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Trade and Synchronization in a Multi-Country Economy*

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Abstract

Countries with strong trade linkages have more synchronized business cycles. However, the standard international business cycle framework cannot replicate this finding, uncovering the trade-comovement puzzle. Modeling trade using more sophisticated micro-level assumptions does not help resolve the puzzle. This happens because for a large class of trade models, under certain macro-level conditions, output comovement is determined by the same factor structure. We show that in such models comovement can be explained by three factors: (i) the correlation between each country's TFP; (ii) the correlation between each country's share of expenditure on domestic goods; and (iii) the correlation between each country's TFP and the partner's share of expenditure on domestic goods. An empirical investigation of the link between trade and each of the three factors shows that the trade-comovement relation is in large part explained by the second factor while in the theoretical model this factor reacts counterfactually to changes in trade costs.

Keywords: Business Cycle Synchronization, International Trade.
JEL Classification: F15; F41; E30.

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1 Introduction

Substantial empirical evidence suggests that countries or regions with stronger trade linkages have more correlated business cycles. Frankel and Rose (1998), Clark and van Wincoop (2001), Calderon, Chong, and Stein (2007), Baxter and Kouparitsas (2004), and Imbs (2004), among others, show that pairs of countries that trade with each other exhibit a high degree of business cycle comovement. These findings have been interpreted as evidence that greater trade integration leads to business cycle synchronization. However, from a theoretical perspective the standard international real business cycle model (IRBC), based on Backus, Kehoe, and Kydland (1994), has difficulties in replicating this empirical fact (see Kose and Yi, 2001 and 2006). In the latter paper, the authors' baseline model explains only one-tenth of the responsiveness of comovement to trade intensity. This has given rise to the so-called trade-comovement puzzle: standard models are unable to generate high output correlations arising from high bilateral trade intensity.

In the conventional IRBC framework, trade is modeled using the Armington specification, which imposes an exogenous trade specialization pattern. In the Armington framework trade adjustments are only at the intensive margin. By contrast, in the new trade theory, trade shares also adjust at the extensive margin: when a country's relative efficiency declines it exports a narrower range of goods. We show that modeling trade linkages using more sophisticated micro-level assumptions does not help resolve the trade-comovement puzzle. This happens because output comovement in an important class of trade models is determined by the same factor structure.

The main contribution of this paper is to show that under certain macro-level assumptions, in a large class of trade models, output correlation between country i and country j, denoted $\text{cor}\left(\widetilde{\mathscr{Y}_i}, \widetilde{\mathscr{Y}_j}\right)$, is explained by three factors: (i) the correlation between each country's total factor productivity (TFP), denoted $\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right)$; (ii) the correlation between each country's share of expenditure on domestic goods, $\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$; and (iii) the correlation between a country's share of expenditure on domestic goods and the partner's TFP, $\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right)$. This relation is summarized by the following equation:

$$\text{cor}\left(\widetilde{\mathscr{Y}_i}, \widetilde{\mathscr{Y}_j}\right) = \beta_1 \,\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right) + \beta_2 \,\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right) + \beta_3 \,\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right). \quad (1)$$

Therefore, the ability of a model to generate higher output synchronization arising from increased bilateral trade depends on the extent to which trade integration affects each of these three factors. This result relates to recent work by Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008) and Arkolakis, Costinot, and Rodriguez-Clare (2012) concerning the welfare gains from trade. These authors show that the real wage (which determines the welfare gains from trade) can be computed as a function of the import penetration ratio and an elasticity parameter that, depending on the particular micro-level assumptions, relates either to preferences or technology. In particular, Arkolakis et al. (2012) show that the gains from trade have the same form in a large class of trade models including the Armington model, Eaton and Kortum (2002), Bernard, Eaton, Jensen, and Kortum (2003), Krugman (1980), and multiple versions of Melitz (2003).

In the context of the IRBC model that concerns us, the labor supply responds to changes in the real wage (which is a function of the share of expenditure on domestic goods). It follows that, in the absence of short-run wealth effects on the labor supply, the share of expenditure on domestic goods, a parameter related to the labor supply elasticity and the trade elasticity are the only determinants of employment and output fluctuations in response to foreign shocks.

Using data on bilateral trade in manufacturing, TFP and output correlations for a panel of 21 OECD countries, we show that the two factors that explain the trade-comovement relation empirically are the correlation between each country-pair's TFP and the correlation between each country-pair's share of expenditure on domestic goods. To investigate the quantitative implications of this result, our starting point is a multi-country model of international trade with endogenous specialization inspired by Eaton and Kortum (2002). We embed it into an IRBC framework by including country specific fluctuations in TFP, allowing for endogenous labor supply, intertemporal substitution in consumption and complete financial markets. We show that the trade-comovement puzzle arises because higher trade integration counterfactually lowers the correlation between each country's share of expenditure on domestic goods.

When we allow for the correlation of TFP shocks to increase with trade, as observed in the data, the model is more successful in replicating the link between trade and comovement. However, we show that even in the presence of correlated shocks, the model still implies a counterfactual relation between bilateral trade and each country pair's correlation of the share of expenditure on domestic goods. The reason for this result is twofold. First, a TFP shock implies a deterioration in the terms-of-trade. Second, the elasticity of the share of expenditure on domestic goods to changes in the terms-of-trade is increasing in the level of bilateral trade. Therefore, following a TFP shock at Home, the Foreign share of expenditure on domestic goods will fall by more, the higher the level of trade integration between Home and Foreign.

In our model, Home TFP improvements are transmitted positively to the trade-partners through a deterioration of Home's terms-of-trade. This is a common feature in most IRBC models. However, this feature is shown to be counterfactual for many countries. For example, Corsetti, Dedola and Leduc (2008 and 2014) show that the terms-of-trade improve conditional on positive TFP shocks, in particular in large countries and relatively less open economies. Raffo (2008) finds that, contrary to what is implied by the standard IRBC model, the terms-of-trade do not have a strong cyclical pattern. We show that with countercyclical terms-of-trade it is not possible for trade integration to raise the correlations of the share of expenditure on domestic goods. Therefore, this provides more evidence in favor of models with acyclical or procyclical terms-of-trade. Ghironi and Melitz (2005) and also Corsetti et al. (2008) develop models that imply procyclical terms-of-trade.

Our work is related to a strand of the literature that extends the IRBC model by changing the micro-level assumptions about trade to improve the model's ability to explain the empirical association between trade and business cycle synchronization. Most closely related to our work is Arkolakis and Ramanarayanan (2009), who develop a two-country IRBC model based on the Eaton and Kortum (2002) framework. Unlike what we do in this paper, they augment their model with vertical specialization and conclude that vertical specialization alone is insufficient to solve the trade-comovement puzzle. They suggest that allowing for variable markups combined with vertical specialization may be helpful to resolve the trade-comovement puzzle. While also focusing on the link between trade and comovement, we differ from their paper in purpose and scope. Our aim is to derive a general framework to understand why standard models fail to account for the link between trade and business cycle synchronization and to identify the sources of the trade-comovement puzzle.

Our paper also contributes to the growing literature on the trade-comovement puzzle. Burstein, Kurz, and Tesar (2008) and di Giovanni and Levchenko (2010) emphasize the relevance of vertical linkages in production to explain the effects of bilateral trade on business cycle synchronization. However, Johnson (2014) finds evidence against the idea that input trade is the missing link to understand the trade-comovement relation, as comovement in value added is not greatly increased with trade integration. In a related study, Liao and Santacreu (2015) build a model based on Ghironi and Melitz (2005), where the dynamics of TFP are driven by extensive margin changes in the number and average productivity of domestic and foreign intermediate producers. Their model is able to produce endogenous TFP correlations consistent with the data. Finally, Drozd, Kolbin, and Nosal (2014) show that the ability of the business cycle model to account for the link between trade and comovement crucially depends on the dynamic properties of the trade elasticity. This happens because the trade elasticity plays a major role in determining how business cycle shocks spill over across borders. In particular, they show that short-run frictions generate a low short-run elasticity and a high long-run elasticity which allows for theory to be closer to the data.

The rest of the paper is organized as follows. Section 2 describes our equilibrium model of trade and the business cycle used to analyze the relation between trade integration and business cycle synchronization. In Section 3 we study in depth the channels through which trade affects business cycle synchronization and, in particular, derive equation (1) that represents the factor structure of GDP comovement. Section 4 presents the main empirical and quantitative findings of the paper. In Section 5 we investigate the robustness of our results to changes in the assumptions about the financial market structure, preferences and production. Finally, Section 6 concludes.

2 The Theoretical Economy

In this Section, we develop a simple model of the world economy to study the link between trade integration and business cycle synchronization. The setup of the model builds on Eaton and Kortum (EK, 2002) and Alvarez and Lucas (2007), but it incorporates intertemporal substitution in consumption and endogenous labor supply. The world economy consists of n countries. Each country is populated with a continuum (unit measure) of identical households. At each date t, the world economy experiences one of finitely many states, or events, $s_t \in \mathcal{S} \equiv [s(1), \ldots, s(k)]$. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events through period t. The probability of any particular event s_{t+1} conditional on history s^t is $\pi(s_{t+1}|s^t)$. The initial realization s_0 is given.

2.1 Technology and Market Structure

Each country has two sectors: a traded intermediate good sector and a non-traded final good sector. The intermediate good sector produces a continuum of traded differentiated inputs using labor. The final good sector is represented by a stand-in firm that produces a non-traded good that is a composite of the intermediate varieties. All markets are perfectly competitive. In what follows we describe each sector in greater detail.

Final Good Sector

The stand-in final good firm in country i makes use of a continuum of differentiated manufactured intermediate commodities indexed by $v \in [0,1]$ that are combined as follows:

$$Y_i\left(s^t\right) = \left[\int_0^1 m_i\left(v, s^t\right)^{\phi} dv\right]^{1/\phi}, \tag{2}$$

where $m_i\left(v, s^t\right)$ is the input of the differentiated intermediate commodity of type v. The parameter $\phi \in (0,1)$ relates to the elasticity of substitution across differentiated intermediate commodities, given by $\sigma = 1/\left(1-\phi\right)$. From (2), it follows that the demand in country i for intermediate variety v satisfies the relation

$$m_i\left(\nu, s^t\right) = \left[\frac{p_i\left(v, s^t\right)}{\mathscr{P}_i\left(s^t\right)}\right]^{-\sigma} Y_i\left(s^t\right), \tag{3}$$

where $\mathscr{P}_i\left(s^t\right) = \left[\int_0^1 p_i\left(v, s^t\right)^{1-\sigma} dv\right]^{1/(1-\sigma)}$ is the *ideal price index* in country i of the composite of intermediate commodities, with $p_i\left(v, s^t\right)$ the price of intermediate variety v in country i. The stand-in final good firm is perfectly competitive and, hence, the final good's price is also $\mathscr{P}_i\left(s^t\right)$.

Intermediate Good Sector

The structure of the intermediate-good sector is as in EK where technology differences are modeled using a probabilistic approach. Countries have differential access to technology, so efficiency varies across commodities and countries. The intermediate commodities are subject to trade barriers that take the form of an *iceberg cost*: to successfully deliver in country j one unit of any differentiated intermediate commodity produced in country i, $\tau_{ji} \geq 1$ units need to be shipped, with $\tau_{ii} = 1$.

Producing one unit of the intermediate commodity v in country i requires $\left[A_i\left(s^t\right)\varphi_{i,v}\right]^{-1}$ units of labor, where $A_i\left(s^t\right)$ follows a mean one, serially correlated discrete Markov process and is independent across countries. Therefore, the cost for intermediate firms in country i to deliver one unit of intermediate commodity v to country j is

$$p_{ji}\left(v, s^t\right) = \left[\frac{W_i\left(s^t\right)}{A_i\left(s^t\right)\varphi_{i,v}}\right]\tau_{ji}, \tag{4}$$

where $W_i\left(s^t\right)$ is the wage rate in country i. There is perfect competition, so country i firms potentially sell the commodity v to country j at price $p_{ji}\left(v, s^t\right)$. The commodity is purchased from the lowest-cost supplier; hence, the price of commodity v in country j is given by

$$p_j\left(v, s^t\right) = \min_{i=1,\dots,n}\left[p_{ji}\left(v, s^t\right)\right]. \tag{5}$$

The variable $\varphi_{i,v}$ determines the efficiency of country i to produce input v. We follow EK and model firms' efficiency using a probabilistic approach: it is assumed that country i's efficiency in producing commodity v is the realization of a random variable φ, which is drawn independently for each v. Country i's efficiency follows a Fréchet distribution:

$$F_i\left(\varphi\right) = \exp\left(-T_i\varphi^{-\theta}\right), \tag{6}$$

where $\theta > 1$ and $T_i > 0$. The parameter θ controls the degree of heterogeneity across firms, with higher θ implying less heterogeneity, and T_i controls the country's overall efficiency. We consider a symmetric world economy and, henceforth, assume that T_i is the same across countries, normalized to unity. The upshot is that in equilibrium the *ideal price index* in country j is

$$\mathscr{P}_j\left(s^t\right) = \kappa\Phi_j\left(s^t\right)^{-1/\theta}, \tag{7}$$

where $\kappa > 0$ is a constant and the random variable

$$\Phi_j\left(s^t\right) = \sum_{i=1}^{n}\left[\frac{W_i\left(s^t\right)\tau_{ji}}{A_i\left(s^t\right)}\right]^{-\theta}, \tag{8}$$

determines the distribution of prices.[1]

[1] $\kappa = \left[\Gamma\left(\frac{1-\sigma+\theta}{\theta}\right)\right]^{1/(1-\sigma)}$, where $\Gamma(.)$ is the Gamma function.

Bilateral Trade Flows

The probability λ_{ji} that country i is the lowest-cost supplier to j for any particular intermediate commodity is given by[2]

$$\lambda_{ji}\left(s^{t}\right)=\left[\frac{\kappa W_{i}\left(s^{t}\right)\tau_{ji}}{A_{i}\left(s^{t}\right)\mathscr{P}_{i}\left(s^{t}\right)}\right]^{-\theta}. \tag{9}$$

Since there are a continuum of intermediate goods, the probability λ_{ji} also corresponds to country j's expenditure on country i's differentiated intermediate goods as a fraction of country j's total expenditure on differentiated intermediate goods. This measure of bilateral trade intensity is closely linked to one of the measures proposed by Frankel and Rose (FR, 1998), which is the sum of a country's bilateral exports divided by the sum of each country's aggregate net income.

2.2 Preferences and Financial Markets

The stand-in household in country i has preferences represented by a utility function of the form introduced by Greenwood et al. (1988), given by

$$u\left(C_{i},L_{i};s^{t}\right)=\ln\left[C_{i}\left(s^{t}\right)-\frac{L_{i}\left(s^{t}\right)^{1+1/\sigma_{n}}}{1+1/\sigma_{n}}\right], \tag{10}$$

where $C_{i}\left(s^{t}\right)$ and $L_{i}\left(s^{t}\right)$ are, respectively, consumption and time spent working by the stand-in household. The parameter σ_{n} corresponds to the Frisch elasticity of labor supply. The choice of preferences does not have wealth effects and therefore excludes intertemporal substitution in the labor choice.[3]

There are complete financial markets. In particular, we assume the existence of a complete set of Arrow securities denominated in units of the numéraire. Thus, the budget constraint of the stand-in agent in country i is

$$\mathscr{P}_{i}\left(s^{t}\right)C_{i}\left(s^{t}\right)+\sum_{s\in\mathcal{S}}\mathcal{Q}\left(s^{t},s\right)B_{i}\left(s^{t},s\right)=W_{i}\left(s^{t}\right)L_{i}\left(s^{t}\right)+B_{i}\left(s^{t-1},s_{t}\right), \tag{11}$$

[2]This probability is obtained by calculating $\lambda_{ji}\left(s^{t}\right)=\text{Prob}\left(p_{ji}\left(v,s^{t}\right)\leq\min\left[p_{jl}\left(v,s^{t}\right);l\neq i\right]\right)$.

