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# Nominal Curvature Design of Circular HSC Columns Confined with Post-Tensioned Steel Straps

**Chau Khun Ma<sup>a\*</sup>, Abdullah Zawawi Awang<sup>a</sup>, Reyes Garcia<sup>b</sup>, Wahid Omar<sup>a</sup>, Kypros Pilakoutas<sup>b</sup>**

<sup>a</sup> Department of Structures and Materials, Faculty of Civil Engineering, Universiti Teknologi Malaysia, Skudai, Johor, 81310, Malaysia, Tel.: +6016-8366360.

<sup>b</sup> Department of Civil and Structural Engineering, The University of Sheffield, Sir Frederick Mappin Building, Mappin Street, Sheffield, S1 3JD, UK.

Email (corresponding author\*): [machaukhun@gmail.com](mailto:machaukhun@gmail.com)

## Abstract

This article proposes new parameters for the practical design of circular high-strength concrete (HSC) columns confined with an innovative Steel Strapping Tensioning Technique (SSTT) using a nominal curvature approach. Previous experimental research has proven the effectiveness of the SSTT at providing active confinement and enhancing the ductility of HSC columns, but to date no practical procedures are available so that the technique can be widely adopted in design practice. The proposed design approach is based on results from segmental analyses of slender SSTT-confined circular columns subjected to eccentric loads. The results obtained from the analyses are used to determine the variables governing the design of such columns. The use of the proposed design parameters predicts conservatively the capacity of small-scale slender HSC circular columns confined using the SSTT, and can be thus used in the practical design of reinforced concrete (RC) structures.

**Keywords:** High-Strength Concrete; Nominal Curvature; RC Columns; Confinement; Steel Straps

# 1 Introduction

Current design guidelines for reinforced concrete (RC) structures promote the use of strong and yet ductile columns able of sustaining large deformations without failure. Many of such columns have small cross sections (compared to the column height) and end restraints that do not prevent column sway. These “slender columns” tend to deform laterally when axial load and flexural moment (first order effects) are applied, thus subjecting the columns to additional (second order) moments. Hence, at large lateral deformations, slender columns can experience global buckling and can fail at lower loads compared to those sustained by short columns. To account for the additional second order effects in design, current design codes [e.g. the Chinese Code GB-50010 (2002); Eurocode 2 (2004)] calculate the total column eccentricity as the sum of the nominal eccentricity (from first order effects) and the additional eccentricity due to slenderness (second order). When such slenderness effects are accounted for in design, larger column sections are normally required to resist the moment which in turn increase the construction costs.

In an attempt to enhance the stiffness and capacity of slender columns (thus reducing sway and second order effects), high-strength concrete (HSC) is extensively utilised nowadays in the design and construction of new RC buildings, particularly in Southeast Asia. While the use of HSC is effective at enhancing the columns’ capacity, previous experimental studies (Shin et al. 1989; Li et al. 1994) also showed that HSC columns can fail in a brittle manner, and thus adequate confinement is required to ensure their ductile behaviour. Internal steel ties/stirrups were used in the past to enhance the ductility of HSC columns (e.g. Canbay et al. 2006, Ho et al. 2010). Nonetheless, the effectiveness of internal stirrups is limited as only the core the columns’ cross section is effectively confined, especially after cover spalling. To increase the effectiveness of the confinement and to prevent spalling, externally bonded Fibre Reinforced Polymer (FRP) wraps were recently proposed as a confining solution for HSC elements (e.g. Idris and Ozbakkaloglu 2013; Lim and Ozbakkaloglu 2014). However, both internal reinforcement and FRP wraps can only provide passive confinement to structural elements. The high initial cost of FRP materials may also discourage their use as confinement solution in low and medium income developing countries. As a result, it is deem necessary to develop more cost-effective confining solutions for HSC columns.

Recently, Ma et al. (2014a) and Lee et al. (2014) investigated experimentally the effectiveness of an innovative active confining technique (referred here as Steel Strapping Tensioning Technique or SSTT) at enhancing the capacity of HSC columns. The SSTT involves the post-tensioning of high-strength high-ductility steel straps around RC elements using air-operated strapping tools similar to those utilised in the packaging industry. After the post-tensioning operation, self-regulated end clips clamp the steel straps and maintain the tensioning force. Contrary to internal stirrups or external FRP wraps, the SSTT provides active confinement to the full cross section of members. The SSTT also has additional advantages such as ease and speed of application, ease of removing or replacing steel straps, and low material and labour costs. Whilst previous tests showed that the use of the SSTT can enhance the deformation capacity of HSC columns by minimum 50% (Ma et al. 2014a; 2014b), no practical design guidelines exist for new SSTT-confined HSC columns so that the confining technique can be widely adopted in practice.

