] and employing Jensen's inequality for the expectation of power and averaging with respect to the channel power. The lower limit of integration is the minimum separation distance between the typical BS and the nearest interfering user. Substituting (1), (2), and (10) and evaluating the piece-wise integral gives the Laplace transform in (8). ■

Corollary 1. *For the realistic case of* $R > r_d'$, $y = R$ *in* (8) *then* $\mathcal{L}_{I_C}\left(s_c\right)$ *reduces to*

$$\mathcal{L}_{I_C}\left(s_c\right) = \exp\left(-\frac{\delta_{c,n}\left(\pi\lambda_b\right)^{1/\delta_{c,n}}}{k_{c,n}\left(1-\delta_{c,n}\right)}\xi_1\left(\alpha_{c,n},k_{c,n},\sqrt{\frac{1}{\pi\lambda_b}}\right)\right), \quad (14)$$

where $\delta_{c,n} = 2/\alpha_{c,n}$.

The Laplace transform of aggregate interference for the D2D links is given in the following theorem.

Theorem 2. *The Laplace transform of interference on the typical D2D receiver from other UEs transmitting in D2D mode using the 3GPP path loss model for UE-UE link and channel inversion power control is given by (9),*

where $k_{d,j} = \left(s_d\mathbb{E}\left[P_d\right]A_{d,j}\rho_d\right)^{-1}, j = \{fs,l,n\}$ *and* $\xi_0\left(a,k,x\right) = {}_2F_1\left(1,1/a;1-1/a;-\left(kx^a\right)^{-1}\right)$.

Proof: The proof follows similar steps to the proof of Theorem 1 with the exception that the interfering UEs include all active D2D UEs $\mathbf{x}_m\in\Phi_d$ and the minimum separation distance between the typical receiver and the interfering UE is zero. ■

Parameter	Value
$\lambda_b,\lambda_u,\zeta,\varepsilon,\beta$	$[1,100,15]/\pi500^2, 0.5, 0.2$
$A_{c,l},A_{c,n}$	$10^{-3.08}, 10^{-0.27}$
$A_{fs},A_{d,l},A_{dn}$	$10^{-3.302}, 10^{-3.08}, 10^{-0.27}$
$\alpha_c,\alpha_{c,l},\alpha_{c,n}$	$3.5, 2.42, 4.28$
$\alpha_d,\alpha_{fs},\alpha_{d,l},\alpha_{d,n}$	$4, 2.27, 4, 4.375$
r_c,r_{fs},r_d',μ	300m, $q(2GHz)$m, 23m, 100m
ρ_c,ρ_d,σ^2	-70dBm,-70dBm,-100dBm

Table I: Simulation parameters

V. Results and Discussion

The first step is to validate our analysis for the D2D and cellular coverage probability using Theorem 1 and 2. For the network simulations, we generate a hexagonal grid cellular network, where the area of each cell is $1/\lambda_b$. The users are distributed uniformly in each realization, where the number of users in each iteration is Poisson distributed with parameter λ_u. We use the values listed in Table I unless stated otherwise.

Figs. 2 and 3 show that the Monte Carlo simulation results for D2D and cellular coverage closely match our theoretical analysis. We also compare our proposed model with the analysis in [2]. By setting the path loss model $l_i(x) = x^{-\alpha_i}$, $A_{i,j} = 1$ and $\alpha_{i,j} = \alpha_i$, $i = \{c,d\}, j = \{n,l,fs\}$ in (13), our model reduces to the reference model in [2]. The Laplace transform of cellular interference for the reference model is given as

$$\mathcal{L}_{I_c}^{ref}\left(s_c\right) = \exp\left(-\frac{\delta_c\left(\pi\lambda_b\right)^{1/\delta_c}}{k_c\left(1-\delta_c\right)}\xi_1\left(\alpha_c,k_c,\left(\pi\lambda_b\right)^{-1/2}\right)\right), \quad (15)$$