[3]Jaimovich and Rebelo (2009) find evidence of a weak wealth effect in labor supply choices. Raffo (2008) shows that this choice of preferences in IRBC models helps reproducing the cyclical properties of quantities, including the correlation and volatility of trade variables with respect to output.

where $B_i\left(s^t, s\right)$ denotes the quantity of bonds purchased by the stand-in household in country i following history s^t, that entitles the holder to receive a payment worth one unit of the numéraire in period $t+1$ if the state of the economy at that date turns out to be $s \in \mathcal{S}$. The price of this state contingent security in units of the numéraire is denoted by $\mathcal{Q}\left(s^t, s\right)$. The Bellman equation characterizing the stand-in household optimal behavior is

$$V_i\left(s^t\right) = \max_{C_i, L_i}\left[u\left(C_i, L_i; s^t\right) + \beta \sum_{s_{t+1}\in\mathcal{S}} \pi\left(s_{t+1}|s^t\right) V_i\left(s^{t+1}\right)\right], \tag{12}$$

with $\beta \in (0,1)$ and subject to the budget constraint (11).

2.3 Equilibrium Conditions

The first-order conditions that characterize the solution to the problem of the stand-in household in country i are given by

$$\mathcal{Q}\left(s^t, s_{t+1}\right)\mu_i\left(s^t\right) = \beta\pi\left(s_{t+1}|s^t\right)\mu_i\left(s^{t+1}\right) \quad \forall\, s_{t+1} \in \mathcal{S}, \tag{13}$$

$$\mu_i\left(s^t\right) = \beta R_t \sum_{s_{t+1}\in\mathcal{S}} \pi\left(s_{t+1}|s^t\right)\mu_i\left(s^{t+1}\right), \tag{14}$$

$$L_i\left(s^t\right) = \left[\frac{W_i\left(s^t\right)}{\mathscr{P}_i\left(s^t\right)}\right]^{\sigma_n}, \tag{15}$$

with $R_t = \left[\sum_{s\in\mathcal{S}} \mathcal{Q}\left(s^t, s\right)\right]^{-1}$ and where

$$\mu_i\left(s^t\right) = \frac{1}{\mathscr{P}_i\left(s^t\right)}\left[C_i\left(s^t\right) - \frac{L_i\left(s^t\right)^{1+1/\sigma_n}}{1+1/\sigma_n}\right]^{-1}, \tag{16}$$

represents the marginal utility of current per capita wealth in units of the numéraire for the stand-in household in country i following history s^t. From equation (9), the real wage in country i can be expressed in terms of the technology shock A_i and the share of expenditure on domestic goods λ_{ii}, as follows

$$\frac{W_i\left(s^t\right)}{\mathscr{P}_i\left(s^t\right)} = \frac{A_i\left(s^t\right)}{\kappa}\left[\frac{1}{\lambda_{ii}\left(s^t\right)}\right]^{1/\theta}. \tag{17}$$

Substituting for the real wage in (15) using (17), it follows that equilibrium hours worked can be expressed as

$$L_i\left(s^t\right) = \left[\frac{A_i\left(s^t\right)}{\kappa}\right]^{\sigma_n} \left[\frac{1}{\lambda_{ii}\left(s^t\right)}\right]^{\sigma_n/\theta}. \tag{18}$$

The market clearing condition in financial markets requires

$$\sum_{i=1}^{n} B_i\left(s^t, s\right) = 0, \quad \forall\, s \in \mathcal{S}. \tag{19}$$

Equilibrium in the market for produced goods in each country i requires total domestic labor income $W_i\left(s^t\right) L_i\left(s^t\right)$ to equal world spending on domestically produced goods, so that

$$W_i\left(s^t\right) L_i\left(s^t\right) = \sum_{j=1}^{n} \lambda_{ji}\left(s^t\right) \left[W_j\left(s^t\right) L_j\left(s^t\right) + D_j\left(s^t\right)\right], \tag{20}$$

where

$$D_j\left(s^t\right) = B_j\left(s^{t-1}, s_t\right) - \sum_{s\in\mathcal{S}} \mathcal{Q}\left(s^t, s\right) B_j\left(s^t, s\right), \tag{21}$$

is country j's trade deficit. Of course, the market clearing condition (19) implies that $\sum_{i=0}^{n} D_i\left(s^t\right) = 0$.

Finally, using (21) to substitute in (11) we have

$$\mathscr{P}_i\left(s^t\right) C_i\left(s^t\right) = W_i\left(s^t\right) L_i\left(s^t\right) + D_i\left(s^t\right). \tag{22}$$

Sequential Competitive Equilibrium

Define the $n \times 1$ wage vector $\mathbb{W}_t = \left[W_1\left(s^t\right), \ldots, W_n\left(s^t\right)\right]'$, the $k \times 1$ security price vector $\mathbb{Q}_t = \left[\mathcal{Q}_1\left(s^t, s\left(1\right)\right), \ldots, \mathcal{Q}_k\left(s^t, s\left(k\right)\right)\right]'$, the $n \times 1$ consumption vector $\mathbb{C}_t = \left[C_1\left(s^t\right), \ldots, C_n\left(s^t\right)\right]'$, the $n \times 1$ employment vector $\mathbb{L}_t = \left[L_1\left(s^t\right), \ldots, L_n\left(s^t\right)\right]'$, and the $n \times k$ matrix that describes the security holdings $\mathbb{B}_t = \left[\mathscr{B}_1\left(s^t\right), \ldots, \mathscr{B}_n\left(s^t\right)\right]'$, where $\mathscr{B}_i\left(s^t\right) \equiv \left[B_i\left(s^t, s\left(1\right)\right), \ldots, B_i\left(s^t, s\left(k\right)\right)\right]$. A sequential competitive equilibrium in the world economy is an allocation $\{[\mathbb{C}_t, \mathbb{L}_t, \mathbb{B}_t]\}_{t=0}^{\infty}$, a wage sequence $\{\mathbb{W}_t\}_{t=0}^{\infty}$ and a price sequence for the Arrow securities $\{\mathbb{Q}_t\}_{t=0}^{\infty}$ such that at every date t: the stand-in household in each country behaves optimally given wages and prices; condition (19) is satisfied, so that financial markets clear; condition (20) is satisfied, so that product markets clear.

In the sequel, we consider a symmetric world economy with ex-ante identical countries, so that we have $\mu_i\left(s^0\right)=\bar{\mu}$ for all $i=1,\ldots,n$. The upshot is that, with complete markets, equation (13) implies that, for each history s^t, $\mu_i\left(s^t\right)=\mu_t$ for all $i,\ldots,n$. This corresponds to the familiar risk-sharing condition conditional on trade costs: the relative marginal utility of consumption between country i and country j is equal to their consumption based real exchange rate, as follows[4]

$$\frac{u'_c\left(C_i,L_i;s^t\right)}{u'_c\left(C_j,L_j;s^t\right)}=\frac{\mathscr{P}_i\left(s^t\right)}{\mathscr{P}_j\left(s^t\right)}. \tag{23}$$

Finally, from (14), μ_t must satisfy the familiar Euler equation

$$\mu_t=\beta R_t E_t\left(\mu_{t+1}\right), \tag{24}$$

with $E_t\left(\mu_{t+1}\right)=\sum_{s_{t+1}\in\mathcal{S}}\pi\left(s_{t+1}|s^t\right)\mu_{t+1}$, the conditional expectation of μ_{t+1} at date t.

Following standard steps, the individual optimality conditions and the market clearing conditions are log-linearized around the symmetric deterministic steady state equilibrium and combined so as to characterize the equilibrium dynamics.[5]

3 Trade and the Channels of Synchronization

In this Section we use the equilibrium conditions obtained in Section 2 to better understand the nature of the trade-comovement puzzle. We begin by constructing a measure of real GDP that is consistent with the methodology used by the National Accounts. Then, we show that comovement in real GDP can be explained by three factors: the correlation between each country's TFP; the correlation between each country's share of expenditure on domestic goods; and the correlation between each country's TFP and the partner's share of expenditure on domestic goods.

[4]Fitzgerald (2012) tests and fails to reject the hypothesis of optimal consumption risk-sharing conditional on trade costs among developed countries in the context of a gravity model with intertemporal trade, similar to the one developed here.

[5]Alvarez and Lucas (2007) establish a very generic result for existence and uniqueness of equilibrium with balanced trade in a model with the EK structure. They also develop a tatônement algorithm to numerically solve for the equilibrium. In our framework, thanks to the symmetry assumption, it is possible to solve analytically for the steady state equilibrium. See Appendix A for details. The log-linear model is described in Appendix B.

3.1 GDP at Constant Prices

The construction of a measure of real GDP consistent with the National Accounts requires the computation of value added at constant prices. Arkolakis and Ramanarayanan (2009) show that measuring real GDP correctly implies that comovement in total factor productivity (TFP) across countries is unrelated to the degree of trade integration.[6] Their result has important implications for our analysis as we show below.

Using the fact that the value added by each intermediate good firm evaluated at current prices is equal to that firm's wage bill, so that $p_i\left(v, s^t\right) y_i\left(v, s^t\right) = W_i\left(s^t\right) L_i\left(v, s^t\right)$, GDP at constant prices in country i is given by

$$\mathscr{Y}_i\left(s^t\right) \equiv \int_{\Omega_i(s^t)} p_i\left(v, s^0\right) y_i\left(v, s^t\right) dv = \int_{\Omega_i(s^t)} \left[\frac{p_i\left(v, s^0\right)}{p_i\left(v, s^t\right)}\right] W_i\left(s^t\right) L_i\left(v, s^t\right) dv, \tag{25}$$

where $\Omega_i\left(s^t\right)$ is the set of differentiated intermediate goods produced in country i in period t, $y_i\left(v, s^t\right)$ is the output of intermediate good v produced by country i, $L_i\left(v, s^t\right)$ is employment in industry v and $p_i\left(v, s^0\right)$ is the price of the intermediate variety v in period 0, which is chosen as the base year to compute GDP at constant prices.

Of course, some goods produced by country i in period t may not have been produced in period 0 and therefore $\Omega_i\left(s^t\right) \neq \Omega_i\left(s^0\right)$. For such goods, $p_i\left(v, s^0\right)$ is undefined. One way to resolve this problem is to use the price at which country i would have been able to sell the good to itself, had it not imported it, $p_{ii}\left(v, s^0\right)$. This interpretation seems the most consistent with the recommendation by the UN System of National Accounts 1993 (SNA 93), which suggests to use the average price changes of similar products as a proxy for the change in price of the new good.[7] Following this convention and making use of (4), equation (25) can be written as follows

$$\begin{aligned} \mathscr{Y}_i\left(s^t\right) &= \int_{\Omega_i(s^t)} \left[\frac{A_i\left(s^t\right)}{A_i\left(s^0\right)} \frac{W_i\left(s^0\right)/\varphi_{i,n}}{W_i\left(s^t\right)/\varphi_{i,n}}\right] W_i\left(s^t\right) L_i\left(v, s^t\right) dv, \\ &= A_i\left(s^t\right) \int_{\Omega_i(s^t)} L_i\left(v, s^t\right) dv = A_i\left(s^t\right) L_i\left(s^t\right), \end{aligned} \tag{26}$$

[6]Kehoe and Ruhl (2008) demonstrate a similar result.

[7]Following Arkolakis and Ramanarayanan (2009), an alternative would be to use the price at which the good was imported at date 0. The two approaches yield the same conclusion about the muted role of trade in measured TFP.

where we use the labor market clearing condition and set $W_i\left(s^0\right)/A_i\left(s^0\right) = 1$, without loss of generality. The upshot is that measured total factor productivity is given by

$$\mathrm{TFP}_{i,t} = \frac{\mathscr{Y}_i\left(s^t\right)}{L_i\left(s^t\right)} = A_i\left(s^t\right), \tag{27}$$

and, hence, it is independent from changes in the terms-of-trade or the level of trade integration.

Finally, combining (26) with (18) we obtain

$$\mathscr{Y}_i\left(s^t\right) = \kappa^{-\sigma_n} A_i\left(s^t\right)^{1+\sigma_n}\left[\frac{1}{\lambda_{ii}\left(s^t\right)}\right]^{\sigma_n/\theta}. \tag{28}$$

3.2 Factor Structure of GDP Comovement

From equation (28) it follows that real GDP fluctuations in country i (in log deviations from steady state) are given by

$$\widetilde{\mathscr{Y}}_{i,t} = \left(1+\sigma_n\right)\widetilde{A}_{i,t} - \left(\frac{\sigma_n}{\theta}\right)\widetilde{\lambda}_{ii,t}. \tag{29}$$

Expression (29) implies that the degree of comovement between any country-pair depends on the correlation between each country's productivity levels $\widetilde{A}_{i,t}$ and on the correlation between each country's share of expenditures on domestic goods $\widetilde{\lambda}_{ii,t}$ (which, in log-deviation from steady state, is equal to the negative of the import penetration ratio). It turns out that expression (29) holds for a large class of trade models, as we establish in Result 1:

Result 1 *Suppose the following macro-level assumptions are satisfied:*

A1. Profits are a constant share of revenue;

A2. The import demand system exhibits constant elasticity of substitution (CES);

A3. Labor supply choices are independent of wealth.

It follows that, irrespective of the micro-level assumptions about trade, output fluctuations are given by equation (29).

See appendix C for proof.

This result builds on the work of Arkolakis et al. (2012), who show that the predictions of a large class of trade models concerning the change in real income associated with any foreign shock only depend on the import penetration ratio and the trade elasticity. The relevant class of models is large and includes many well-known trade models such as the Armington model, Eaton and Kortum (2002), Bernard et al. (2003) extension of EK to imperfect competition, Krugman (1980), and multiple versions of Melitz (2003).

From equation (29), we obtain the following three-factor model for the output correlation between countries i and j

$$\text{cor}\left(\widetilde{\mathscr{Y}}_i, \widetilde{\mathscr{Y}}_j\right) = \beta_1 \,\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right) + \beta_2 \,\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right) + \beta_3 \text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right), \tag{30}$$

where $\text{cor}\,(x, z)$ denotes the correlation between two variables x and z. The coefficients β_1 and β_2 are positive while β_3 is negative.[8] The equation implies three channels through which trade can increase business cycle synchronization, summarized in the following result:

Result 2 *The output correlation for each country-pair may be expressed as the sum of three factors, as in equation* (30). *It follows that there are three channels through which an increase in bilateral trade may increase business cycle synchronization: (i) increased bilateral trade resulting in a higher correlation between each country's technology shocks; (ii) increased bilateral trade resulting in a*

[8] The factor loadings are given by

$$\beta_1 \equiv (1+\sigma_n)^2 \left[\frac{\text{var}\left(\widetilde{A}_i\right)}{\text{var}\left(\widetilde{\mathscr{Y}}_i\right)}\right], \qquad \beta_2 \equiv \left(\frac{\sigma_n}{\theta}\right)^2 \left[\frac{\text{var}\left(\widetilde{\lambda}_{ii}\right)}{\text{var}\left(\widetilde{\mathscr{Y}}_i\right)}\right] \qquad \text{and}$$

$$\beta_3 \equiv -2\,(1+\sigma_n)\,(\sigma_n/\theta) \left[\frac{\text{std}\left(\widetilde{A}_i\right)\text{std}\left(\widetilde{\lambda}_{ii}\right)}{\text{var}\left(\widetilde{\mathscr{Y}}_i\right)}\right].$$

where var and std denote, respectively, the variance and standard deviation of a variable x. See Appendix D for detailed derivations. The intuition for the sign of the coefficients is as follows. (i) If two countries have more correlated TFPs, their GDPs will also be more correlated, and this effect is stronger when the relative volatility of TFP, $\text{var}\left(\widetilde{A}_i\right)/\text{var}\left(\widetilde{\mathscr{Y}}_i\right)$, is high. (ii) The correlation of the share of expenditure on domestic goods is positively related to GDP correlations because an increase in the domestic expenditure on domestic goods is associated with a lower real wage and a lower real GDP; hence, if the domestic expenditure shares are positively correlated, then, all else equal, GDPs will be positively correlated, and this effect is stronger when the relative volatility of the expenditure share, $\text{var}\left(\widetilde{\lambda}_{ii}\right)/\text{var}\left(\widetilde{\mathscr{Y}}_i\right)$, is high. (iii) Finally, the correlation of the share of expenditure on domestic goods with foreign TFP is negatively related to the GDP correlation because, if the Home expenditure on domestic goods rises when foreign TFP rises, home GDP is lower while foreign GDP is higher, all else equal.

higher correlation between each country's share of expenditure on domestic goods; and (iii) increased bilateral trade reducing the correlation between the share of expenditure on domestic goods and foreign technology shocks.

See appendix D for proof.