This article proposes new parameters for the practical design of circular high-strength concrete (HSC) columns confined with an innovative Steel Strapping Tensioning Technique (SSTT) using a nominal curvature approach. To achieve this, the proposed design approach uses results from segmental analyses of slender SSTT-confined columns subjected to eccentric loads. The results obtained from the analyses are used to determine the basic parameters that govern the design of such columns. The proposed nominal curvature approach is validated using experimental results available in the literature. The practical design approach proposed in this study is expected to contribute towards a wider use of the cost-effective SSTT in new HSC structures by providing guidelines that can reduce considerably the computational time during design.

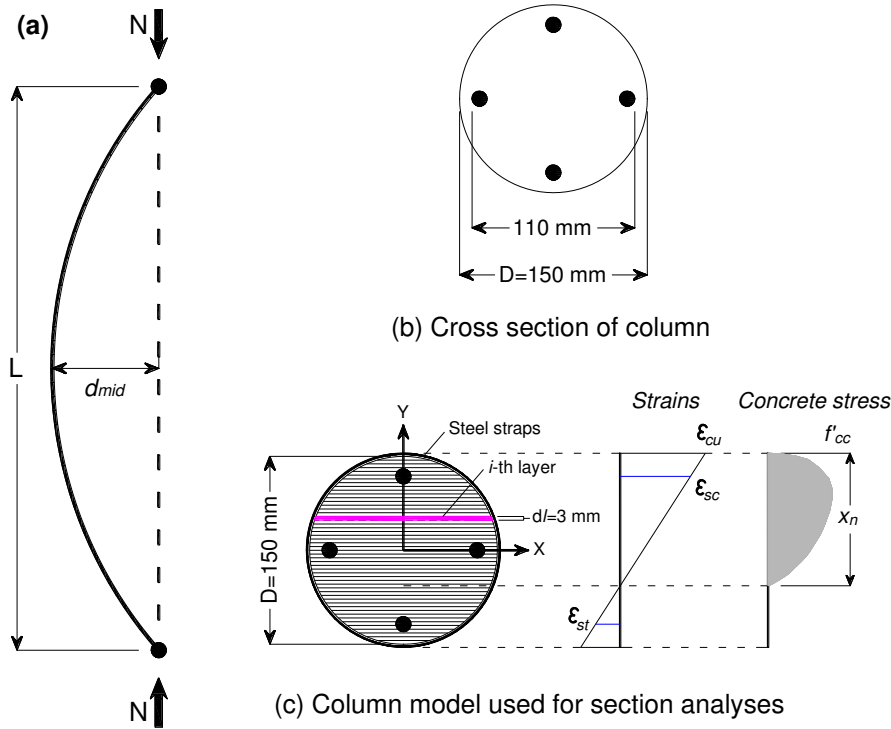
## 2 Analysis of SSTT-confined columns

### 2.1 General deflection of columns

The deflection  $\delta$  of a hinged column can be mathematically approximated using a half-sine wave as (see Fig. 1a):

$$\delta = -\delta_{mid} \text{Sin}\left(\frac{\pi}{L}x\right) \quad (1)$$

where  $\delta_{mid}$  is the lateral displacement at the column mid-height;  $x$  is the distance along the column axis from the first column end; and  $L$  is the column length (height).



**Fig. 1.** (a) Column model with hinged ends, (b)-(c) theoretical segmented column model used for calculation of moment-curvature relationships

The column curvature  $\phi$  at a height  $x$  can be obtained by differentiating Equation (1) twice:

$$\phi = \delta_{mid} \frac{\pi^2}{L^2} \sin\left(\frac{\pi}{L} x\right) \quad (2)$$

By replacing Equation (2) into (1),  $\delta_{mid}$  and  $\phi_{mid}$  are defined by:

$$\delta_{mid} = \frac{L^2}{\pi^2} \phi_{mid} \quad (3)$$

The flexural moment  $M_{mid}$  at the critical mid-height section of a column can be then approximated as:

$$M_{mid} = N(e + \delta_{mid}) = N\left(e + \frac{L^2}{\pi^2} \phi_{mid}\right) \quad (4)$$

where  $N$  is the axial load on the column; and  $e$  is the load eccentricity at column ends. The moment computed using the latter equation leads to stresses at the column's mid-height section. Such moment and corresponding stresses can be determined using conventional section analysis, provided the constitutive models of concrete and steel reinforcement are known.

