Monograph:
Adnan, M. and Fowkes, A.S. (2009) A Novel Macroscopic Dynamic Loading Model and its Properties. Working Paper. Leeds Institute of Transport Studies (Unpublished)

Leeds Institute for Transport Studies Working Paper 593

ITS Working Paper 593

October 2009

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Muhammad Adnan and Anthony Fowkes

Working Paper No. 593
Institute for Transport Studies

A Novel Macroscopic Dynamic Loading Model and its Properties

Muhammad Adnan[1] **and** ***Anthony Fowkes***[2]

[1] PhD Student, Institute for Transport Studies, University of Leeds, England. Phone: +44(0) 113 3431788 E-mail: m.adnan04@leeds.ac.uk
[2] Reader in Transport Econometrics, Institute for Transport Studies, University of Leeds, England, Phone: +44(0) 113 3435340, E-mail: A.S.Fowkes@its.leeds.ac.uk

Abstract

Existing macroscopic dynamic loading models (Linear travel time, Divided linear travel time and Point-Queue models), which are based on representation of link properties as a whole and fully comply with the requirements of the dynamic traffic assignment (DTA) procedure, are widely used in many DTA studies. This is because of their lower implementation and computational costs. DTA literature consistently suggesting alternative models but many of them do not comply with the required properties for their use in DTA and if the model fulfils these requirements, the computational and implementation issues in these models are such demanding that their general use is very limited (e.g. Cell Transmission model). This paper presents a novel model (Adnan-Fowkes model), which utilise the similar modelling framework through which Point-Queue model was developed and at the same time addresses the drawbacks exist in the Point-Queue and Liner travel time models which are due to their simple mathematical formulation. The proposed model is developed with a simple mathematical construction and it is as easy to implement as Point-Queue and Linear travel time models. The paper comprehensively discusses the properties which are desirable for the use of any model in DTA and analytically illustrates that the Adnan-Fowkes model is successfully fulfilling all these requirements. Numerical experiments are also conducted for comparison of the behaviour of the Adnan-Fowkes model along with the Point-Queue and Linear travel time models. The results of these experiments provide more useful insight for the Adnan-Fowkes model and support the characteristics of this model which are mentioned analytically in earlier sections of this paper.

1. Introduction

A new model is proposed in this paper that addresses the drawbacks of underestimation of travel time of the “bottleneck” (or “point-queue”) model (Heydecker and Addison 1998, Mun 2001) and overestimation of travel time when the “linear travel time” model (Friesz et al 1993, Astarita 1996, Mun 2001, Mun 2007) is used for dynamic loading of vehicles on the link. The bottleneck model allows incorporation of congestion effects only when inflow rate exceeds capacity and the outflow rate obtained from the model at this point is equal to capacity of the link. This involves a major simplification of reality, since increasing congestion will cause increasing travel times before full capacity of outflow is reached. On the other hand, the linear travel time model immediately assumes that travel times are rises as soon as there is any traffic i.e. if only two or three vehicles are travelling on the link, these vehicles will also effect the travel time of entering vehicles and cause overestimated travel times in almost free flow traffic conditions. This has been termed as double- counting effect in the literature (Nie and Zhang 2005). A model is proposed in this paper which would behave between the bottleneck and linear travel time models and allow more realistic incorporation of congestion effects. A divided linear travel time model developed by Mun (2001) in order to address the drawbacks of Point-Queue and Linear travel time model is also illustrated and compared with the model proposed in this paper to describe the fundamental difference between them and to show which is more appropriate for the measurement of travel times. In addition to this it is also shown that, like point-queue, linear travel time and divided linear travel time models, the proposed model also fulfils the desirable properties for dynamic traffic assignment (DTA). We shall refer to the new model as the Adnan-Fowkes model.

Empirical findings, (Jang et al 2005) regarding the variation of travel time with the traffic level, have suggested that travel time follows a convex path. However, macroscopic models reported in the literature, which are based on the variables that represents the characteristics of the link as a whole, follows a linear travel time function dependent on vehicles traversing on the link. This situation has arisen because non-linear forms of travel time function violate an important desirable condition for DTA i.e. the

FIFO (first in, first out) condition (Nie and Zhang 2005, Mun 2001). The Adnan-Fowkes model, proposed in this paper, also follows a linear travel time function in such a manner that it approximates a convex path due to its definition of the outflow rate, but retains the FIFO property. Figure 1 shows the behaviour of different loading models with each other, it is clear from the figure that linear travel time model is overestimating travel time at the stage when link is not significantly busy and on the other hand Point-Queue model is underestimating travel time at the stage when link is busy but not overloaded. The other two models i.e. Divided linear travel time and Adnan-Fowkes models behaving between these two extreme models. This paper is structured as follows; section 2 illustrates the model formulation along with the analytical comparison with other three models, section 3 describes the properties of the Adnan-Fowkes model, section 4 presents the investigation of the models against the desirable properties for the DTA and section 5 presents numerical implementation of all four models for their comprehensive comparison. Finally, last section concludes the paper.

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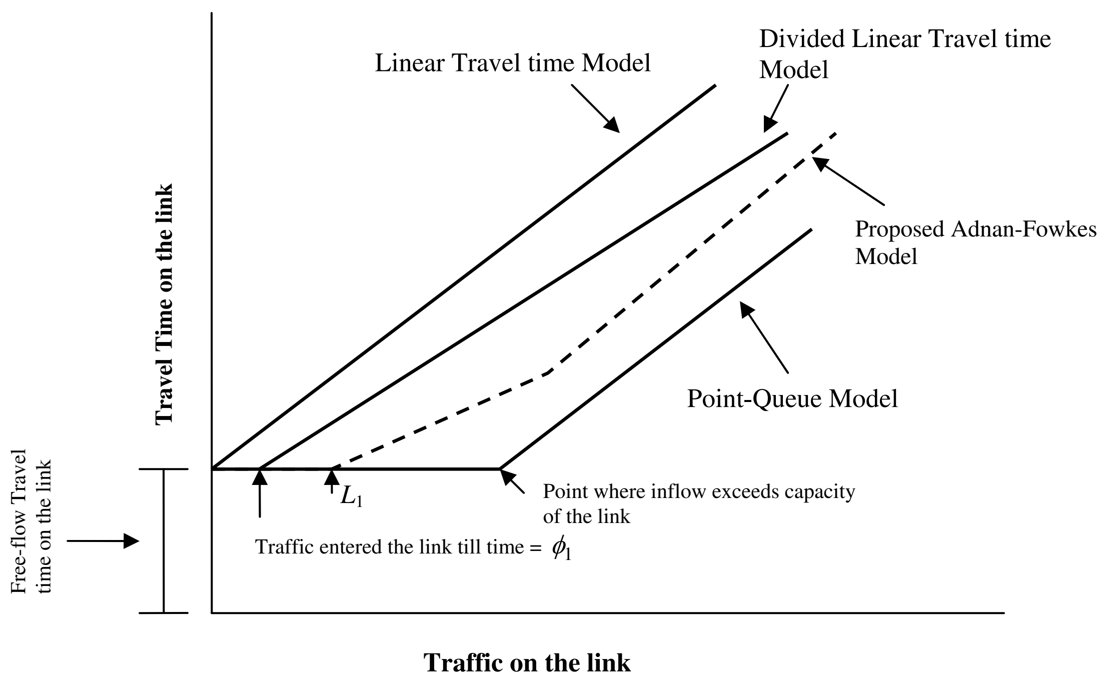


