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# The Study of Chain-like Materials for Use in Biomedical Ultrasound

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**Abstract**—Wave propagation in chain-like materials has been studied previously at low frequencies. The present study has generated these waves at higher frequencies with components  $>200$  kHz, using chains of 1 mm diameter spheres. Resonant ultrasonic horns at 73 kHz have been used as sources of narrowband excitation, which transform into a train of broadband impulses that have the characteristics of solitary waves. These have potential applications in biomedical ultrasound as high amplitude, wide bandwidth impulses.

**Keywords**—non-linear; chains; solitary waves.

## I. INTRODUCTION

There has been much recent interest in the use of chain-like structures for use with acoustic and ultrasonic signals [1,2]. Consider a chain of spheres, such as that shown in Fig.1, which is subject to an externally-applied pre-compression force  $F_0$ . An input signal, in the present case a windowed sinusoidal tone-burst ( $F_{IN}$ ), is applied at one end. The signal propagates along the chain, and on exit it will be modified by interaction with the spheres. In particular, an interesting phenomena can be highlighted, namely the Hertzian contact between each pair of spheres. This leads to non-linearity, the nature of which is dependent upon the relative values of  $F_{IN}$  and  $F_0$ ; thus, if  $F_{IN} \gg F_0$ , the non-linearity is maximized. This non-linearity will lead to the creation of increased bandwidth. In addition, the use of a chain of spheres leads to the potential existence of non-linear normal modes of vibration of the system [3,4]. The nature of the resultant signals is, in fact, highly dependent on additional factors such as the size, shape and number of particles.

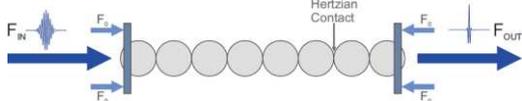


Fig. 1. Schematic diagram of a chain of spheres.

Provided that  $F_{IN}$  is sufficiently large, non-linearity will lead to the generation of harmonics, which means that the wave transmitted from one sphere to the next is modified as it travels. Thus, an impulsive input could narrow in time, and

could also be focused using variable delays. This led to the concept of a “sound bullet” [5]. Alternatively, a sine wave input would be distorted by harmonic generation to a new periodic waveform. In a chain of finite length, the signals would also reflect between each end of the chain, further changing the detected signal at the output ( $F_{OUT}$  in Fig. 1).

In the present work, the aim was to generate a set of impulses at high amplitude, primarily for therapeutic High Intensity Focused Ultrasound (HIFU) and drug delivery applications. These should also have as wide a bandwidth as possible. To achieve this, it was decided to use a high input amplitude in the form of a sinusoidal windowed tone-burst, and to use this to drive signals through a chain under negligible pre-compression ( $F_{IN} \gg F_0$ ). Note that recent studies have tended to concentrate on using an impulsive input [6]. This study focusses on maximizing the ease of generating large values of  $F_{IN}$ , using resonant excitation at higher frequencies; this could perhaps be extended towards the frequencies used in HIFU and other applications. The result should then be the maximization of harmonic generation and hence increased bandwidth. It was also of interest to establish whether a solitary wave could be generated. This requires both non-linearity and dispersion to exist, and in fact this is a natural occurrence within such chains. Solitary waves have interesting properties (solitons will pass through another without interaction for example, and can travel long distances without changing their fundamental characteristics), and tend to travel along the chains with particular characteristics. It is these characteristics which this research intends to exploit.

A different strategy has been adopted in the results below to those of other researchers, by focusing on the generation of a series of harmonics and sub-harmonics in a chain of spheres, using a narrow bandwidth input. This leads to the generation of solitary wave impulses, and these would be amplified after multiple reflections between the two ends of the chain. Thus, the approach would be an interaction between the generation of different frequency components [7,8], and the natural normal modes of the chain itself [3,4]. This would need to be

considered carefully in the experimental design, so that the conditions might be created for optimal generation of solitary travelling wave impulses.