## 2.2 Constitutive models for SSTT-confined HSC and reinforcement

To account for the effect of the active confinement provided by the steel straps to HSC, the constitutive relationship proposed by Awang et al. (2013) is used in the analyses. The model is based on the equations proposed originally by Popovics (1973). However, Awang et al. (2013) calibrated the model using an extensive experimental database of uniaxial compressive tests (140 concrete cylinder specimens) and extended its applicability to HSC columns confined with the SSTT. According to Popovics, the concrete stress  $f_{ci}$  at a given strain  $\varepsilon_{ci}$  is defined by:

$$f_{ci} = \frac{f'_{cc} x^r}{r-1+x^r} \quad (5)$$

where  $x = \varepsilon_{ci} / \varepsilon'_{cc}$ ;  $\varepsilon_{cc}$  is the axial compressive strain of concrete;  $\varepsilon'_{cc}$  and  $f'_{cc}$  are the strain and concrete strength of confined concrete, respectively; whereas  $f_{co}$  is the unconfined concrete compressive strength. In the above equation,  $r = E_c / (E_c - E'_{sec})$ , where  $E_c$  is the tangent modulus of elasticity of concrete and  $E'_{sec}$  is the secant modulus of elasticity of the confined concrete at peak stress. For the analyses carried out in this study, these values are assumed to be  $E_c = 4700 \sqrt{f'_{cc}}$  (MPa) and  $E'_{sec} = f'_{cc} / \varepsilon'_{cc}$  (MPa).

According to Awang's empirical model, the values  $f'_{cc}$  and  $\varepsilon'_{cc}$  of SSTT-confined concrete can be calculated as:

$$f'_{cc} = f_{co} \cdot 2.62(\rho_v)^{0.4} \quad (6)$$

$$\varepsilon'_{cc} = \varepsilon_{co} \cdot 11.60(\rho_v) \quad (7)$$

$$\varepsilon'_{cu} = \varepsilon_{co} \cdot (8.9\rho_v + 0.51) \quad (8)$$

where  $\rho_v$  is the volumetric confinement ratio of steel straps ( $\rho_v = V_s f_y / V_c f_{co}$ , where  $V_s$  and  $V_c$  are the volumes of straps and confined concrete, respectively, and  $f_y$  is the yield strength of the straps);  $\varepsilon_{co}$  is the ultimate strain of unconfined HSC (assumed equal to 0.004); and  $\varepsilon'_{cu}$  is the ultimate strain of confined HSC. The constitutive relationship defined by the above equations was calibrated using data from 100×200 mm HSC cylinders confined with SSTT confinement ratios ranging from 0.076 to 1.50. Note also that the above equations assume that a minimum amount of straps (at least  $\rho_v = 0.076$ ) always exist around the element.

A simplified bilinear tensile stress-strain ( $f_s$ - $\varepsilon_s$ ) model is adopted to model the behaviour of the longitudinal column reinforcement:

$$f_s = \varepsilon_s E_s \quad \text{for } 0 \leq \varepsilon_s \leq \varepsilon_y \quad (9)$$

$$f_s = f_y \quad \text{for } \varepsilon_s > \varepsilon_y \quad (10)$$

where  $E_s$  is the elastic modulus of the steel; and  $\varepsilon_s$  and  $\varepsilon_y$  are the corresponding strain and yield strain.

### 2.3 Moment-curvature relationships

The moment-curvature relationship of a SSTT-confined HSC column can be determined using the material properties described in the previous section and a given cross section geometry. Hence, a circular column with a cross section diameter  $D=150$  mm cast with theoretical HSC of  $f_{co}=60$  MPa is assumed for the analyses carried out in this article (see Fig. 1b). A yield strength  $f_y=460$  MPa and an elastic modulus  $E_s=200$  GPa are assumed for the longitudinal column reinforcement, which is assumed as concentrated at the locations shown in Fig. 1b. A free concrete cover of 20 mm is also assumed in the analyses.