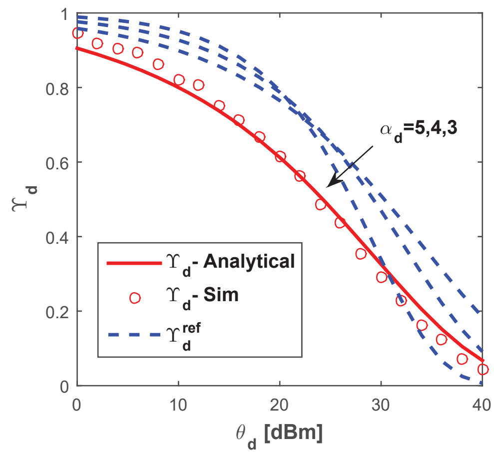


Figure 2: D2D coverage probability.

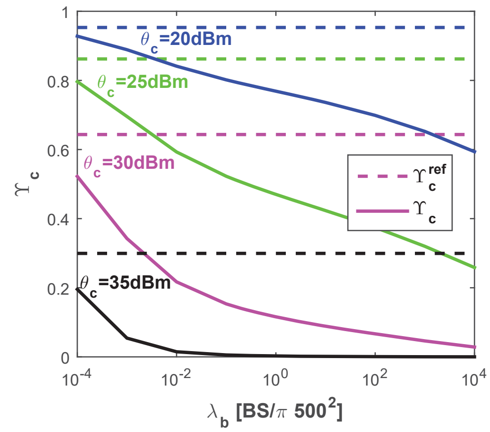


Figure 4: Effect of BS intensity on the cellular coverage probability.

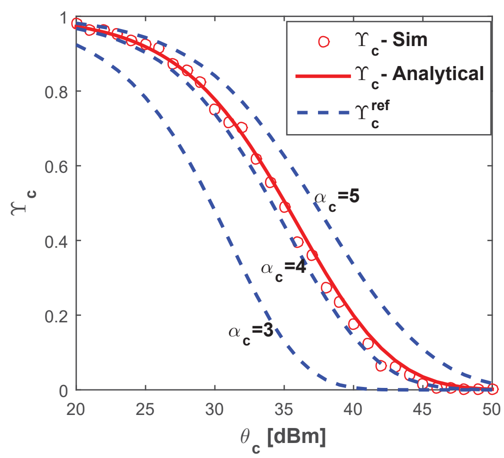


Figure 3: Cellular coverage probability.

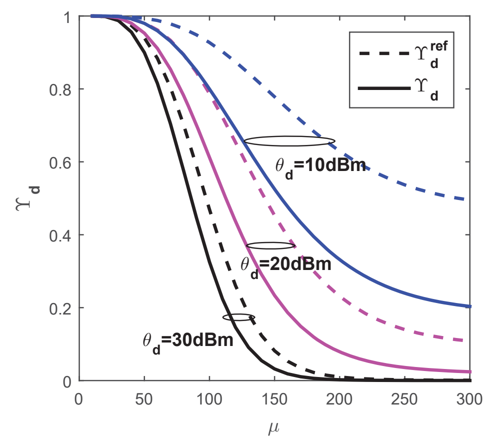


Figure 5: Effect of mode selection threshold on the D2D coverage probability.

where $k_c = \left(s_c \mathbb{E}\left[P_c^{ref}\right] \rho_c\right)^{-1}$. Similarly, the D2D interference for the baseline model is given as

$$\mathcal{L}_{I_d}^{ref}\left(s_d\right) = \exp\left(-\frac{\pi\lambda_d}{\text{sinc}(\delta_d)} k_d^{\delta_d}\right), \quad (16)$$

where $k_d = s_d \mathbb{E}\left[P_d^{ref}\right] \rho_d$. The plots reveal that the D2D coverage with the 3GPP path loss model significantly deviates from the simplistic approach in [2]. This is because of the piece-wise nonlinearity in the path loss model described in (5). The cellular coverage however, follows a similar trend when the network is sparse ($\lambda_b = 1/\pi 500^2$) as the cellular path loss exponent remains fairly constant for the users. We can see from (14) that the Laplace transform of cellular interference is essentially equal to (15) when $\alpha_{c,n} = \alpha_c$ and $A_{c,n} = 1$.