Figure 1: Behaviour of different Loading Models

2. Model Formulation

The Adnan-Fowkes model is illustrated in Figure 1, where it is compared to other three models. It can be viewed as an extension of point-queue model, however, instead of two states (free-flow and fully-congested flow) we are proposing three states (free-flow, partially-congested flow and fully-congested flow) within the model. In addition to that we are using two outflow controlling parameters which constrain the behaviour of the model in such a manner that it not only removes the overestimation error in the linear travel time model under less congested environment but also removes the underestimation error in the Point-Queue model when the link is moderately congested but has not yet reached at its full capacity. The Adnan-Fowkes model is given by eqns. (1) and (2) as follows:

$$v(t) = \begin{cases} u(t-\phi) + z(t) & u(t-\phi) + z(t) < L_1 \\ \dfrac{L_1 + (n-1)\left[u(t-\phi) + z(t)\right]}{n} & L_1 \le u(t-\phi) + z(t) < L_2 \\ C & u(t-\phi) + z(t) \ge L_2 \end{cases} \tag{1}$$

and

$$L_2 = \frac{nC - L_1}{n-1} \tag{2}$$

or, equivalently,

$$L_1 = nC - (n-1)L_2 \tag{2a}$$

$$\frac{d\,z(t)}{dt} = \begin{cases} -z(t) & u(t-\phi) + z(t) < L_1 \\ \dfrac{u(t-\phi) - (n-1)\,z(t) - L_1}{n} & L_1 \le u(t-\phi) + z(t) < L_2 \\ u(t-\phi) - C & u(t-\phi) + z(t) \ge L_2 \end{cases} \tag{3}$$

where, $u(t-\phi)$ is the inflow rate at time $t-\phi$

ϕ as the free flow travel time on the link

$z(t)$ represents the number of vehicles in the queue at the end of the link

$v(t)$ represents outflow rate at time t

L_1 represents the link inflow $(L_1 \le C)$ below which travel time on the link equals free flow journey time and,

L_2 represents the link inflow that first causes outflow to reach the capacity level of the link.

For comparison, the bottleneck model, the most widely used model in DTA because of its simplicity, is given in this notation as:

$$v(t)=\begin{cases} u(t-\phi) & z(t)=0 \text{ and } u(t-\phi)<C \\ C & otherwise \end{cases} \tag{4}$$

and

$$\frac{d\,z(t)}{dt}=\begin{cases} 0 & z(t)=0 \text{ and } u(t-\phi)<C \\ u(t-\phi)-C & otherwise \end{cases} \tag{5}$$

The travel time $R(t)$ for vehicle entering at t , for the bottleneck model, can be given as

$$R(t)=\phi+\frac{z(t+\phi)}{C} \tag{6}$$

This says that travel time is equal to free-flow travel time up to the point where a queue starts to form, where after travel time increase linearly with the amount of queuing traffic. Matching inflow to outflow will then maintain journey times at whatever level had then been reached. Equation (6) is retained in the Adnan-Fowkes model. Of course, because the amount of queuing is different in the two models, actual values for $R(t)$ will differ between the two models.

An inconsistency is noted in the analytical formulation of this model which violates flow conservation requirement at a particular situation. For instance, suppose a link in which traffic is heavily loaded and after that there is no further inflow, queue at the end of the link is start dissipating (using the second state of the model in which outflow is equal to the capacity, C), now a situation will come when queue ($z(t)$) is less than the capacity C, but this model always predict outflow equals to C when there is queue, even though this queue is less than C. This is the violation of flow conservation in which total inflow to the link is not equal to the total outflow and number of vehicles on the link for this particular situation. This problem has been overcome in the Adnan-Fowkes model by properly representing all the states that allows dissipation of queues.

The third model being considered is the linear travel time model. This is given by

$$R(t) = \phi + \frac{x(t)}{C} \tag{7}$$

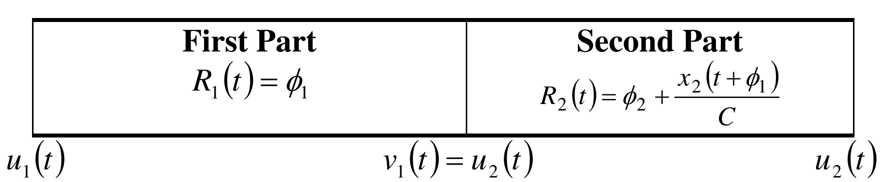
where, $x(t)$ is the traffic on the link, and is calculated using equations (8) and (9), which are described as flow conservation and flow propagation functions.

$$x(t) = \int u(t)dt - \int v(t)dt \tag{8}$$

$$v(t + R(t)) = \frac{u(t)}{\frac{d\varphi(t)}{dt}} = \frac{u(t)}{1 + \frac{dR(t)}{dt}} \tag{9}$$

Evidently, with the linear travel time model, travel times start at the free-flow travel time when there is no traffic on the link, then rise linearly as any traffic at all is added. Furthermore, equation (9) says that if journey times remain constant from one period to the next, that implies that inflow at time *t* exactly matches outflow at time $t + \varphi(t)$. Where, $\varphi(t)$ is the exit time of vehicles that entered at time *t*.

The fourth model being considered is the divided linear travel time model. According to Mun (2001), the link is divided into two parts one is the area where traffic can propagate with free-flow speed and the other is the one where the linear travel time model is applied. He found out that when the linear travel time model is discretised for its implementation, the ratio of the length of analysis time interval (Δt) to free flow travel time (ϕ) is in the range of 0.8 ~1 the outflow profile obtained from this model is much smoother. Therefore, he suggested that in the second part of the link, the free flow travel time is equivalent to length of analysis time interval. This can be better understood from figure 2.



A divided linear travel time model: where, $\phi = \phi_1 + \phi_2$

Travel time of the first part of the link: $R_1(t) = \phi_1 = \phi - \Delta t$

Travel time of the second part of the link: $R_2(t) = \phi_2 + \frac{x_2(t+\phi_1)}{C} = \Delta t + \frac{x_2(t+\phi_1)}{C}$

Figure 2: Schematic representation of divided linear travel time model

where, ϕ_1 and ϕ_2 are the free flow travel time of the first and second part of the link respectively, $x_2(t+\phi_1)$ is the amount of traffic on the second part of the link and Δt is the length of analysis time interval (discretised time step, e.g. 1 min or 0.5 min). Accordingly, the total link travel time is then,

$$R(t) = R_1(t) + R_2(t) = \phi + \frac{x_2(t+\phi_1)}{C} \tag{10}$$

For determination of number of vehicles and outflow, this model also utilise flow conservation and propagation equations (8) and (9) for second part of the link. The model respects FIFO principle and consistent with all other requirements of DTA (Mun 2001). Similar to linear travel time model, this model also follows the assumption of linearity in estimation of travel time, thus non-linear behaviour of travel time increase with congestion is not addressed. However, overestimation problem of the linear travel time model in uncongested condition is successfully addressed to an extent by using only that proportion of traffic on the link for measuring queuing delay which is exist in the second part of the link. It can be seen from the illustration that, this model (divided linear travel time model) do not incorporate any congestion effects till time reaches at ϕ_1 and after this time this model starts incorporating congestion effects irrespective of the link inflow rate. This suggests that division of the linear travel time model in this model is based on time (which should be less than free-flow travel time), however, in the Adnan-Fowkes model division of the states is based on link inflows (similar to the point-queue model). This is illustrated further in numerical experiment section.

3. Derivation of the Adnan-Fowkes Model

At this point we should pause to give some insight into the Adnan-Fowkes model and check that the three states of the model join correctly together.

We first check the state boundary conditions for eqn. (1).

When $u(t-\phi) + z(t) = L_1$,

$$\text{STATE } 2 = \frac{u(t-\phi) + z(t) + (n-1)[u(t-\phi) + z(t)]}{n} = u(t-\phi) + z(t) = \text{STATE } 1$$

and, when $u(t-\phi)+z(t)=L_2$,

using equation (2a)

$$\text{STATE} \quad 2 = \frac{\{nC - (n-1)[u(t-\phi)+z(t)]\} + (n-1)[u(t-\phi)+z(t)]}{n} = C = \text{STATE} \quad 3$$

The special property of the above presented model is that when L_1 assumed equal to *C*, equation (2) gives L_2 equal to *C* as well regardless of the value of *n* and the model eventually collapses into Point-Queue model. It would be interesting to suggest value of *n* for which the model provides plausible results, however, its true value needs to be calibrated through examination of real data. Table 1 shows the model behaviour by assuming different values of L_1 and *n*.