A cross section of  $0.5 \times 15$  mm and an elastic modulus of 200 GPa are assumed for each confining steel strap, which correspond to typical properties of commercial packaging straps used in Southeast Asia. To achieve a desired confined concrete strength using the SSTT, the strap spacing, number of strap layers, yield strength of the straps and concrete strength can be varied to change the volumetric ratio  $\rho_v$ . Nonetheless, Ma et al. (2014a) suggested values of  $\rho_v$  ranging from 0.09 and 0.50 for practical confining applications. Although values  $\rho_v > 0.50$  can be theoretically achieved, the number of straps around elements is restricted by several practical aspects such as a) the clear spacing between straps necessary to secure the metal clips using the jaws (1-2 mm), b) the number of strap layers than can be secured using a single clip (maximum two layers), and c) the yield strength of the steel straps. As a consequence, SSTT volumetric ratios  $\rho_v=0.09, 0.25$  and  $0.50$  are used in this study to assess the effect of very light, moderate and relatively heavy confinement that can be applied in practice.

Conventional section analyses are carried out using the stress-strain model for SSTT-confined HSC section (described in the previous section) and theoretical column model shown in Fig. 1b-c. The axial load,  $N_{step}$  and flexural moment,  $M_{step}$  at successively incremental loading steps are given by:

$$N_{step} = \sum_{i=0}^n f_{ci} y_i dl + (\sigma_{si} - f_{ci}) A_{si} \quad (11)$$

$$M_{step} = \sum_{i=0}^n (f_{ci} y_i dl) P_i + (\sigma_{si} - f_{ci})(R - d_{si}) A_{si} \quad (12)$$

where  $y_i$  is the width of  $i$ -th layer;  $dl$  is the thickness of each layer ( $dl=3$  mm according to Fig. 1c);  $\sigma_{si}$  is the stress of the longitudinal column bar at the  $i$ -th layer;  $A_{si}$  is the corresponding cross-sectional area of the longitudinal column bar;  $P_i$  is the distance from the centre point of the  $i$ -th layer to the neutral axis;  $R$  is the column radius ( $R=D/2$ ); and  $d_{si}$  is the distance between longitudinal tensile bars and the extreme concrete fibre. As such, Equations (11) and (12) are used in this article to generate the moment-curvature relationships of the SSTT-confined columns analysed in subsequent sections of this article.

### 3 Nominal curvature approach

#### 3.1 Background

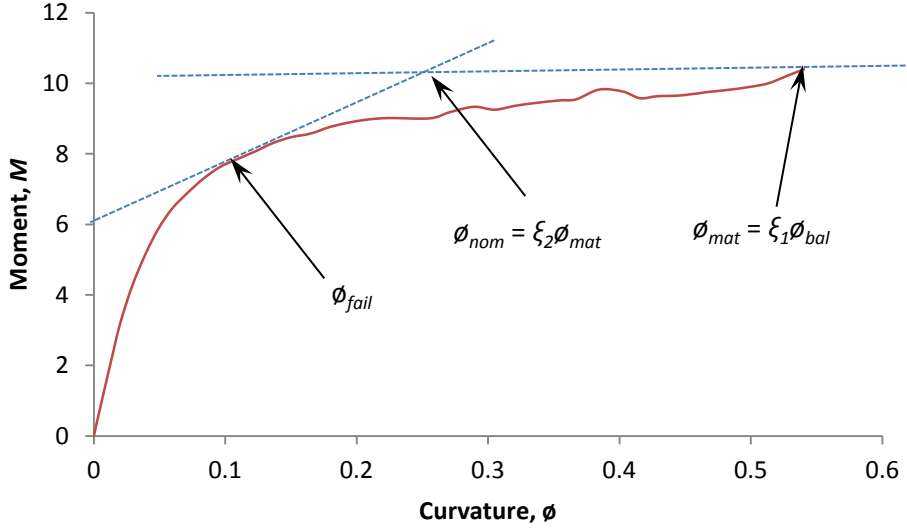
Existing design standards (e.g. BS-8110, 1997; GB-50010, 2002; CEN, 2004) simplify the design procedure of slender RC columns by amplifying the first-order flexural moment to calculate the second-order moment. Hence, the design is similar to that of short columns, but the second-order deflection is accounted for by assuming that the total eccentricity is the sum of the applied eccentricity and the additional eccentricity due to the column slenderness. From Equation (3), the nominal eccentricity  $\delta_{nom}$  (i.e. column lateral displacement) can be defined as:

$$\delta_{nom} = \frac{L^2}{\pi^2} \phi_{nom} \quad (13)$$

where  $\phi_{nom}$  is the nominal curvature at the column mid-height.

