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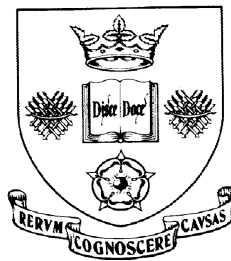


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**Central Bank Preferences, Distribution Forecasts and  
Economic Stability in a Small Open-Economy**

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### **Abstract:**

Buffering unforeseen shocks and forecasting the evolution of the economy are key monetary policy issues. This paper relates the Central Bank's choice of the inflation index that wishes to stabilise to the quality of its forecasts and the expected economic stability. The framework is a Markov jump-linear-quadratic system for optimal policy with model uncertainty. Comparing CPI and domestic inflation targeting, the latter implies considerably less variability in the distribution forecast of the economic dynamics. Furthermore, domestic inflation targeting stands out for more expected economic stability and less sensitiveness to interest rate smoothing. Finally, the prediction accuracy of the interest rate behaviour significantly improves with domestic inflation targeting.

**Key words:** Multiplicative uncertainty; Markov jump linear quadratic systems; small open-economy; optimal monetary policy; inflation index.

**JEL:** E52, E58, F41

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# Central Bank Preferences, Distribution Forecasts and Economic Stability in a Small Open-Economy

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July 18, 2009

## Abstract

Buffering unforeseen shocks and forecasting the evolution of the economy are key monetary policy issues. This paper relates the Central Bank's choice of the inflation index that wishes to stabilise to the quality of its forecasts and the expected economic stability. The framework is a Markov jump-linear-quadratic system for optimal policy with model uncertainty. Comparing CPI and domestic inflation targeting, the latter implies considerably less variability in the distribution forecast of the economic dynamics. Furthermore, domestic inflation targeting stands out for more expected economic stability and less sensitiveness to interest rate smoothing. Finally, the prediction accuracy of the interest rate behaviour significantly improves with domestic inflation targeting.

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# 1 Introduction

Buffering unforeseen shocks and forecasting the evolution of the economy are key monetary policy issues. While the former is a classical theme, the relevance of the latter became apparent with the advent of inflation targeting. This new monetary regime highlighted how long and variable lags in the transmission of the policy action, along with the exposure to exogenous disturbances, require an operating procedure based on *distribution forecast targeting*. Central bank proficiency at forecasting also matters in shaping the expectations of the private sector, thus enhancing monetary policy effectiveness. This is the so called expectations channel. Its relevance in the monetary policy transmission mechanism is well captured by the consolidated view that successful monetary policy is, mainly, the management of the market expectations, as Woodford (2001) initially put it<sup>1</sup>. On the operational side, another key monetary policy issue is the choice of the price index to target (Bernanke et al. 1999). Intuitively, a price index should measure the cost of living. Yet, economists have long recognized that such a price index may not meet the purposes of conducting monetary policy (Mankiew and Reis 2003).

Starting from these considerations, this paper relates the accuracy of the central bank distribution forecasts and the expected perturbing impact of unforeseen shocks to the choice of the inflation index to stabilize in a small open-economy. The focus of the analysis is on the choice between the domestic price index and the consumer price (CPI) index. These indexes differ in that the former refers to the goods produced domestically while the latter to the consumption goods produced domestically and imported. This disparity implies a different sensitivity of the indexes to exchange rate movements and shocks stemming from the rest of the world. Indeed, the CPI exhibits a direct sensitivity through the price of foreign goods imported for consumption. In contrast, the sensitivity of the domestic price index is mediated by foreign goods used as inputs to produce domestic goods. This different sensitivity of the two indexes gets reflected in the extent and timing of the central bank response to exchange rate movements and foreign shocks, and motivates the interest to study how the indexes perform in terms of forecasts' accuracy and expected economic stability.

Aiming to portray a real-world monetary policy scenario, the current analysis en-

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<sup>1</sup>Theoretically, the New-Keynesian model embedding agents' forward looking behavior shows the major role played by the expectations channel. In practice, the rising trend over the last decade in central banks' transparency, in particular the publication of the internal distribution forecasts explained by Monetary Policy Reports, signals the importance attributed to this channel (see, among others, Blinder et al. 2001, Fracasso, Genberg and Wyplosz 2003, Geraats 2002, 2005).

compasses both *additive* and *multiplicative* uncertainty. The modeling strategy for the various sources of uncertainty follows the Svensson and Williams (2007) approach based on Markov jump-linear-quadratic systems. In this framework, I consider the distribution forecasts of the macrovariables determined by the optimal monetary policy response to several exogenous shocks in the presence of model uncertainty. It is worthy of note that forecasting the evolution of the economy requires specifying correct economic dynamics. Thus, this paper uses a monetary policy transmission mechanism with realistic lags and inertia in the private sector behavior.

The main contribution of the analysis lies in showing that the stabilization of CPI inflation tends to be inversely related to both the accuracy of the distribution forecasts of the other macrovariables and the expected perturbing impact of several shocks. Thus, the current work unveils domestic inflation targeting (henceforth DIT) as the policy that performs best at forecasting accuracy and shocks buffering for most of the macrovariables. The intuition is based on the combined action of two factors: the *level of policy activism* corresponding to the choice of the inflation index to stabilize, and the *consideration of model uncertainty* on the part of the central bank. Since under CPI inflation targeting (henceforth CPI IT) there is more policy activism than under DIT, when the central bank decides the optimal policy and takes into account model uncertainty a more active policy results in more volatility and shock sensitivity for most of the macrovariables.

This finding relates to three strands of the monetary policy literature: i. central bank transparency and the publication of its distribution forecasts; ii. optimal monetary policy with model uncertainty and exogenous shocks; and iii. the choice of the inflation measure to target.

Regarding central bank transparency, the possibility to increase the overall forecasting accuracy through the choice of the inflation index, in particular for the interest rate, can foster the credibility of the central bank. More credibility, in turn, enhances the ability of the distribution forecasts to shape the expectations of the private sector. This suggests that under DIT there are larger benefits associated with the publication of the distribution forecasts for the interest rate and the related distribution forecasts for the other macrovariables.

The second link with the literature is about a new aspect of the relation between optimal monetary policy and model uncertainty. Starting with the Brainard's (2007) seminal contribution, some authors among which Söderström (2002) have investigated how the optimal monetary policy response to the state variables of the economy attenuates

or increases in the presence of model uncertainty. It has not been investigated, however, if and to what extent accounting for the presence of model uncertainty matter in the choice of the inflation index to stabilize. In this respect, the current paper contributes to the literature by showing that with model uncertainty the choice of the inflation measure significantly affects the expected forecast accuracy and economic stability.

Finally, the paper relates to the open question in monetary policy of which inflation measure to target. All over the world, inflation targeting central banks tend to choose the CPI as the index to target. Yet this is more and more a contentious choice as argued in an increasing number of contributions from central banks practice and academic literature. Clearly, the CPI bears the advantage of being an index the private sector is more sensitive to and familiar with. Thus, targeting CPI inflation favours central bank accountability. This statistic, however, has various downsides. One problem pointed at by Batini, Levine and Pearlman (2005) is that policy rules which include the CPI may lead to economic indeterminacy. A second problem is that the CPI index is quite exposed to shocks that turn out to be temporary. In this case the central bank tends not to react because interventions in the presence of lags between the instrument and the goal can increase, rather than reduce, the variability of CPI<sup>2</sup>.

Along with these shortcomings, CPI IT does not seem to offer a clear advantage in terms of welfare. Indeed, adopting a welfare perspective, various scholars reached contrasting conclusions on the inflation measure to target. Aoki (2001) and Benigno (2004) examine a model with two sectors that differ in their degree of price stickiness and show that monetary policy should target inflation in the sticky-price sector. In an open economy this prescription suggests one should target domestic inflation as it tends to be stickier than CPI inflation. Mankiew and Reis (2003) show that in a two-sector economy the price index maximising economic stability is positively related to the sectoral price sensitivity to the business cycle and the sectoral degree of price stickiness, while it is negatively related to the volatility of idiosyncratic shocks and to the weight of the sectoral price in the CPI . As a result, a stability price index is substantially different from the CPI. Gali and Monacelli (2005) argue that DIT dominates both CPI IT and an

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<sup>2</sup>Heikensten (1999) and Rosemberg (2004) discuss how this made it difficult for the Riksbank to explain its behavior to the private sector, sometimes requiring to motivate policy decisions using other price indexes less exposed to temporary shocks. Rosemberg also notes that at some occasions the actual monetary policy has de facto been based on a different index. Similarly, Macklem (2001) maintains that while the Bank of Canada's inflation-control target is specified in terms of CPI inflation, operationally, the Bank uses a measure of trend or "core" inflation as short term guide for its monetary policy actions. Further along the line, Young Ha (2002) and Guender (2003) introduced a case for choosing domestic inflation as it is less exposed to temporary shocks.

exchange-rate peg. They base their argument on the "excess smoothness" induced in the exchange rate by CPI targeting or an exchange rate peg which prevents relative prices from adjusting sufficiently fast. Different results are obtained by Kirsanova, Leith and Wren-Lewis (2006) and De Paoli (2004) who find that central bank preferences should include the terms of trade gap together with the output gap and domestic price inflation.

These diverse findings can be explained by different assumptions at the basis of the private sector behavior. From a central bank operational perspective, however, it is difficult to assess the most appropriate assumptions to model the behavior of the private sector. This is due to considerable uncertainty on the true model of the economy. Furthermore, the relationship between optimal monetary policy for a small open-economy and welfare in the presence of realistic transmission lags is still largely unexplored. It is arguably premature to directly use welfare models for policy prescriptions. Moving from these remarks, the current work adopts an operational perspective that abstracts from welfare considerations. It explores how alternative price indexes perform in terms of distribution forecast accuracy and amplitude of business cycle fluctuations when the central bank considers model uncertainty in the optimal policy design.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 presents and describes the results using impulse distribution forecasts under alternative central bank inflation indexes. Section 4 discusses the paper's findings in relation to transparency in monetary policy and the publication of future policy intentions. Conclusions are in section 5. Finally, the Appendix describes the state-space form of the model.

## 2 The model

The model consists of a linear-quadratic setup for optimal monetary policy nested into a non-certainty equivalence framework. As to the agents' behavior, preferences and constraints are modeled to have a realistic transmission mechanism of the monetary policy. This is a necessary condition to have proper dynamics and, consequently, realistic forecasts. Non-certainty equivalence, the second component of the framework, is a necessary condition to study how *multiplicative* uncertainty affects the optimal monetary policy. Since the model has also forward looking variables, non-certainty equivalence is modeled by using the general approach developed by Svensson and Williams (2007).

The characterization of the behavior of the private sector follows Flamini (2007) and



can be summarized in five main assumptions<sup>3</sup>. First, the economy is populated by four optimizing agents: a representative firm both for the sector that produces and retails domestic goods and for the sector that imports and retails foreign goods (henceforth the domestic and the import sector respectively), a representative household and a central bank. Second, the domestic and import sectors are connected. Indeed, the domestic one employs import goods as an intermediate input and the import sector, in turn, may employ domestic goods to retail foreign goods creating incomplete pass-through. Third, both sectors are characterized by monopolistic competition and sticky prices. The latter assumption with respect to the import sector determines delayed pass-through. Fourth, realistic persistence in the behavior of the firms and households is captured, respectively, by inflation indexation and habit formation in consumption. Fifth, in line with the empirical evidence observed by central banks, a two-period lag for monetary policy to affect domestic inflation and a one-period lag to affect the aggregate demand are introduced, respectively, by predetermined pricing and consumption decisions.

These ingredients map into aggregate demands and supplies for the two sectors and an uncovered interest parity relation. Finally, the model is closed with an intertemporal loss function modelling the preferences of the central bank and exogenous relations to capture the behavior of the rest of the world.