The model proposed here now requires to be tested against the desirable properties for DTA and also a numerical comparison will be carried out with the other models i.e. bottleneck, linear travel time model in order to investigate its consistency and behaviour with the already existing models. The section below describes desirable properties for the DTA and model behaviour against them.

Table 1: Model Behaviour with different values of L_1 and *n*

L_1	n	L_2 (from equation 2)	$v(t)$ (from equation 1)	*Comments*
C	*>1*	*C*	$\begin{cases} u(t-\phi) \\ C \end{cases}$	2nd state in equation 1 is inactive, and Model collapses to Point-Queue model.
0.5C	2	*1.5C*	$\begin{cases} u(t-\phi) \\ 0.25C+0.5\,u(t-\phi) \\ C \end{cases}$	All three states are active and model may gives behaviour as half-way between linear travel time and Point-Queue models.
0.5C	*3*	*1.25C*	$\begin{cases} u(t-\phi) \\ 0.167C+0.667\,u(t-\phi) \\ C \end{cases}$	All three states are active here as well and again it behaves half-way between linear travel time and Point-Queue model.
0.5C	5	*1.125C*	$\begin{cases} u(t-\phi) \\ 0.1C+0.8\,u(t-\phi) \\ C \end{cases}$	All three states are active here as well and again it behaves half-way between linear travel time and Point-Queue model. Showing the range of *n* in which model is behaving plausibly.
0.5C	*100*	*1.005C*	$\begin{cases} u(t-\phi) \\ 0.005C+0.99u(t-\phi) \\ C \end{cases}$	All three states are active here as well and again it behaves half-way between linear travel time and Point-Queue model. Range between L_1 and L_2 is squeezes with increase in *n*. Model again collapsing towards Point-Queue model.

4. Examination of Model against Desirable Properties for DTA

There are several requirements identified from the literature that appropriate dynamic loading models should meet for their application to dynamic traffic assignment (DTA) (Mun (2007), Heydecker and Addison (2006), Carey (2004) and Mun (2001)). These are as follows

- Flow Conservation
- Flow Propagation
- First-in-First Out (FIFO)
- Causality
- Reasonable Outflow behaviour
- Positivity, existence and uniqueness

The following paragraphs discuss the above mentioned requirement in detail

- *Flow Conservation:*

Conservation of traffic on the link is considered as the important requirement for loading models as one cannot imagine that a vehicle that entered a link will disappear and do not exit from the link at all or in other terms total outflow exceeds from the total inflow to the link at any time. Mathematically this is expressed as

$$U(t-\phi) = V(t) + z(t) \tag{11}$$

where, $U(t-\phi)$ and $V(t)$ are *accumulated* inflow and outflow at time $t-\phi$ and t respectively. If we consider that at initial time t_0 link is empty, then the above equation ensures that difference between the cumulative inflow and outflow is the amount of vehicles that joined the queue at the end of the link $z(t)$. If cumulative inflow and outflow terms are considered at time t then there difference is represented as $x(t)$(i.e. amount of vehicles traversing on the link), and eqn. (11) is then equivalent to eqn. (8). If eqn. (11) is differentiated with respect to time t, it can be written as

$$\frac{d\ z(t)}{dt} = u(t-\phi) - v(t) \tag{12}$$

Use of eqn. (12) along with eqn. (1) gives eqn.(3), which is already considered as part of the proposed model. Thus Adnan-Fowkes model conserve the flow at any time t according to eqn. (11). The proof of this is as follows

Proof

Taking the right side of eqn. (12) and substituting the appropriate value from that states given by eqn. (1) we have:

In STATE 1, $u(t-\phi)-v(t)=u(t-\phi)-u(t-\phi)-z(t)=-z(t)$ as shown RHS of eqn. (3)

In STATE 2, $u(t-\phi)-v(t)=u(t-\phi)-[\{L_1+(n-1)(u(t-\phi)+z(t))\}/n]$
$= [u(t-\phi)-(n-1)z(t)-L_1]/n$

In STATE 3, $u(t-\phi)-v(t)=u(t-\phi)-C$

Next, we check that eqn. (3) is continuous by checking the state boundaries.

When $u(t-\phi)+z(t)=L_1$,

STATE 2 = $[u(t-\phi)-(n-1)z(t)-L_1]/n \;=\; -z(t)$ = STATE 1

When $u(t-\phi)+z(t)=L_2$,

STATE 2 = $[u(t-\phi)-(n-1)z(t)-L_1]/n$
$=[u(t-\phi)-(n-1)z(t)-(nC-(n-1)L_2)]/n = u(t-\phi)-C$ = STATE 3

- *Flow Propagation:*

In dynamic settings, the flow on the link should propagate in a manner that is consistent with the speed of the vehicle. The minimum time taken for a vehicle to traverse the link should not be shorter than the free flow travel time. Mathematically this is expressed as follows

$$U(t)=V(\varphi(t)) \tag{13}$$

where, $\varphi(t)$ is the link exit time for vehicles that entered at time t. Differentiating eqn. (13) with respect to entry time t gives

$$u(t)=v(\varphi(t))\frac{d\varphi(t)}{dt} \tag{14}$$

Eqn. (14) is similar to eqn.(8), this suggests that linear travel time model directly uses the flow propagation eqn. for calculating outflow rate. In the literature of DTA, this eqn. is considered as time-flow consistency equation because it ensures the consistency between the three important ingredients i.e. inflow, outflow and travel time when FIFO holds. The proposed model, in which eqn. (1) defines the outflow rate function along with the definition of travel time function in a similar manner as defined in the Point-Queue model. Therefore, the proposed model does not require eqn. (13) at any stage. However, the notion behind the flow propagation is articulated with the use of a free-flow travel time ϕ, as the minimum travel time that is required to traverse on the link. Outflow models in which travel time is taken as a function of outflow rate, do not have this free-flow travel time term because of which these model are not able to describe spatial propagation of flow on the link (Mun 2007).

- *FIFO:*

As we are dealing here with macroscopic dynamic loading models, in which behaviour of group of vehicles is modelled, it is entirely necessary to make sure that FIFO condition is not violated. However, in microscopic models in which each vehicle treated as a separate entity, FIFO can be violated by permitting overtaking, as in reality. This is because in macroscopic models, it is considered that vehicles that enter the link at the same time will exit the link after experiencing the same travel time. It is suggested in the literature (Astarita 1996, Mun 2001) that as far as $d\varphi(t)/dt \geq 0$ is satisfied, FIFO principle is intact. Additionally, violation of this condition will gives negative outflow if eqn. (14) is used for its calculation, which is in contrast to reality and also violates flow conservation property. The condition, $d\varphi(t)/dt \geq 0$, suggested that rate of change of travel time on link at any time t should be greater than -1 (i.e. $d\,R(t)/dt \geq 0$) as exit time $\varphi(t)$ is the combination of $R(t) + t$. In the proposed model, If we differentiate equation (6), then we can get

$$\frac{d\,R(t)}{dt} = \frac{1}{C}\,\frac{d\,z(t+\phi)}{dt} \tag{15}$$

For showing that the model fulfils FIFO, it requires that all states of the model in equation (3) should be greater than or equal to – *C*.

Proof

Eqn. (3) which represents the rate of change of traffic in the queue at time *t,* can be reformulated to represents the same change of rate at time $t+\phi$ for the consistency of time dimension. This can be given as

$$\frac{d\, z(t+\phi)}{dt}=\begin{cases} -z(t+\phi) & u(t)+z(t+\phi)<L_1 \\ \dfrac{u(t)-(n-1)\,z(t+\phi)-L_1}{n} & L_1\le u(t)+z(t+\phi)<L_2 \\ u(t)-C & u(t)+z(t+\phi)\ge L_2 \end{cases} \quad (16)$$

If STATE 1 is considered in equation (16), which is constrained by the inflow L_1 and by definition this should be less than or equal to *C,* therefore, boundary condition for STATE 1 should follow $u(t)+z(t+\phi)<C$, which suggests that $z(t+\phi)<C$, this can be written as $-z(t+\phi)>-C$. So, FIFO is respected in the STATE 1. The proof for STATE 3 is also very simple to illustrate for this property, i.e. inflow rate should always follow $e(t)\ge 0$, which suggests that the minimum possible value of STATE 3 is – *C*, so FIFO is maintained here as well. The STATE 2 of equation (16) is constrained with two boundary conditions; i.e. $u(t)+z(t+\phi)\ge L_1$ and $u(t)+z(t+\phi)\le L_2$, therefore, the proof is first illustrated for the 1st boundary condition and then for the 2nd boundary condition.