According to Jiang and Teng (2013), the curvatures  $\phi_{nom}$ ,  $\phi_{fail}$ , and  $\phi_{mat}$  (shown in Fig. 2) need to be determined to assess the nominal curvature of a column that experiences stability failure under a constant axial load  $N$ . In Fig. 2,  $\phi_{fail}$  is the curvature of the critical section at column failure, whereas  $\phi_{mat}$  is the maximum curvature that the critical section of the column can resist when subjected to  $N$ .





**Fig. 2.** Definition of the nominal curvature of a column

The value  $\phi_{nom}$  can be calculated using the curvature at balanced failure  $\phi_{bal}$ . Using strain compatibility, the curvature at balanced failure can be defined as:

$$\phi_{bal} = \frac{\varepsilon_{cu}}{x_n} = \frac{\varepsilon_{cu} + \varepsilon_y}{d} \quad (14)$$

where  $x_n$  is the neutral axis depth of the column section;  $\varepsilon_y$  is the reinforcement strain at yield;  $d$  is the effective column depth; and the rest of the terms are as defined before.

The axial load corresponding to a balanced failure is defined by  $N_{bal}$ . However, failures can occur at other axial load levels and therefore a factor  $\zeta_1$  is used in practical design to define the point at material failure (see Fig. 2). Likewise, a factor  $\zeta_2$  accounts for the difference between  $\phi_{nom}$  and  $\phi_{mat}$  as shown in Fig. 2. Hence, the following equation relates the curvatures  $\phi_{nom}$  and  $\phi_{bal}$ :

$$\phi_{nom} = \zeta_1 \zeta_2 \phi_{bal} \quad (15)$$

Therefore, the values  $N_{bal}$ ,  $\zeta_1$  and  $\zeta_2$  have to be determined to calculate the nominal curvature of a slender column.

### 3.2 Calculation of $N_{bal}$ for slender SSTT-confined columns

Previous research (e.g. Cheng et al. 2002; Jiang 2008) has shown that the level of confinement influences the magnitude of the balanced load of a column. This is particularly true for SSTT-confined columns as the magnitude of the active confinement applied to the column can be pre-selected during the design

stage. To develop an expression for calculating  $N_{bal}$  in SSTT-confined columns, a parametric analytical study is conducted considering the following variables, which consider typical values used in the practical design of circular HSC columns of buildings in Southeast Asia:

- Column longitudinal reinforcement ratios  $\rho_s=0.02, 0.03$  and  $0.04$
- Cover depth ratios  $d/D=0.75, 0.80$  and  $0.85$
- SSTT-confinement ratios  $\rho_v=0.09, 0.25$  and  $0.50$

Table 1 summarises the parameters used in the parametric study and the corresponding  $N_{bal}$  calculated using section analyses according to the procedure described in Section 2.3. In all analyses shown in Table 1, the steel yield strain was equal to 0.0023. The results indicate that  $\rho_v$  is the main parameter that affects the values  $N_{bal}$ , whereas the parameters  $\rho_s$  and  $d/D$  have a marginal effect on  $N_{bal}$ .