Equation (30) and Result 2 provide the basis for the empirical analysis that follows. We use bilateral trade data on manufactures, manufacturing output and employment for a panel of 21 OECD countries over the period 1988 – 2007. The share of expenditure on domestic goods λ_{ii} (equivalently, one minus the import penetration ratio) can be calculated from bilateral trade data. We compute TFP for each country as the ratio between real manufacturing GDP and employment in manufacturing, calculate the correlations for each term of equation (30) and estimate the three-factor structure. We evaluate our model by testing if the estimated factor loadings have the expected signs and are statistically significant, and by judging the model's goodness of fit.

If the fit of the empirical model is judged to be good, we can dissect the trade-comovement puzzle by examining the channels through which trade leads to business cycle synchronization in the theoretical model and contrast it with the empirical results. By inspecting how bilateral trade intensity affects each of the three factors we are able to identify if country-pairs that trade more experience stronger output comovement because of: (i) higher correlation between each country's TFP; (ii) higher correlation between each country's share of expenditure on domestic goods; or (iii) lower correlation between the share of expenditure on domestic goods and foreign TFP.

Finally, notice that the three factors are not independent. In particular, an increased correlation between each country's TFP may imply an increased correlation between each country's share of expenditure on domestic goods. We investigate this possibility using the theoretical model and allowing for TFP correlation to vary with the level of trade integration in a way that is consistent with the data. The next Section describes the quantitative results.

4 Quantitative Results

This Section examines the ability of our model to replicate the trade-comovement relation by confronting the model with the data. We first estimate a regression in the spirit of FR using both empirical data and simulated data from the theoretical model. To this aim, we solve the model for a world economy where the number of countries n is set equal to four, implying six distinct country-pairs. This allows us to construct a symmetric world economy matching the minimum, maximum and median bilateral trade flows observed in the data and reported in Table 1. In the baseline model, the correlation of the TFP shocks is fixed for all country pairs at a level that matches the median correlation observed in the empirical data (the details about the calibration are explained in Appendix F).

After establishing that our model with uncorrelated technology shocks is unable to replicate the link between trade and comovement observed in the data we proceed in two steps. First, we dissect the relation between trade and business cycle synchronization by analyzing the three-factor model of equation (30) and assessing how trade affects each factor. Second, we allow for the presence of endogenous TFP correlations and show that although the model is now more successful to explain the trade-comovement relation, it still implies a counterfactual relation between trade integration and $\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$.

4.1 The FR Regression

As a starting point we estimate by ordinary least squares (OLS) the following regression in the spirit of FR using both empirical and simulated data:

$$\text{cor}\left(\widetilde{\mathscr{Y}_i}, \widetilde{\mathscr{Y}_j}\right) = b_0 + b_1 \log\left(\text{Bilateral Trade}\right)_{ij} + \varepsilon_{ij}, \tag{31}$$

where $\text{cor}\left(\widetilde{\mathscr{Y}_i}, \widetilde{\mathscr{Y}_j}\right)$ is the correlation between detrended log GDP in country i and country j, and $\log\left(\text{Bilateral Trade}\right)_{ij}$ is log of the country-pair's bilateral trade intensity, defined as the sum of bilateral manufacturing imports from country i to country j and from country j to country i, as a fraction of the two countries' total manufacturing output averaged over the entire period. We estimate equation (31) using manufacturing data, obtained from the OECD STAN database over

Table 1: Descriptive statistics (empirical data)

	Bilateral trade	Detrended GDP correlation	Detrended TFP correlation
Minimum	0.001	−0.557	−0.636
Maximum	0.388	0.991	0.900
Median	0.015	0.645	0.350
Std. deviation	0.042	0.390	0.377

The number of observations is 210 country-pairs. See Appendix E for details.
GDP and TFP are linearly detrended.

the 1988 – 2007 period. We focus on manufacturing data because our empirical work requires panel data on bilateral trade flows and the manufacturing data allows us to extend the time-series length of the panel substantially. However, the FR result about trade and comovement is robust across alternative measures of output and trade. Details about the data are included in Appendix E.

The first two columns of Table 2 show the OLS estimates of the FR regression. The first column only includes the bilateral trade covariate and shows that an increase in trade intensity leads to higher business cycle synchronization. The estimated b_1 coefficient is statistically significant and equal to 0.103. In terms of magnitude, the slope coefficient estimate implies that a country pair with twice the trade intensity of another country pair will have a 0.071 higher GDP correlation. However, in this regression the R^2 is small, indicating that bilateral trade only explains 10% of the cross-section variation in business cycle comovement. Therefore, in the second column we also include fixed effects for each individual country.[9] The estimated b_1 coefficient in the second regression remains positive and significant with a value of 0.074, indicating that there are no severe problems with omitted variable bias. Reassuringly, the adjusted R^2 is greatly increased at 73%, suggesting that the fixed effects capture most of the variation due to unobserved covariates.

The third column in Table 2 reports the slope coefficient implied by the baseline model. In this

[9] As the unit of observation is the country-pair, we identify a fixed effect for each individual country. In particular, fixed effects regression that we estimate is

$$\text{cor}\left(\widetilde{\mathscr{Y}}_i, \widetilde{\mathscr{Y}}_j\right) = b_0 + b_1 \log\left(\text{Bilateral Trade}\right)_{ij} + f_i + f_j + \varepsilon_{ij},$$

with f_i and f_j the country fixed effects.

Table 2: Frankel and Rose regressions (data and model)

dependent variable: cor $\left(\widetilde{\mathscr{Y}}_i, \widetilde{\mathscr{Y}}_j\right)$	empirical data		model
	1.	2.	3.
log (Bilateral Trade)	0.103***	0.074***	0.007
	(4.87)	(3.79)	
constant	0.963***	1.391***	0.412
	(10.36)	(4.64)	
country dummies	no	yes	—
Adj. R^2	0.10	0.72	—
observations	210	210	—
Wald test:			
$b_{1,\text{data}} = b_{1,\text{baseline model}}$	20.60 (0.00)	11.80 (0.00)	—

*p<0.10, ** p<0.05, *** p<0.01.
Coefficient estimates reported with $t-$stat in parentheses.
Wald test reported with $p-$value in parentheses.

case b_1 is equal to 0.007, which is at least an order of magnitude smaller than the slope coefficient from the data. The Wald test is performed to test if the coefficient estimated using empirical data is the same as the implied coefficient obtained from the model. The null hypothesis is, of course, overwhelmingly rejected. This constitutes the trade-comovement puzzle documented in Kose and Yi (2006): the theoretical model is not able to replicate the empirical relation between trade and business cycle synchronization. Thus, our first finding is that a model that represents trade using the more sophisticated EK structure is not more successful at explaining the relationship between trade and business cycle comovement than the standard IRBC model.

4.2 Dissecting the Trade-comovement Puzzle

We use the factor structure of GDP comovement obtained in equation (30) to dissect the sources of the trade-comovement puzzle. We begin by examining if the factor structure derived for the theoretical economy holds empirically. The results in the first column of Table 3 are encouraging.

Table 3: The factor structure of GDP comovement (empirical data)

	dependent variable			
	$\text{cor}\left(\widetilde{\mathscr{Y}_i}, \widetilde{\mathscr{Y}_j}\right)$	$\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right)$	$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$	$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right)$
	1.	2.	3.	4.
$\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right)$	0.135*			
	(1.96)			
$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$	0.320***			
	(5.15)			
$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right)$	−0.282*			
	(−1.84)			
$\log\left(\text{Bilateral Trade}\right)$		0.061***	0.074***	−0.014
		(2.87)	(3.25)	(−1.41)
constant	0.266***	0.560***	0.937***	−0.130***
	(5.61)	(5.61)	(9.42)	(−3.04)
Adj. R^2	0.17	0.03	0.04	0.00
observations	210	210	210	210

*p<0.10, ** p<0.05, *** p<0.01.
Coefficient estimates reported with $t-$stat in parentheses.

Each factor loading has the expected sign and the three factors explain 17% of the variation in comovement observed in the data. Thus, the factor structure implied by equation (30) provides a satisfactory empirical model of GDP comovement and, hence, we can use it to inspect the sources of the trade comovement puzzle.

Having established that the fit of the three-factor model is good and consistent with theory, the next step is to study how each factor responds to changes in bilateral trade intensity. This allows us to study empirically the channels through which trade leads to higher business cycle synchronization, and contrast these results with the theoretical predictions (within the framework of Result 2). To

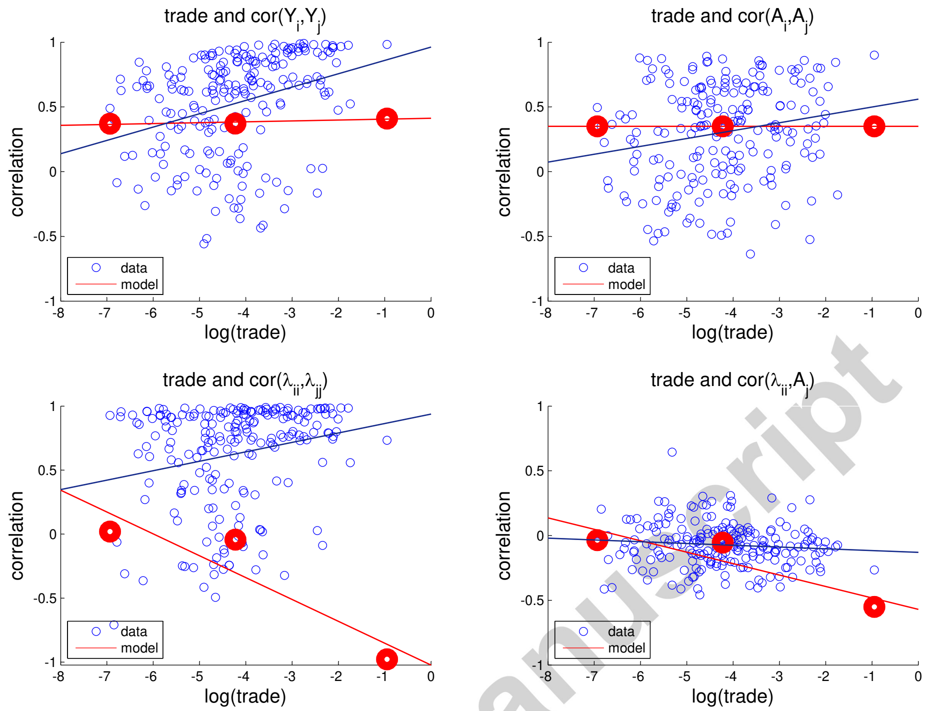

Figure 1: trade and comovement (baseline model)

do this, we estimate by OLS the following three equations:

$$\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right) \quad = \quad b_0 + b_1 \ \log\left(\text{Bilateral Trade}\right)_{ij} + \varepsilon_{ij}, \tag{32}$$

$$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right) \quad = \quad b_0 + b_1 \ \log\left(\text{Bilateral Trade}\right)_{ij} + \varepsilon_{ij}, \tag{33}$$

$$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right) \quad = \quad b_0 + b_1 \ \log\left(\text{Bilateral Trade}\right)_{ij} + \varepsilon_{ij}. \tag{34}$$

The last three columns of Table 3 report the estimation results. The results suggest that higher bilateral trade intensity is associated with (i) a higher correlation between each country's technology shocks, and (ii) a higher correlation between each country's share of expenditure on domestic goods. By contrast, there is no significant relation between trade and the correlation between the share of expenditure on domestic goods and foreign TFP. Therefore, we conclude that empirically the first and second factors are responsible for the positive association between trade and business cycle

Table 4: Trade and business cycle synchronization

	dependent variable			
	$\text{cor}\left(\widetilde{\mathscr{Y}_i}, \widetilde{\mathscr{Y}_j}\right)$	$\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right)$	$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$	$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right)$
	1.	2.	3.	4.
	Panel A – empirical data			
$\log\left(\text{Bilateral Trade}\right)$	0.103***	0.061***	0.074***	−0.014
	(4.87)	(2.87)	(3.25)	(−1.41)
constant	0.963***	0.560***	0.937***	−0.130***
	(10.36)	(5.61)	(9.42)	(−3.04)
	Panel B – model: baseline			
$\log\left(\text{Bilateral Trade}\right)$	0.007	0.000	−0.171	−0.088
constant	0.412	0.350	−1.022	−0.570
	Panel C – model: endogenous TFP correlations			
$\log\left(\text{Bilateral Trade}\right)$	0.065	0.061	−0.115	−0.035
constant	0.606	0.560	−0.796	−0.343
Wald test (endogenous TFP correlations):				
$b_{1,\text{data}} = b_{1,\text{model}}$	3.23 (0.07)	0.00 (1.00)	69.44 (0.00)	4.78 (0.03)
$b_{1(\text{data,fixed effects})} = b_{1,\text{model}}$	0.22 (0.64)			

*p<0.10, ** p<0.05, *** p<0.01.
Coefficient estimates reported with $t-$stat in parentheses.
Wald test reported with $p-$value in parentheses.

comovement. In particular, the second factor is the most important: countries with strong trade linkages exhibit a higher correlation of the share of expenditures on domestic goods.

These findings are in sharp contrast to what is implied by the theoretical economy. This is illustrated in Figure 1. The world economy consists of four countries, so that each country has three trade partners and the calibration of trade costs is done such that the bilateral trade intensities correspond to the minimum, the maximum and the median observed in the empirical data. The first panel of the figure shows the relation between GDP correlation and bilateral trade implied by the model. In turn, panels two, three and four look at each factor in isolation.

The first panel illustrates the trade-comovement puzzle and the three subsequent panels dissect the puzzle exploiting the factor structure of GDP comovement. Looking at the second panel, there is no association between trade and the correlation of the TFP shocks. This is by design, as in the baseline model TFP correlations are fixed. The third panel reveals that the link between trade and the correlation between each country's share of expenditure on domestic goods is counterfactually negative. This is an important factor to explain the trade-comovement puzzle. A productivity shock in country i implies an increase in λ_{ii}. It also raises the real wage in country j, $W_j\left(s^t\right)/P_j\left(s^t\right)$, because country j enjoys lower import prices. From equation (17), this in turn implies that λ_{jj} must fall. This effect is more pronounced when trade integration is high. Thus, the model implies a negative link between trade integration and $\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$. This counterfactual implication is key to understanding the trade-comovement puzzle. Finally, looking at the fourth panel, the small positive association between trade and comovement is driven by the third component: an increase in trade is associated with a lower correlation between a country's TFP and the foreign country's share of expenditure on domestic goods. This is at odds with the empirical results which show that the third channel is not statistically significant.[10]

These results are also shown in Table 4. Panel A reports the empirical regression between GDP correlation and bilateral trade, and also the regressions studying the association between trade intensity and each separate factor. Panel B shows the same regression coefficients as in Panel A, but reported for our baseline model and, thus, contains the results described above. In particular, the failure of the model to account for the relation between trade and the correlation of each country's share of expenditure on domestic goods is evident.

[10]Recall from Result 2 that there are three channels through which trade intensity may increase business cycle correlations: (i) increased bilateral trade resulting in a higher correlation between each country's technology shocks; (ii) increased bilateral trade resulting in a higher correlation between each country's share of expenditure on domestic goods; and (iii) increased bilateral trade reducing the correlation between the share of expenditure on domestic goods and foreign technology shocks. In Figure 1 we show that the first channel is absent by design and the second channel is at odds with Result 2. The third channel is the only one that works in the direction postulated in Result 2. However, this is the channel which is not significant empirically.

4.3 Endogenous TFP Correlations

In the baseline model, the correlation of the TFP shocks is fixed for all country pairs at a level that matches the median correlation in the data. However, this is counterfactual as there is evidence that countries with stronger trade linkages also have more correlated TFP shocks.[11] Therefore, we now depart from the baseline model and allow for the correlation of the TFP shocks to vary with trade. We do this by using the estimated regression equation (32) to predict the level of TFP correlations given the corresponding level of bilateral trade and calibrating the covariance matrix of the TFP shocks accordingly.

The results are shown in Panel C and D of Table 4. In Panel C, the same regression coefficients as in Panel A are reported for the theoretical economy with endogenous TFP correlations; Panel D, reports a Wald test for the null hypothesis that the coefficient estimated using empirical data is equal to the coefficient implied by the theoretical model with endogenous TFP correlations.