The STATE 2 of the model is given by

$$\frac{d\, z(t+\phi)}{dt}=\frac{u(t)+z(t+\phi)-L_1-n\,z(t+\phi)}{n} \quad (17)$$

Using 1st boundary condition for this STATE, the smallest value $u(t)+z(t+\phi)$ can take is equal to L_1 , therefore, substitution of this in (17) it can be written as

$$\frac{d\, z(t+\phi)}{dt}=-z(t+\phi)$$

Now, the 1st boundary condition for this STATE suggest that $z(t+\phi)\le L_1$, and by definition of L_1 it is known that $(L_1\le C)$ then the comparison of these suggests that $z(t+\phi)\le C$, insertion of –ve sign will gives $-z(t+\phi)\ge -C$. So, FIFO is intact for the

STATE 2 using 1st boundary condition. To prove respect of FIFO for STATE 2 for the 2nd boundary condition, consider equation (17) again and substitute the value of L_1 from eqn. (2a). This gives the following

$$\frac{d\,z(t+\phi)}{dt} = \frac{u(t)+z(t+\phi)-nC+(n-1)L_2-n\,z(t+\phi)}{n}$$

$$= \frac{u(t)+z(t+\phi)-L_2}{n} - C + L_2 - z(t+\phi)$$

$$= \frac{u(t)}{n} - \frac{L_2 - z(t+\phi)}{n} + L_2 - z(t+\phi) - C$$

$$= \frac{u(t)}{n} + \left[L_2 - z(t+\phi) - \frac{L_2 - z(t+\phi)}{n} \right] - C$$

$$= \frac{u(t)}{n} + \left(L_2 - z(t+\phi)\right)\left(1 - \frac{1}{n}\right) - C \qquad (18)$$

The 2nd boundary condition of the STATE 2 i.e. $u(t)+z(t+\phi) \le L_2$, suggesting that $z(t+\phi) \le L_2$, this means the quantity $(L_2 - z(t+\phi)) \ge 0$, furthermore it is known that $u(t) \ge 0$ and $n \ge 1$. Therefore, the first two terms in the R.H.S of equation (18) are always positive or equal to zero. This suggests that equation (18) can be written as $\frac{d\,z(t+\phi)}{dt} \ge -C$. Thus, FIFO is preserved for the STATE 2 using 2nd boundary condition as well.

- *Causality*

In the DTA literature, causality is termed as the dependency of the upstream vehicles on the downstream vehicles when travel time is estimated for the upstream vehicles. The dynamic loading model is required to meet this condition, as it is unacceptable and far away from reality that travel times of the vehicles which are at downstream of the link is affected by upcoming vehicles in the link. It has been shown in the literature that outflow models, in which outflow is taken as function of vehicles on the link, exhibit violation of causality (Astarita 1996). In the proposed model as it can be seen that travel time for vehicles entering at time *t* is dependent on the vehicles that are in the queue at the end of the link (equation 6), therefore, future inflow into the link is not involved in the calculation of travel time of the vehicles at current time *t*.

- *Reasonable Outflow behaviour*

This requirement is describe as it is generally accepted that the outflow rate increases as the amount of traffic on the link increases until it reaches the outflow capacity of the link and there is no capacity constraints on the following links. It has been shown in the literature that some non-linear models behave unreasonably when the traffic on the link exceeds certain levels, i.e. the outflow rate decreases as the amount of traffic on the link increases.

In the proposed model, three states of the outflows are described. In the first state there is no constraint on the outflow as link is operated on free-flow condition. The second state in which the outflow is constrained by imposing a limits other than the capacity of the link, however, in this state outflow is not constrained in a manner that it causes decrease of outflow with increase of inflow as it can be seen that the outflow rate in the second state is still a function of inflow rate (see equation 1). The incorporation of this second state is basically playing a vital role in distinguishing the model from other models that already used for DTA. Furthermore, this state is giving a more realistic behaviour and in accordance with what is suggested in US Highway Capacity Manual. According to which the facility is in the state of level service "C" when the ratio of flow to capacity is in between 0.5~0.83. Here, level of service "C" means operating speeds are in the range of 2/3 to 3/4 of maximum. The third state ensures that outflow rate will remain up to the capacity of the link and the link at this stage will operate in fully congested state.

- *Positivity, existence and uniqueness*

It is required for the DTA that the three important terms should be positive i.e. Inflow rate which is the given quantity, amount of traffic at the end of the link and outflow rate both of them calculated through equation (1) and equation (6).

$$u(t) \geq 0, \qquad x(t) \geq 0 \quad \textit{and} \qquad v(t) \geq 0 \qquad\qquad \forall t$$

Existence means that for any pattern of inflows and outflow it is always possible to obtain a travel time for vehicles entering at time t. Uniqueness here mans that travel time is unique and continuous with respect to entry time. In addition, computational efficiency of the loading model is also considered as important as more computational efforts are required to achieve equilibrium, therefore the model that has high computational efficiency would be more preferable than others. The proposed model shown above fulfils these properties as well and also due to its simple mathematical construction it demand less computational efforts.

5. Numerical Experiments:

We evaluate the proposed model along with the Point-Queue and linear travel time model for four different scenarios (four different inflow profiles) which is first used by Nie and Zhang (2005) in their study for comparison of different loading models.

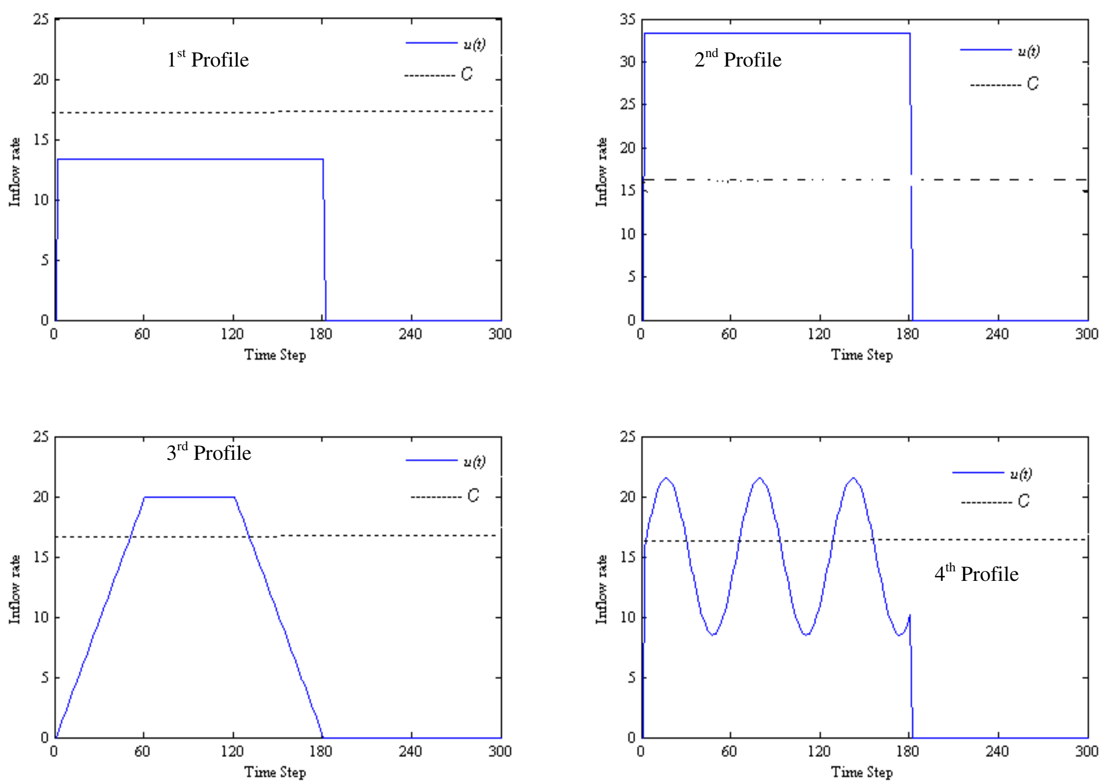


Figure 3: Four Inflow profile scenarios for Model Evaluation

Figure 3 shows the four different inflow profiles we are using to evaluate behaviour of different models. The first inflow profile represents piece-wise constant inflow in light traffic congestion, second profile represents piece-wise constant inflow in heavy traffic, third profile represents slowly varying inflow in moderately-congested traffic, and the last one represents fast varying inflow in moderately congested traffic. The last two inflow profiles will able to capture the transition from light to heavy congested or vice versa. The capacity (C) of the link is assumed equal to 1000 vehicles/hour (16.67 vehicle/minute), free flow travel time (ϕ) is assumed equal to 10 minutes and one time step is considered equal to 1 minute