## II. EXPERIMENTAL DESIGN

An ultrasonic horn was used as the source of high amplitude input forces. This had a primary resonance at 73 kHz plus others at higher frequencies. It was driven by an amplified tone-burst of 20 cycles at 73 kHz. This was chosen so as to allow the vibrations at the tip of the horn to build up to a high amplitude, while keeping the overall time duration of the input signal to a reasonable value. The motion at the horn tip under these conditions is shown in Fig. 2(a), as measured using a Polytec vibrometer. It can be seen that the amplitude at the tip (in this case measured as a particle velocity waveform) builds up close to a maximum value, and then decays over a similar timescale. Note that the frequency response is thus a resonant peak, with a finite width at half-maximum value of  $\sim 4$  kHz, as shown in Fig. 2(b).

The tip of the horn was positioned so as to just contact the first sphere in a chain via a layer of ultrasonic couplant, as shown in the schematic diagram of Fig. 3. The chain of six stainless steel ball-bearings, each of 1 mm diameter, was held horizontally within a cylindrical channel, with the spheres touching under minimal pre-compression  $F_0$ . At the far end, a plate containing an aperture was used to contain the chain.

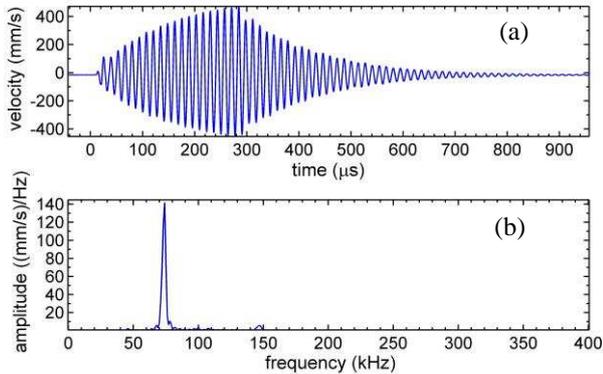


Fig. 2. (a) Waveform of the motion of the vibrating horn tip, as measured using a vibrometer. Excitation was a tone-burst of 20 cycles at 73 kHz. (b) Spectrum of the waveform shown in (a).

The particle velocity waveform of the end sphere could then be measured using the vibrometer. Note that the amplitude and frequency of  $F_{IN}$  could be varied using the waveform generator and power amplifier. The signals were recorded using a digital oscilloscope for later analysis.

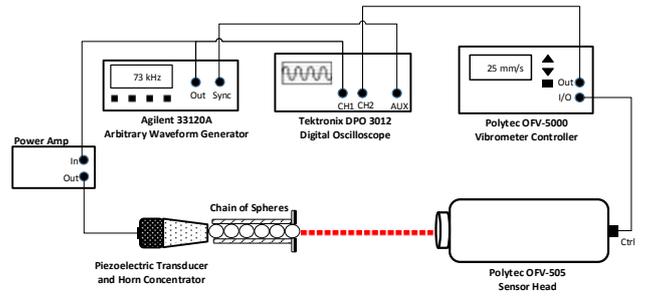


Fig. 3. Schematic diagram of the apparatus

## III. THEORY

Nesterenko [1] has described the processes that would be expected in these chains. The model is based on an analysis which looks at the motion of the centre of each sphere. Assuming that the displacements of the centre of successive spheres are  $u_1, u_2, \dots, u_n$ , it is possible to derive a model which describes the expected motion of the final sphere, as a function of the input waveform. Hertzian contact leads to an expression describing the force  $F$  between adjacent spheres, which is of the form:

$$F = \frac{2E_s}{3(1-\nu_s^2)} \left( \frac{R_1 R_2}{R_1 + R_2} \right)^{1/2} [(R_1 + R_2) - (x_2 - x_1)]^{3/2} \quad (1)$$

Here,  $E$  is the Young's modulus and  $\nu$  the Poisson's ratio of each sphere, and  $R_1$  and  $R_2$  are radii of the spheres with centre coordinates  $x_1$  and  $x_2$ . Note that the force  $F$  depends on the displacements of the sphere centers to the power  $3/2$ , the source of the non-linear behavior.