Once we allow for endogenous TFP correlations, the model is able to better replicate the trade-comovement relation. The implied coefficient in the regression between GDP correlation and trade intensity is 0.065, which is roughly 65% of the empirical counterpart. In fact, at the 5% level, we are unable to reject the hypothesis that the estimated coefficient is equal to the value implied by the model. The $p-$value for this null hypothesis is 7%. If we consider the estimated coefficient from the fixed effect regression, the $p-$value is even higher, at 64% (reported in the last row of Table 4). Thus, allowing for endogenous TFP correlations helps to resolve the trade-comovement puzzle. This finding is consistent with those by Kose and Yi (2006). However, given the factor structure of GDP comovement derived earlier, we can inspect the mechanism further and look at how each of the three factors that explain GDP comovement is affected by bilateral trade.

In what follows we focus on columns two, three and four of Panel C in Table 4. The results in column two simply clarify the calibration strategy: in the regression between TFP correlations and trade, the model implied coefficients are the same as the empirical coefficients by construction. Our main finding is reported in column three: the model with endogenous TFP correlations still implies a negative association between bilateral trade and the correlation between each country's

[11] See, for instance, Kose and Yi (2006) and Liao and Santacreu (2015).

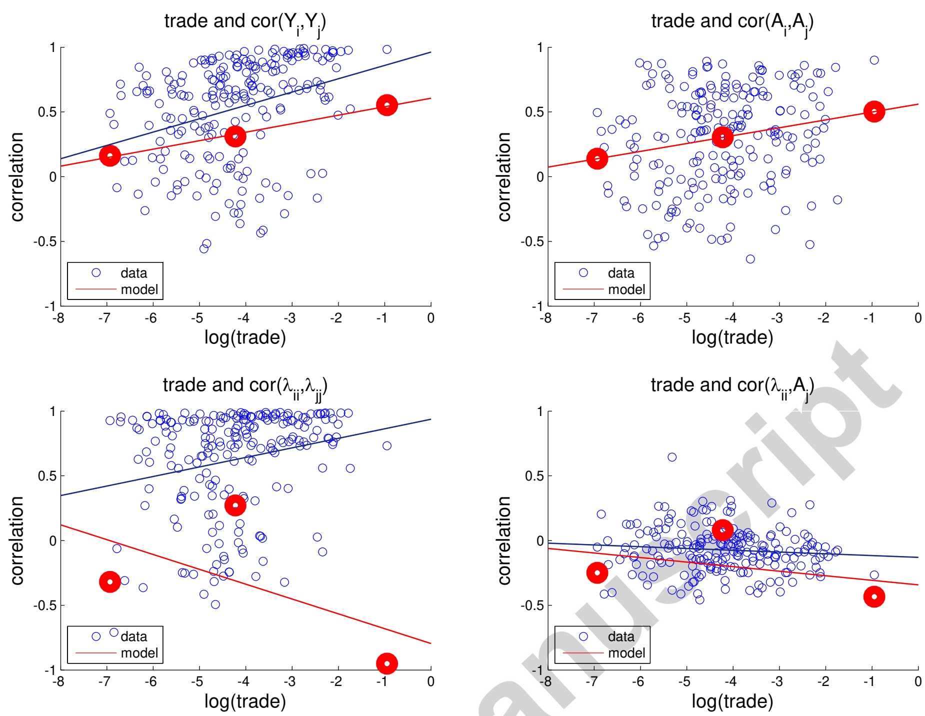

Figure 2: trade and comovement (endogenous TFP correlations)

share of expenditure on domestic goods, with a coefficient equal to -0.115.[12] This is counterfactual since empirically there is a positive link between trade and the correlation between each country's share of expenditure on domestic goods, with a regression coefficient equal to 0.074. Finally, the behavior of the third factor is consistent with the empirical data: higher trade intensity lowers the correlation between foreign TFP shocks and the share of expenditure on domestic goods.[13]

The main empirical result is well illustrated in Figure 2. Although allowing for TFP correlations to increase with bilateral trade as in the data helps the model to account for the trade comovement relationship (shown in the first graph), it leads to a strong counterfactual negative association between the correlation of the share of expenditure on domestic goods and bilateral trade (shown in the third graph). This negative association is smaller than in the baseline model with uncorrelated

[12] Interestingly, the relation between bilateral trade and cor $\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$ implied by the model is not monotonic. In Section 4.5 we look more carefully at the determinants of this relationship in the theoretical model.

[13] This last mechanism is less important to us because the link between trade intensity and the correlation between foreign TFP shocks and the share of expenditure on domestic goods is not statistically significant in the empirical regression.

Table 5: Decomposing the trade-comovement puzzle

	Panel A: baseline model			
overall gap $= -0.096$	$b_{1,\text{model}} - b_{1,\text{data}}$	β	$\beta \times (b_{1,\text{model}} - b_{1,\text{data}})$	$\beta \times (b_{1,\text{model}} - b_{1,\text{data}})$ in % of overall gap
$\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right)$	−0.061	0.135	−0.008	8%
$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$	−0.245	0.320	−0.078	81%
$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right)$	−0.074	−0.282	0.021	−22%
	Panel B: endogenous TFP correlations			
overall gap $= -0.038$	$b_{1,\text{model}} - b_{1,\text{data}}$	β	$\beta \times (b_{1,\text{model}} - b_{1,\text{data}})$	$\beta \times (b_{1,\text{model}} - b_{1,\text{data}})$ in % of overall gap
$\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right)$	0.000	0.135	0.000	0%
$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$	−0.189	0.320	−0.060	158%
$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right)$	−0.021	−0.282	0.006	−16%

shocks, which suggests that the presence of correlated TFP shocks helps but it is not enough to resolve the trade-comovement puzzle.

4.4 Trade-comovement puzzle: counterfactuals

As a final step to understand the trade-comovement puzzle, we develop develop a decomposition that assesses the quantitative importance of each of the three channels, based on the factor structure (30).[14] We compute the difference between the model implied b_1 coefficients in equations (32), (33) and (34), and their respective empirical counterparts. These differences are shown in the first column of Table 5. Next, we multiply these differences by the factor loading β_r, for each factor, $r = 1, 2, 3$, in equation (30). This product is reported in the third column. It provides an estimate of how much the gap between the empirical trade-comovement coefficient in the FR regression (31) and its theoretical counterpart can be closed if we consider the counterfactuals in which we exactly match the relation between one of the three factors and bilateral trade, everything

[14] We thank an anonymous referee for suggesting this decomposition analysis.

else staying the same.

In Panel A of Table 5 considers the decomposition for the baseline model. The OLS coefficient in the FR regression (31) is 0.103 and the corresponding coefficient from the baseline model is 0.007. Thus, the overall gap to overcome is −0.096. Looking at the first row of Panel A, we find that the counterfactual relation between trade and the TFP correlation is contributing 0.008 points to the trade-comovement gap (corresponding to 8% of the overall gap). In the second row of Panel A, we find that the counterfactual relation between trade and the correlation in the share of expenditure in domestic goods yields 0.078 points to the trade-comovement gap (corresponding to 81% of the overall gap). In turn, the third factor is not contributing to the trade-comovement puzzle and, instead, lowers by 22% the gap between the empirical relation and the theoretical counterpart.

In Panel B of Table 5, we consider the model with endogenous TFP correlations. When we do this, we eliminate entirely the contribution of the first factor to the trade-comovement puzzle and we find that the overall gap falls (in absolute value) from −0.096 to −0.038. Thus, we close roughly 2/3 of the gap, which is substantially more than the 8% contribution from the first factor, reported in Panel A. This happens for two reasons. First, as we make the TFP correlations endogenous, the other factors are also affected. In particular, the contribution of the first factor changes from −0.078 to −0.060 (as reported in the second row of Panel B). Second, this exercise indicates that the theoretical model attributes too much importance to the first factor relative to the data (while the empirical factor loading is $\beta_1 = 0.135$, its theoretical counterpart is 1.041). Thus, despite closing 2/3 of the overall gap in the trade-comovement link, the model still implies a strong counterfactual relation between trade and the correlation of the share of expenditure in domestic goods. This is another way to illustrate the main result in our paper: when we allow for endogenous TFP correlations, the trade-comovement relationship implied by the model is closer to its empirical counterpart, but for the wrong reasons.

These findings suggest that both empirical and theoretical research should examine further the positive association between trade and the comovement in the share of expenditure on domestic goods. Empirically, this channel is the most important to explain why trade leads to business cycle synchronization. Therefore, we argue that this should constitute a litmus test for theoretical models: stronger trade linkages should lead to synchronization in the share of expenditure in domestic goods.

4.5 Inspecting the Mechanism: Bilateral Trade and cor $\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$

The relation between bilateral trade and cor $\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$ implied by the model with endogenous TFP correlations is not monotonic. As shown in the third graph of Figure 2, as we move from low levels of bilateral trade to intermediate levels, cor $\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$ increases. After that, as we move to high levels of bilateral trade, it falls again. Therefore, it is important to inspect the mechanism more carefully.[15] We do this by looking at impulse response functions (IRFs) conditional on a TFP shock, for each component of $\widetilde{\lambda}_{ii}$.[16] In particular, we start from equation (9) to obtain

$$\frac{\lambda_{j1}\left(s^t\right)}{\lambda_{jj}\left(s^t\right)} = \left[\frac{W_1\left(s^t\right)}{W_j\left(s^t\right)}\right]^{-\theta} \tau_{j1}^{-\theta}. \tag{35}$$

In log-deviations from steady state equation (35) takes the form

$$\widetilde{\lambda}_{jj,t} = \theta\left(\widetilde{W}_{1,t} - \widetilde{W}_{j,t}\right) + \widetilde{\lambda}_{j1,t}, \tag{36}$$

where an increase in $\left(\widetilde{W}_{1,t} - \widetilde{W}_{j,t}\right)$ in equation (36) represents an improvement in the terms-of-trade of country 1 vis-à-vis country j, while $\widetilde{\lambda}_{j1,t}$ corresponds to country j's import ratio from country 1.[17] A positive technology shock in country 1 (hereafter, the Home country) raises Home's share of expenditure on domestic goods, $\widetilde{\lambda}_{11,t}$. We now ask, under what circumstances does the share of expenditure on domestic goods also increase in other countries?

From equation (36) we see that, for the share of expenditure on domestic goods to increase in countries $j = 2, 3, 4$ (henceforth, the Foreign countries) conditional on a positive TFP shock at Home, either the Home terms-of-trade must improve or the Foreign's import penetration ratio must increase. The IRFs in Figure 3 show that these two terms move in opposite directions: a positive

[15] In Appendix G we examine the implications of this non-monotonicity when the country pairs with low bilateral trade intensity are more predominant.

[16] We obtain the IRFs for the baseline model with exogenous TFP correlations to avoid confounding factors in the interpretation of the transmission mechanism of TFP shocks. Thus, the IRFs show the response of each variable following a Home TFP shock, with Foreign's TFP constant.

[17] From equation (4), changes in the price of country i exports are given by $\widetilde{p}_{ji,t} = \widetilde{W}_{i,t}$, and so $\left(\widetilde{W}_{i,t} - \widetilde{W}_{j,t}\right)$ captures changes in the terms-of-trade.

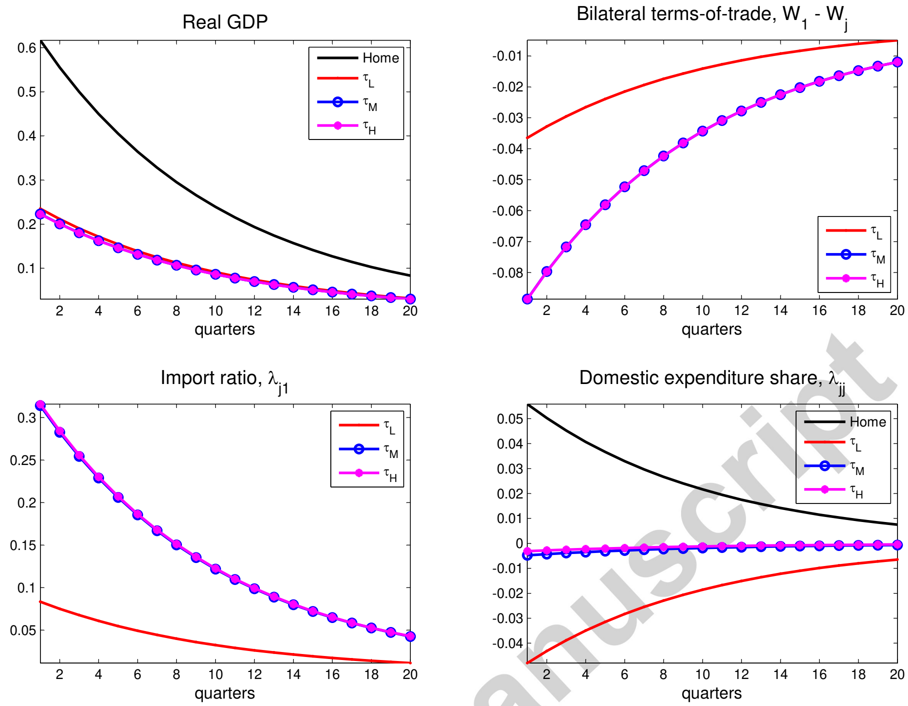

Figure 3: inspecting the mechanism (IRF to a TFP shock at Home)

TFP shock at Home worsens Home's terms-of-trade while raising the Foreign's import penetration ratio. The overall effect on Foreign's share of expenditure on domestic goods is ambiguous.

From the IRFs in the bottom-right panel of Figure 3 we see that for the Foreign countries with high and median trade costs the two effects almost cancel out and, hence, a TFP shock at Home has almost no effect on those countries' share of expenditure on domestic goods. Instead, for the Foreign country with low trade costs the deterioration in the Home's terms-of-trade dominates and, hence, the Foreign's share of expenditure on domestic goods falls. In Appendix B we show that the direct impact of changes in the terms-of-trade on the Foreign import ratio from Home depends on the bilateral trade flow in the following way:

$$\frac{\partial \widetilde{\lambda}_{j1,t}}{\partial \left(\widetilde{W}_{1,t} - \widetilde{W}_{j,t}\right)} = -\theta \left(1 - \bar{\lambda}_{j1}\right), \tag{37}$$

and, combining (36) with (37), we obtain

$$\frac{\partial \widetilde{\lambda}_{jj,t}}{\partial \left(\widetilde{W}_{1,t} - \widetilde{W}_{j,t} \right)} = \theta \bar{\lambda}_{j1}. \tag{38}$$

Therefore, the deterioration in Home's terms-of-trade implies counterfactually that, conditional on a positive Home TFP shock, the correlation between the Home and the Foreign share of expenditure on domestic goods falls with increased trade integration (measured by $\bar{\lambda}_{j1}$). The strength of this effect depends on θ: with higher θ, implying less firm heterogeneity, the counterfactual effect is larger.[18]

The parameter θ is calibrated to 3.60 following empirical evidence in Bernard et al. (2003). The upshot is that given the median bilateral trade flow of 1.5%, the elasticity of the domestic expenditure share to changes in the terms-of-trade is only 5.4%. Instead, for the maximum bilateral trade flow observed in the data, which is 39%, the same elasticity is 140.4%. Hence, the fall in the share of expenditure on domestic goods is much higher for the country with low bilateral trade costs. This happens despite the deterioration in Home's terms-of-trade vis-à-vis the Foreign economies with median and high trade costs being greater than that vis-à-vis the Foreign economies with low trade costs. Given such a large difference in the relative elasticities, allowing for correlated TFP shocks is not enough to overthrow this result.

The inability of the model to generate the observed relation between trade and the synchronization of the share of expenditure on domestic goods follows from the behavior of the terms-of-trade. Indeed, our model possesses the feature, common to most IRBC models, that Home TFP gains are transmitted positively to the trade partners through a deterioration of Home's terms-of-trade.[19] This property of the IRBC models is often counterfactual as shown in Corsetti et al. (2008) and, using an alternative identification scheme, in Corsetti et al. (2014). However, the elasticity of the

[18] It is interesting to note that this elasticity does not depend on the elasticity of substitution across varieties, σ.

[19] An exception to this feature is Ghironi and Melitz (2005) who construct a model where the entry of new producers and varieties in the Home economy following a TFP shock may dampen or even reverse the deterioration in the terms-of-trade. This is an important example, as the Ghironi and Melitz (2005) model is consistent with the factor decomposition for cross-country output correlation proposed in this paper. Thus, it is in principle possible to develop microfoundations which are consistent with our factor model, and be able to reproduce the empirical trade-comovement relation. For example, Cacciatore (2014) builds on Ghironi and Melitz (2005) and adds search and matching frictions in the labor market. These labor market frictions turn out to be important to explain business cycle synchronization.

share of expenditure on domestic goods to changes in the terms-of-trade is increasing in the level of trade integration. The upshot is that the only mechanism compatible with increased comovement in the share of expenditure on domestic goods following greater trade integration requires procyclical terms-of-trade.