## 2.1 The household

The economy is made up of a continuum of consumers/producers indexed by  $j \in [0, 1]$  sharing the same preferences and living forever. Intertemporal utility for the representative household is given by

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau U(C_{t+\tau}, \check{C}_{t+\tau-1}), \quad (1)$$

where  $\delta$  is the intertemporal discount factor,  $C_t$  is total consumption of household  $j$ , and  $\check{C}_t$  is the total aggregate consumption. Preferences over total consumption feature habit formation which is modeled as in Abel (1990) by the following instantaneous utility function

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<sup>3</sup>A terse description of the private sector behaviour is reported here as it allows a clear presentation of the *model uncertainty* considered by the central bank and modeled in sections 2.5-2.6. For details on the derivation of the structural relations referring to the private sector see Flamini (2007).

$$U(C_{t+\tau}, \check{C}_{t+\tau-1}) = \frac{(C_{t+\tau}/\check{C}_{t+\tau-1})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}, \quad (2)$$

where  $\iota \geq 0$  captures habit persistence and  $\sigma > 0$  is the intertemporal elasticity of substitution. Total consumption,  $C_t$ , is a Cobb-Douglas function of domestic good consumption,  $C_t^d$ , and import good consumption,  $C_t^i$ ,

$$C_t \equiv C_t^{d(1-w)} C_t^{iw}, \quad (3)$$

where  $w$  determines the steady state share of imported goods in total consumption and  $C_t^d, C_t^i$  are Dixit-Stiglitz aggregates of continuum of differentiated domestic goods and import goods (henceforth indexed with  $d$  and  $i$  respectively),

$$C_t^h = \left[ \int (C_t^h(j))^{1-\frac{1}{\vartheta}} dj \right]^{\frac{1}{1-\vartheta}}, \quad h = d, i,$$

where  $\vartheta > 1$  is the elasticity of substitution between any two differentiated goods and, for the sake of simplicity, is the same in both sectors<sup>4</sup>. The flow budget constraint for consumer  $j$  in any period  $t$  is given by

$$\frac{B_t}{1 + I_t} + \frac{B_t^*}{1 + I_t^*} S_t + P_t^c C_t = B_{t-1} + B_{t-1}^* S_t + D_t^d + D_t^i,$$

where  $B$  and  $B^*$  are two international bonds issued on a discount basis and denominated in domestic and foreign currency with interest rates  $I_t$  and  $I_t^*$  respectively,  $S_t$  is the nominal exchange rate, expressed as home currency per unit of foreign currency,  $D_t^d$  and  $D_t^i$  are the dividends distributed by the domestic and the import sector and, finally,  $P^c$  is the overall Dixit-Stiglitz price index for the minimum cost of a unit of  $C_t$  and is given by

$$P_t^c = \frac{P_t^{iw} P_t^{d(1-w)}}{w^w (1-w)^{(1-w)}}, \quad (4)$$

with  $P^d, P^i$  denoting, respectively, the Dixit-Stiglitz price index for goods produced in the domestic and import sector.

Assuming a no-Ponzi schemes condition, utility maximization subject to the budget constraint and the limit on borrowing gives the Euler equation and the Uncovered Interest Parity, respectively

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<sup>4</sup>Following Corsetti and Pesenti (2004), the intratemporal elasticity of substitution between domestic and import goods is set equal to one. This assumption ensures the stationarity of the model.

$$c_t = \beta c_{t-1} + (1 - \beta) c_{t+1|t} - (1 - \beta) \sigma (i_t - \pi_{t+1|t}^c), \quad \beta \equiv \frac{\iota(1 - \sigma)}{1 + \iota(1 - \sigma)} < 1, \quad (5)$$

$$i_t - i_t^* = s_{t+1|t} - s_t + v_t, \quad (6)$$

where for any variable  $x$ , the expression  $x_{t+\tau|t}$  stands for the rational expectation of that variable in period  $t + \tau$  conditional on the information available in period  $t$  and, by means of a log-linearization, the variables  $c_t$ ,  $\pi_t^c$ ,  $i_t$ ,  $i_t^*$ ,  $(s_{t+1|t} - s_t)$  and  $v_t$  are log-deviations from their respective constant steady state values; finally,  $c_t$  denotes total aggregate consumption, obtained considering that in equilibrium total consumption for agent  $j$  is equal to total aggregate consumption, i.e.  $C_t = \check{C}_t$ ,  $\pi_t^c$  denotes CPI inflation (measured as the log deviation of gross CPI inflation from the constant CPI inflation target), and  $v_t$  is a risk premium shock added to capture financial market volatility and it is modeled with a stationary univariate AR(1) process

$$v_{t+1} = \gamma_v v_t + \xi_{t+1}^v.$$

### 2.1.1 Domestic consumption of goods produced in the domestic sector

Preferences captured by equation (3) imply that the (log deviation of the) domestic demand for goods produced in the domestic sector,  $c_t^d$ , is given by

$$c_t^d = c_t - (p_t^d - p_t^c),$$

which, considering the (log-linearized version of the) price index equation (4), can be rewritten as

$$c_t^d = c_t + wq_t, \quad (7)$$

where  $q_t \equiv p_t^i - p_t^d$  is the (log-deviation of the) terms of trade.

Then, solving equation (5) for  $c_t$  and combining it with equation (7) I obtain

$$c_t^d = -\sigma (1 - F_1 L)^{-1} \rho_t - \sigma (1 - F_1 L)^{-1} wq_t + wq_t, \quad (8)$$

where  $F_1 < 1$  is the smaller root of the characteristic polynomial of equation (5) and

$$\rho_t \equiv \sum_{\tau=0}^{\infty} (i_{t+\tau|t} - \pi_{t+\tau+1|t}^d) \quad (9)$$

can be interpreted as the long real interest rate.

### 2.1.2 Aggregate demand for goods produced in the domestic sector

Total aggregate demand for the good produced in the domestic sector is

$$\widehat{Y}_t^d = C_t^d + Y_t^{d,d} + Y_t^{d,i} + C_t^{*d}, \quad (10)$$

where  $Y_t^{d,d}$ ,  $Y_t^{d,i}$  and  $C_t^{*d}$  denote the quantity of the (composite) domestic good which is used as an input in the domestic sector, as an input in the import sector and which is demanded by the foreign sector, respectively.

Both sectors are assumed to share the same Leontief technology and each one features a continuum of unit mass of firms, indexed by  $j$ , that produce differentiated goods  $Y_t^d(j)$  and  $Y_t^i(j)$  in the domestic and import sector respectively. Furthermore, the two sectors differ for the input used: the domestic sector uses a composite input consisting of the domestic (composite) good itself and the (composite) import good provided by the import sector; the import sector uses a composite input consisting of the foreign good  $Y_t^*$  and the domestic (composite good). Thus the technologies in the domestic and import sector are given respectively by

$$Y_t^d(j) = f \left[ A_t^d \min \left\{ \frac{Y_t^{d,d}}{1-\mu}, \frac{Y_t^{d,i}}{\mu} \right\} \right], \quad Y_t^i(j) = f \left[ A_t^i \min \left\{ \frac{Y_t^*}{1-\mu^i}, \frac{Y_t^{d,i}}{\mu^i} \right\} \right], \quad \mu, \mu^i \in [0, 1], \quad (11)$$

where  $f$  is an increasing, concave, isoelastic function,  $A_t$  is an exogenous (sector specific) economy-wide productivity parameter,  $(1-\mu)$  and  $\mu$  denote, respectively, the shares of the domestic good and import good in the composite input required to produce the differentiated domestic good  $j$ , and  $(1-\mu^i)$  and  $\mu^i$  denote, respectively, the shares of the foreign good and domestic good in the composite input required to provide the differentiated import good  $j$ .

Thus the quantities of the (composite) domestic good used as an input in the domestic

and import sector are

$$Y_t^{d,d} = \frac{1}{A_t^d} (1 - \mu) f^{-1} \left( \widehat{Y}_t^d \right), \quad Y_t^{d,i} = \frac{1}{A_t^i} \mu^i f^{-1} \left( \widehat{Y}_t^i \right), \quad (12)$$

where  $\widehat{Y}_t^i$  denotes the demand of the import good. Finally, log-linearizing equation (10) around the steady state values yields

$$\widehat{y}_t^d = \kappa_1 (\mu^i) c_t^d + \kappa_2 (\mu^i) \widehat{y}_t^i + \kappa_3 (\mu^i) c_t^{*d}, \quad (13)$$

where  $\kappa_1' (\mu^i)$ ,  $\kappa_3' (\mu^i) < 0$  and  $\kappa_2' (\mu^i) > 0$ .

Next, as in Svensson (2000),  $c_t^{*d}$  is exogenous and given by

$$c_t^{*d} = \overline{\beta}_y^* y_t^* + \theta^* w^* q_t, \quad (14)$$

where  $c_t^*$  denotes (log) foreign real consumption,  $\theta^*$  and  $w^*$  denote, respectively, the foreign atemporal elasticity of substitution between domestic and foreign goods and the share of domestic goods in foreign consumption. Furthermore, the output-gap in the domestic sector  $y_t^d$  is defined as

$$y_t^d \equiv \widehat{y}_t^d - y_t^{d,n},$$

where  $y_t^{d,n}$  denotes the log deviation of the natural output in the domestic sector from its steady state value, and in both sectors the log-deviation of the natural output from its steady state value is exogenous, stochastic and follows

$$y_{t+1}^{h,n} = \gamma_y^{h,n} y_t^{h,n} + \eta_{t+1}^{h,n}, \quad 0 \leq \gamma_y^{h,n} < 1, \quad h = d, i, \quad (15)$$

where  $\eta_{t+1}^{h,n}$  is a serially uncorrelated zero-mean shock to the natural output level (a productivity shock). Finally, in line with the central banks' view of the approximate one-period lag necessary to affect aggregate demand, I assume that consumption decisions are predetermined one period in advance. Accordingly, repeating the same derivation with preferences maximized on the basis of one period ahead information results in the aggregate demand in the domestic sector. This relation, expressed in terms of the output-gap, is given by

$$y_{t+1}^d = \beta_y y_t^d - \beta_\rho \rho_{t+1|t} + \beta_q q_{t+1|t} - \beta_{q-1} q_t + \beta_{y^*} y_t^* + \beta_{y^n} y_t^{d,n} + \eta_{t+1}^d - \eta_{t+1}^{d,n}, \quad (16)$$

where  $\eta_{t+1}^d$  is a serially uncorrelated zero-mean demand shock. In (16) all the coefficients

are positive and functions of the structural parameters of the model.

### 2.1.3 Aggregate demand of goods produced in the import sector

Aggregate demand for import goods is given by

$$\widehat{Y}_t^i = C_t^i + Y_t^{i,d} \quad (17)$$

where  $Y_t^{i,d}$  denotes the amount of the import good used as an input in the domestic sector. Log-linearizing (17) around the steady state results in

$$\widehat{y}_t^i = (1 - \widetilde{\kappa}) \widehat{c}_t^i + \widetilde{\kappa} \widehat{y}_t^d. \quad (18)$$

Finally, the same assumptions used to derive the aggregate demand for the domestic sector goods yield

$$y_{t+1}^i = \beta_y y_t^i - \beta_\rho \rho_{t+1|t} - \beta_q q_{t+1|t} + \beta_{q-1} q_t + \beta_{y^*} y_t^* + \beta_{y^n} y_t^{i,n} + \eta_{t+1}^i - \eta_{t+1}^{i,n}, \quad (19)$$

where all the coefficients are positive and depend on the structural parameters of the model, and  $\eta_{t+1}^i$  is a serially uncorrelated zero-mean demand shock.

## 2.2 Firms

In both sectors, the aggregate supply is derived according to the Calvo (1983) staggered price model while inflation inertia is introduced as in Christiano, Eichenbaum and Evans (2005) and also by the presence of the terms of trade as shown in Benigno (2004). Beyond the use of different inputs, the two sectors differ in the firms decision timing.

### 2.2.1 Domestic sector

In the domestic sector, the representative consumer/producer  $j$  produces the variety  $j$  of the domestic good,  $Y_t^d(j)$ , with a composite input whose price is  $W_t$ . Since all the varieties use the same technology, there is a unique input requirement function for all  $j$  given by  $\frac{1}{A_t^d} f^{-1} [Y_t^d(j)]$  and the variable cost of producing the quantity  $Y_t^d(j)$  is  $W_t \frac{1}{A_t^d} f^{-1} [Y_t^d(j)]$ . Furthermore, since there is a Dixit-Stiglitz aggregate of domestic

goods, the demand for variety  $j$  is

$$Y_t^d(j) = \widehat{Y}_t^d \left( \frac{P_t^d(j)}{P_t^d} \right)^{-\vartheta},$$

where  $P_t^d(j)$  is the nominal price for variety  $j$  and  $\vartheta$  is the elasticity of substitution between different varieties. As shown in equation (11), the composite input is a convex combination of both aggregates of domestic and import goods. Thus the price of the input is given by  $W_t \equiv (1 - \mu) P_t^d + \mu P_t^i$ .