Using Eq. (1), it is possible to derive a set of equations which describe the dynamic behavior of each sphere in the chain. This can then be used to predict the motion of the final sphere, and hence the output at the far end. For the last sphere, the equation can be written as

$$m \frac{d^2 u_N}{dt^2} = \frac{2\sqrt{R}}{3} \left[ \frac{\theta_m}{\sqrt{2}} (u_{N-1} - u_N)^{3/2} - 2\theta_r u_N^{3/2} \right] + \lambda (\dot{u}_{N-1} - \dot{u}_N) H(u_{N-1} - u_N) - \lambda \dot{u}_N H(u_N). \quad (2)$$

Here,  $\theta_r$  is the effective Young's modulus for interaction between the last sphere of radius  $R$  and the end wall of any containment, and  $\lambda$  is a damping coefficient to represent losses (due to friction, viscous damping etc) in the system. Similar expressions can be derived for the interaction between the first sphere and the horn, and between other spheres within the chain. It is possible to predict the behavior for any input waveform. The equations demonstrate that the subsequent behavior of the chain will depend on the size, elastic properties and number of spheres. Further analysis shows that the resultant propagation is expected to have an upper frequency limit, or cut-off frequency, beyond which solitary wave behavior is not expected.

## IV. EXPERIMENTAL RESULTS

Experiments have been performed in which the input signal at 73 kHz was steadily increased in amplitude, and the output waveform recorded in each case using the vibrometer. The spectrum of the received signal was also recorded, together

with the input waveform from the vibrating horn tip, at each excitation level. The evolution of solitary wave transients could then be observed as a function of increased input amplitude. Three examples are shown in Figs. 4-6.

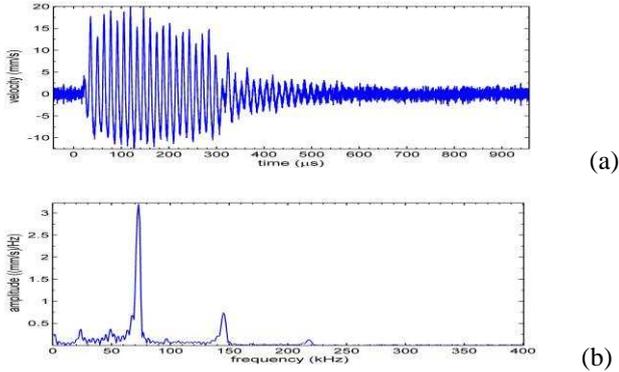


Fig.4. (a) Waveform and (b) spectrum of an experimental signal recorded for a chain of 6 ball-bearings of 1 mm diameter. The input force  $F_{IN}$  was small, having a particle velocity amplitude of 0.15 m/s at 73 kHz.

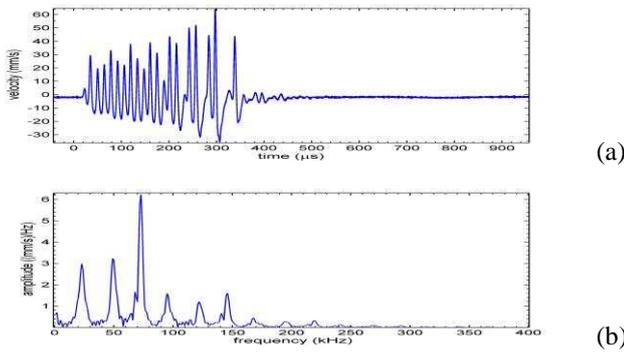


Fig.5. As Fig. 4, but for an increased input force  $F_{IN}$  with a particle velocity amplitude of  $0.41 \text{ ms}^{-1}$  at 73 kHz.

It can be seen that both the waveform and spectra change with input amplitude level. At the lowest amplitudes, Fig. 4, the measured waveform in the chain exhibits the characteristics of weakly non-linear ultrasound propagation, where most of the signal's energy distributed at the fundamental frequency with the presence of several harmonics and a low amplitude sub-harmonic. As the input amplitude increases, Fig. 5, a set of shorter transients starts to develop, with a spectrum that contains distinct harmonics and sub-harmonics frequency peaks, with the maximum peak at the excitation frequency. At the highest input amplitude, Fig. 6, the impulses are now very distinct, and the maximum peak amplitude is at a sub-harmonic of the input in the frequency spectrum. This demonstrates that the increased input amplitude is an important parameter, as the creation of impulses becomes more consistent with the  $F_{IN} \gg F_0$  condition.