**Table 1.** Parameters considered for predicting the balanced failure of SSTT-confined HSC circular section

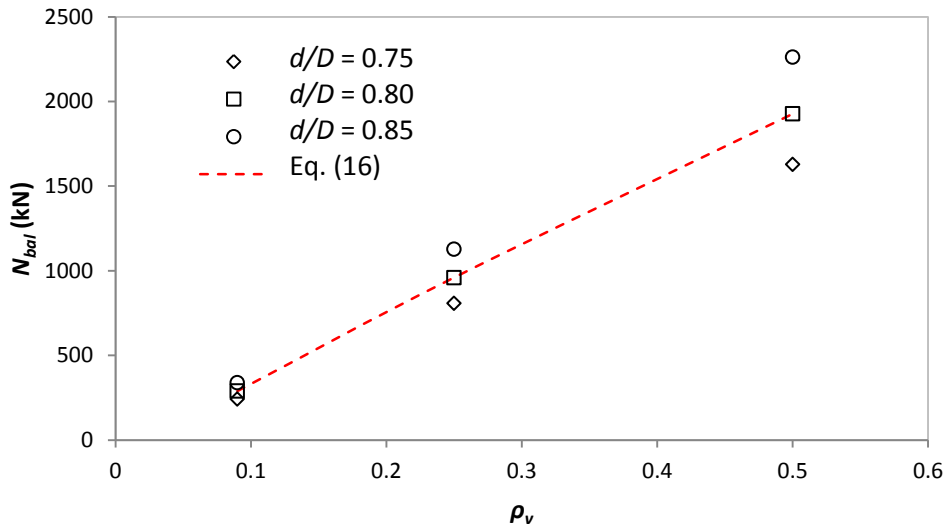
Steel ratio, $\rho_s$	Cover depth ratio, $d/D$	SSTT-confinement ratio, $\rho_v$	Concrete strain, $\varepsilon_{cc}$	Balanced curvature, $\phi_{bal}$ ( $\text{mm}^{-1}$ )	Neutral axis depth, $x_n$ (mm)	Balanced load, $N_{bal}$ (kN)
<b>0.02</b>	0.75	0.09 <sup>a</sup>	0.00520	0.0000682	76.3	242.9
		0.25	0.01094	0.0001204	90.9	806.9
		0.50	0.01984	0.0002013	98.6	1628.8
	0.80	0.09 <sup>a</sup>	0.00520	0.0000625	83.2	289.0
		0.25	0.01094	0.0001103	99.1	959.7
		0.50	0.01984	0.0001845	107.5	1926.6
	0.85	0.09 <sup>a</sup>	0.00520	0.0000577	90.1	338.7
		0.25	0.01094	0.0001018	107.4	1126.6
		0.50	0.01984	0.0001703	116.5	2261.7
<b>0.03</b>	0.75	0.09 <sup>a</sup>	0.00520	0.0000682	76.3	242.9
		0.25	0.01094	0.0001204	90.9	806.9
		0.50	0.01984	0.0002013	98.6	1628.8
	0.80	0.09 <sup>a</sup>	0.00520	0.0000625	83.2	289.0
		0.25	0.01094	0.0001103	99.1	959.7
		0.50	0.01984	0.0001845	107.5	1926.6
	0.85	0.09 <sup>a</sup>	0.00520	0.0000577	90.1	338.7
		0.25	0.01094	0.0001018	107.4	1126.6
		0.50	0.01984	0.0001703	116.5	2261.7
<b>0.04</b>	0.75	0.09 <sup>a</sup>	0.00520	0.0000682	76.3	242.9
		0.25	0.01094	0.0001204	90.9	806.9
		0.50	0.01984	0.0002013	98.6	1628.8
	0.80	0.09 <sup>a</sup>	0.00520	0.0000625	83.2	289.0
		0.25	0.01094	0.0001103	99.1	959.7
		0.50	0.01984	0.0001845	107.5	1926.6
	0.85	0.09 <sup>a</sup>	0.00520	0.0000577	90.1	338.7
		0.25	0.01094	0.0001018	107.4	1126.6
		0.50	0.01984	0.0001703	116.5	2261.7

<sup>a</sup>  $\rho_v=0.09$  represents “unconfined” concrete (Ma et al, 2014a)

Fig. 3 shows the effect of strap confining ratio  $\rho_v$  on  $N_{bal}$  for a SSTT-confined HSC column sections (as described in section 2.3) with values  $d/D=0.75, 0.80$  and  $0.85$ . The results in Fig. 3 indicate that  $N_{bal}$  increases approximately in a linear manner with  $\rho_v$ . Hence, the following simplified expression is proposed to calculate  $N_{bal}$  of SSTT-confined HSC columns:

$$N_{bal} = (3.8\rho_v + 0.05) f_{cu} A_c \quad (16)$$

where  $f_{cu}$  is the concrete compressive strength of plain concrete; and  $A_c$  is the cross section area of the column. Fig. 3 shows that Equation (16) (dashed line) is sufficiently accurate to assess  $N_{bal}$ .