Then, I assume (i) that the consumer/producer chooses in any period the new price with probability  $(1 - \alpha)$  or keeps the previous period price indexed to past inflation with probability  $\alpha$ , and (ii) that the price at period  $t + 2$  is chosen 2 periods in advance. The latter assumption is motivated by the fact that domestic sector firms take both production and retailing decisions. The implication is that monetary policy needs a two-period lag to affect domestic inflation. This is in line with the central banks' experience of an approximate two-period lag for monetary policy to have the highest impact on inflation. It follows that the decision problem for firm  $j$  at time  $t$  is

$$\max_{\widetilde{P}_{t+2}^d} E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \widetilde{\lambda}_{t+\tau+2}^d \left\{ \frac{\widetilde{P}_{t+2}^d \left( \frac{P_{t+\tau+1}^d}{P_{t+1}^d} \right)^\zeta}{P_{t+2+\tau}^d} \widehat{Y}_{t+\tau+2}^d \left[ \frac{\widetilde{P}_{t+2}^d \left( \frac{P_{t+\tau+1}^d}{P_{t+1}^d} \right)^\zeta}{P_{t+2+\tau}^d} \right]^{-\vartheta}}{f^{-1} \left[ \widehat{Y}_{t+\tau+2}^d \left( \frac{\widetilde{P}_{t+2}^d \left( \frac{P_{t+\tau+1}^d}{P_{t+1}^d} \right)^\zeta}{P_{t+2+\tau}^d} \right)^{-\vartheta} \right]} \right\} - \frac{W_{t+\tau+2}}{P_{t+\tau+2}^d} \frac{1}{A_{t+\tau+2}^d} \quad (20)$$

where  $\widetilde{\lambda}_t^d$ ,  $\widetilde{P}_{t+2}^d$  and  $\zeta$  denote, respectively, the marginal utility of domestic goods, the new price chosen in period  $t$  for period  $t + 2$  and the degree of indexation to the previous period inflation rate<sup>5</sup>. Following Svensson (2000), I set  $\delta = 1$  to ensure the natural-rate hypothesis. Finally, assuming that the purchasing power parity holds in the long run,

<sup>5</sup>Recalling that consumption decisions are predetermined one period in advance, the marginal utility of domestic goods  $\widetilde{\lambda}_t^d$  is obtained by the following first-order condition with respect to  $C_{t+1}^d$

$$E_t U_d(C_{t+1}^d, C_{t+1}^i) = E_t [\lambda_{t+1} P_{t+1}^d] \equiv E_t \widetilde{\lambda}_{t+1}^d,$$

where  $\lambda_t$  is the marginal utility of nominal income in period  $t$ .

the log-linearized version of the Phillips curve for the domestic sector turns out to be

$$\pi_{t+2}^d = \frac{1}{1 + \zeta} \left[ \zeta \pi_{t+1}^d + \pi_{t+3|t}^d + \frac{(1 - \alpha)^2}{\alpha(1 + \omega\vartheta)} (\omega y_{t+2|t}^d + \mu q_{t+2|t}^d) \right] + \varepsilon_{t+2} \quad (21)$$

$$= \phi_\pi \pi_{t+1}^d + (1 - \phi_\pi) \pi_{t+3|t}^d + \phi_y^d y_{t+2|t}^d + \phi_q^d q_{t+2|t}^d + \varepsilon_{t+2}, \quad (22)$$

where  $\omega$  in (21) is the output elasticity of the marginal input requirement function and  $\varepsilon_{t+2}$  is a zero-mean i.i.d. cost-push shock. In (22) all the implicitly defined coefficients are positive.

### 2.2.2 Import sector

Similar to the domestic sector, variety  $j$  of the import goods,  $Y_t^i(j)$ , is produced by the representative consumer/producer  $j$  with a composite input whose price is  $F_t$ . Since the input requirement function is  $\frac{1}{A_t^i} f^{-1}[Y_t^i(j)]$ , the variable cost of producing the quantity  $Y_t^i(j)$  is  $F_t \frac{1}{A_t^i} f^{-1}[Y_t^i(j)]$ . Furthermore, considering that the input is a convex combination of the aggregate of domestic goods and of the foreign good, with price  $P_t^* S_t$ , where  $P_t^*$  is the price in foreign currency of the foreign good, it follows that  $F_t \equiv \mu^i P_t^d + (1 - \mu^i) P_t^* S_t$ .

Now relaxing the assumption that pricing decisions are predetermined and keeping all the remaining assumptions used to derive the Phillips curve in the domestic sector results in

$$\pi_t^i = \frac{1}{1 + \zeta} \left[ \zeta \pi_{t-1}^i + \pi_{t+1|t}^i + \frac{(1 - \alpha^i)^2}{\alpha^i(1 + \omega\vartheta)} (\omega y_t^i + q_t^i) \right] \quad (23)$$

$$= \phi_\pi \pi_{t-1}^i + (1 - \phi_\pi) \pi_{t+1|t}^i + \phi_y^i y_t^i + \phi_q^i q_t^i, \quad (24)$$

where  $\alpha^i$  has the same meaning as its analogous variable in the domestic sector,  $q_t^i$  denotes (the log deviation of) the price of the composite input in the import sector expressed in terms of the import goods price,  $p_t^i$ , and is defined as

$$q_t^i \equiv (1 - \mu^i) (s_t + p_t^*) + \mu^i p_t^d - p_t^i, \quad (25)$$

where  $p_t^*$  is the (log) foreign price level. Relaxing the assumption of predetermined pricing decisions is motivated by the fact that the import sector only acts as a retailer for the foreign goods and, in practice, retailers do not set their price before they take effect as much as producers do. It is worthy of note that  $\alpha^i$  and  $\mu^i$  determine, respectively, the



speed and the degree of completeness of the pass-through<sup>6</sup>.

## 2.3 CPI inflation and the uncovered interest parity

CPI-inflation,  $\pi_t^c$ , is given by

$$\pi_t^c = (1 - w) \pi_t^d + w \pi_t^i, \quad (26)$$

where  $w$  is the steady state share of imported goods in total consumption and determines the degree of openness of the economy. In order to eliminate the non-stationary nominal exchange rate, it is convenient to express the Uncovered Interest Parity in terms of  $q_t^i$  obtaining

$$q_{t+1|t}^i - q_t^i = (1 - \mu^i) r_t - (1 - \mu^i) (i_t^* - \pi_{t+1|t}^*) - (\pi_{t+1|t}^i - \pi_{t+1|t}^d) - (1 - \mu^i) v_t, \quad (27)$$

where  $r_t$  is the short term real interest rate defined as  $r_t \equiv i_t - \pi_{t+1|t}^d$ .

## 2.4 The public sector and the rest of the world

The behavior of the central bank consists of minimizing the following loss function:

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau [\mu^c \pi_{t+\tau}^{c2} + \mu^d \pi_{t+\tau}^{d2} + \lambda y_{t+\tau}^{d2} + \nu (i_{t+\tau} - i_{t+\tau-1})^2], \quad (28)$$

where  $\mu^c$ ,  $\mu^d$ ,  $\lambda$  and  $\nu$  are weights that express the preferences of the central bank for CPI and domestic inflation targets, the output stabilization target, and the instrument smoothing target, respectively.

The rest of the world is exogenous and described by stationary univariate AR(1) processes for foreign inflation and income

$$\pi_{t+1}^* = \gamma_\pi^* \pi_t^* + \varepsilon_{t+1}^*, \quad (29)$$

$$y_{t+1}^* = \gamma_y^* y_t^* + \eta_{t+1}^*, \quad (30)$$

where the coefficients are non-negative and less than unity, and the shocks are white noises. Finally, the foreign sector monetary policy is set according to the following Taylor

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<sup>6</sup>About the relevance of these factors in the determination of low exchange rate pass-through see Devereux and Yetman (2008), Burstein, Neves and Rebelo (2003) and Corsetti and Dedola (2005).

rule

$$i_t^* = f_\pi^* \pi_t^* + f_y^* y_t^* + \xi_t^*, \quad (31)$$

where the coefficients are positive, and  $\xi_{it}^*$  is a white noise.

## 2.5 Optimal monetary policy with model uncertainty

I now assume that the central bank is uncertain on the *persistence* in the behaviors of firms and households, the *degree of price stickiness* and the *speed* and *completeness* of the pass-through. This assumption is modeled assuming that the central bank has a prior belief on the probability distribution of the deep parameters underlying these phenomena. A similar approach is followed by Kimura and Kurozumi (2007) who show in a forward-looking model how deep parameter uncertainty can lead to a more aggressive optimal monetary policy in a closed economy.

The uncertainty on the persistence in the household behavior is modeled by assuming uncertainty over habit formation in consumption preferences, captured by the parameter  $\iota$  in equation (2). This choice is useful to model central bank uncertainty on the degree of backward and forward looking household behavior. This can be seen in equation (5) where the parameter  $\beta$  is now uncertain due to  $\iota$ . It turns out that the uncertainty on this basic feature of the household behavior impacts on many coefficients in the aggregate demands. Indeed, considering equations (16) and (19) uncertain habit formation implies that, for any period  $t$ , not only the coefficient of the previous period output gap becomes uncertain, i.e.  $\beta_y$ , but also several other coefficients become uncertain. Specifically, these are the coefficients for the previous period terms of trade,  $\beta_{q-1}$  and  $\beta_{q-1}^i$ , foreign output  $\beta_{y^*}$  and  $\beta_{y^*}^i$ , and the natural output in the domestic sector and import sectors,  $\beta_{y^n}$  and  $\beta_{y^n}^i$ .

The remaining sources of multiplicative uncertainty of the model are located in the supply side. Here the setup features sticky prices *à la* Calvo (1983) and indexation to the previous period inflation rate for the firms that cannot optimally update the price in the current period. In this framework, by assuming that the central bank is uncertain on the firms' degree of backward-looking indexation,  $\zeta$ , in equation (20), the central bank turns out to be uncertain on the degree of inertia and forward-looking behavior in the inflation process. This assumption is motivated by a fair amount of disagreement in empirical evidence and theoretical works<sup>7</sup>. Importantly, uncertain inertia in the firms behavior, turns out to affect all the coefficients of the aggregate supplies and adds to

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<sup>7</sup>See Kimura and Kurozumi (2007) as to the contrasting results disseminated in the literature.

the uncertainty on the degree of price stickiness. The latter is modeled by introducing an uncertain probability of optimally updating the prices in the current period, that is by assuming that the variables  $(1 - \alpha)$  and  $(1 - \alpha^i)$  in (21) and (23) are stochastic. With uncertainty on  $\zeta$ ,  $\alpha$ ,  $\alpha^i$ , the central bank is uncertain in any period  $t$  about the slope of the aggregate supply in both sectors,  $\phi_y^d$  and  $\phi_y^i$ . The slope of the Phillips curve, i.e. the response of inflation to fluctuations in resource utilization, is a relationship which seems difficult to pin down in a statistically significant way (Holmberg 2007). Furthermore, the last two decades point to a flattening of the Phillips curve whose causes are not yet fully understood. Anchoring inflation expectations via better monetary policy seems a prominent candidate to explain this phenomenon (Mishkin 2007, Boivin and Giannoni 2006, and Roberts 2006), yet changes in the price-setting behaviour could also be important and dependent on the level and variability of inflation<sup>8</sup>. Bean (2007) also argues that the flattening of the Phillips curve is observationally equivalent to a downward sloping Phillips curve shifting to the left as the natural rate of unemployment fell with monetary policy simultaneously ensuring that inflation remained stable. This view implies that the uncertainty about the natural rate of unemployment makes it hard to pin down the slope of the Phillips curve. All in all, these factors surround the slope of the Phillips curve with a fair amount of uncertainty.

Also, uncertainty on  $\zeta$ ,  $\alpha$  and  $\alpha^i$  makes uncertain the impact of the terms of trade on domestic inflation,  $\phi_q^d$ , and the impact of the input price in the import sector on import inflation,  $\phi_q^i$ . These uncertain impacts capture the imperfect knowledge of the central bank on how the exchange rate affects the economy<sup>9</sup>. As to the input price in the import sector, which is a function of the exchange rate, the uncertain coefficient  $\phi_y^i$  determines the uncertainty on the speed of the pass through. Finally, uncertainty on the completeness of the pass-through is modelled by assuming that  $\mu^i$  is a random variable. The existence of the pass-through uncertainty is commonly known. For example, Cassino, Drew and McCaw (1999) point out that the pass-through has been quite variable over time in New Zealand. Pass-through changes can be associated with changes in the use of imported inputs or in the composition of a country's import basket if the component products have distinct pass-through elasticities (Campa and Goldberg 2006 and 2005); with changes in monetary stability and the persistence of exogenous shocks (Devereux and Engel 2001, Devereux, Engel and Storgaard 2004, and Devereux and Yetman 2008); with changes

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<sup>8</sup>See, among others, Cogley and Sbordone (2005) and Rubio-Ramirez and Villaverde (2007).