The horn could also be driven at different resonant frequencies. Fig. 7 shows an example when a frequency of 112 kHz was used at the maximum input particle velocity amplitude (0.55 m/s). Although the signals are smaller in amplitude, the fundamental frequency has been retained, and there is also a strong sub-harmonic at 56 kHz. Finally, results are presented in Fig. 8 for a horn frequency of 207 kHz. The input amplitude is now much smaller, and hence the detected signal (Fig. 8(a)) has a much reduced signal to noise ratio. However, it can be seen that the main feature is a single arrival with a decaying oscillation centered around 70 kHz (Fig. 8(b)). This represents a sub-harmonic at 1/3 of the original input frequency.

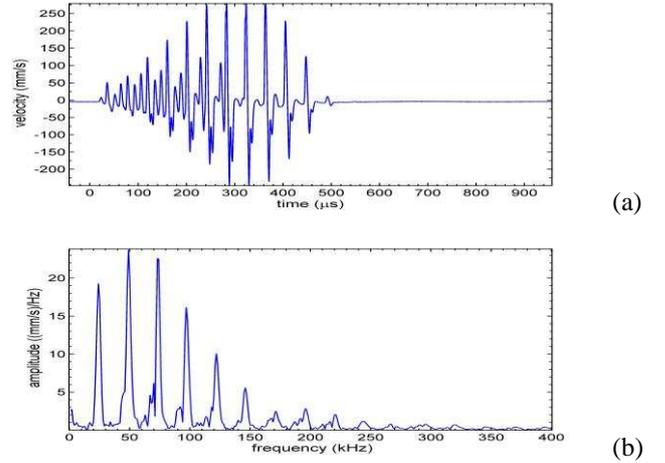


Fig.6. As Fig. 4, but for the largest input force  $F_{IN}$  at 73 kHz.

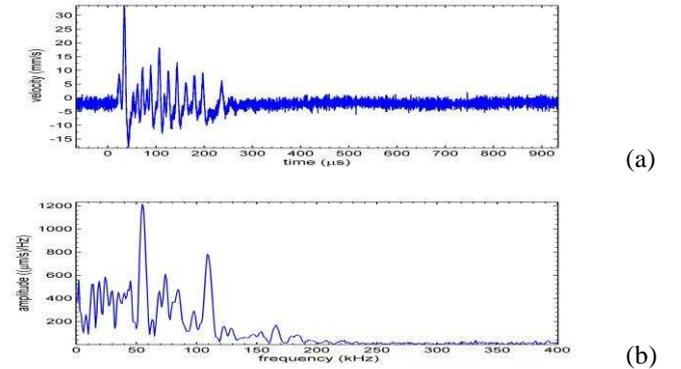


Fig.7. (a) Waveform and (b) spectrum of an experimental signal recorded for a chain of 6 ball-bearings of 1 mm diameter at 112 kHz.

## V. DISCUSSION

Consider first the data collected using a 73 kHz excitation frequency (Figs. 4-6). Here, the process at low input energies (Fig. 4) starts as expected with the generation of harmonics due to non-linear behavior at the interface between each pair of spheres. The signal is still dominated by the fundamental at 73 kHz, and two harmonics at 146 kHz and 219 kHz are present.

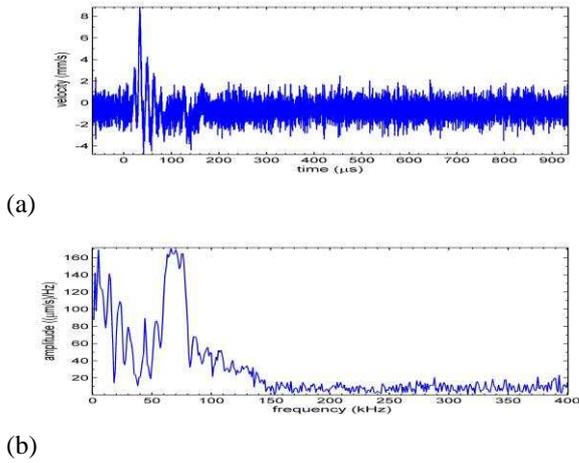


Fig.8. (a) Waveform and (b) spectrum of an experimental signal recorded for a chain of 6 ball-bearings of 1 mm diameter. The input force  $F_{IN}$  was at 207 kHz.