The behavior of cellular coverage with the increasing BS density is studied with the help of Fig. 4. The reference cellular coverage is not affected by the change in λ_b. This is due to the channel inversion power control, as the cell size goes small, the interference power also decreases accordingly. This ideal behavior is not observed in reality with the 3GPP path loss model and we see that as λ_b grows, the chances of having LoS interference links also increases, which intensifies the interference power. Hence, the probability of cellular coverage drops with the increase in λ_b.

Fig. 5 shows the effect of varying the mode selection threshold on the D2D coverage probability. The D2D coverage decays more steeply than the reference when the mode selection threshold is increased. This is because as μ increases, number of D2D transmitters increases. In the case of the proposed model, it also implies that the density of interfering UEs with free space path loss and LoS also increases.

A. *Average throughput*

The average throughput of the network under discussion is the sum of the rates of all active links normalized over the transmission bandwidth and unit area. It is expressed as

$$\mathcal{T} = \lambda_c(1-\beta)R_c + \lambda_d \beta R_d, \text{ bps/Hz/m}^2 \quad (17)$$

where R_c and R_d are the expected rates of the cellular and D2D links respectively and β is the spectrum resource partition factor. Using Shannon's capacity formulation, the rates per unit bandwidth can easily be expressed as

$$
\begin{aligned}
R_c &= \mathbb{E}\left[\frac{1}{N}\log_2\left(1+\Upsilon_c\right)\right] \\
&= \frac{\lambda_b}{\lambda_c}(1-\exp(-\lambda_c/\lambda_b))\, R'_c, \qquad (18)
\end{aligned}
$$

where $R'_c = \mathbb{E}\left[\log_2\left(1+\Upsilon_c\right)\right]$ premultiplied by the term equal to $\mathbb{E}\left[1/N\right]$, which is the expectation taken over the number of users N including the tagged UE. The D2D rate is similarly expressed as $R_d = \mathbb{E}\left[\log_2\left(1+\Upsilon_d\right)\right]$. The average throughput is then given as

$$
\mathcal{T} = \lambda_b(1-\beta)(1-\exp(-\lambda_c/\lambda_b))R'_c + \lambda_d\beta R_d \qquad (19)
$$

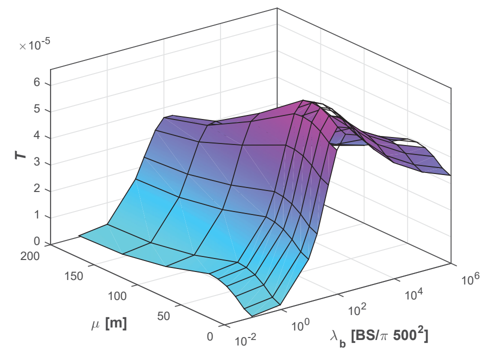
The expectation for R'_c and R_d are computed as $\mathbb{E}\left[\log_2\left(1+\Upsilon_i\right)\right] = \int_{z>0}(1+z)^{-1}\Upsilon_i(z)\,dz$.

We wish to see how the variation in λ_b and μ impacts the throughput of our proposed model and the reference model as the throughput is a function of both the D2D and cellular coverage. Fig. 6a shows that for the proposed coverage model, the average throughput first increases with an increase in λ_b and μ and attains a maximum value at a point (λ_b^*, μ^*) after which it decays. The increase with respect to μ is attributed to the fact that initially, the activation of more D2D users offloads cellular traffic and enables spatial frequency reuse. However, after a certain value of μ, the interference due to further activation of D2D UEs becomes dominant and reduces the average throughput. Recall from Fig. 4 that the increase in λ_b results in a decrease in cellular coverage and hence the cellular rate, but this decrease is initially overcome with the increase in λ_b. But after a certain value of λ_b, the throughput beings to decrease. This value of $\lambda_b = \lambda_b^*$ is irrespective of the value of μ. This is because, R'_c is the only term in (18) which depends on λ_b and it is independent of μ. This, however, is not the case for μ as it appears in both terms in (19). The optimal point (λ_b^*, μ^*) is obtained numerically and is equal to $(200/\pi 500^2, 40\text{m})$.