<sup>9</sup>See Leitemo and Söderström (2007) for the impact on optimal monetary policy of the policymakers' fear of misspecification in the determination of the exchange rate.

in the market share and in the differentiation degree of goods of the exporting country (Bacchetta and van Wincoop 2005). Thus, pass-through uncertainty derives from the limited knowledge on the role played by these and other factors that, both at the micro and macro level, seem to determine the pass-through.

## 2.6 Certainty non-equivalence and model uncertainty

To illustrate the introduction of uncertainty on structural parameters in a non-certainty equivalence environment, it is convenient to rewrite the model in State-space form. From the central bank standpoint the problem is to find the expected interest rate path that minimizes its loss given the law of motion of the economy:

$$\text{Min}_{\{i_{t+\tau|t}\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} Y_{t+\tau}' K Y_{t+\tau}$$

subject to

$$\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = \begin{bmatrix} A_{11,t+1} & A_{12,t+1} \\ A_{21,t} & A_{22,t} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_{1,t+1} \\ B_{2,t} \end{bmatrix} i_t + \begin{bmatrix} B_{1,t+1}^1 \\ B_{2,t}^1 \end{bmatrix} i_{t+1|t} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix},$$

$$Y_t \equiv C_{Z,t} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + C_{i,t} i_t,$$

where the target variables, the predetermined variables, and the forward looking variables are, respectively

$$\begin{aligned} Y_t &= (\pi_t^c, \pi_t^d, y_t^d, i_t - i_{t-1})', \\ X_t &= (\pi_t^d, \pi_{t+1|t}^d, \pi_{t-1}^i, \pi_t^*, y_t^d, y_t^i, y_t^*, i_t^*, y_t^{d,n}, y_t^{i,n}, i_{t-1}, q_{t-1}, q_{t-1}^i, v_t)', \\ x_t &= (\pi_t^i, q_t^i, \rho_t, \pi_{t+2|t}^d)', \end{aligned}$$

and where  $K$  captures the central bank's preferences, a diagonal matrix with the diagonal  $(\mu^c, \mu^d, \lambda, \nu)$  and off-diagonal elements equal to zero. Following Svensson and Williams (2007) I assume that the matrices

$$A_{11,t}, A_{12,t}, B_{1,t}, B_{1,t}^1, A_{21,t}, A_{22,t}, B_{2,t}, B_{2,t}^1, C_{Z,t}, C_{i,t}, \quad (32)$$

are random, each free to take  $n_j$  different values in period  $t$  corresponding to the  $n_j$  modes indexed by  $j_t \in \{1, 2, \dots, n\}$ . This means that, for example,  $A_{11,t} = A_{11,j_t}$ . The modes are drawn initially from a discrete stationary distribution which is assumed to be uniform. A uniform distribution captures the assumption that the central bank only knows a band for each uncertain deep parameter. For each uncertain parameter, say  $\xi$ , a benchmark value is chosen,  $\bar{\xi}$ , and the lower and upper bound of the support of the distribution are set equal to  $\bar{\xi} - x\bar{\xi}$  and  $\bar{\xi} + x\bar{\xi}$  respectively, where the coefficient  $x$  modules the variance of the distribution and therefore the amount of uncertainty.

After the initial draw from the stationary distribution the modes follow a Markov process with constant transition probabilities given by

$$P_{jk} \equiv \Pr \{j_{t+1} = k | j_t = j\} = \frac{1}{n}, \quad j, k \in \{1, 2, \dots, n\}.$$

Furthermore, I assume that model uncertainty and shocks to the economy are independent so that modes  $j_t$  and innovations  $\varepsilon_t$  are independently distributed. Finally, I assume that the central bank does not know how the structural parameters co-move together, should they be dependent. So in any period the realization of each parameter is independent of the realizations of the other parameters<sup>10</sup>. As to the central bank knowledge before choosing the instrument-plan  $\{i_{t+\tau|t}\}_{\tau=0}^{\infty}$  at the beginning of period  $t$ , the information set consists of the probability distribution of  $\varepsilon_t$ , the transition matrix  $[P_{jk}]$ , the  $n_j$  different values that each of the matrices can take in any mode, and finally the realizations of  $X_t, j_t, \varepsilon_t, X_{t-1}, j_{t-1}, \varepsilon_{t-1}, x_{t-1}, \dots$ .

Since the model cannot be solved analytically and embeds also forward looking variables, I use the numerical methods developed by Svensson and Williams (2007) to find the equilibrium in the presence of multiplicative uncertainty under commitment in a timeless perspective (see Woodford 2003 and Svensson and Woodford 2005)<sup>11</sup>. Numerical methods, in turn, require a calibration which is presented in the following section.

## 2.7 Calibration

Two groups of parameters need to be calibrated to solve the model. The first consists of the parameters that are assumed to be known with certainty, while the second the

<sup>10</sup>If, for example, each uncertain parameter can take  $d$  values in any period and there are  $m$  uncertain parameters, then the number of modes is  $n = d^m$ . In this work  $d = 5$  and  $m$  can be either 1 or 2 or 5 depending on the uncertainty cases described below.

<sup>11</sup>The implementation of these methods, which are used to obtain the figures and tables shown below is carried out coding in Matlab.

benchmark values for the uncertain parameters.

The choice of the parameters assumed to be known with certainty follows Svensson (2000) as the current model is similar in structure to the Svensson's one. These parameters, with respect to the domestic economy, are the output elasticity of the marginal input requirement function,  $\omega = 0.8$ ; the elasticity of substitution between varieties of the same type of good  $\vartheta = 1.25$ ; the intertemporal elasticity of substitution,  $\sigma = 0.5$ ; the share of import good in the composite input to produce the domestic good,  $\mu = 0.1$ ; the share of import goods in domestic consumption,  $w = 0.3$ . With respect to the foreign sector, the elasticity of substitution between domestic and import goods for foreign consumers is  $\theta^* = 2$ ; the share of the domestic good in foreign consumption is  $w^* = 0.15$ ; the income elasticity of foreign real consumption is  $\bar{\beta}_y^* = 0.9$ ; and the coefficients for the foreign Taylor rule are  $f_{\pi^*} = 1.5$ , and  $f_{y^*} = 0.5$ . Finally, the exogenous cost push and demand shocks have variances  $\sigma_{\pi}^2 = \sigma_y^2 = 1$ ; the natural output shocks have variances  $\sigma_{y^{d,n}}^2 = \sigma_{y^{i,n}}^2 = 0.5$  and AR(1)-parameter  $\gamma_y^{d,n} = \gamma_y^{i,n} = 0.96$ , and finally the risk premium, foreign inflation and output have AR(1) process-parameter  $\gamma_{y^*} = \gamma_{\pi^*} = \gamma_v = 0.8$  and variances  $\sigma_{\nu}^2 = \sigma_{\pi^*}^2 = \sigma_{y^*}^2 = 0.5$ .

The benchmark values of the uncertain parameters follow Banerjee and Batini (2003) as to the measure of habit formation in the utility function,  $\bar{\iota} = 0.8$  and Smets and Wouters (2005) as to the degree of indexation to the previous period inflation rate,  $\bar{\zeta} = 0.66$ . The probability on not optimally updating the price in the current period in the domestic and import sector,  $\bar{\alpha}$ , and  $\bar{\alpha}^i$ , are set equal to 0.5 following Svensson (2000) and Flamini (2007), respectively. Finally, the value of the share of domestic good in the composite input to supply the import good,  $\bar{\mu}^i$ , is set to 0.35 consistently with Flamini (2007) and such that the lower and upper bound of the support of the  $\mu^i$  distribution are realistic for the uncertainty level considered in the analysis; specifically the lower and upper bounds are 0.245 and 0.405.

### 2.7.1 Robustness check

The current model is also similar in spirit to the Leitemo and Söderström (2005) model. Although the latter is not microfounded, its parametrization for the exogenous disturbances provides a valid alternative to check for the robustness of the results. In the Leitemo and Söderström model, the cost-push shock and the demand shock are AR(1) processes and their AR(1)-coefficients,  $\gamma_{\pi}$  and  $\gamma_y$ , are set equal to 0.3 (this is a difference with the previous calibration where the AR(1)-coefficients for these two shocks are im-

licitly set equal to zero). The variances for these shocks are  $\sigma_y^2 = 0.656$  and  $\sigma_\pi^2 = 0.389$ , while the variance for the shocks to the risk premium, foreign inflation, and foreign output gap are  $\sigma_\nu^2 = 0.844$ ,  $\sigma_{\pi^*}^2 = 0.022$ , and  $\sigma_{y^*}^2 = 0.083$ , respectively<sup>12</sup>. For the risk premium AR(1)-coefficient  $\gamma_\nu$ , Leitemo and Söderström considers the interval  $[0, 1]$ . In the current analysis, having to choose one value,  $\gamma_\nu$  is set equal to 0.5.

### 3 Central bank preferences and distribution forecasts

We can now appreciate how the possibility of moving from *mean forecast targeting* to *distribution forecast targeting* dramatically enriches the monetary policy analysis. In this work, distribution forecasts are determined by the simultaneous presence of model uncertainty and exogenous shocks. Their relevance lies in shedding light on two important aspects of the economic outlook associated with different monetary policies. The first is the expected volatility of the macrovariables at any future time period. The second is the joint impact of shocks and model uncertainty upon economic stability along with the specific contribution of model uncertainty.

#### 3.1 Overview with a cost push shock and general uncertainty

Figures 1-2 illustrate the unconditional distribution forecasts of the impulse responses to a (one standard deviation) cost-push shock in the presence of general uncertainty, which encompasses uncertainty on the pass-through,  $(\mu_j^i, \alpha_j^i)$ , on the persistence in the private sector's behaviour,  $(\iota_j, \zeta_j)$ , and on the slope of the domestic AS,  $(\alpha_j)$ . In each figure, the first and second column report the distribution forecasts of the main macrovariables under the optimal policies of domestic and CPI inflation targeting respectively. Assuming an uncertainty level of 30% on all the uncertain parameters, these figures have been generated by drawing an initial mode of the Markov chain from its stationary distribution, simulating the chain for a sequence of periods forward, and then repeating this procedure for 1000 simulations runs<sup>13</sup>. Thus these figures display mean (dashed line), and quantiles (grey bands), of the empirical distribution. In particular, the dark, medium and light grey band show the 30%, 60%, and 90% probability bands, respectively. Figures 1-2 consider high and low preferences for interest rate smoothing respectively<sup>14</sup>. In the former, the

<sup>12</sup>Leitemo and Roisland (2002) find these variances with a structural VAR on the Norwegian economy.

<sup>13</sup>The results presented in this and the next sections are robust to smaller and larger uncertainty levels.

<sup>14</sup>Specifically, the interest rate smoothing preferences parameter,  $\nu$ , in equation (28), is 0.05 in Figure 1 and 0.002 in Figure 2.

central bank carries out a mild monetary policy in which there is almost no attempt to buffer the shock. This case starts to reveal the impact of model uncertainty and alternative inflation indexes on the distribution forecasts. It thus provides a benchmark. In the latter case, low preferences for interest rate smoothing, the monetary policy is more realistic and the different impact of model uncertainty on the distribution forecasts linked to alternative preferences on the inflation index is fully revealed.

Figure 1 features a high preference for interest rate smoothing. Here visual inspection shows that the volatility of the macrovariables distribution and the perturbing impact of the shock tend to be higher under CPI IT. In Figure 2, switching to a low preference for interest rate smoothing the previous result is strongly amplified: DIT implies much less volatility of the projections of the economy, in particular of the interest rates, and a surprisingly better ability to absorb the cost-push shock. As we would have expected, under CPI IT the optimal monetary policy attempts to absorb the cost-push shock using the exchange rate. This is reflected in the initial decrease of import inflation,  $\pi^i$ , shown in the sixth row, second column. What is interesting here is that the policy manoeuvre has a limited impact on absorbing the shock on CPI inflation. Indeed, the initial path of CPI inflation is only marginally lower with CPI IT than with DIT. This can be seen in the last row comparing the distribution forecasts in both cases. Furthermore, in terms of volatility of the distribution forecast, CPI inflation,  $\pi^c$ , does not seem to be less volatile under CPI IT. On the basis of these results, it could be argued that shifting from CPI IT to DIT would imply a small cost in terms of higher CPI inflation versus a large benefit in terms of the volatility of the distribution forecast of all the other variables and the expected perturbing impact of the shock. Nonetheless, different central banks can attribute different values to the distribution forecasts accuracy of various macrovariables and their expected shock sensitivity. An extreme case could be, for example, a central bank that is only concerned in keeping CPI inflation as close as possible to its long run value. For this central bank, CPI IT would be the most appropriate policy. Thus, the usefulness of these findings primarily lies in showing what the domestic and CPI inflation targeting policies can offer in terms of distribution forecasts accuracy and expected economic stability.