**Fig. 3.** Effect of SSTS confinement ratio  $\rho_v$  on  $N_{bal}$  for a SSTS-confined HSC circular column

### 3.3 Calculation of factors $\xi_1$ and $\xi_2$ for slender SSTS-confined columns

Eurocode 2 (CEN 2004) proposes the following equation for calculating the factor  $\xi_1$  in the design of slender RC columns:

$$\xi_1 = \frac{N_{uo} - N_u}{N_{uo} - N_{bal}} \leq 1 \quad (17)$$

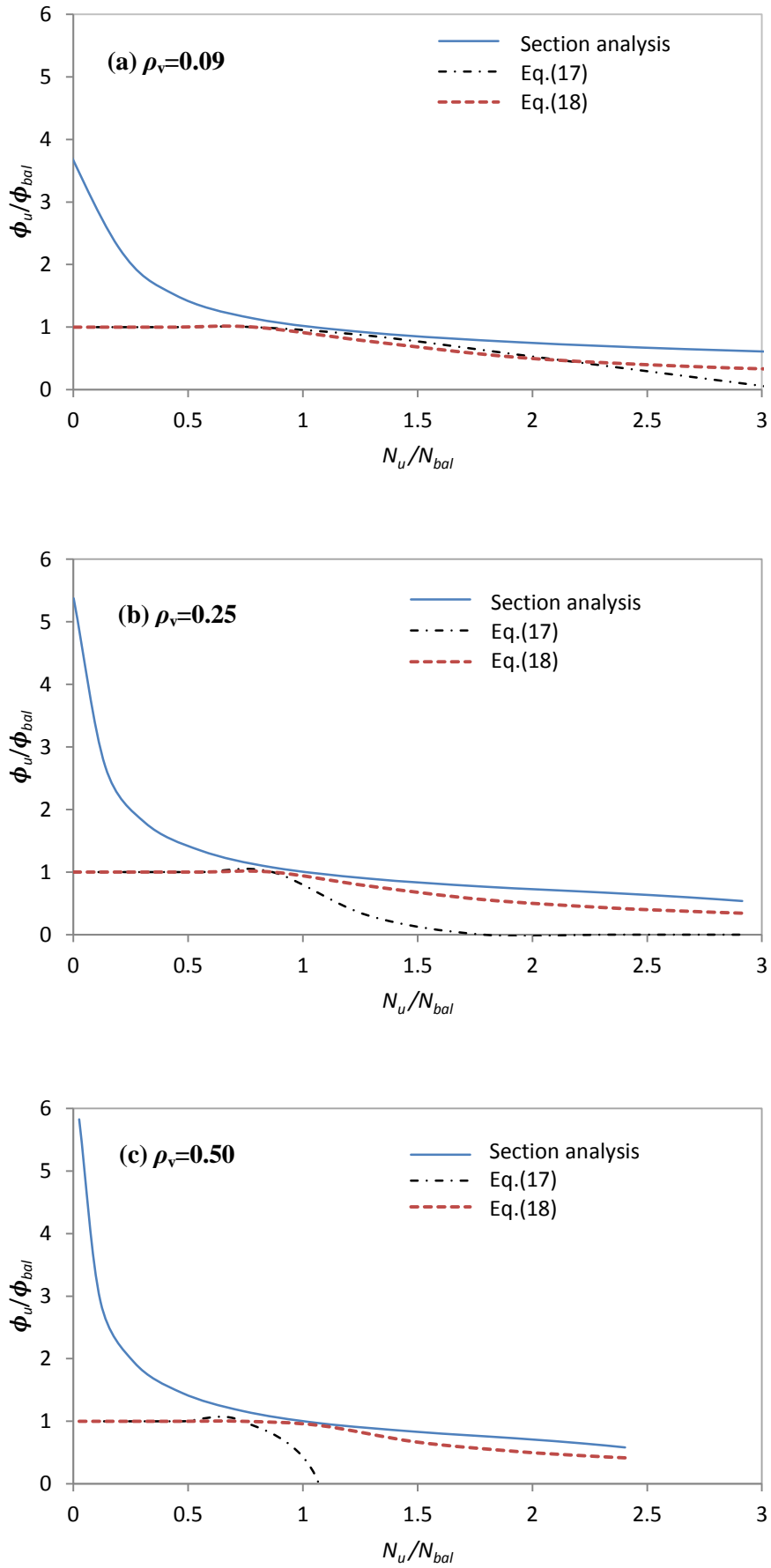
where  $N_{uo}$  is the axial load capacity of an RC section concentrically compressed and the rest of the variables are as defined before.