5 Sensitivity Analysis

In this Section we investigate the sensitivity of our findings to changes in the assumptions about the structure of financial markets, the intertemporal substitution in labor supply choices, and the inclusion of physical capital in production.

5.1 Financial Autarky

Our baseline model assumed complete financial markets. However, Heathcote and Perri (2002) show that IRBC models with financial autarky match better some key business cycle moments, compared to the same models under complete markets. The factor structure of GDP comovement obtained earlier holds irrespective of the structure of financial markets, allowing us to investigate how each factor that determines the link between trade and comovement behaves under the assumption of financial autarky and, hence, balanced trade period by period. With financial autarky, $D_i\left(s^t\right)=0$ for each history s^t, and equations (11) and (20) become

$$\mathscr{P}_i\left(s^t\right) C_i\left(s^t\right) = W_i\left(s^t\right) L_i\left(s^t\right), \tag{39}$$

$$W_i\left(s^t\right) L_i\left(s^t\right) = \sum_{j=1}^{n} \lambda_{ji}\left(s^t\right)\left[W_j\left(s^t\right) L_j\left(s^t\right)\right], \tag{40}$$

respectively. Equations (23) and (24) are no longer relevant. We solve the version of the model in which the correlation of the TFP shocks varies with trade. The results are shown in Panel A of Table 6 and should be compared to the results in Table 4.

The results under financial autarky are very similar to those under complete markets. In particular, the business cycle synchronization effect of trade is entirely driven by the correlation of the TFP shocks and the effect of trade on the second factor, $\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$, is again counterfactual. The muted role of the financial market structure is unsurprising in a model featuring GHH preferences. When intertemporal substitution in labor supply is absent, capital flows do not affect the economy's aggregate labor supply through the income effect. For this reason, in what follows we explore what happens if we allow for intertemporal substitution in labor supply choices.

5.2 Intertemporal Substitution in Labor Supply

We allow for intertemporal substitution in labor supply by introducing preferences of the form

$$u\left(C_i, L_i; s^t\right) = \ln C_i\left(s^t\right) - \frac{L_i\left(s^t\right)^{1+1/\sigma_n}}{1+1/\sigma_n}. \tag{41}$$

We refer to the iso-elastic preferences in (41) as the KPR preferences.[20] Instead of (15), the new intratemporal condition determining the choice between leisure and consumption is

$$L_i\left(s^t\right) = \left[\frac{1}{C_i\left(s^t\right)} \frac{W\left(s^t\right)}{\mathscr{P}_i\left(s^t\right)}\right]^{\sigma_n}. \tag{42}$$

Under financial autarky, equation (39) holds, and consumption corresponds to labor income. Given the preferences form (41), after an increase in the real wage the income and substitution effects exactly offset one another, with the upshot that the aggregate labor supply is endogenously fixed and there are no transmission dynamics.[21] Therefore, we will focus on the economy under complete markets.

With the alternative preferences, the factor structure (30) still holds approximately.[22] Thus, we examine how allowing for intertemporal substitution in labor supply changes the link between the level of bilateral trade and each factor. The model is solved under the assumption of endogenous TFP correlations. The results are shown in Panel B of Table 6 and should be compared to the results in Table 4.

With KPR preferences the model is less successful at replicating the relation between trade and

[20] After King, Plosser, and Rebelo (1988).
[21] This is easily seen from combining conditions (39) and (42).
[22] The factor structure holds exactly for the approximate solution obtained using log-linearization methods, as we do in this paper. See appendix H for details.

Table 6: Trade and business cycle synchronization (additional experiments)

	dependent variable			
	$\text{cor}\left(\widetilde{\mathscr{Y}}_i, \widetilde{\mathscr{Y}}_j\right)$	$\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right)$	$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$	$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right)$
	1.	2.	3.	4.
	Panel A – model: financial autarky			
$\log\left(\text{Bilateral Trade}\right)$	0.066	0.061	−0.128	−0.060
constant	0.604	0.560	−0.848	−0.422
	Panel B – model: KPR preferences			
$\log\left(\text{Bilateral Trade}\right)$	0.053	0.061	−0.090	−0.013
constant	0.471	0.560	−0.696	−0.268
	Panel C – model: capital in production			
$\log\left(\text{Bilateral Trade}\right)$	−0.015	−0.021	−0.159	−0.069
constant	0.396	0.336	−0.974	−0.436

In both panel A and panel B, the model is solved assuming endogenous TFP correlations.
In panel B and panel C, the model is solved under complete markets.

comovement. The OLS coefficient of the regression between output correlation and trade is lowered, equal to 0.053. This is less than the imposed OLS coefficient of the regression between TFP correlations and trade. Thus, under KPR preferences the labor adjustment goes opposite the effect of TFP synchronization, eliminating some of its effect on GDP comovement. Thus, intertemporal substitution in labor supply is unhelpful to solve the trade-comovement puzzle. The reason for this is found by inspecting Figure 4, which shows the IRFs for employment under the two alternative preference forms.[23] Under GHH preferences, there is no intertemporal substitution in labor supply. Therefore, following a positive TFP shock in country 1, employment increases in all the countries. This is because the lower price of goods imported from country 1 raises the real wage everywhere. This positive effect is more pronounced when trade costs are low, implying a positive link between

[23]The IRFs are for the model without endogenous TFP correlations, since the goal is to show the effect of intertemporal substitution in labor supply.

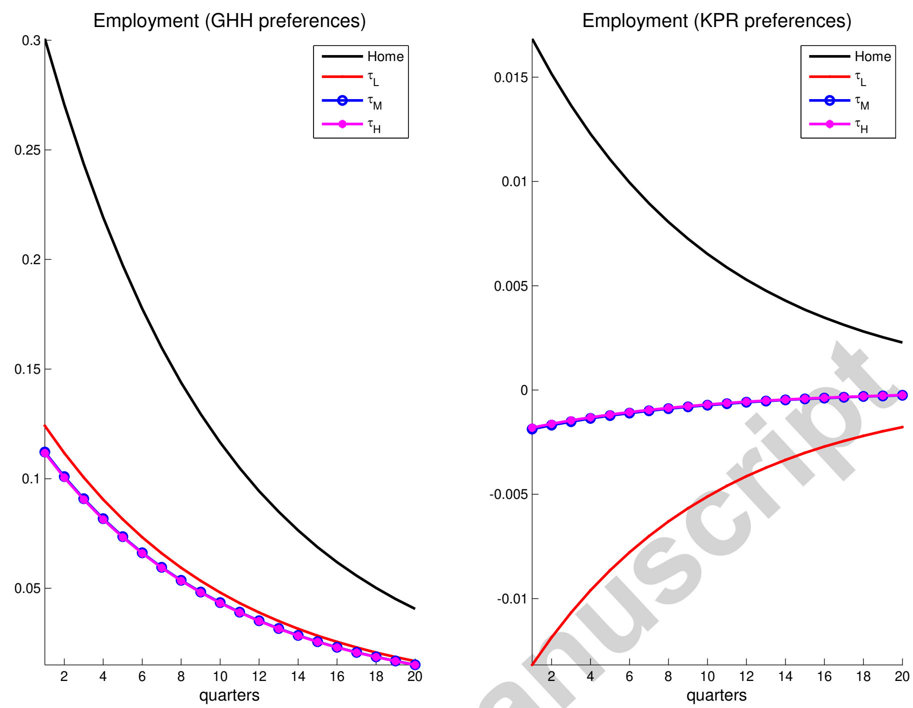

Figure 4: employment (alternative preferences)

employment comovement and bilateral trade.

Instead, with intertemporal substitution in labor supply and complete markets, a positive TFP shock in country 1 leads to an increase in foreign consumption implied by the risk-sharing condition. Therefore, under complete markets, intertemporal substitution in labor supply generates negative comovement in employment. This counterfactual feature is not specific to our model and is a very general and well known property of the standard IRBC model (Baxter and Crucini, 1995). Under complete markets, the increase in foreign consumption following the TFP shock in country 1, implied by the risk-sharing condition, dwarfs the substitution effect implied by the increase in the foreign real wage. This dampening effect is more pronounced when trade costs are low, implying a negative link between employment comovement and bilateral trade.

By contrast, with GHH preferences, there is no income effect on labor supply, which results in a positive international comovement of employment increasing with the level of bilateral trade. Thus, for endogenous labor supply to help solve the trade-comovement puzzle, GHH preferences are a desirable feature. Moreover, the model with GHH preferences also yields substantially larger employment volatility for the same labor supply elasticity parameter. This can be seen from the magnitudes of the IRF coefficients in Figure 4, which are much larger for the GHH model. These results are consistent with work by Raffo (2008), showing that GHH preferences help matching the business cycle dynamics of consumption and net exports.

5.3 Capital in Production

Our baseline model does not include physical capital in production, similarly to the set-up in EK and Ghironi and Melitz (2005). Here, we establish that the factor structure (30) also holds in the model with capital in production and we investigate how the trade-comovement link is affected by the inclusion of productive capital.

Let each intermediate commodity be produced using the following Cobb-Douglas technology

$$y_i\left(v, s^t\right) = Z_i\left(s^t\right)\left[\varphi_{i,v} K_i\left(v, s^{t-1}\right)^{\alpha} L_i\left(v, s^t\right)^{1-\alpha}\right], \tag{43}$$

where $\alpha \in (0,1)$ is the capital income share and $Z_i\left(s^t\right)$ is an aggregate productivity shock. The capital is rented from the household sector and the rental rate for capital is $r_i\left(s^t\right)$. As a result, the marginal cost of producing intermediate commodity v in country i to be delivered in country j is

$$p_{ji}\left(v, s^t\right) = \left[\frac{r_i\left(s^t\right)^{\alpha} W_i\left(s^t\right)^{1-\alpha}}{\varkappa Z_i\left(s^t\right)\varphi_{i,v}}\right]\tau_{ji}, \tag{44}$$

with $\varkappa = \alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}$. With competitive factor and good's markets, the capital income and labor income shares are, respectively, given by

$$\alpha \mathscr{P}_i\left(s^t\right) Y_i\left(s^t\right) = r_i\left(s^t\right) K_i\left(s^{t-1}\right), \tag{45}$$

$$\left(1-\alpha\right)\mathscr{P}_i\left(s^t\right) Y_i\left(s^t\right) = W_i\left(s^t\right) L_i\left(s^t\right). \tag{46}$$

The household sector purchases the non-traded final good for either consumption or investment in physical capital, and the stand-in household's budget constraint is now given by

$$\mathscr{P}_i\left(s^t\right) C_i\left(s^t\right) + \mathscr{P}_i\left(s^t\right) I_i\left(s^t\right) = W_i\left(s^t\right) L_i\left(s^t\right) + r_i\left(s^t\right) K_i\left(s^{t-1}\right) + D_i\left(s^t\right), \tag{47}$$

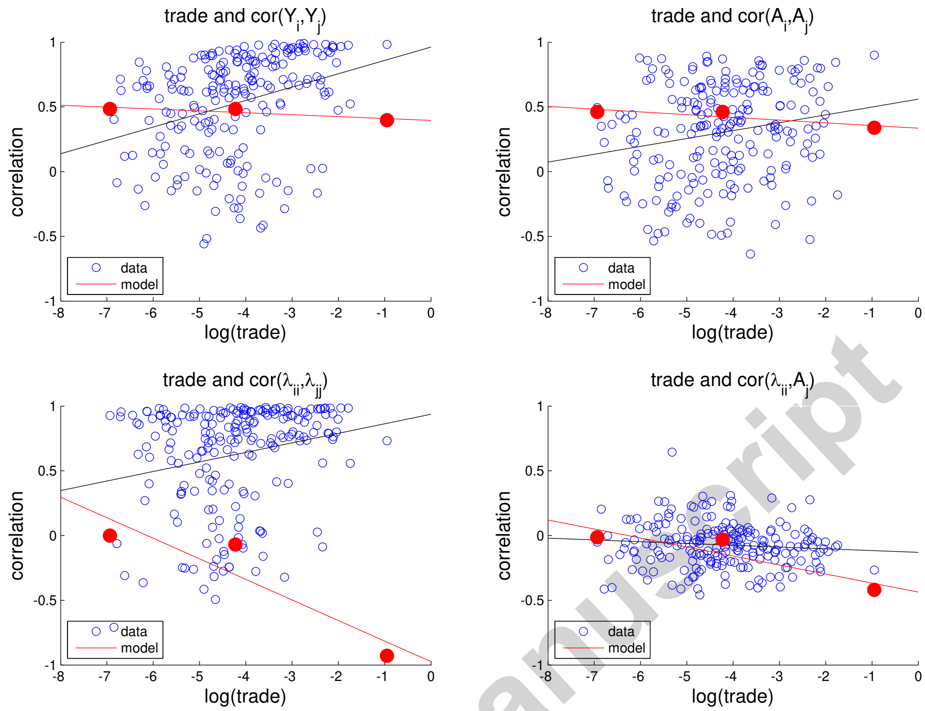

Figure 5: trade and comovement (capital in production)

where $I_i\left(s^t\right) = K\left(s^t\right) - (1-\delta) K\left(s^{t-1}\right)$, with $\delta \in (0,1)$, the capital depreciation rate. Therefore, with investment in physical capital there is an additional Euler equation, given by

$$\mu_i\left(s^t\right) = \beta \sum_{s_{t+1}\in\mathcal{S}} \pi\left(s_{t+1}|s^t\right) \mu_i\left(s^{t+1}\right)\left[r_i\left(s^{t+1}\right) + (1-\delta)\right]. \tag{48}$$

In the set-up with capital, the share of expenditure by country j in country i goods is given by

$$\lambda_{ji}\left(s^t\right) = \left[\frac{\kappa r_i\left(s^t\right)^{\alpha} W_i\left(s^t\right)^{1-\alpha}\tau_{ji}}{\varkappa Z_i\left(s^t\right)\mathscr{P}_i\left(s^t\right)}\right]^{-\theta}, \tag{49}$$

and the ideal price index in country j is still given by (7), but with

$$\Phi_j\left(s^t\right) = \sum_{i=1}^{n}\left[\frac{r_i\left(s^t\right)^{\alpha} W_i\left(s^t\right)^{1-\alpha}\tau_{ji}}{\varkappa Z_i\left(s^t\right)}\right]^{-\theta}. \tag{50}$$

Finally, total domestic income must be equal to total expenditure in domestic goods, as follows

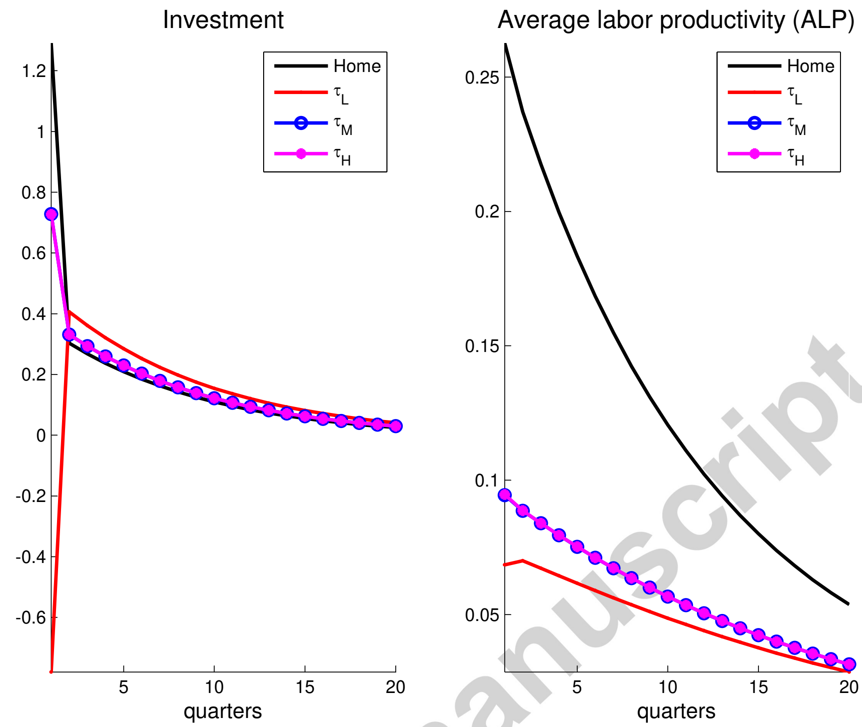

Figure 6: investment and productivity (capital in production)

$$W_i\left(s^t\right)L_i\left(s^t\right)+r_i\left(s^t\right)K_i\left(s^{t-1}\right)=\sum_{j=1}^{n}\lambda_{ji}\left(s^t\right)\left[W_j\left(s^t\right)L_j\left(s^t\right)+r_j\left(s^t\right)K_j\left(s^{t-1}\right)+D_j\left(s^t\right)\right]. \quad (51)$$

The other equilibrium conditions are as in the baseline model. We establish the following result:

Result 3 *The factor structure of real GDP comovement given in equation* (30) *holds in the model with capital in production, with the difference that* $\widetilde{A}_{i,t}$ *is not the exogenous total factor productivity but, instead, is the aggregate average productivity of labor (APL), and is an endogenous variable that depends on the capital-labor ratio, as follows*

$$\widetilde{A}_{i,t}=\widetilde{Z}_{i,t}+\alpha\left(\widetilde{K}_{i,t-1}-\widetilde{L}_{i,t}\right). \quad (52)$$

In Appendix I, Result 3 is proven, and all the equilibrium conditions in log-linear form are provided.