It is also worth noting that switching from high to low interest rate smoothing, the overall volatility of the economy does not increase much with DIT while it gets huge with CPI IT. Thus, the domestic inflation index stands out also for much less sensitiveness to abrupt changes of the interest rate.



### 3.2 Preferences and volatility of the $i$ and $y^d$ distribution forecasts with a cost-push shock in various uncertainty cases

On the basis of the previous analysis with high and low interest smoothing preferences, a natural question to ask is whether the volatility of the macrovariables is monotonous in the preferences for smoothing. This is relevant given the uncertainty on the smoothing preferences of the central bank and, more in general, the time varying degree of activism in monetary policy possibly related to central bank judgment. To address this question, Figure 3 focuses on the cost-push shock case and presents the standard deviation of the distribution forecasts of the nominal interest rate and the domestic output-gap for the periods considered above and for interest rate smoothing values in the set  $V = \{0.002, 0.005, \dots, 0.04\}$ <sup>15</sup>. Explaining this figure, each sub plot reports two surfaces that describe the standard deviation of the distribution forecasts under CPI and DIT. The first and the second row refer to the interest rate and the output gap, respectively, while the columns refer to four uncertainty cases, specifically uncertainty (i) on the pass-through, (ii) on the persistence of the behaviour of households and firms, (iii) on the degree of price flexibility in the domestic sector (AS slope uncertainty), and (iv) on all the previous sources, i.e. general uncertainty.

A first result considering the interest rate (first row) is that either the CPI IT surface is always above the DIT surface (in the uncertainty on the pass-through, on the persistence in the behaviour of households and firms, and general uncertainty cases, first, second, and forth column respectively), or the two surfaces tend to overlap with the DIT one slightly above the CPI one for small preferences on interest rate smoothing (in the cases of uncertainty on the slope of the Phillips curve in the domestic sector, third column). This shows that under the pass-through, persistence, and general uncertainty cases the CPI IT policy results systematically in a larger standard deviation for the distribution forecast of the interest rate than DIT. Instead, when we consider the case of uncertainty on the degree of price flexibility in the domestic sector, the standard deviation associated with DIT tends to be higher than the one associated with CPI IT. Moving to the second row describing the variability of the distribution forecast of the output gap in the domestic sector we obtain similar results.

Second, the volatility of the distribution forecasts of the interest rate and the output gap tend to be monotonically increasing in the preference for not smoothing the interest rate. Yet, it is interesting to note that, decreasing interest rate smoothing, the volatility

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<sup>15</sup>Section 3.4. and 3.5 will extend the analysis to other macrovariables and shocks.

under CPI IT tends to increase more than under DIT.

The relevance of these findings lies in unveiling DIT as a policy that leads to less variability of the distribution forecasts of the interest rate and of the output gap in the presence of a cost-push shock. Also, it is less sensitive to interest rate smoothing than the CPI IT policy. Since the interest rate smoothing preference is inversely linked to the preferences for the other target variables<sup>16</sup>, it follows that with DIT the central bank can stabilize the output gap and inflation with a lower cost in terms of a rough path of the interest rate.

In order to quantitatively compare the policies associated with the two surfaces it is informative to compute the ratio of the means (along all the smoothing preferences values and the periods considered) of the standard deviations in the two policy cases, i.e.

$$R^\sigma \equiv \frac{\text{mean} \left|_{\nu,t} \text{std}_{\nu,t}^c(\text{variable})\right.}{\text{mean} \left|_{\nu,t} \text{std}_{\nu,t}^d(\text{variable})\right.},$$

where  $\text{std}_{\nu,t}^h(\text{variable})$ ,  $h = c, d$ , denote the standard deviation of the distribution forecast of the considered variable for period  $t$ , and smoothing preferences value  $\nu$ , and  $c$  and  $d$  denote CPI and DIT, respectively. Table 2 considers the nominal interest rate and the domestic output gap and presents the statistics  $R^\sigma$  for various uncertainty types.

INSERT TABLE 2 HERE

This analysis shows that for the nominal interest rate, in almost all uncertainty cases, domestic inflation preferences dominate CPI inflation preferences. Furthermore, when we focus on the more representative case of general uncertainty, which includes all the previous cases, the mean of the standard deviation under CPI IT is 2.79 times larger than under DIT. Considering the output gap, DIT dominates CPI IT in all the uncertainty cases except the one of uncertainty on the slope of the aggregate supply where they tend to be equivalent. In the general uncertainty case the average variability of the distribution forecast for the output gap with the CPI policy is 1.48 times larger than with the other policy.

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<sup>16</sup>To see this, just scale the weights in the loss function.

### 3.3 Preferences and expected stability of $i$ and $y^d$ with a cost push shock in various uncertainty cases

An interesting question to ask is how central bank's preferences rank in terms of the expected perturbing impact of exogenous shocks on the economy. The medians of the distribution forecasts provide a *prima facie* answer. Figure 4 illustrates the medians of the distribution forecasts of the nominal interest rate and the domestic output gap, first and second row respectively. The columns refer to the uncertainty cases. These medians (for the considered periods and values of interest rate smoothing preferences) are illustrated by two surfaces for the CPI and DIT policies. Denoting these surfaces as median surfaces, the *distance* of the median surface from the zero plane provides a measure of the expected median perturbing impact of the shock. Interestingly, Figure 4 shows that the distance of the median surface from the zero plane under CPI IT is always larger than under DIT.

Recalling that the model variables are log deviations from their steady state values, this result shows that with DIT the nominal interest rate and the output gap are expected to deviate less from the long-run equilibrium after a cost push shock than with CPI IT.

In order to quantitatively compare the distance of the two surfaces from the zero plane it is informative to introduce the ratio of the means (of the absolute values) of the medians in the two policy cases for all the smoothing preferences values and the period considered, that is

$$R^M \equiv \sum_{\nu \in V} \sum_{t=0}^T \frac{|\text{median}_{\nu,t}^c(\text{variable})|}{|\text{median}_{\nu,t}^d(\text{variable})|},$$

Table 3 reports  $R^M$  with respect to the nominal interest rate and the output gap.

INSERT TABLE 3 HERE

It shows that if the central bank chooses the CPI IT policy, the expected median deviation from the steady state value for the nominal interest rate and the output gap in the general uncertainty case are, respectively, 2.26 and 2.06 times larger than if it chooses the DIT policy.

### 3.4 A broad perspective with general uncertainty

To complete the picture of the relation between the choice of the inflation index, distribution forecasts accuracy and expected economic stability, I extend the analysis to other

macrovariables and shocks. The other macrovariables considered are CPI and domestic inflation,  $\pi^c$  and  $\pi^d$  respectively, the short term real interest rate,  $r$ , and the real exchange rate,  $q$ . The other (one standard deviation) shocks considered are a shock to the aggregate demand, the foreign interest rate, the natural output, the risk premium, and the foreign output. The results are reported in Tables 4-5 for the *general uncertainty* case.

INSERT TABLES 4-5 HERE

To discuss the results it is useful to define three levels of dominance in terms of intervals for the ratios  $R^\sigma$  and  $R^M$ . These levels of dominance are

$$\begin{aligned} \text{Weak Dominance} & \iff 0.9 < R \leq 1.\bar{1} \\ \text{Dominance} & \iff 0.5 < R \leq 0.9 \text{ or } 1.\bar{1} < R \leq 2 \\ \text{Strong Dominance} & \iff 0 < R \leq 0.5 \text{ or } R > 2 \end{aligned}$$

Results in Tables 4a show that, with regard to less volatility, DIT is strongly dominant in 8 cases, dominant in 8 cases, weakly dominant in 4 cases, weakly dominated in 6 cases, dominated in 10 cases, and strongly dominated in 0 cases. Thus, abstracting from the *weak dominance* cases, DIT is *strongly dominant* or *dominant* in 44.4% of the cases, while it is dominated in 27.7% of the cases. Considering expected shock sensitivity, Table 5a shows similar results: DIT is *strongly dominant* or *dominant* in 47.2% of the cases, while it is *dominated* in 22.2% of the cases<sup>17</sup>. Interestingly, DIT strongly dominates in terms of volatility and expected shock sensitivity in approximately one fifth and one third of the cases respectively, yet it is never strongly dominated. Checking for the robustness of these results, the analysis based on the Leitemo and Söderström (2005) calibration corroborates the previous findings. Results on Table 4b show that in terms of less volatility, DIT is strongly dominant or dominant in the 63.8% while it is dominated in the 16.6% of the cases. In terms of shock sensitivity, DIT is strongly dominant or dominant in 61.1% of the cases and is dominated in 11.1% of the cases.

It is worth noting that the cases in which DIT is dominated tend to pertain to CPI inflation, as we would expect, and also to the real exchange rate. As to the former, except for the cost-push shock, both the distribution forecasts of domestic and CPI inflation are not very sensitive to exogenous disturbances. Thus the two policies tend to be similar in their ability to stabilize inflation even if each one is better at stabilizing its own measure of inflation<sup>18</sup>. As to the latter, the real exchange rate, with a demand, natural output,

<sup>17</sup>DIT is strongly dominant in 12 cases, dominant in 5 cases, weakly dominant in 3 cases, weakly dominated in 8 cases, dominated in 8 cases, and strongly dominated in 0 cases.

<sup>18</sup>The impulse response distribution forecasts for the complete set of shocks are available upon request.

risk premium, and foreign output shock, CPI IT performs better as is shown in Table 4a,b. This is due to the fact that it aims to stabilize both domestic and import inflation, which determine the real exchange rate.

Shocks to the risk premium, foreign interest rate and foreign output gap deserve a final comment. In these cases the shocks impact on the nominal exchange rate via the uncovered interest parity. Then, if the central bank does not react, the shock propagates to CPI inflation. Thus with CPI IT the central bank has to respond to these shocks. Yet, the central bank may not be willing to react to shocks that affect the nominal exchange rate. Leitemo and Söderström (2005) maintain that it should not. Their argument is that there is uncertainty about how the exchange rate is determined and the effect of exchange rate movements on the economy. This implies that rules with the exchange rate are more sensitive to model uncertainty. Thus a monetary policy developed in the context of an exchange rate model could perform poorly if that model is incorrect. Empirical evidence in this respect is not conclusive. Lubik and Schorfheide (2007) find that Australia and New Zealand did not react to movements in the exchange rate while Canada and the UK did.

These results on overall forecasts' accuracy and economic instability are based on two factors. More policy activism under CPI IT than under DIT and the presence of model uncertainty.

More policy activism under CPI IT is determined by two differences between CPI inflation and domestic inflation. First, monetary policy can affect CPI inflation before domestic inflation via import inflation. This is due to the fact that the domestic sector produces and retails domestic goods, while the import sector only retails foreign goods. The presence of production decisions matters in that it implies a longer lag for monetary policy to affect domestic inflation than CPI inflation via the output gap. This is the policy transmission that occurs through the aggregate demand channel and/or the switching demand exchange rate channel. Similarly, the exchange rate and the price of the foreign goods in foreign currency affect domestic inflation with a lag via  $q_t$  in the AS for the domestic sector, while they affect directly import inflation via  $q_t^i$  in the AS for the import sector<sup>19</sup>.

The second difference between CPI and domestic inflation is that the former is more exposed to foreign shocks. Shocks to the risk premium, the foreign interest rate, or foreign output and inflation lead to exchange rate volatility via the uncovered interest

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<sup>19</sup>The impact of the exchange rate on the domestic price of the foreign good is amply documented in the literature and usually referred to as the Direct Exchange Rate channel.

parity. Since import sector inputs are more intensive in foreign goods than domestic sector inputs, movements in the exchange rate exert a stronger impact on CPI inflation than domestic inflation. Thus, to avoid that exchange rate volatility lead to CPI inflation volatility, the central bank has to intervene promptly. This means that under CPI IT, in the presence of foreign shocks there is a more pronounced trade-off between CPI inflation and interest rate volatility.

These differences among the inflation indices imply that CPI inflation is more sensitive to monetary policy and external shocks than domestic inflation. As a result, they tend to foster more policy activism under CPI IT as shown by Tables 6a,b that extend Tables 5a,b to the case of no model uncertainty<sup>20</sup>. Focusing on the nominal interest rate (fourth column), Tables 6a,b show that DIT tends to dominate CPI IT. When model uncertainty is introduced, more activism under CPI IT implies that this policy leads to more overall volatility in the distribution forecasts and more economic instability than DIT. The specific impact of model uncertainty on economic instability is presented and discussed in the next section by contrasting Tables 5a,b with 6a,b.