1st Inflow Profile Case:

In this case inflow rate is planned in such a manner that it produce low congestion situation on the link. This case is analysed here because it will activate only free-flow travel state for the Point-queue model and for our proposed model two states will be activated i.e. free-flow and mild congestion flow as inflow rate is constant and always under capacity. The results obtained are shown in figure 4.

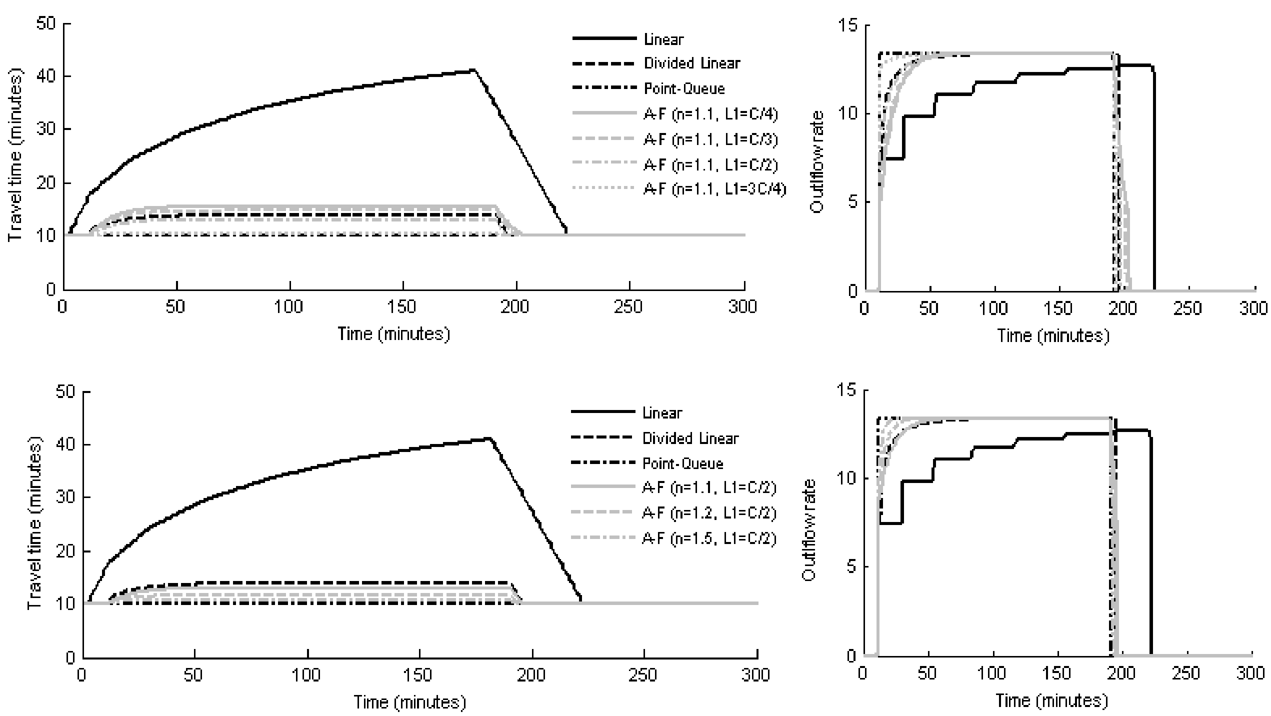


Figure 4: 1st Case Travel time and Outflow Profiles for dynamic loading Models

Figure 4 clearly reflects the overestimation behaviour of the linear travel time model, as this model immediately incorporating the effects of congestion for vehicles upstream caused by the vehicles down stream. On the other hand, point-queue model shows that the link is always at a free-flow state (link traverse time is always equal to free-flow travel time i.e.10 minutes), suggesting underestimation of travel time. Divided linear travel time model, which is developed to overcome the overestimation problem in linear travel time model, is behaving well and successful in overcoming the overestimation problem. This is because only part of the traffic existing on the link is considered for estimating congestion effects. Furthermore, this model is not showing congestion effects for the first few initial time steps, this is due to the assumption of the vacant link at the start of simulation and also the manner in which this model works i.e. dividing the link into two parts. So, the vehicles which first entered the link have to traverse with a free-flow speed in the first part of the link. The component responsible for consideration of the congestion effects is active at the time when vehicles reach at the second part of the link. Adnan-Fowkes model (presented as A-F model in the figure 4) is experimented with five different combinations of values of L_1 and *n*. As inflow rate in this case is always under capacity, therefore, only two initial states of this model are active dependent on the chosen value of L_1 . If L_1 considered greater than 0.8 *C* (i.e. constant inflow rate of this inflow profile), then in this circumstances only first state of the model will be active. Lower values of *n* are responsible for larger gap between L_1 and L_2 which suggests that 2nd state in the model will be activated for greater range of the inflow rate. Also lower values of *n* decrease the outflow rate at the 2nd state and hence causing more vehicles in the queue at the end of the link which are causing more congestion or increase in travel time of the incoming vehicles. Adnan-Fowkes model, which is developed to overcome the underestimation error in the point-queue model is behaving according to the expectations.

It can be seen from figure 4 that divided linear travel time model and Adnan-Fowkes model are behaving similar to each other but there is a fundamental difference in the construction of these two models. Divided linear travel time model can only avoid consideration of congestion effects up to the time steps (tick of clock) equivalent to the free-flow travel time for the first part of the link irrespective of the amount of inflow rate. This suggest that if inflow rate is considerably lower after the free flow travel time as well, then this model also incorporates the congestion effects (similar to linear travel time model) and therefore travel time for the vehicles upstream is increased significantly which is not desirable. However, Adnan-Fowkes model follows more appropriate approach in this regard as it uses the mechanism in which inflow rate of the link is the main factor for controlling the consideration of congestion effects. This suggests that if inflow rate is considerably lower then this model always predict travel time equal to free flow travel time of the link. Figure 5 represents obtained results when travel time is differentiated with respect to time for Point-Queue and Adnan-Fowkes model. It has been noted that derivative of travel time is always greater than -1 for this inflow profile; therefore, all models here are respecting FIFO condition.