Next, we look at how the quantitative results are affected by the inclusion of capital in production. In the results we show, the capital share is set to $\alpha = 0.33$ and the depreciation rate to $\delta = 0.05$. The results are reported in Panel C of Table 6. The main finding, is that the trade-comovement puzzle is more pronounced in the model with capital in production. In particular, the correlation between trade and output synchronization is negative and the association between the APL correlation and trade also turns negative. Figure 5 shows how all three factors behave in a way not consistent with the data.

The reason for this has to do with the resource-shifting problem that has been identified by Kose and Yi (2006), whereby greater trade linkages lead to capital and other resources shifting to the country that receives the favorable productivity shock. This is illustrated in the first panel of Figure 6, that shows the IRF for investment after a TFP shock in the Home country. We find that, on impact, investment in Home increases, while investment in the nearest trade partner (for which bilateral trade costs are low) falls by roughly half the same amount. Instead, investment in the countries not closely integrated with the Home country increases. The reduction in investment in the nearest trade partner implies that average labor productivity falls relative to the average labor productivity in the other countries, since APL is a function of the capital-labor ratio. This result, which amplifies the trade-comovement puzzle in the model with capital in production is shown in the second panel of Figure 6.

6 Conclusion

Substantial empirical evidence suggests that countries or regions with stronger trade linkages have more correlated business cycles. However, from a theoretical perspective the IRBC model has difficulties in replicating this empirical fact. This has given rise to the so-called trade-comovement puzzle: standard models are unable to generate high output correlations arising from high bilateral trade intensity.

In this paper, we examine the sources of the trade-comovement puzzle. We show that within a large class of trade models (that includes the Armington model; Eaton and Kortum, 2002; Bernard et al., 2003; Krugman, 1980; and multiple versions of Melitz, 2003), there are three channels through which bilateral trade may increase business cycle synchronization: (i) if trade raises the correlation between each country's technology shocks; (ii) if trade leads to higher correlation between each country's share of expenditure on domestic goods; and (iii) if trade lowers the correlation between the share of expenditure on domestic goods and foreign TFP. Although the baseline model used to obtain this decomposition does not include capital in production, we show that a model with capital in production yields the same factor structure, with aggregate ALP replacing TFP.

Using bilateral trade data in manufactures for a panel of 21 OECD countries we find that the first and second channels are supported by the data: higher bilateral trade intensity is associated with higher TFP correlations and with a higher correlation between each country's share of expenditure on domestic goods. Consistent with the results of Kose and Yi (2006), we find that allowing for correlated TFP shocks is part of the solution to the trade-comovement puzzle. However, we show that even allowing for correlated shocks, the model still implies a counterfactual relation between bilateral trade and each country's correlation of the share of expenditure on domestic goods. Therefore, simply allowing for correlated shocks is not a satisfactory solution to the trade-comovement puzzle. Instead, our findings suggests that both the empirical and theoretical research should examine the sources of the positive association between trade and the comovement in the share of expenditure on domestic goods.

This invites more research to uncover new trade related transmission mechanisms of productivity shocks. A candidate ingredient for such models includes endogenous firm entry dynamics in settings with firm heterogeneity, in line with the framework of Ghironi and Melitz (2005). In their model, following a transitory productivity shock in the Home country, there is an increase in the number of new domestic firms. This raises domestic labor demand, softening the terms-of-trade deterioration, and may even lead to their improvement. Moreover, the transitory productivity shock also lowers the productivity threshold for firms to export; thus, the average productivity of exporters falls, which increases the average price of Home's exports. This is reinforced further if productivity shocks are combined with efficiency gains in setting up new firms, as shown in Corsetti, Martin, and Pesenti (2007), or if there are labor market frictions, as in Cacciatore (2014).

Another candidate mechanism to solve the trade-comovement puzzle, as shown in Drozd et al. (2014), are models featuring dynamic trade elasticities and, in particular, large long-run elasticities and small short-run elasticities. This is because, with large long-run elasticities, it is possible to match the cross-sectional dispersion in bilateral trade flows with less dispersion of the trade cost parameters. This makes high bilateral trade intensity more compatible with synchronization in the share of expenditure on domestic goods because it is possible to obtain high bilateral trade flows with relatively high trade costs (weakening the resource-shifting channel). As argued by Drozd et al. (2014), a model of dynamic trade elasticities can be micro-founded using product market search frictions à la Drozd and Nosal (2012). Therefore, future work should evaluate the quantitative implications of both mechanisms (endogenous entry dynamics and search frictions) to match the data, in the context of the factor structure representation we propose in this paper.

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APPENDIX

A Steady State Equilibrium

Let $\bar{X}$ denote the steady state level for the variable X. We consider a symmetric deterministic steady state equilibrium with $\bar{\lambda}_{ii}$, $\bar{W}_i$, $\bar{\mathscr{P}}_i$, $\bar{C}_i$ and $\bar{L}_i$ equal for all i. Moreover, in the symmetric equilibrium there are no trade deficits, so that $\bar{D}_i = 0$ for all i. The symmetry assumption makes it possible to solve analytically for the steady state. We measure everything in units of labor and, hence, $\bar{W} = 1$. Combining equations (7), (8) and (9) yields that in a symmetric steady state equilibrium, $\bar{\lambda}_{ii}$ is given by

$$\bar{\lambda} = \left(\sum_{j=1}^{n} \tau_{ij}^{-\theta} \right)^{-1} . \tag{A.1}$$

Substituting in (18) for $\bar{\lambda}$ using (A.1) yields

$$\bar{L} = \kappa^{-\sigma_n} \left(\sum_{j=1}^{n} \tau_{ij}^{-\theta} \right)^{\sigma_n/\theta} . \tag{A.2}$$

Making use of (9) to solve for the price level yields

$$\bar{\mathscr{P}} = \kappa \left(\sum_{j=1}^{n} \tau_{ij}^{-\theta} \right)^{-1/\theta} . \tag{A.3}$$

Using the fact that in the steady state equilibrium there are no trade deficits so that $\bar{\mathscr{P}}\bar{C} = \bar{W}\bar{L}$, yields the steady state level of consumption

$$\bar{C} = \kappa^{-(1+\sigma_n)} \left(\sum_{j=1}^{n} \tau_{ij}^{-\theta} \right)^{(1+\sigma_n)/\theta} . \tag{A.4}$$

Finally, equation (16) yields the steady state marginal utility of wealth

$$\bar{\mu} = \left[\bar{\mathscr{P}} \left(\bar{C} - \frac{\bar{L}^{1+1/\sigma_n}}{1 + 1/\sigma_n} \right) \right]^{-1} . \tag{A.5}$$

B Log-linear Equilibrium Conditions

Following standard steps, the individual optimality conditions and the market clearing conditions are log-linearized around the symmetric deterministic steady state equilibrium and combined so as to characterize the equilibrium dynamics. We consider all the variables in log-deviation except for the trade deficit which is considered in levels since the trade deficit must be zero in steady state. Let $\widetilde{X}_{i,t}$ denote the variable $X_i\left(s^t\right)$ in log-deviation from steady state and $\mathscr{D}_{i,t} = \left(\bar{\mathscr{P}}\bar{C}\right)^{-1} D\left(s^t\right)$ denote the ratio between the trade deficit at date t and the steady state income. The numéraire is taken to be labor in country 1, so that $\widetilde{W}_{1,t} = 0$ for all t. The log-linear equilibrium conditions are the following

$$\widetilde{\mu}_t + \widetilde{\mathscr{P}}_{i,t} = -\bar{\mathscr{P}}\bar{\mu}\left(\bar{C}\widetilde{C}_{i,t} - \bar{L}^{1+1/\sigma_n}\widetilde{L}_{i,t}\right), \quad \forall i = 1, \ldots, n, \tag{B.1}$$

$$\widetilde{\mu}_t = \widetilde{R}_t + E_t\left(\widetilde{\mu}_{t+1}\right), \tag{B.2}$$

$$\widetilde{\mathscr{P}}_{i,t} + \widetilde{A}_{i,t} = \bar{\lambda}\sum_{j=1}^{n}\widetilde{W}_{j,t}\tau_{ij}^{-\theta}, \tag{B.3}$$

$$\widetilde{\lambda}_{ji,t} = -\theta\left(\widetilde{W}_{i,t} - \widetilde{\mathscr{P}}_{j,t} - \widetilde{A}_{j,t}\right), \tag{B.4}$$

$$\widetilde{L}_{i,t} = \sigma_n\left(\widetilde{W}_{i,t} - \widetilde{\mathscr{P}}_{i,t}\right), \tag{B.5}$$

$$\widetilde{W}_{i,t} + \widetilde{L}_{i,t} = \sum_{j=1}^{n}\bar{\lambda}_{ji}\left(\widetilde{\lambda}_{ji,t} + \widetilde{W}_{j,t} + \widetilde{L}_{j,t} + \mathscr{D}_{j,t}\right), \tag{B.6}$$

$$\widetilde{\mathscr{P}}_{i,t} + \widetilde{C}_{i,t} = \widetilde{W}_{i,t} + \widetilde{L}_{i,t} + \mathscr{D}_{i,t}, \tag{B.7}$$

$$\sum_{i=0}^{n}\mathscr{D}_i\left(s^t\right) = 0. \tag{B.8}$$

Finally, in Section 4 we make use of equation (37). To obtain this equation, combine (B.3) and (B.4) to obtain

$$\begin{aligned}\widetilde{\lambda}_{ji,t} &= -\theta\left(\widetilde{W}_{i,t} - \bar{\lambda}\sum_{l=1}^{n}\widetilde{W}_{l,t}\tau_{jl}^{-\theta}\right) \\ &= -\theta\left[\left(\widetilde{W}_{i,t} - \widetilde{W}_{j,t}\right) + \bar{\lambda}\widetilde{W}_{j}\sum_{l=1}^{n}\tau_{jl}^{-\theta} - \bar{\lambda}\sum_{l=1}^{n}\widetilde{W}_{l,t}\tau_{jl}^{-\theta}\right] \\ &= -\theta\left[\left(\widetilde{W}_{i,t} - \widetilde{W}_{j,t}\right) - \bar{\lambda}\sum_{l=1}^{n}\left(\widetilde{W}_{l,t} - \widetilde{W}_{j,t}\right)\tau_{jl}^{-\theta}\right],\end{aligned} \tag{B.9}$$

where we have used the fact that $\bar{\lambda} = \left(\sum_{l=1}^{n}\tau_{jl}^{-\theta}\right)^{-1}$.

The upshot is that

$$\begin{aligned}\frac{\partial\widetilde{\lambda}_{j1,t}}{\partial\left(\widetilde{W}_{1,t} - \widetilde{W}_{j,t}\right)} &= -\theta\left(1 - \bar{\lambda}\tau_{j1}^{-\theta}\right) \\ &= -\theta\left(1 - \bar{\lambda}_{j1}\right),\end{aligned} \tag{B.10}$$

where we have used the fact that $\bar{\lambda}_{ji} = \bar{\lambda}\tau_{ji}^{-\theta}$. This corresponds to equation (37) in the main text.

C Proof of Result 1

If assumptions A1 and A2 are satisfied, the results in Arkolakis et al. (2012) imply that the following gravity equation holds

$$\lambda_{ji}\left(s^t\right) = \left[\frac{\kappa W_i\left(s^t\right)\tau_{ji}}{A_j\left(s^t\right)\mathscr{P}_j\left(s^t\right)}\right]^{-\theta}, \tag{C.1}$$

where κ is a constant parameter and $\mathscr{P}_j\left(s^t\right) = A\left(s^t\right)^{-1}\left[\int_0^1 p_j\left(n, s^t\right)^{1-\sigma} dn\right]^{1/(1-\sigma)}$ is the aggregate price level. Solving for the real wage we obtain

$$\frac{W_i\left(s^t\right)}{\mathscr{P}_j\left(s^t\right)} = \frac{A\left(s^t\right)}{\kappa}\left[\frac{1}{\lambda_{jj}\left(s^t\right)}\right]^{1/\theta}. \tag{C.2}$$

From assumption A3, it follows that the labor supply is a function only of the real wage, so that

$$L_i\left(s^t\right) = \left[\frac{W_i\left(s^t\right)}{\mathscr{P}_j\left(s^t\right)}\right]^{\sigma_n}, \tag{C.3}$$

where σ_n is the labor supply elasticity. The value added by each intermediate firm is equal to the sum of that firm's wage bill and profits, so that

$$p_i\left(v,s^t\right)y_i\left(v,s^t\right)=W_i\left(s^t\right)L_i\left(v,s^t\right)+\Pi_i\left(v,s^t\right), \tag{C.4}$$

where $\Pi_i\left(v,s^t\right)$ are profits for the intermediate firm producing variety v.[24] From assumption A2 profits are a constant share γ of revenues, so that $\Pi_j\left(v,s^t\right)=\gamma p_i\left(v,s^t\right)y_i\left(v,s^t\right)$. It follows that

$$\mathscr{Y}_i\left(s^t\right)\equiv\int_{\Omega_i(s^t)}p_i\left(v,s^0\right)y_i\left(v,s^t\right)dv=(1-\gamma)^{-1}\int_{\Omega_i(s^t)}\left[\frac{p_i\left(v,s^0\right)}{p_i\left(v,s^t\right)}\right]W_i\left(s^t\right)L_i\left(v,s^t\right)dv. \tag{C.5}$$

Making use of (C.1) and (C.2) yields

$$\mathscr{Y}_i\left(s^t\right)=\kappa^{-\sigma_n}A_i\left(s^t\right)^{1+\sigma_n}\left[\frac{1}{\lambda_{ii}\left(s^t\right)}\right]^{\sigma_n/\theta}, \tag{C.6}$$

where we have substituted for prices following the same steps described in Section 3.1. The upshot is that output in log-deviations from steady state is given by equation (29) as had to be shown.