### 3.5 Model uncertainty and expected economic instability

Tables 5a,b describe the ability of domestic and CPI inflation targeting to stabilize the economy in the presence of exogenous shocks (additive uncertainty) and model uncertainty (multiplicative uncertainty). While considering simultaneously these sources of uncertainty matters to investigate optimal monetary policy in a more realistic framework, it is interesting to assess the specific contribution of model uncertainty. In fact, most of the literature on monetary policy abstracts from model uncertainty, even though its existence poses a major challenge to real-world monetary policy. It is therefore relevant to analyze the extent to which accounting for model uncertainty affects the results on economic instability.

Tables 6a abstracts from model uncertainty and shows that domestic and CPI inflation

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<sup>20</sup>With no multiplicative uncertainty, at any point in time a distribution forecast boils down to *degenerate distribution forecast*, which is the probability distribution of a random variable with always the same value. Characterized by a support of only one value, say  $k_0$ , the degenerate distribution is localized at a point  $k_0$  on the real line and its probability mass function is given by

$$f(k; k_0) = \begin{cases} 1, & \text{if } k = k_0 \\ 0, & \text{if } k \neq k_0. \end{cases}$$

Thus, a degenerate distribution does satisfy the definition of random variable and  $k_0$  is both the mean and the median of the distribution.

targeting are dominant or strongly dominant in 41.6% and 13.8% of the cases respectively, whereas weak dominance occurs in the remaining 44.4% of the cases. Moving with Table 5a from additive uncertainty only to both additive and multiplicative uncertainty dramatically increases the differences in the policies' performance. We observe that the weak dominance cases fall to 30.5% and that the cases in which domestic and CPI inflation targeting are dominant or strongly dominant rise to 47.2% and 22.2%, respectively.

These results are again corroborated by the analysis based on the Leitemo and Söderström (2005) calibration. Table 6b shows that domestic and CPI inflation targeting are strongly dominant or dominant in the 46.6% and 10% of the cases, while weak dominance occurs in the remaining 43.3% of the cases. Taking into account model uncertainty with Table 5b, the weak dominance cases fall to 33.3% and the strong dominance and dominance cases rise to 56.6% with DIT but remain at the 10% with CPI IT.

Three remarks are in order. First these results show that without model uncertainty DIT already leads to more overall economic stability than CPI IT. This result is determined by the larger policy activism that occurs with CIP IT. Interestingly, the mechanism that generates more economic stability under DIT is consistent with the mechanism that governs economic stability in Mankiew and Reis (2003). Indeed, Mankiew and Reis show that a price index for economic stability should weigh more the sectors whose price level is more predetermined and are less exposed to idiosyncratic shocks. As discussed before, in the current model the presence of import inflation in CPI inflation implies that domestic inflation is more predetermined and less exposed to foreign shocks than CPI inflation. As a result, less policy activism occurs that, in turn, leads to more economic stability. Second, these results show that considering the presence of multiplicative uncertainty does matter in assessing the performance of different policies in terms of expected economic stability. Multiplicative uncertainty sharpens the difference in the performance of the two policies by reducing the number of weak dominance cases. Hence, considering multiplicative uncertainty allows the policy makers taking more informed decisions on the convenience of the two policies in a larger number of cases. Finally, these findings reveal that multiplicative uncertainty magnifies the tendency of DIT to dominate CPI IT in terms of economic stability for several macrovariables.

## 4 Forecast accuracy and future policy inclinations

On the basis of the previous findings, an inflation targeting central bank that chooses *domestic* instead of *CPI* inflation stabilization improves the quality of several forecasts, in particular for the interest rate. Interestingly, this result relates to transparency in monetary policy and bears a significant policy implication. In particular, it relates to transparency on future policy intentions and to the recent debate on the instrument-rate assumption underlying projections of target variables. The debate arises from two alternatives facing monetary policy which imply different levels of transparency: either publishing the optimal instrument-rate plan and the corresponding projections of the economy, or publishing the projections of the economy based on a specific assumption on the interest rate, e.g. the assumption of a constant interest rate or an interest rate path given by market expectations<sup>21</sup>. The first alternative has been pioneered by the Reserve Bank of New Zealand and then adopted by the Norges Bank, the Swedish Riksbank, and the Czech National Bank. Yet, most of the inflation targeting central banks has so far opted for the second alternative being reluctant or very cautious in fully disclosing future policy intentions.

In this respect it can be argued that the more accurate and reliable the central bank distribution forecasts, the more the central bank can affect the private sector expectations if it chooses to be transparent and disclose future policy intentions. Hence, by showing the existence of a relationship between alternative inflation indexes and the quality of the distribution forecasts, this paper suggests that the choice of DIT can increase the benefits associated with transparency on future policy inclinations.

## 5 Conclusions

This paper argues that the choice of the inflation index to target significantly affects forecasts' accuracy and economic stability. In particular this work first shows that when model uncertainty is considered in the policy design, choosing the domestic inflation index reduces the volatility of the distribution forecasts for most of the macrovariables. On the other hand, if the central bank has a special interest in reducing the volatility of CPI inflation, and in some cases of the real exchange rate, regardless of the other macrovariables, then choosing the CPI suits this goal better.

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<sup>21</sup>For a discussion on these alternatives, see Blinder and Wyplosz (2004), Goodhart (2005), Honkapohja and Mitra (2005), Mishkin (2004), Moessner and Nelson (2008), Qvigstad (2006), Rudebusch and Williams (2007), Svensson (2006, 2008), Woodford (2005) and Holmsen et. al (2008).



Second, DIT tends to dominate CPI IT in terms of expected economic stability. Furthermore, accounting for model uncertainty magnifies this tendency and markedly improves the policymakers' ability to take informed decisions on the convenience of each index.

Finally, the paper shows that preferences on smoothing the interest rate do not affect much the behavior of the economy under DIT, while they do affect it under CPI IT. Arguably, central banks may not have any preferences on smoothing the interest rate (see Rudebusch 2002, 2006). Yet, if they do, the relevance of this result lies in allowing the central bank to stabilize the output gap and inflation with lower costs in terms of a rough path of the interest rate.

These findings also present an additional reason in favor of publishing the central bank optimal instrument-rate plan and the corresponding projections of the economy. In fact, by targeting domestic inflation instead of CPI inflation it is possible to obtain more accurate forecasts of the economy's dynamics, in particular of the interest rate. If the accurateness of the forecasts increases the potential accountability of the central bank and its credibility, it can also improve the effectiveness of monetary policy via the management of the private sector's expectations. The policy implication is that the choice of DIT can increase the benefits associated with transparency on future policy intentions. Thus, under DIT, these results support the alternative to publish the projections of the economy corresponding to the optimal interest rate path expected by the central bank.

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## Tables

<b>Structural parameter</b>	<b>Benchmark value</b>
$\bar{\iota}$	0.8
$\bar{\zeta}$	0.66
$\bar{\alpha}$	0.5
$\bar{\alpha}^i$	0.5
$\bar{\mu}^i$	0.35

Table 1. Benchmark values for the uncertain parameters

<b>Uncertainty type</b>	$i$	$y^d$
Pass-through	3.68	2.27
Persistence private sector behavior	1.16	1.11
Domestic AS slope	0.91	1.01
General	2.79	1.48

Table 2.  $R^\sigma$  : Ratio of the means under DIT and CPI IT of the standard deviations of the distribution forecasts for interest rate smoothing preferences  $\nu \in \{0.002, 0.005, \dots, 0.04\}$  and periods  $t \in \{0, 1, \dots, 15\}$ . Shock: cost-push. First calibration.

<b>Uncertainty type</b>	$i$	$y^d$
Pass-through	1.74	1.63
Persistence private sector behavior	1.82	1.47
Domestic AS slope	2.01	1.64
General	2.26	2.06

Table 3.  $R^M$  : Ratio of the means under DIT and CPI IT of the standard deviations of the distribution forecasts for interest rate smoothing preferences  $\nu \in \{0.002, 0.005, \dots, 0.04\}$  and periods  $t \in \{0, 1, \dots, 15\}$ . Shock: cost-push. First calibration.

Shock	$\pi^c$	$\pi^d$	$y^d$	i	r	q
Cost-push	1.08	1.15	1.48	2.79	2.62	1.44
Demand	0.89	1.16	0.95	1.05	1.05	0.82
Foreign interest rate	0.77	1.32	1.18	2.91	2.77	1.01
Natural output	0.87	1.11	0.98	0.97	0.97	0.75
Risk premium	0.71	0.90	0.82	2.09	2.05	0.77
Foreign output	0.76	1.16	0.94	2.22	2.26	0.88

**Table 4a.**  $R^\sigma$  : Ratio of the means under DIT and CPI IT of the standard deviations of the distribution forecasts for interest rate smoothing preferences  $\nu \in \{0.002, 0.005, \dots, 0.04\}$  and periods  $t \in \{0, 1, \dots, 15\}$ . Uncertainty case: general. First calibration.

Shock	$\pi^c$	$\pi^d$	$y^d$	i	r	q
Cost-push	0.91	1.05	2.06	2.26	1.13	3.15
Demand	0.98	1.10	0.99	0.98	0.99	1.25
Foreign interest rate	0.74	5.18	4.55	5.09	6.06	1.08
Natural output	0.93	1.38	0.97	0.85	0.90	0.81
Risk premium	0.57	2.66	0.88	2.51	2.42	0.81
Foreign output	0.58	4.50	2.42	1.61	1.47	0.80

**Table 5a.**  $R^M$  : Ratio of the means under DIT and CPI IT of the medians of the distribution forecasts for interest rate smoothing preferences  $\nu \in \{0.002, 0.005, \dots, 0.04\}$  and periods  $t \in \{0, 1, \dots, 15\}$ . Uncertainty case: general. First calibration.

Shock	$\pi^c$	$\pi^d$	$y^d$	i	r	q
Cost-push	0.96	1.01	1.42	1.64	0.94	1.17
Demand	1.02	1.05	0.99	1.01	1.01	1.09
Foreign interest rate	0.82	4.26	2.70	6.72	8.49	0.97
Natural output	1.23	1.32	0.98	0.92	0.97	1.01
Risk premium	0.68	2.27	0.87	3.15	3.56	1.01
Foreign output	0.65	3.71	0.74	1.54	1.37	0.99

**Table 6a.**  $R^M$  : Ratio of the means under DIT and CPI IT of the medians of the distribution forecasts for interest rate smoothing preferences  $\nu \in \{0.002, 0.005, \dots, 0.04\}$  and periods  $t \in \{0, 1, \dots, 15\}$ . Uncertainty case: no model uncertainty. First calibration.

Shock	$\pi^c$	$\pi^d$	$y^d$	i	r	q
Cost-push	1.05	1.13	1.23	1.79	1.22	2.04
Demand	0.86	1.16	0.91	0.94	1.17	1.01
Foreign interest rate	0.76	1.31	1.19	2.23	1.35	2.91
Natural output	0.87	1.12	0.99	0.89	1.12	0.95
Risk premium	0.74	1.33	1.19	2.67	1.38	3.35
Foreign output	0.77	1.15	0.94	1.75	1.19	2.22

**Table 4b.**  $R^\sigma$  : Ratio of the means under DIT and CPI IT of the standard deviations of the distribution forecasts for interest rate smoothing preferences  $\nu \in \{0.002, 0.005, \dots, 0.04\}$  and periods  $t \in \{0, 1, \dots, 15\}$ . Uncertainty case: general. Second calibration.