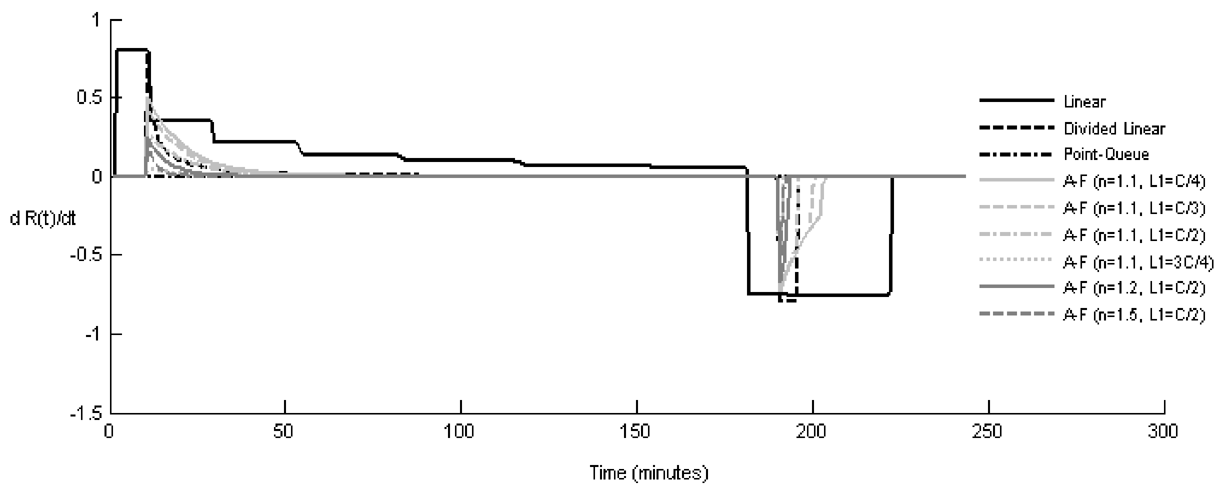


Figure 5: Derivative of Travel time with respect to time for Inflow 1

2nd Inflow Profile Case:

The 2nd case in which heavily congested condition is simulated through a constant piece-wise inflow profile whose inflow rate is always twice as greater than the capacity of the link up till 180 time-steps. This case is simulated in order to see the travel time and outflow behaviour of the two models under consideration which would be mainly due to the queues at the bottleneck and less dependent on the variation of inflow rate.

The results obtained for this experiment are shown in figure 6. It can be seen that again linear travel time model is overestimating the travel time compared to other models. However, the degree of overestimation is significantly less in this case compared to the light traffic congestion case (shown for the 1st inflow profile case). This suggests that impact of double counting effect in estimating travel time from this model is much less in heavy congestion conditions. This can be explained through the outflow rate profile, as in this case the outflow rate is increases very rapidly, thus causing less traffic on the link for measurement of travel time.

Divided linear travel time, point-queue and Adnan-Fowkes models are again producing very similar results in this case. In the point-queue model, under this inflow profile case, 2nd state is always active which says that outflow from the model equals the capacity of the link. The same behaviour is noted for Adnan-Fowkes model as well, even variation in the values of L_1 and n are not causing any difference. This is because, inflow rate is taken here as double of the capacity and all combination of L_1 and n examined here gives value of L_2 lower than the $2C$. As a result of this, first and second state of Adnan-Fowkes model is always inactivated and model behaving equivalent to the point-queue model. Therefore, for the point-queue and Adnan-Fowkes models, link is at free-flow state only up to the few initial time steps (i.e. equivalent to free flow travel time), which is the notion on which divided linear travel time model is built. This is the main reason of the similar behaviour of these three models. In figure 7 travel time for this case is differentiated with respect to time t, the results show that all models respect FIFO condition for this profile as well.

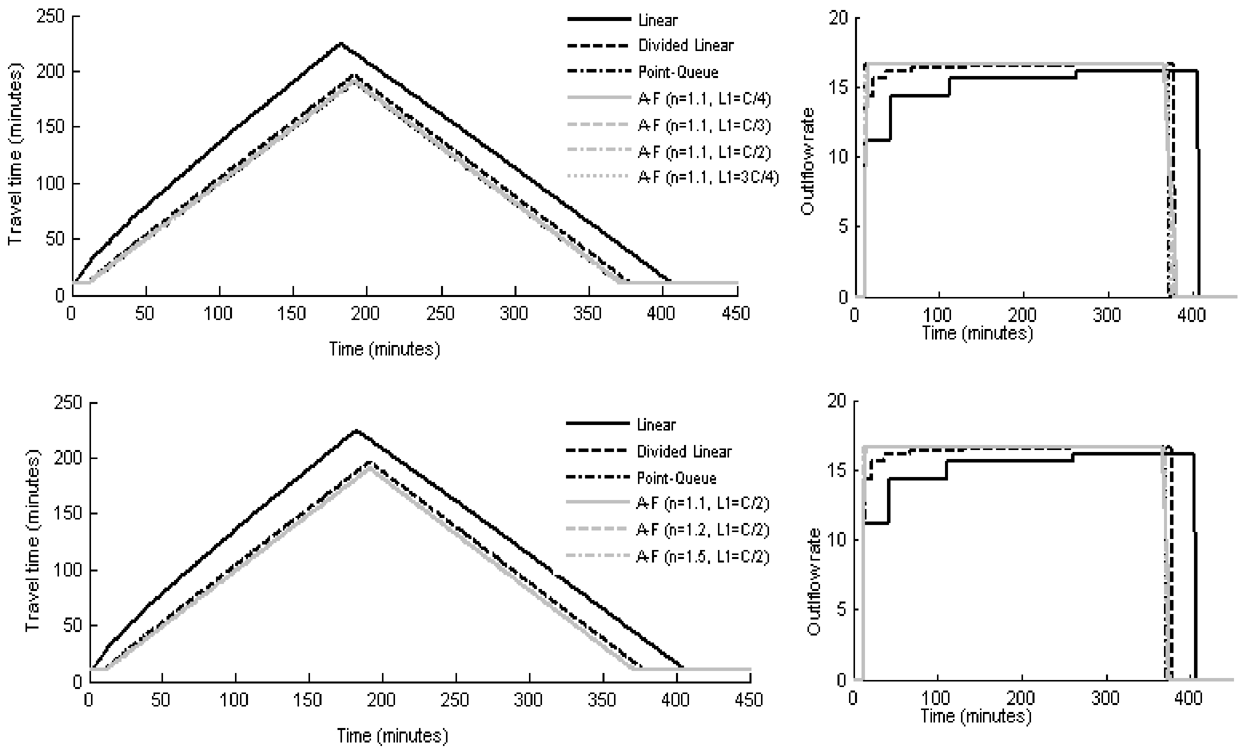


Figure 6: 2nd Case Travel time and Outflow Profiles for dynamic loading models

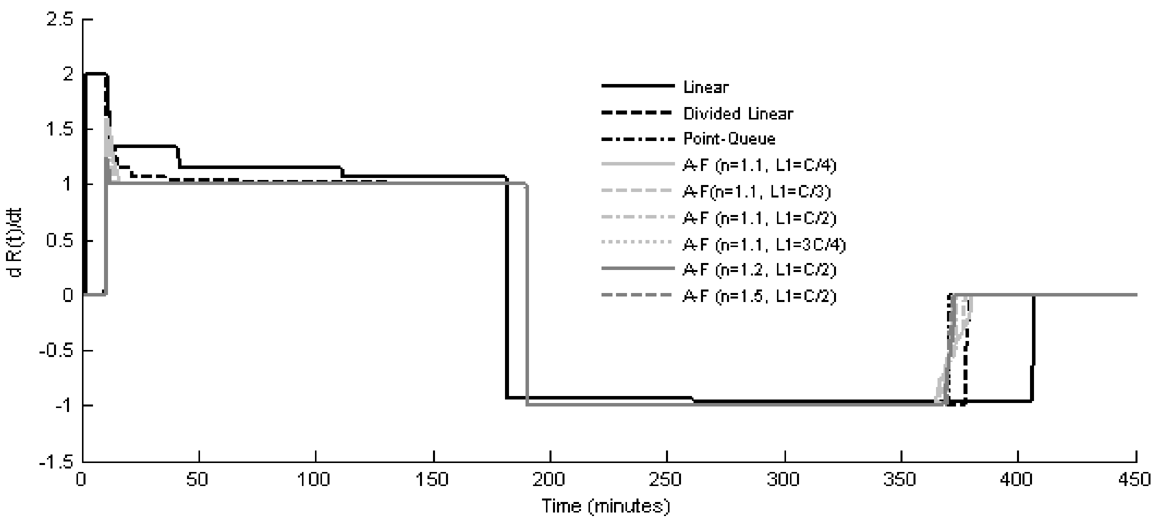


Figure 7: Derivative of Travel time with respect to time for case 2

3rd Inflow Profile Case:

This case is investigated in order to show the behaviour of the models for peak hour traffic. The inflow gradually increases from 0 to 1.2*C* using 60 time-steps, after which inflow rate is constant for another 60 time-steps and then it decreases to 0 for next 60 time-steps. The results for this case are shown in Figure 8.