D Proof of Result 2

From equation (29), GDP fluctuations in country i (in log-deviations from steady state) are given by

$$\widetilde{\mathscr{Y}}_{i,t}=(1+\sigma_n)\widetilde{A}_{i,t}-\left(\frac{\sigma_n}{\theta}\right)\widetilde{\lambda}_{ii,t}. \tag{D.1}$$

It follows that the covariance between the logarithm of output in country i and in country j is

$$\text{cov}\left(\widetilde{\mathscr{Y}}_i,\widetilde{\mathscr{Y}}_j\right)=\vartheta_1\,\text{cov}\left(\widetilde{A}_i,\widetilde{A}_j\right)+\vartheta_2\,\text{cov}\left(\widetilde{\lambda}_{ii},\widetilde{\lambda}_{jj}\right)+\vartheta_3\left[\text{cov}\left(\widetilde{\lambda}_{ii},A_j\right)+\text{cov}\left(\widetilde{\lambda}_{jj},A_j\right)\right], \tag{D.2}$$

where $\vartheta_1=(1+\sigma_n)^2$, $\vartheta_2=(\sigma_n/\theta)^2$ and $\vartheta_3=-(1+\sigma_n)(\sigma_n/\theta)$. In a symmetric world economy $\text{var}(\mathscr{Y}_i)$, $\text{var}\left(\widetilde{A}_i\right)$ and $\text{var}\left(\widetilde{\lambda}_{ii}\right)$ are the same for all countries and $\text{cov}\left(\widetilde{\lambda}_{ii},\widetilde{A}_j\right)=\text{cov}\left(\widetilde{\lambda}_{jj},\widetilde{A}_j\right)$. The upshot is that by dividing each side of equation (D.2) by $\text{var}(\mathscr{Y}_i)$, and dividing and multiplying the first term of the RHS by $\text{var}\left(\widetilde{A}_i\right)$, the second term by $\text{var}\left(\widetilde{\lambda}_{ii}\right)$ and the third term by $\text{std}(A_i)\times$

[24]Note that in our theoretical framework profits are zero because we are considering a model with perfect competition.

std $\left(\widetilde{\lambda}_{ii}\right)$ yields the equation

$$\text{cor}\left(\widetilde{\mathscr{Y}_i}, \widetilde{\mathscr{Y}_j}\right) = \beta_1 \,\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right) + \beta_2 \,\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right) + \beta_3 \text{cor}\left(\widetilde{\lambda}_{ii}, A_j\right), \tag{D.3}$$

where the factor loadings are given by

$$\beta_1 \equiv (1+\sigma_n)^2 \left[\frac{\text{var}\left(\widetilde{A}_i\right)}{\text{var}\left(\widetilde{\mathscr{Y}_i}\right)}\right], \quad \beta_2 \equiv \left(\frac{\sigma_n}{\theta}\right)^2 \left[\frac{\text{var}\left(\widetilde{\lambda}_{ii}\right)}{\text{var}\left(\widetilde{\mathscr{Y}_i}\right)}\right] \quad \text{and}$$

$$\beta_3 \equiv -2\left(1+\sigma_n\right)\left(\sigma_n/\theta\right) \left[\frac{\text{std}\left(\widetilde{A}_i\right) \text{std}\left(\widetilde{\lambda}_{ii}\right)}{\text{var}\left(\widetilde{\mathscr{Y}_i}\right)}\right].$$

Result 2 follows immediately from equation (D.3), where the factors (i), (ii) and (iii) are as follows: (i) the correlation between each country's TFP; (ii) the correlation between each country's share of expenditure on domestic goods; (iii) the correlation between the country's share of expenditure on domestic goods and the partner country's TFP (equal to the negative of the correlation between the country's import penetration ratio and the partner country's technology shocks).

E Data

We consider a sample of 21 OECD countries composed of Austria (AT), Belgium (BE), Denmark (DK), France (FR), Germany (DE), Italy (IT), Netherlands (NL), Norway (NO), Sweden (SW), Switzerland (CH), Canada (CA), Japan (JP), Finland (FI), Greece (GR), Ireland (IR), Korea (KO), Portugal (PT), Spain (SP), and New Zealand (NZ) United Kingdom (UK) and the United States (US), over the period 1988–2007. The variables definitions and data sources are as follows:

Real Output

Real output is measured using manufacturing GDP and CPI data from the OECD STAN database. Bilateral correlations are calculated using the linearly detrended log real output.

Country's Share of Expenditure on Domestic Goods

The measure λ_{ii} is the fraction of total manufacturing expenditure of country i on manufacturing goods made in country i. The expenditure on goods made at home is measured as the gross manufacturing output (converted to dollars using current exchange rates) less total manufacturing exports. Total manufacturing expenditure is measured as the sum of expenditure on manufacturing goods made at home and expenditure on total manufacturing imports. All data are obtained from the OECD STAN database. Bilateral correlations between λ_{ii} and λ_{jj} are calculated from the raw series of λ's.

Trade Intensity

We measure trade intensity between each country pair i and j, labeled $\left(\text{Bilateral Trade}\right)_{ij}$, by normalizing bilateral trade—that is, the sum of each country's manufacturing imports from the other—by the sum of nominal manufacturing GDP in the two countries, averaged over the entire period. Manufacturing imports data, denominated in dollars, are taken from the OECD STAN database. We normalize trade by nominal GDP (converted in dollars using current exchange rates), also from the STAN database.

Total Factor Productivity (TFP)

We measure TFP as the ratio between real manufacturing output and manufacturing employment obtained from the STAN database.

F Calibration of the Model

The number of countries n is set equal to four, implying six distinct country-pairs. Having six distinct country-pairs allows us to construct a symmetric world economy matching the minimum, maximum and median bilateral trade flows observed in the data.

The list of technology parameters that have to be determined includes the following: the elasticity of substitution between intermediate inputs σ; the parameter that controls the level of heterogeneity in productive efficiency θ; and the six trade-cost parameters τ_{ij} for each distinct country pair.[25] The first two parameters are chosen based on evidence in Bernard et al. (2003), who choose the parameters θ and σ matching the productivity and size advantage of exporters in the U.S. plant-level data. The parameter θ is chosen to match the productivity advantage of exporters,

[25] We assume $\tau_{ij} = \tau_{ji}$ and $\tau_{ii} = 1$.

and the parameter σ corresponds to the price elasticity of demand for differentiated intermediate commodities and therefore relates to the size advantage of exporting establishments. The values estimated by Bernard et al. (2003) for θ and σ are, respectively, 3.60 and 3.79.

The trade-cost parameters τ_{ij} are chosen to match the minimum, maximum and median bilateral trade flows observed in the OECD Structural Analysis (STAN) database. To construct a symmetric world economy, the matrix of trade costs must be symmetric, of the form

$$\mathcal{T} = \begin{bmatrix} 1 & \tau_L & \tau_M & \tau_H \\ \tau_L & 1 & \tau_H & \tau_M \\ \tau_M & \tau_H & 1 & \tau_L \\ \tau_H & \tau_M & \tau_L & 1 \end{bmatrix}, \tag{F.1}$$

with $1 < \tau_L < \tau_M < \tau_H$. We choose τ_L and τ_H to match, respectively, the maximum and minimum bilateral trade flow observed in our data and τ_M to match the median trade flow. In a symmetric world economy the bilateral trade flow between two countries i and j is given by λ_{ij}. Let $\lambda_{\min}$, $\lambda_{\max}$ and λ_{med} denote the minimum, maximum and median bilateral trade flows. Then, from the manipulation of equation (9) we obtain, in the symmetric world economy with $n = 4$ and $\bar{W} = 1$ (labor is the numéraire) the following relations:

$$\begin{aligned} &1 - \lambda_{\min} - \lambda_{\max} - \lambda_{\text{med}} = \left(1 + \tau_L^{-\theta} + \tau_M^{-\theta} + \tau_H^{-\theta}\right)^{-1}, \\ &\tau_L = \left(\frac{\lambda_{\max}}{1 - \lambda_{\min} - \lambda_{\max} - \lambda_{\text{med}}}\right)^{-1/\theta}, \quad \tau_H = \left(\frac{\lambda_{\min}}{1 - \lambda_{\min} - \lambda_{\max} - \lambda_{\text{med}}}\right)^{-1/\theta}, \\ &\tau_M = \left(\frac{\lambda_{\text{med}}}{1 - \lambda_{\min} - \lambda_{\max} - \lambda_{\text{med}}}\right)^{-1/\theta}. \end{aligned} \tag{F.2}$$

Thus, given targets for $\lambda_{\min}$, $\lambda_{\max}$ and λ_{med} it becomes possible to set values for each of the three trade costs τ_L, τ_H and τ_M. The resulting matrix of trade costs is given by

$$\mathcal{T} = \begin{bmatrix} 1.000 & 1.127 & 2.801 & 5.948 \\ 1.127 & 1.000 & 5.948 & 2.801 \\ 2.801 & 5.948 & 1.000 & 1.127 \\ 5.948 & 2.801 & 1.127 & 1.000 \end{bmatrix}. \tag{F.3}$$

The remaining technology parameters that need to be chosen are the parameters of the stochastic process for the technology shocks, $\widetilde{A}_{i,t}$, a finite-state Markov process with states and transition probabilities set to approximate the continuous autoregressive model given by (up to a constant)

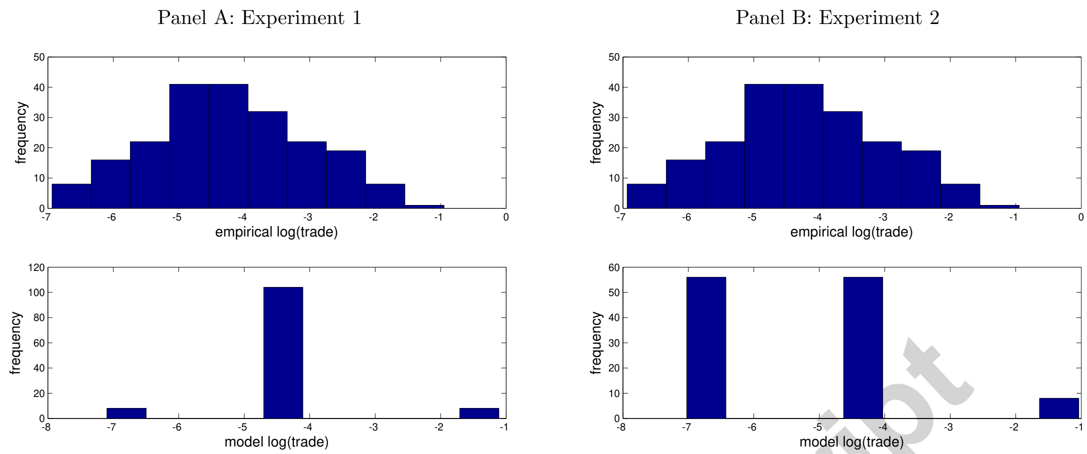

Figure 7: trade and comovement (16 country model)

$$\widetilde{A}_{i,t} = \rho \widetilde{A}_{i,t-1} + \mathcal{E}_t,$$

where $\mathcal{E}_t$ is a normally distributed and zero-mean shock with standard deviation σ_ϵ. We allow the shocks to be correlated across countries but in the baseline model set the correlation coefficient to be the same for all country-pairs. This correlation coefficient is chosen to match the median bilateral correlation of TFP observed in our sample. In one of our additional quantitative experiments we allow the correlation of TFP shocks across countries to vary with the level of trade between each country-pair. We choose values for σ_ϵ and ρ to match the standard deviation and the autocorrelation of output fluctuations in the US. The only preference parameter that must be chosen is the labor supply elasticity σ_n. In our baseline calibration we set $\sigma_n = 1$ consistent with the estimates of the labor supply elasticity using macro data.

G Results Based on Model With 16 countries

In this section we consider a calibrated world economy of 16 countries (120 trade partners), and conduct two alternative experiments: Experiment 1, where each country has bilateral trade intensity with 13 partners equal to the median bilateral trade intensity in the data, it has bilateral trade intensity with one partner equal to the minimum bilateral trade intensity in the data, and it has

Table 7: Trade and business cycle synchronization (16 country model)

	dependent variable			
	$\text{cor}\left(\widetilde{\mathscr{Y}}_i, \widetilde{\mathscr{Y}}_j\right)$	$\text{cor}\left(\widetilde{A}_i, \widetilde{A}_j\right)$	$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{\lambda}_{jj}\right)$	$\text{cor}\left(\widetilde{\lambda}_{ii}, \widetilde{A}_j\right)$
	1.	2.	3.	4.
	Panel A – model: 16 countries (experiment 1)			
log (Bilateral Trade)	0.064	0.061	−0.061	0.006
constant	0.604	0.560	−0.334	−0.023
	Panel B – model: 16 countries (experiment 2)			
log (Bilateral Trade)	0.064	0.061	−0.042	0.008
constant	0.598	0.566	−0.294	−0.006

In both Panel A and Panel B the model is solved assuming endogenous TFP correlations.
In Panel B the model is solved under complete markets.

bilateral trade intensity with one partner equal to the maximum bilateral trade intensity in the data; Experiment 2, where each country has low bilateral trade with 7 countries, the median bilateral trade with 7 countries, and high bilateral trade with just one partner.

Panel A of Figure 7 shows the distribution of log bilateral trade used in Experiment 1 and contrasts it to the actual empirical distribution. The results concerning trade and comovement are shown in Panel A of Table 7 and should be contrasted to Table 4. The findings are virtually identical to the baseline results. This happens because the distribution of the logarithm of bilateral trade is roughly symmetric (as shown in Figure 7), meaning that most country-pairs have bilateral trade in logs equal to the median value, with symmetric tales.[26] However, findings may change if we oversample countries with low bilateral trade intensity.

This is the purpose of Experiment 2. Panel B of Figure 7 shows the distribution of log bilateral trade used in Experiment 2, where each country has low bilateral trade with 7 countries, median bilateral

[26] Of course, bilateral trade in levels is strongly right skewed, with most country pairs trading very little.

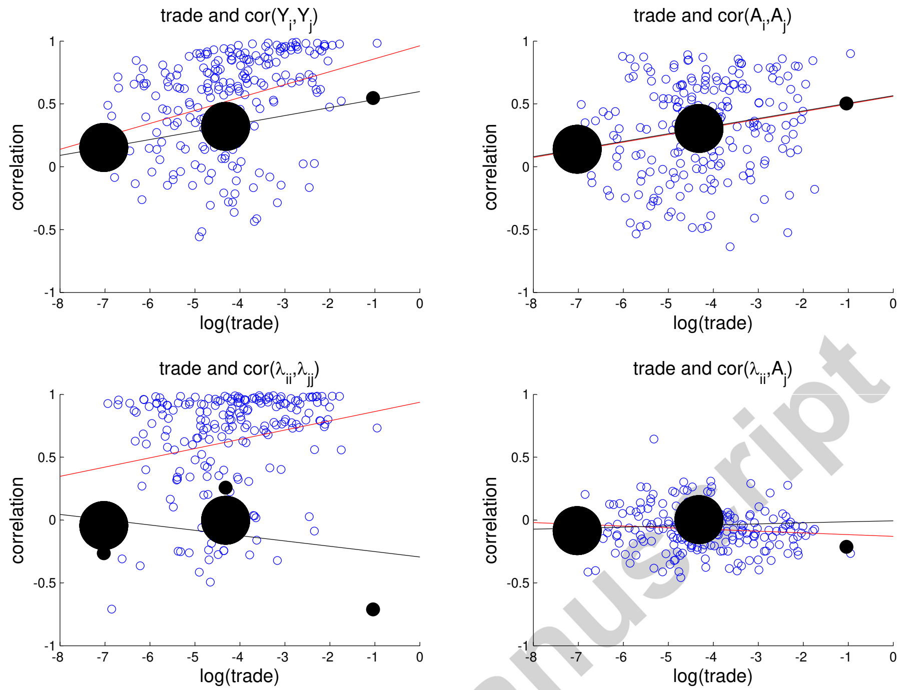

Figure 8: trade and comovement (16 country model)

trade with another 7 countries and high bilateral trade with a single country. This experiment is relevant if the relationship between log bilateral trade and comovement is non-linear and if bilateral trade bellow the median is very predominant in the data, which to a certain extent is true (the empirical distribution of log bilateral trade is not exactly symmetric and is slightly right skewed).

By oversampling the minimum bilateral trade, we are giving the best possible chance to this non-linearity. The results are shown in Panel B of Table 7. They reveal that the non-linearity is unimportant to explain the link between bilateral trade in logs and output correlation and, in particular, the FR coefficient is virtually unaffected, at 0.064. The only factor for which the non-linearity plays a role concerns the correlation of the share of expenditure on domestic goods. However, the relation with trade is still counterfactual, at -0.042.

These findings are represented graphically in Figure 8, that reproduces the scatter plots in Figure 2, but with the dots of the model based observations represented in different sizes, proportional to the frequency of their sampling. The regression lines represented correspond to the estimated regression equations and, hence, take into account the oversampling of countries with low bilateral trade flows. It is apparent from the Figure that the results are not strongly affected by the implied non-linearity.

H Factor Structure With Alternative Preferences

The purpose of this Section is to show that when the preferences allow for intertemporal substitution in labor supply the factor structure (30) still holds for the model solved using log-linearization methods around the deterministic steady state.

When the preferences are of the form in (41), the optimality condition determining the choice between leisure and consumption can be written in log-linear form, as follows

$$\widetilde{L}_{i,t} = \sigma_n \left(\widetilde{W}_{i,t} - \widetilde{\mathscr{P}}_{i,t} - \widetilde{C}_{i,t} \right). \tag{H.1}$$

where $\widetilde{X}_{i,t}$ denotes the variable $X\left(s^t\right)$ in log-deviation from steady state.