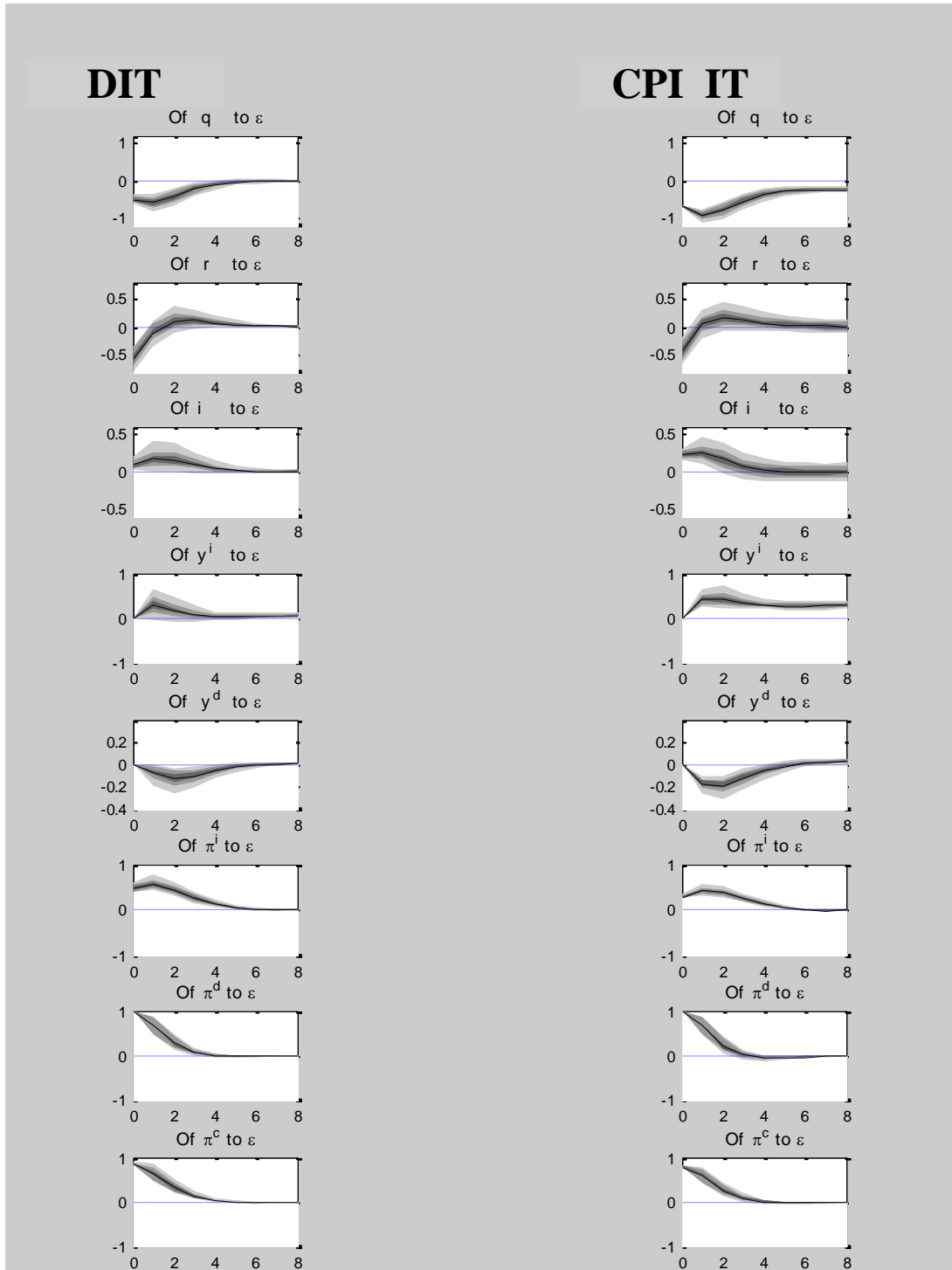
Shock	$\pi^c$	$\pi^d$	$y^d$	i	r	q
Cost-push	0.93	1.06	1.46	1.56	1.27	1.50
Demand	0.99	1.17	0.99	1.01	1.18	0.93
Foreign interest rate	0.76	5.59	4.15	5.62	5.64	5.02
Natural output	0.93	1.38	0.97	0.99	1.40	0.85
Risk premium	0.63	4.11	3.35	2.79	4.16	2.91
Foreign output	0.59	4.50	2.42	1.03	4.39	1.60

**Table 5b.**  $R^M$  : Ratio of the means under DIT and CPI IT of the medians of the distribution forecasts for interest rate smoothing preferences  $\nu \in \{0.002, 0.005, \dots, 0.04\}$  and periods  $t \in \{0, 1, \dots, 15\}$ . Uncertainty case: general. Second calibration.

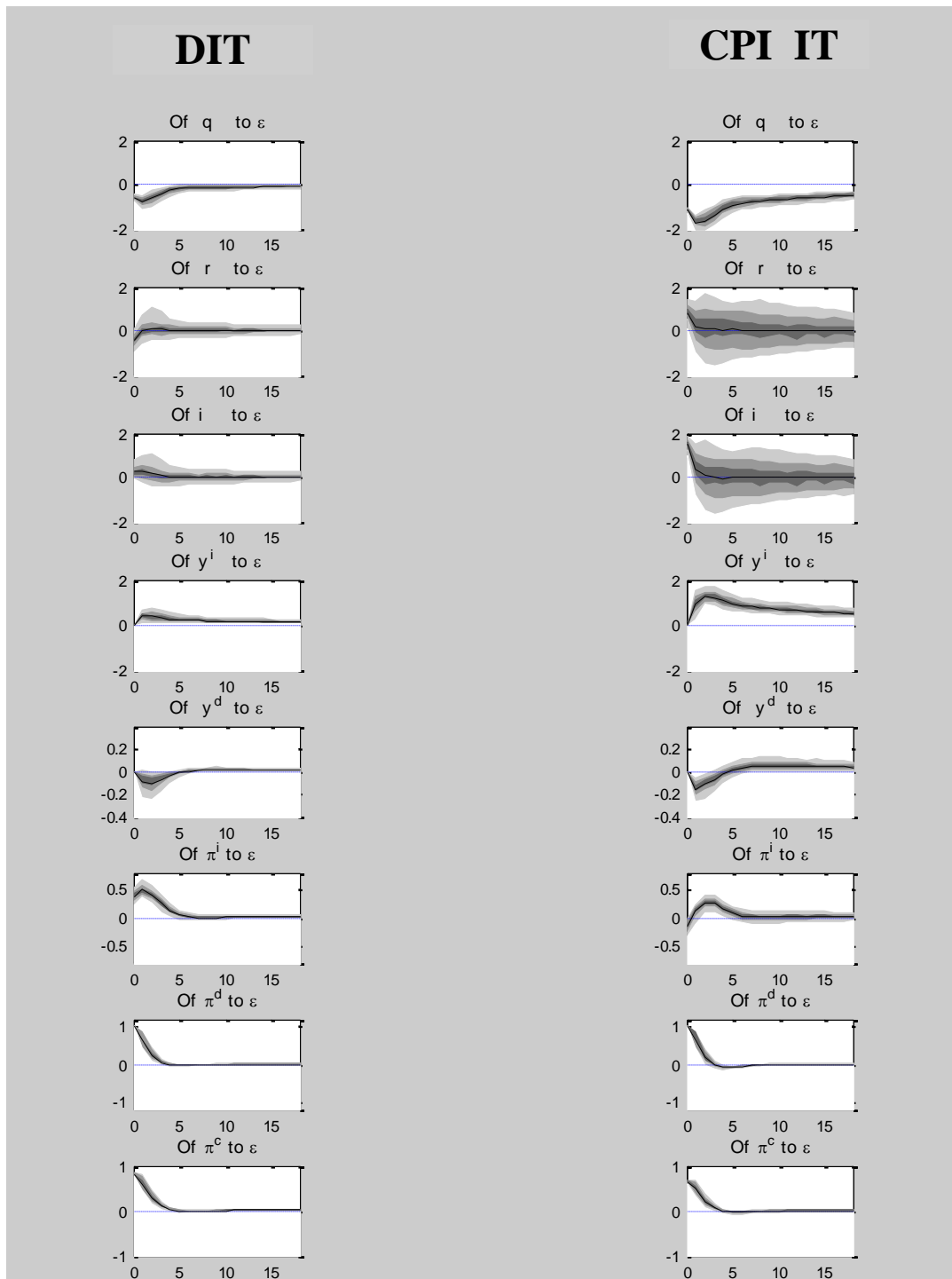
Shock	$\pi^c$	$\pi^d$	$y^d$	i	r	q
Cost-push	0.98	1.01	1.18	1.39	1.03	1.27
Demand	1.05	1.06	0.97	1.01	1.06	0.98
Foreign interest rate	0.82	4.26	2.70	6.92	4.26	6.72
Natural output	1.23	1.32	0.98	0.99	1.34	0.92
Risk premium	0.71	3.00	1.07	4.46	3.00	3.81
Foreign output	0.65	3.71	0.74	1.05	3.74	1.54

**Table 6b.**  $R^M$  : Ratio of the means under DIT and CPI IT of the medians of the distribution forecasts for interest rate smoothing preferences  $\nu \in \{0.002, 0.005, \dots, 0.04\}$  and periods  $t \in \{0, 1, \dots, 15\}$ . Uncertainty case: no model uncertainty. Second calibration.