It is very clear from figure 8 that linear travel time model again producing significantly higher travel times and in this case degree of overestimation of travel time is significantly higher compared to travel times obtained from other models. Point-queue model has also shown free-flow state and consideration of congestion effects, suggesting its both states are active in this situation. However, underestimation problem of this model in initial stages (i.e. travel time is equal to link's free-flow travel time) is clearly evident. Divided linear travel time model and Adnan-Fowkes models show reasonable estimation of travel times. It has been noted that outflow profile of the divided linear travel time model never reaches capacity (C) of the link at any time (similar to linear travel time model). This situation may raise the question regarding the meaning of the term C used in these models (i.e. linear and divided linear travel time model) because inflow exceeds C at some points in time (see inflow profile for this case) but outflow never reaches C. Adnan-Fowkes and Point-Queue models do not raise this question as outflow from the link reaches capacity (C) of the link at points in time when link is overloaded. Adnan-Fowkes model is more flexible with the introduction of two more parameters (i.e. L_1 and n) in their modelling framework, that certainly provide more ease for adjustment of travel time profile obtained from this model with real data. The interesting point here is that value of n is playing a major role in defining the degree of convexity of the travel time profile, while L_1 ensures the starting point after which effect of congestion is considered for the incoming vehicles. This trend can be seen in outflow profile as well.

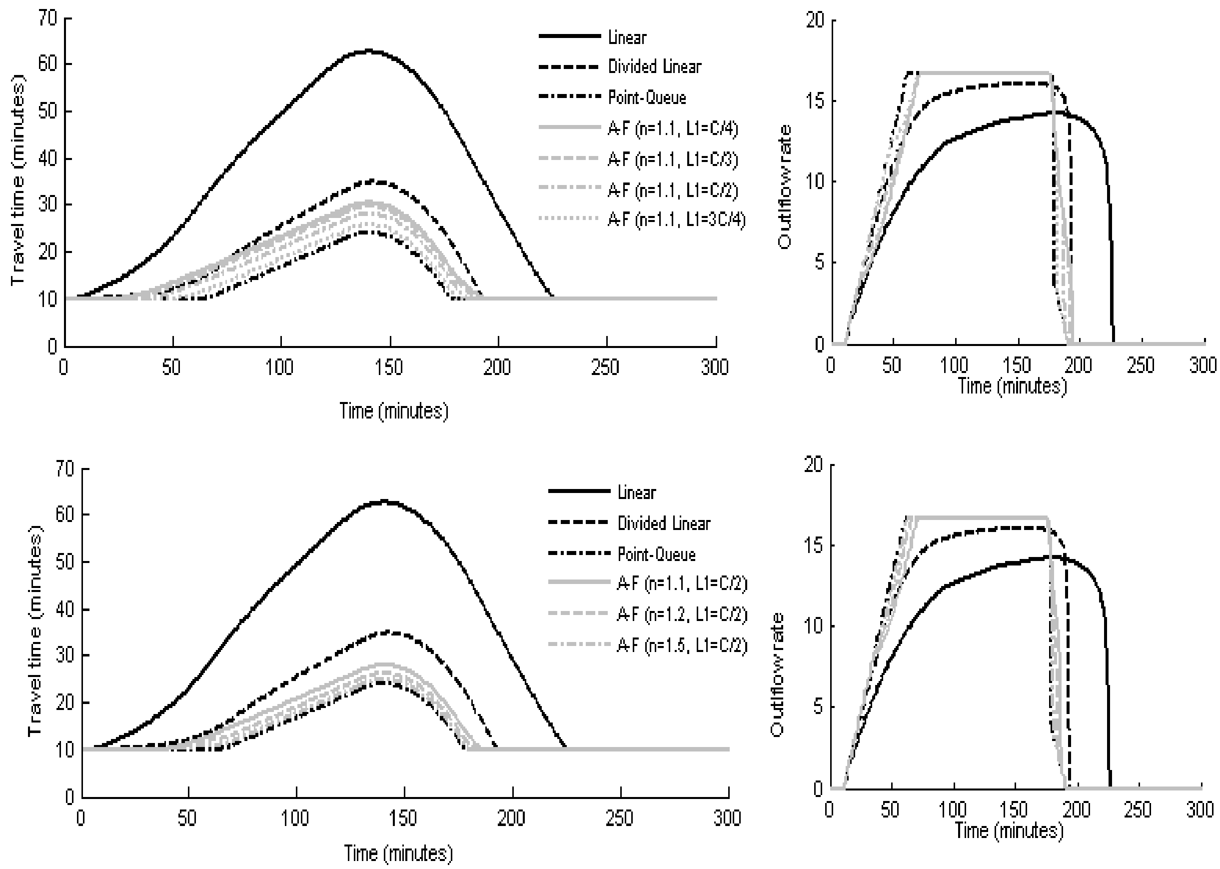


Figure 8: 3rd Case Travel time and Outflow Profiles for dynamic loading models

For this case as well, travel time is differentiated with respect to time t and the obtained results are shown in figure 9. The figure shows that FIFO condition is intact for this inflow profile as well as value of $dR(t)/dt$ is always greater than -1.

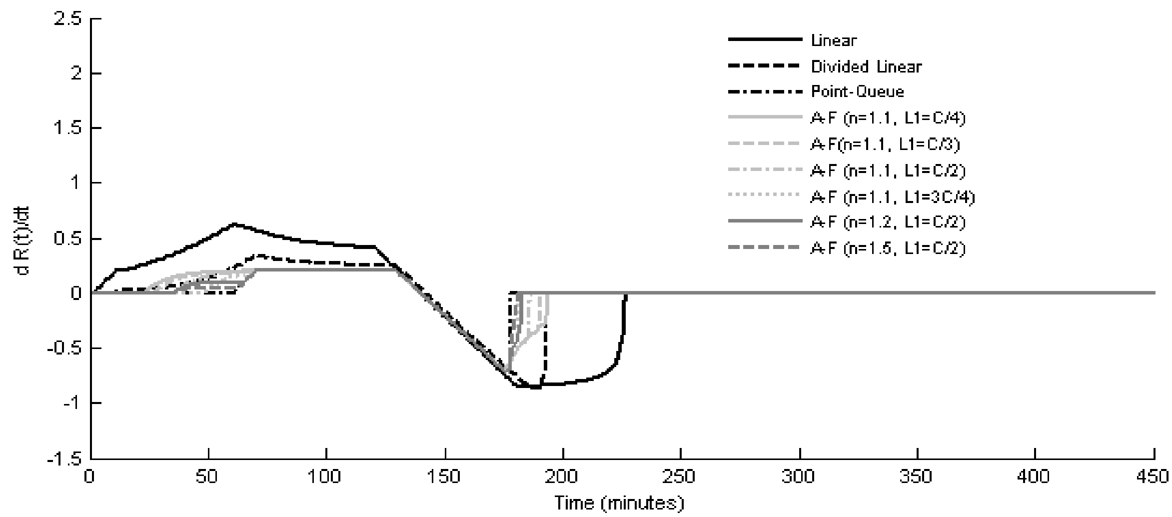


Figure 9: Derivative of Travel time with respect to time

4th Inflow Profile Case:

In this case behaviour of the models is analysed for the fast varying inflow profiles. Inflow profile is based on sinusoidal function and the inflow is varied in such a manner that it fluctuates across the capacity of the link i.e. at some instant inflow is under capacity and at some other instant inflow is over capacity. The highest value inflow can take is up to $1.3C$ and lowest value inflow can take is around $0.48C$. This case is also important to analyse as it has been noted in the literature that some loading models (non-linear models) are not able to exhibit respect of FIFO condition due to sudden change in inflow profiles (which is the main property of this case). The results obtained for this case are summarised in Figure 10. The same trend is noted here for the variation of values of L_1 and n as revealed in the above analysed cases. i.e. lower values of L_1 and n are responsible for higher congestion on the link. The outflow profiles obtained here for our proposed model are much smoother than the outflow profile obtained from Point-Queue model especially when inflow is in transition from under capacity to over capacity and vice versa. The main point noted here that our proposed model, even for all the cases; always estimate the travel time either greater or equivalent to Point-Queue model. There is no point in time it is observed that our proposed model is giving travel time lower than the Point-Queue model. This suggest that our proposed model successfully addresses the drawback of underestimation of travel time of Point-Queue model and also the area in which the Point-Queue model is subjected to produce plausible results (congested condition), our proposed model would also approximate Point-Queue model in those areas. Additionally for linear travel time model another problem is noted apart from its overestimation problem. This is regarding unsmooth nature of its outflow profile. Therefore, use of this model may cause some serious problems when more than one link is considered in the network as outflow from the previous link will serve as inflow to the next link. Further to that it has been noted that travel time profiles and outflow profiles try to replicate the features of inflow profiles (i.e. travel time and outflow profiles fluctuates with fluctuation of the inflow profile). However, the degree of fluctuation of profiles obtained for linear travel time model is much higher compared to the results obtained for other models. The plot which represents derivative of travel time (figure 11) is always greater than -1, thus FIFO is intact for this inflow scenario as well.

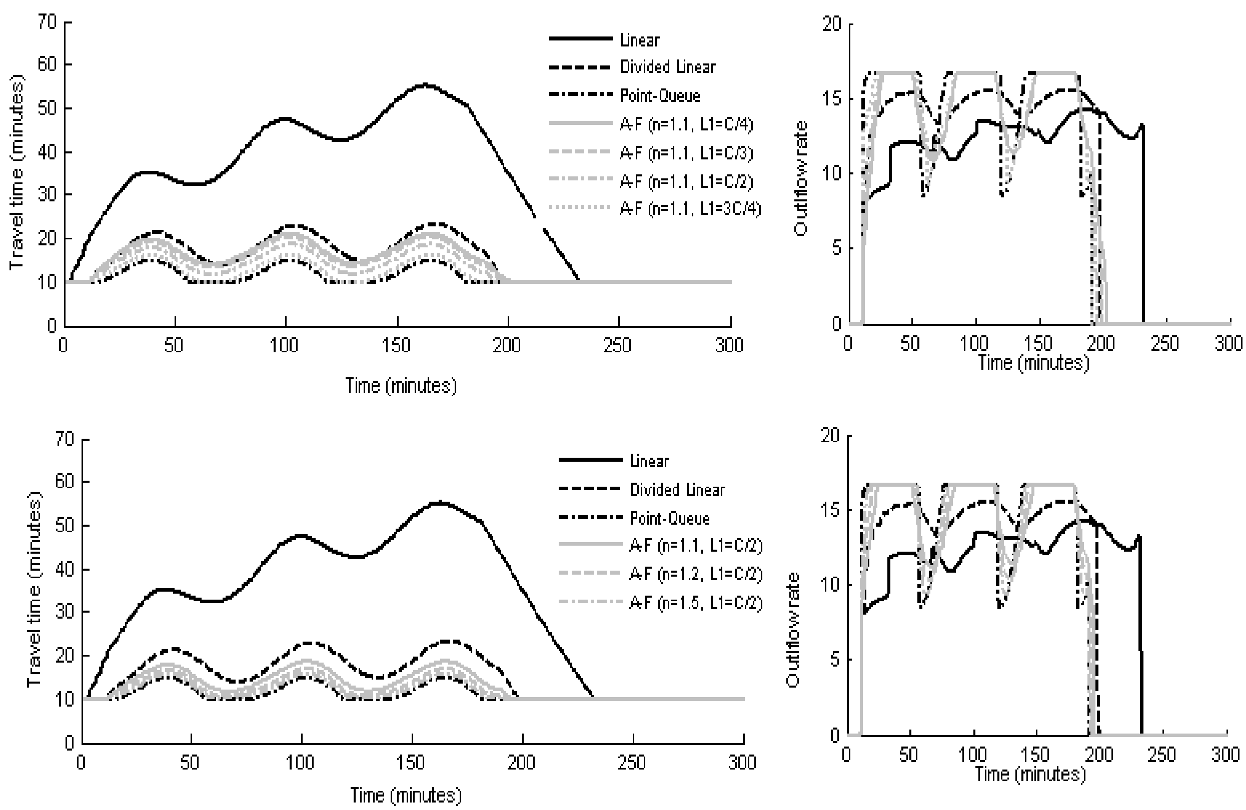


Figure 10: 4th Case Travel time, Outflow Profiles for different dynamic loading models

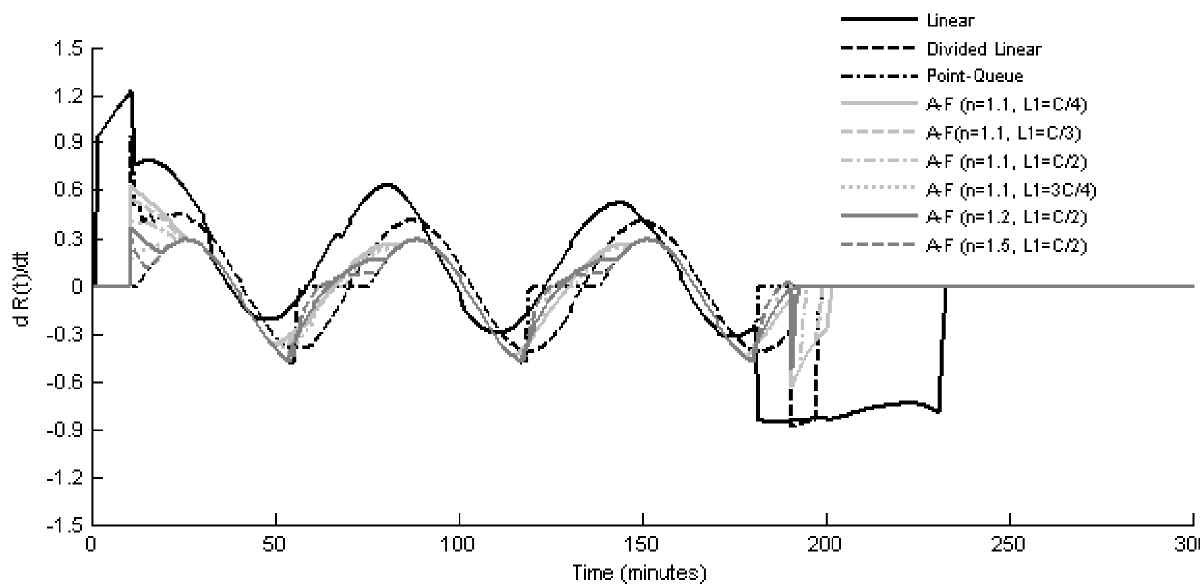


Figure 11: Derivative of travel time with respect to time for all dynamic loading models

6. Conclusion:

This paper reported a novel macroscopic dynamic loading model which utilise the similar modelling framework through which well known Point-Queue model was developed. The Adnan-Fowkes model considers three states of flow on the link instead of two states, which distinguishes it from the Point-Queue model and at the same time this model addresses the overestimation problem found in another well known model i.e. linear travel time model. The paper presented different properties of this model along with its full derivation in both terms i.e. analytically and numerically. Additionally, analytical proofs are provided for the proposed model against desirable properties required for any model for its use in DTA. These proofs suggests that model is fulfilling all the requirement for its use in DTA and can be a good candidate to challenge the existing macroscopic dynamic loading models, which are used in DTA due to their lower implementation and computational costs. The numerical experiments for all the considered models clearly suggest that examination of real data is necessary in order to justify the selection of proper model. Further to that this examination allows calibration and estimation of parameters involved in the models. There is no such study exists that is focused on the examination of these models with real data, therefore, in the future research efforts should be made to analyse the appropriateness of the model with real data.

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