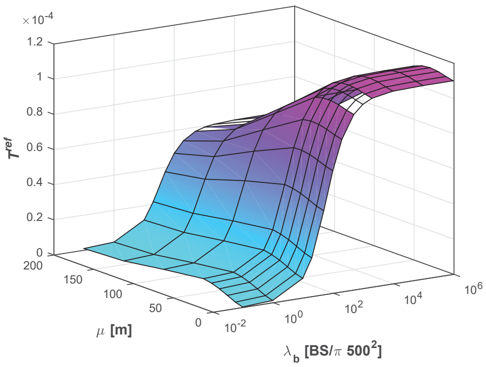
For the purpose of comparison, the average throughput for the reference model is displayed in Fig. 6. A similar trend is observed with the variation in μ but a striking difference is seen for the variation in λ_b. This is because, due to a simplistic path loss assumption, the cellular coverage, and hence the cellular rate, does not change with respect to λ_b and as λ_b grows, (19) converges to $\lambda_c(1-\beta)R_c^{'ref} + \lambda_d\beta R_d^{ref}$ as $\lim\limits_{\lambda_b\to\infty} \lambda_b(1-\exp(-\lambda_c/\lambda_b)) = \lambda_c$.

VI. Conclusion

This paper analyzes the cellular networks overlaid with D2D communication in the UL using path loss models recommended by the 3GPP and compares the coverage and throughput with the baseline model in [2], which uses a simple power law path loss model and does not differentiate between the LoS, NLoS and free space regimes. The realistic path loss model significantly impacts the coverage and throughput results. A major difference is that our theoretical results confirm that as the density of the BSs grows, there is no perfect interference cancellation as suggested by the reference model.



(a) Existence of an optimal (μ^*, λ_b^*) pair with 3GPP recommended path loss model.



(b) Reference model [2].

Figure 6: Average network throughput for various values of mode selection threshold and BS intensity.

References

[1] A. Asadi, Q. Wang, and V. Mancuso, "A survey on device-to-device communication in cellular networks," *IEEE Communications Surveys & Tutorials*, vol. 16, no. 4, pp. 1801–1819, 2014.

[2] X. Lin, J. G. Andrews, and A. Ghosh, "Spectrum sharing for device-to-device communication in cellular networks," *IEEE Transactions on Wireless Communications*, vol. 13, no. 12, pp. 6727–6740, 2014.

[3] H. ElSawy, E. Hossain, and M.-S. Alouini, "Analytical modeling of mode selection and power control for underlay D2D communication in cellular networks," *IEEE Transactions on Communications*, vol. 62, no. 11, pp. 4147–4161, 2014.

[4] T. Bai and R. W. Heath, "Coverage and rate analysis for millimeter-wave cellular networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, pp. 1100–1114, 2015.

[5] X. Zhang and J. G. Andrews, "Downlink cellular network analysis with multi-slope path loss models," *CoRR*, vol. abs/1408.0549, 2014. [Online]. Available: http://arxiv.org/abs/1408.0549

[6] M. Ding, D. Lopez-Perez, G. Mao, P. Wang, and Z. Lin, "Will the area spectral efficiency monotonically grow as small cells go dense?" *arXiv preprint arXiv:1505.01920*, 2015.

[7] M. Di Renzo, "Stochastic geometry modeling and analysis of multi-tier millimeter wave cellular networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 9, pp. 5038–5057, 2015.

[8] "3GPP TR 36.828: Further enhancements to lte time division duplex (TDD) for (DL-UL) interference management and traffic adaptation," 2012.

[9] "3GPP TR 36.843: Study on LTE device to device proximity services; radio aspects," 2014.

[10] M. Haenggi, *Stochastic geometry for wireless networks*. Cambridge University Press, 2012.

[11] G. E. Andrews, R. Askey, and R. Roy, "Special functions (encyclopedia of mathematics and its applications vol 71)," 1999.