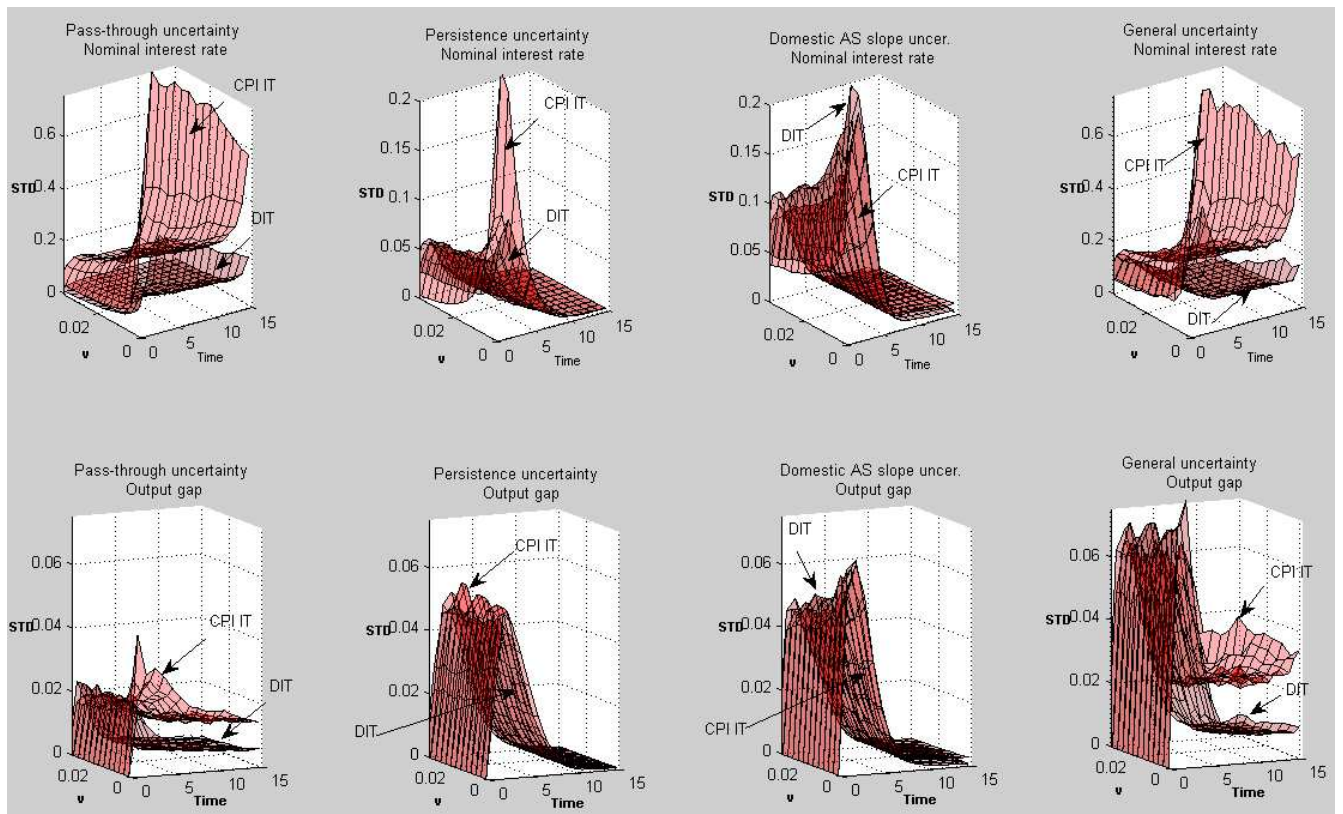




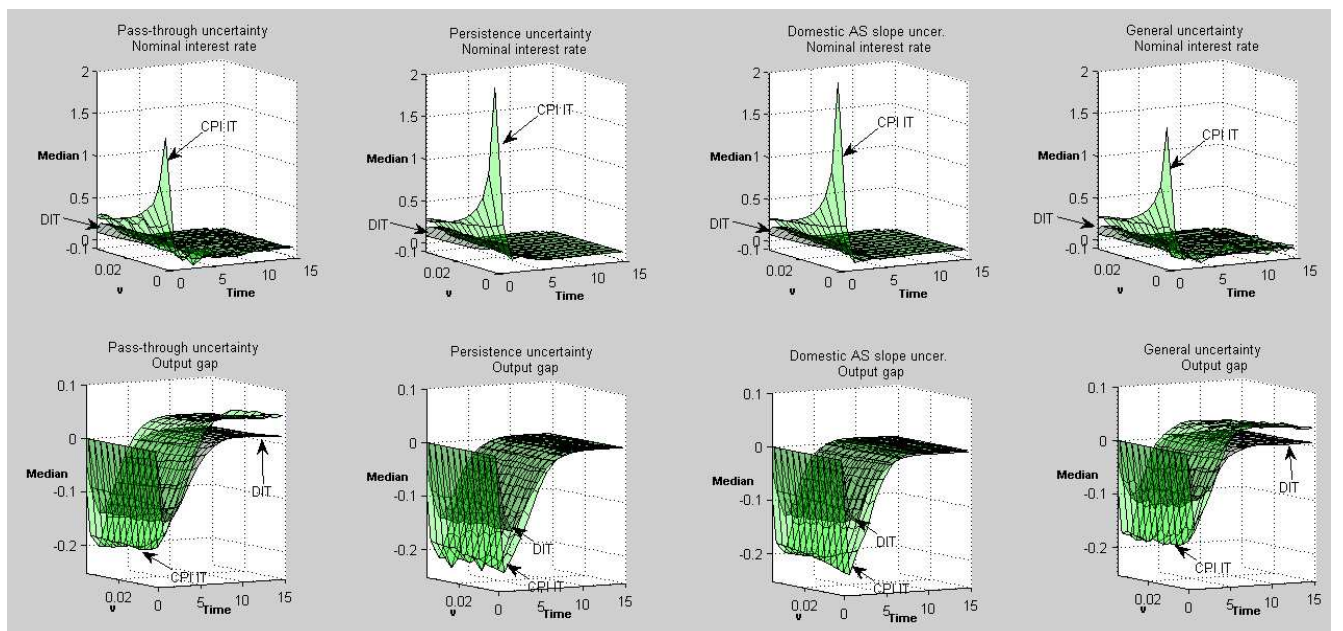
**Figure 1:** Unconditional distribution forecasts of the impulse responses to a cost-push shock in the general uncertainty case and for high smoothing preferences, i.e.  $\nu = 0.05$ . First and second column report, respectively, the distribution forecasts under the DIT and CPI IT policies. Solid lines: Mean responses. Dark/medium/light grey bands: 30/60/90% probability bands. First calibration.



**Figure 2:** Unconditional distribution forecasts of the impulse responses to a cost-push shock in the general uncertainty case and for low smoothing preferences, i.e.  $\nu = 0.002$ . First and second column report, respectively, the distribution forecasts under the DIT and CPI IT policies. Solid lines: Mean responses. Dark/medium/light grey bands: 30/60/90% probability bands. First calibration.



**Figure 3.** STD of the impulse response distribution to a cost-push shock under DIT and CPI IT for  $v \in \{0.002, 0.005, \dots, 0.04\}$  and  $t \in \{0, 1, \dots, 15\}$ . Variables:  $i$  and  $y^d$ , first and second row respectively. Uncertainty cases: pass-through, persistence in the behaviour of the private sector, slope of the domestic AS, and general, first, second, third and fourth column respectively. First calibration.



**Figure 4.** Median of the impulse response distribution to a cost-push shock under DIT and CPI IT for  $v \in \{0.002, 0.005, \dots, 0.04\}$  and  $t \in \{0, 1, \dots, 15\}$ . Variables:  $i$  and  $y^d$ , first and second row respectively. Uncertainty cases: pass-through, persistence in the behaviour of the private sector, slope of the domestic AS, and general, first, second, third and fourth column, respectively. First calibration.

# Central Bank Preferences, Distribution Forecasts and Economic Stability in a Small Open-Economy

## Appendix: State-space form

In order to apply the Svensson and Williams (2007) approach to solve the central bank optimization problem and compute the distribution forecasts, it is necessary to write the model in state-space form. Accordingly, first note that

$$\pi_{t+2|t+1}^d = \pi_{t+2|t}^d + \frac{\zeta_j}{1 + \zeta_j} \varepsilon_{t+1},$$

$$\rho_{t+1|t} = \rho_t - i_t + \pi_{t+1|t}^d, \quad (33)$$

$$q_{t+1|t} = q_t + \pi_{t+1|t}^i - \pi_{t+1|t}^d. \quad (34)$$

Then, take the expectation in period  $t$  of equation (21) and solve it for  $\pi_{t+3|t}^d$ ,

$$\pi_{t+3|t}^d = (1 + \zeta_j) \pi_{t+2|t}^d - \zeta_j \pi_{t+1|t}^d - \xi_j [\omega y_{t+2|t}^d + \mu q_{t+2|t}]. \quad (35)$$

where  $\xi_j \equiv \frac{(1-\alpha_j)^2}{\alpha_j(1+\omega\vartheta)}$ . Next, lead equation (16) one period and take the expectation in period  $t$ . Then apply the same procedure to (33), (34) and (23) and substitute for  $\rho_{t+2|t}$ ,  $q_{t+2|t}$  and  $\pi_{t+2|t}^i$  in the equation for  $y_{t+2|t}^d$ . This gives

$$\begin{aligned} y_{t+2|t}^d &= \beta_{yj} y_{t+1|t}^d - \beta_\rho \rho_{t+1|t} + \beta_\rho i_{t+1|t} - (\beta_\rho + \beta_q) \pi_{t+2|t}^d + (\beta_q - \beta_{q-1,j}) q_t \\ &\quad + [\beta_q (2 + \zeta_j) - \beta_{q-1,j}] \pi_{t+1|t}^i - (\beta_q - \beta_{q-1,j}) \pi_{t+1|t}^d - \beta_q \zeta_j \pi_t^i - \beta_q \xi_j^i \omega y_{t+1|t}^i \\ &\quad - \beta_q \xi_j^i q_{t+1|t}^i + \beta_{y^*j} y_{t+1|t}^* + \beta_{y^n j} y_{t+1|t}^{d,n}, \end{aligned}$$

where  $\xi_j^i \equiv \frac{(1-\alpha_j^i)^2}{\alpha_j^i(1+\omega\vartheta)}$ . Finally, substitute for  $y_{t+2|t}^d$  and  $q_{t+2|t}$  in (35). This gives

$$\begin{aligned} \pi_{t+3|t}^d &= [1 + \zeta_j + \xi_j \omega (\beta_\rho + \beta_q) + \xi_j \mu] \pi_{t+2|t}^d + [\xi_j \mu - \zeta_j + \xi_j \omega (\beta_q - \beta_{q-1,j})] \pi_{t+1|t}^d \\ &\quad + (\xi_j \omega \beta_q \xi_j^i + \xi_j \mu \xi_j^i) q_{t+1|t}^i - \xi_j [\omega (\beta_q (2 + \zeta_j) - \beta_{q-1,j}) + \mu (1 + \zeta_j) + \mu] \pi_{t+1|t}^i \\ &\quad - \xi_j \omega \beta_{yj} y_{t+1|t}^d + \xi_j \omega \beta_\rho \rho_{t+1|t} - \xi_j \omega \beta_\rho i_{t+1|t} + \xi_j \zeta_j (\omega \beta_q + \mu) \pi_t^i \\ &\quad + \omega \xi_j \xi_j^i (\omega \beta_q + \mu) y_{t+1|t}^i - \omega \xi_j \beta_{y^*j} y_{t+1|t}^* - \omega \xi_j \beta_{y^n j} y_{t+1|t}^{d,n} - \xi_j [\omega (\beta_q - \beta_{q-1,j}) + \mu] q_t. \end{aligned}$$

It follows that under the Svensson (2003) calibration we have

$$A = \begin{bmatrix} e_2 \\ e_n \\ e_{nX+1} \\ \gamma_\pi^* e_4 \\ \beta_{yj} e_5 - \beta_\rho A_{nX+3} + \beta_q (A_{12} + A_{nX+1} - A_1) - \beta_{q-1,j} A_{12} + \beta_{y^*j} e_7 + \beta_{y^n j} e_9 \\ \beta_{yj} e_6 - \beta_\rho^i A_{nX+3} + \beta_q^i (A_{12} + A_{nX+1} - A_1) + \beta_{q-1,j}^i A_{12} + \beta_{y^*j}^i e_7 + \beta_{y^n j}^i e_{10} \\ \gamma_y^* e_7 \\ f_\pi^* \gamma_\pi^* e_4 + f_y^* \gamma_y^* e_7 \\ \gamma_y^{d,n} e_9 \\ \gamma_y^{i,n} e_{10} \\ e_0 \\ e_{12} + A_3 - e_1 \\ e_{nX+2} \\ \gamma_v e_{nX} \\ -\xi_j^i (\omega e_6 + e_{nX+2}) - \zeta_j e_3 + (1 + \zeta_j) e_{nX+1} \\ e_{nX+2} + \mu_j^i e_2 - A_{nX+1} - (1 - \mu_j^i) (e_8 - A_4) - (1 - \mu_j^i) A_{nX} \\ e_{nX+3} + e_2 \\ A_n \end{bmatrix},$$

where  $e_i$ ,  $i = 0, \dots, n$  stands for a  $1 \times n$  row vector that for  $i = 0$  has all the elements equal to zero and for  $i \neq 0$  has element  $i$  equal to unity and all other elements equal to zero;  $A_i$  stands for row  $i$  of the matrix  $A$ ,  $nX = 14$  is the number of predetermined variables,  $nx = 4$  is the number of non-predetermined variables,  $n = nX + nx$ , and

$$\begin{aligned} A_n &= [1 + \zeta_j + \xi_j \omega (\beta_\rho + \beta_q) + \xi_j \mu] e_n + [\xi_j \mu - \zeta_j + \xi_j \omega (\beta_q - \beta_{q-1,j})] A_1 \\ &+ (\xi_j \omega \beta_q \xi_j^i + \xi_j \mu \xi_j^i) A_{nX+2} - \xi_j [\omega (\beta_q (2 + \zeta_j) - \beta_{q-1,j}) + \mu (1 + \zeta_j) + \mu] A_{nX+1} \\ &- \xi_j \omega \beta_y A_5 + \xi_j \omega \beta_\rho A_{nX+3} + \xi_j \zeta_j (\omega \beta_q + \mu) e_{nX+1} \\ &+ \omega \xi_j \xi_j^i (\omega \beta_q + \mu) A_6 - \omega \xi_j \beta_{y^*j} A_7 - \omega \xi_j \beta_{y^n j} A_9 - \xi_j [\omega (\beta_q - \beta_{q-1,j}) + \mu] A_{12}. \end{aligned}$$

Finally the vectors  $B$  and  $B^1$  are given by

$$\begin{aligned} B &= e'_5 \beta_\rho + e'_6 \beta_\rho^i + e'_{11} + e_{nX+2} (1 - \mu_j^i) - e'_{nX+3} \\ &+ e'_n [-\xi_j \omega \beta_\rho (1 + \beta_{yj}) + \xi_j \xi_j^i (\omega \beta_q + \mu) (1 - \mu_j^i) + \xi_j \xi_j^i \omega (\beta_q \omega + \mu) \beta_\rho^i] \end{aligned}$$

$$B^1 = -e'_n \xi_j \omega \beta_\rho.$$

Under the Leitemo and Söderström (2005) calibration  $nX = 16$ , the vector of predetermined variable is

$$X_t = \left( \pi_t^d, \pi_{t+1|t}^d, \pi_{t-1}^i, \pi_t^*, y_t^d, y_t^i, y_t^*, i_t^*, y_t^{d,n}, y_t^{i,n}, i_{t-1}, q_{t-1}, q_{t-1}^i, \varepsilon_t, \eta_t, v_t \right)',$$

and

$$A_1 = e_2 + A_{nX-2}$$

$$A_5 = \beta_{yj}e_5 - \beta_\rho A_{nX+3} + \beta_q (A_{12} + A_{nX+1} - A_1) - \beta_{q-1,j}A_{12} + \beta_{y^*j}e_7 + \beta_{y^n j}e_9 + A_{nX-1}$$

$$A_6 = \beta_{yj}e_6 - \beta_\rho^i A_{nX+3} + \beta_q^i (A_{12} + A_{nX+1} - A_1) + \beta_{q-1,j}^i A_{12} + \beta_{y^*j}^i e_7 + \beta_{y^n j}^i e_{10} + A_{nX-1}$$

$$A_{nX-2} = \gamma_\pi e_{nX-2}$$

$$A_{nX-1} = \gamma_y e_{nX-1}$$

At this point the Svensson and Williams (2007) numerical methods to find the distribution forecasts are implemented in Matlab.