If the model is solved using log-linearization methods, the resulting approximate policy functions are linear functions of the state vector, s_t. In particular, the approximate policy functions for the choice of hours worked and consumption in country i are given by

$$\widetilde{L}_{i,t} = h'_{il} s_t, \tag{H.2}$$

$$\widetilde{C}_{i,t} = h'_{ic} s_t, \tag{H.3}$$

where $h_{il} = \left(h^1_{il}, \ldots, h^m_{il}\right)'$ and $h_c = \left(h^1_{ic}, \ldots, h^m_{ic}\right)'$ are $n-$dimensional parameter vectors and $s_t = \left(s^1_t, \ldots, s^m_t\right)'$ is the $n-$dimensional vector of state variables. Since the equilibrium conditions do not include endogenous predetermined variables, the state vector s_t only includes exogenous stochastic random variables (the TFP levels in each country).[27]

For simplicity, but without loss of generality, suppose s_t is a scalar random variable (say, TFP in country 1). Then we have that $\widetilde{C}_{i,t} = h_{ic} s_t$ and $s_t = \left(\widetilde{L}_{i,t}/h_{il}\right)$. Substituting in equation (H.1), we

[27] The only endogenous predetermined variables in the model are the levels of debt in each country, $B_i\left(s^t, s\right)$. However, these variables are not part of the equilibrium conditions. Instead, the equilibrium conditions include the trade deficits in each country, $D_i\left(s^t\right) = B_i\left(s^{t-1}, s_t\right) - \sum_{s\in\mathcal{S}} \mathcal{Q}\left(s^t, s\right) B_i\left(s^t, s\right)$, that are not predetermined variables.

obtain

$$\widetilde{L}_{i,t} = \sigma_n \left[\widetilde{W}_{i,t} - \widetilde{\mathscr{P}}_{i,t} - h_{ic} \left(\widetilde{L}_{i,t}/h_{il} \right) \right], \tag{H.4}$$

and solving for $\widetilde{L}_{i,t}$ yields

$$\begin{aligned} \widetilde{L}_{i,t} &= \frac{\sigma_n}{1 + \sigma_n \left(h_{ic}/h_{il} \right)} \left(\widetilde{W}_{i,t} - \widetilde{\mathscr{P}}_{i,t} \right), \\ &= \varsigma \left(\widetilde{W}_{i,t} - \widetilde{\mathscr{P}}_{i,t} \right), \\ &= \varsigma \left(\widetilde{A}_{i,t} - \theta^{-1} \widetilde{\lambda}_{ii,t} \right). \end{aligned} \tag{H.5}$$

with $\varsigma = \sigma_n \left(1 + \sigma_n \left(h_{ic}/h_{il}\right)\right)^{-1}$, and where the final equation makes use of equation (17) in log-deviations from steady state.

From equation (26), we have that $\widetilde{\mathscr{Y}}_{i,t} = \widetilde{A}_{i,t} + \widetilde{L}_{i,t}$ and making use of equation (H.5) to substitute for $\widetilde{L}_{i,t}$, we obtain

$$\widetilde{\mathscr{Y}}_{i,t} = (1 + \varsigma) \widetilde{A}_{i,t} - \left(\frac{\varsigma}{\theta} \right) \widetilde{\lambda}_{ii,t}, \tag{H.6}$$

which has the same structure as equation (D.1). Therefore, the same factor structure as in (30) applies to the model with intertemporal substitution in labor supply, when the model is solved using log-linear methods.

I Capital in Production

In this Section we provide a detailed description of the version of the model that includes physical capital in production which is examined in Section 5.3. First, we show how to measure GDP at constant prices in this version of the model. Second, we show that the factor structure of GDP correlations still obtains. Finally, we provide a detailed list of the model's log-linear equilibrium conditions.

GDP at Constant Prices (Capital in Production)

We first show how to construct a measure of real GDP consistent with the National Accounts, analogous to the one derived in Section 3.1 but for the version of the model with physical capital.

As before, GDP at constant prices in country i is

$$\begin{aligned}\mathscr{Y}\left(s^t\right) &\equiv \int_{\Omega_i(s^t)} p_i\left(v,s^0\right) y_i\left(v,s^t\right) dv, \\ &= Z_i\left(s^t\right) \int_{\Omega_i(s^t)} p_i\left(v,s^0\right) \varphi_{i,v} K_i\left(v,s^{t-1}\right)^{\alpha} L_i\left(v,s^t\right)^{1-\alpha} dv, \\ &= Z_i\left(s^t\right) K_i\left(s^{t-1}\right)^{\alpha} L_i\left(s^t\right)^{1-\alpha} \int_{\Omega_i(s^t)} p_i\left(v,s^0\right) \varphi_{i,v} \left[\frac{K_i\left(v,s^{t-1}\right)}{K_i\left(s^{t-1}\right)}\right]^{\alpha} \left[\frac{L_i\left(v,s^t\right)}{L_i\left(s^t\right)}\right]^{1-\alpha} dv,\end{aligned} \tag{I.1}$$

with $K_i\left(s^t\right) = \int_{\Omega_i(s^t)} K_i\left(v,s^t\right) dv$ and $L_i\left(s^t\right) = \int_{\Omega_i(s^t)} L_i\left(v,s^t\right) dv$, the aggregate capital stock and hours worked, respectively.

Next, notice that

$$p_i\left(v,s^0\right) = \frac{r_i\left(s^0\right)^{\alpha} W_i\left(s^0\right)}{\varkappa Z\left(s^0\right) \varphi_{i,v}}, \tag{I.2}$$

is the same for all varieties v, except for the time-invariant idiosyncratic productivity shock $\varphi_{i,v}$. Thus, the product $c_t = p_i\left(v,s^t\right)\varphi_{i,v}$, is the same for all varieties v. It follows that, from the first-order conditions solving the intermediate producer problem, we have that

$$\begin{aligned}K_i\left(v,s^{t-1}\right) &= \left[\frac{\alpha p\left(v,s^t\right)\varphi_{i,v} Z_i\left(s^t\right)}{r_i\left(s^t\right)}\right]^{1/(1-\alpha)} L_i\left(v,s^t\right), \\ &= \left[\frac{\alpha c_t Z_i\left(s^t\right)}{r_i\left(s^t\right)}\right]^{1/(1-\alpha)} L_i\left(v,s^t\right).\end{aligned} \tag{I.3}$$

From the above condition, it follows that $\left(K_i\left(v,s^{t-1}\right)/L_i\left(v,s^t\right)\right) = \left(K_i\left(s^{t-1}\right)/L_i\left(s^t\right)\right)$ is the same for all varieties v and, equivalently, we have that

$$\frac{K_i\left(v,s^{t-1}\right)}{K_i\left(s^{t-1}\right)} = \frac{L_i\left(v,s^t\right)}{L_i\left(s^t\right)}. \tag{I.4}$$

Using (I.4) in (I.1), we obtain

$$\begin{aligned}\mathscr{Y}\left(s^t\right) &= Z_i\left(s^t\right) K_i\left(s^{t-1}\right)^{\alpha} L_i\left(s^t\right)^{1-\alpha} \int_{\Omega_i(s^t)} p_i\left(v,s^0\right)\varphi_{i,v} \left[\frac{K_i\left(v,s^{t-1}\right)}{K_i\left(s^{t-1}\right)}\right] dv, \\ &= Z_i\left(s^t\right) K_i\left(s^{t-1}\right)^{\alpha} L_i\left(s^t\right)^{1-\alpha} c_0,\end{aligned} \tag{I.5}$$

where we have used the fact that $K_i\left(s^{t-1}\right) = \int_{\Omega_i(s^t)} K_i\left(v,s^{t-1}\right) dv$, and where $c_0 = p_i\left(v,s^0\right)\varphi_{i,v}$ is a positive constant which is irrelevant for GDP fluctuations and thus can be ignored.

Factor Structure of GDP Comovement (Capital in Production)

In this Section, we show that the factor structure of GDP comovement applies to the model with capital in production. To see this, use equation (I.5) and consider the average labor productivity in the model with capital, given by

$$\frac{\mathscr{Y}_i\left(s^t\right)}{L_i\left(s^t\right)} = Z_i\left(s^t\right)\left[\frac{K_i\left(s^{t-1}\right)}{L_i\left(s^t\right)}\right]^{\alpha} \equiv A_i\left(s^t\right). \tag{I.6}$$

This measure of productivity corresponds to our empirical measure of TFP and also to the measure used in the baseline model (27). However, in the version of the model with capital, it is more appropriate to refer to it as average labor productivity (ALP). Most importantly, notice that while in the baseline model $A_i\left(s^t\right)$ is exogenous, here it is endogenous and depends on the capital-labor ratio.

Next, from equation (49), we have that

$$\begin{aligned}\lambda_{ii}\left(s^t\right) &= \left[\frac{\kappa r_i\left(s^t\right)^{\alpha} W_i\left(s^t\right)^{1-\alpha}}{\varkappa Z_i\left(s^t\right)\mathscr{P}_i\left(s^t\right)}\right]^{-\theta}, \\ &= \left[\frac{\kappa\left(r_i\left(s^t\right)/W_i\left(s^t\right)\right)^{\alpha} W_i\left(s^t\right)}{\varkappa Z_i\left(s^t\right)\mathscr{P}_i\left(s^t\right)}\right]^{-\theta}.\end{aligned} \tag{I.7}$$

From the first-order conditions solving the intermediate producer problem, we have that

$$r_i\left(s^t\right) = \alpha p_i\left(v, s^t\right)\varphi_{i,v} Z_i\left(s^t\right)\left(L_i\left(v, s^t\right)/K_i\left(v, s^{t-1}\right)\right)^{1-\alpha}, \tag{I.8}$$

$$W_i\left(s^t\right) = (1-\alpha)\, p_i\left(v, s^t\right)\varphi_{i,v} Z_i\left(s^t\right)\left(K_i\left(v, s^{t-1}\right)/L_i\left(v, s^t\right)\right)^{\alpha}, \tag{I.9}$$

for each variety v. Combining the two equations above it follows that the capital-labor ratio is the same for all firms $\left(K_i\left(v, s^{t-1}\right)/L_i\left(v, s^t\right)\right) = \left(K_i\left(s^{t-1}\right)/L_i\left(s^t\right)\right)$, with

$$\frac{r_i\left(s^t\right)}{W_i\left(s^t\right)} = \frac{\alpha}{1-\alpha}\frac{L_i\left(s^t\right)}{K_i\left(s^{t-1}\right)}, \tag{I.10}$$

where $\left(L_i\left(s^t\right)/K_i\left(s^{t-1}\right)\right)$ is the aggregate labor-capital ratio in country i. Plugging (I.10) in (I.7) and making use of equation (I.6), yields

$$\lambda_{ii}\left(s^t\right) = \left[\frac{\kappa W_i\left(s^t\right)}{(1-\alpha)\, Z_i\left(s^t\right)\left(K_i\left(s^{t-1}\right)/L_i\left(s^t\right)\right)^{\alpha}\mathscr{P}_i\left(s^t\right)}\right]^{-\theta}, \\ = \left[\frac{\kappa W_i\left(s^t\right)}{(1-\alpha)\, A_i\left(s^t\right)\mathscr{P}_i\left(s^t\right)}\right]^{-\theta}. \tag{I.11}$$

Using (I.11) to solve for the real wage, yields

$$\frac{W_i\left(s^t\right)}{\mathscr{P}\left(s^t\right)} = \frac{(1-\alpha)\, A_i\left(s^t\right)}{\kappa}\left[\frac{1}{\lambda_{ii}\left(s^t\right)}\right]^{1/\theta}, \tag{I.12}$$

an equation analogous to (17), but where $A\left(s^t\right)$ now corresponds to the aggregate average labor productivity. Finally, combining (I.6) and (I.12), with the optimality condition (15), we obtain

$$\mathscr{Y}\left(s^t\right) = A\left(s^t\right) L\left(s^t\right) = A\left(s^t\right)\left(W\left(s^t\right)/\mathscr{P}\left(s^t\right)\right)^{\sigma_n}, \\ = \left((1-\alpha)/\kappa\right)^{\sigma_n} A_i\left(s^t\right)^{1+\sigma_n}\left[\frac{1}{\lambda_{ii}\left(s^t\right)}\right]^{\sigma_n/\theta}, \tag{I.13}$$

which again, is analogous to (28). From (I.13) it follows that

$$\widetilde{\mathscr{Y}}_{i,t} = (1+\sigma_n)\,\widetilde{A}_{i,t} - \left(\frac{\sigma_n}{\theta}\right)\widetilde{\lambda}_{ii,t}. \tag{I.14}$$

This equation has the same structure as equations (29) and (D.1). Thus, it follows that the factor structure of GDP comovement given by (30) also holds for the model with capital, with the only difference that, from equation (I.6), the aggregate average labor productivity in logs, is given by

$$\widetilde{A}_{i,t} = \widetilde{Z}_{i,t} + \alpha\left(\widetilde{K}_{i,t} - \widetilde{L}_{i,t}\right), \tag{I.15}$$

which is no longer exogenous, but depends on the capital-labor ratio. This proves Result 3.

Log-linear Equilibrium Conditions (Capital in Production)

The log-linear equilibrium conditions for the model with capital in production are the following

$$\widetilde{\mu}_t + \widetilde{\mathscr{P}}_{i,t} = -\bar{\mathscr{P}}\bar{\mu}\left(\bar{C}\widetilde{C}_{i,t} - \bar{L}^{1+1/\sigma_n}\widetilde{L}_{i,t}\right), \quad \forall\, i = 1,\ldots,n, \tag{I.16}$$

$$\widetilde{\mu}_t = \widetilde{R}_t + E_t\left(\widetilde{\mu}_{t+1}\right), \tag{I.17}$$

$$E\left(\widetilde{r}_{i,t+1}\right) = E_t\left[\frac{\widetilde{\mu}_t - \widetilde{\mu}_{t+1}}{1-(1-\delta)\beta}\right], \tag{I.18}$$

$$\widetilde{\mathscr{P}}_{i,t} + \widetilde{Z}_{i,t} = \bar{\lambda}\sum_{j=1}^{n}\left[\alpha\widetilde{r}_{j,t} + (1-\alpha)\widetilde{W}_{j,t}\right]\tau_{ij}^{-\theta}, \tag{I.19}$$

$$\widetilde{\lambda}_{ji,t} = -\theta\left[\alpha\widetilde{r}_{i,t} + (1-\alpha)\widetilde{W}_{i,t} - \widetilde{\mathscr{P}}_{j,t} - \widetilde{Z}_{j,t}\right], \tag{I.20}$$

$$\widetilde{L}_{i,t} = \sigma_n\left(\widetilde{W}_{i,t} - \widetilde{\mathscr{P}}_{i,t}\right), \tag{I.21}$$

$$\widetilde{r}_{i,t} - \widetilde{W}_{i,t} = \widetilde{L}_{i,t} - \widetilde{K}_{i,t-1}, \tag{I.22}$$

$$(1-\alpha)\left(\widetilde{W}_{i,t} + \widetilde{L}_{i,t}\right) + \alpha\left(\widetilde{r}_{i,t} + \widetilde{K}_{i,t-1}\right) = \sum_{j=1}^{n}\bar{\lambda}_{ji}\left[\widetilde{\lambda}_{ji,t} + (1-\alpha)\left(\widetilde{W}_{j,t} + \widetilde{L}_{j,t}\right) + \alpha\left(\widetilde{r}_{j,t} + \widetilde{K}_{j,t-1}\right) + \mathscr{D}_{j,t}\right], \tag{I.23}$$

$$\widetilde{\mathscr{P}}_{i,t} + \left(1-\delta\bar{K}/\bar{Y}\right)\widetilde{C}_{i,t} + \left(\delta\bar{K}/\bar{Y}\right)\widetilde{I}_t = (1-\alpha)\left(\widetilde{W}_{i,t} + \widetilde{L}_{i,t}\right) + \alpha\left(\widetilde{r}_{i,t} + \widetilde{K}_{i,t-1}\right) + \mathscr{D}_{i,t}, \tag{I.24}$$

$$\widetilde{K}_{i,t} = \delta\widetilde{I}_{i,t} + (1-\delta)\widetilde{K}_{i,t-1}, \tag{I.25}$$

$$\sum_{i=0}^{n}\mathscr{D}_i\left(s^t\right) = 0, \tag{I.26}$$

$$\widetilde{A}_{i,t} = \widetilde{Z}_{i,t} + \alpha\left(\widetilde{K}_{i,t-1} - \widetilde{L}_{i,t}\right). \tag{I.27}$$