There is a hint of a lower frequency peak in the spectrum of Fig. 4(b). This corresponds, in fact, to the resonance of the full set of six spheres, i.e. to the frequency expected from reflection of signals between the two ends of the chain ( $\sim 24$  kHz).

As the input drive signal increases in amplitude, Fig. 5, the time waveform of Fig. 5(a) starts to exhibit the development of separate impulsive signals later in the waveform; these seem to develop in time. The corresponding spectrum (Fig. 5(b)) demonstrates a strong link to the low frequency component seen in Fig. 4. In fact, the observed frequency peaks correspond to a set of harmonics of the lowest peak at 24 kHz. This is because the signal now develops a set of impulses which reflect within the chain, developing a periodic waveform determined by the chain characteristics (i.e. effectively a Fourier series). At the highest input amplitudes, Fig. 6, the time waveform contains a set of prominent impulses, reflecting within the chain. These build up with time, and then decay once the input signal itself stops. These are thought to be travelling solitary wave impulses, each of which has a broad bandwidth, separated in time by two transits through the chain of 6 spheres.

It is possible to use the theory outlined briefly in Section III to predict what would be expected from the Hertzian contact problem within a chain of spheres. The initial results of a simulation performed with a 20 cycles of tone-burst at 73 kHz is shown in Fig. 9. This is for negligible values of the pre-compression force  $F_0$ , and thus corresponds approximately to the results shown in Fig. 6. It can be seen that, while some details differ, the main wave behavior is predicted – a set of impulses that build up over time, with the main frequency components being both sub-harmonics and harmonics of the original drive frequency. The lowest frequency peak corresponds to that expected from the round-trip travel time of the impulses, as in the experiment.

The experiments at higher excitation frequencies (presented in Figs 7 and 8) show that the generation of harmonics was much more difficult under these conditions. This might be expected from a smaller degree of non-linearity at the lower excitation amplitudes used.

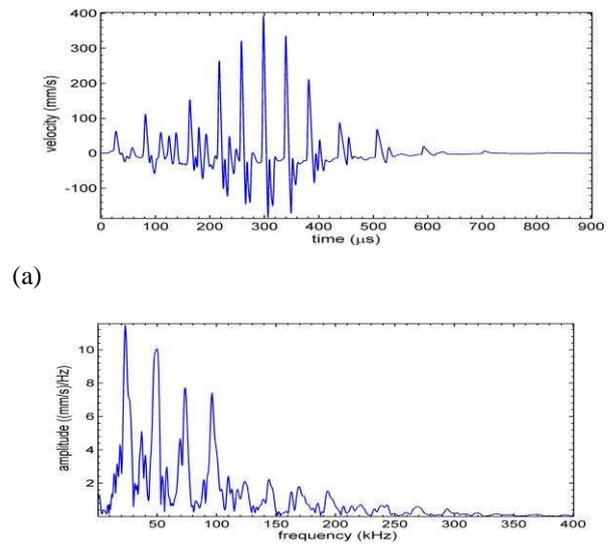


Fig.9. Theoretical predictions showing (a) the waveform and (b) the corresponding spectrum for a chain of 6 ball-bearings of 1 mm diameter. The input force  $F_{IN}$  used for the prediction was the same as that used experimentally in Fig. 6 at 73 kHz.

## VI. CONCLUSIONS

It has been demonstrated that a narrow-bandwidth input at a frequency of 73 kHz can be used to generate solitary wave impulses in a chain of 6 spheres. These have many potential uses in biomedical ultrasound, particularly for therapeutic use.

## ACKNOWLEDGMENT

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