More recently, Jiang (2008) proposed calculating  $\xi_1$  of FRP-confined RC columns using Equation (18). FRP confinement represents the state-of-the-art in external confinement of RC columns and is therefore considered here for comparison:

$$\xi_1 = \frac{N_{bal}}{N_u} \leq 1 \quad (18)$$

Fig. 4 compares the prediction of Equations (17) and (18) with the “exact” results from segmental analyses for an assumed section with  $\rho_v=0.09$ , 0.25 and 0.50 and constant  $\rho_s=0.02$ . In this figure, the SSTS-confinement ratio  $\rho_v=0.09$  is assumed as “unconfined” concrete according to findings by Ma et al. (2014a), whereas  $\rho_s=0.02$  is the minimum longitudinal reinforcement ratio recommended in Eurocode 2

(CEN, 2004). The load  $N_{\text{bal}}$  used in the calculations was the exact value from the numerical analysis to avoid any discrepancies introduced by approximate equations. Note that the axial loads and curvatures shown in Fig. 4 were normalised by  $N_{\text{bal}}$  and  $\phi_{\text{bal}}$ , respectively.



**Fig. 4.** Variation of factor  $\zeta_1$  as a function of axial load for a STT-confined HSC section ( $\rho_s=0.02$ )

Fig. 4a-c indicate that, as expected, the curvature decreases with the level of axial load. For SSTT-confined HSC columns, the predictions given by Equation (17) differ considerably from the “exact” section analyses results for large ultimate loads or for moderate and relatively high levels of confinement ( $\rho_v=0.25$  and  $0.50$ ). On the other hand, Equation (18) gives more consistent values of normalised curvature ratios for  $N_u/N_{bal}>1$  to  $1.50$ . It is worth mentioning that the accuracy of  $\zeta_1$  in predicting the capacity of slender columns needs to be considered along with  $\zeta_2$ . The inconsistencies observed in Fig. 4a-c when  $N_u/N_{bal}<1$  are due to the slenderness effect. As  $\zeta_1$  is for short columns, only values  $\phi_u/\phi_{bal}<1$  need to be assessed. The values  $\phi_u/\phi_{bal}>1$  correspond to slender columns. Compared to Equation (17), Equation (18) is more accurate in estimating the normalised axial load of the section. Note that  $\zeta_1$  in Fig. 4 is capped to a value of 1 as short columns fail due to material failure, and thus only  $\zeta_1$  needs to be used in design. Conversely, both  $\zeta_1$  and  $\zeta_2$  have to be considered in the design of slender columns. Whilst Jiang (2008) has proven that in all cases  $\zeta_2<1$ , the value  $\zeta_2$  strongly depends on the end eccentricity and the column slenderness. However, when  $\zeta_1$  is limited to 1, the end eccentricity has negligible effect on  $\zeta_2$  and therefore  $\zeta_2$  can be assumed to be a function of slenderness only. For the case of SSTT-confined sections, the effect of the confinement provided by the external steel straps has to be accounted for too. Recently, Jiang and Teng (2013) proposed to design slender FRP-confined circular RC columns using the  $\zeta_2$  factor given by:

$$\zeta_2 = 1.15 + 2.1 (\rho_v^2 - \rho_v) - 0.01 \frac{l}{D} \leq 1 \quad (19)$$

where all variables are as defined before. To simplify the design, the Chinese code GB-50010 (2002) only considers the slenderness ratio to compute  $\zeta_2$ :

$$\zeta_2 = 1.15 - 0.01 \frac{l}{D} \leq 1 \quad (20)$$

Fig. 5 shows the influence of the factor  $\zeta_2$  on the nominal curvature as a function of the slenderness and confinement ratios. It is shown that the nominal curvature is affected by both the slenderness ratio and confinement ratio, which suggests that Equation (19) is more appropriate for SSTT-confined sections. Nonetheless, the effectiveness of Equation (19) in design can only be assessed when combined with  $\zeta_1$  and ultimate load resistance  $N_u$  as shown in the following section.

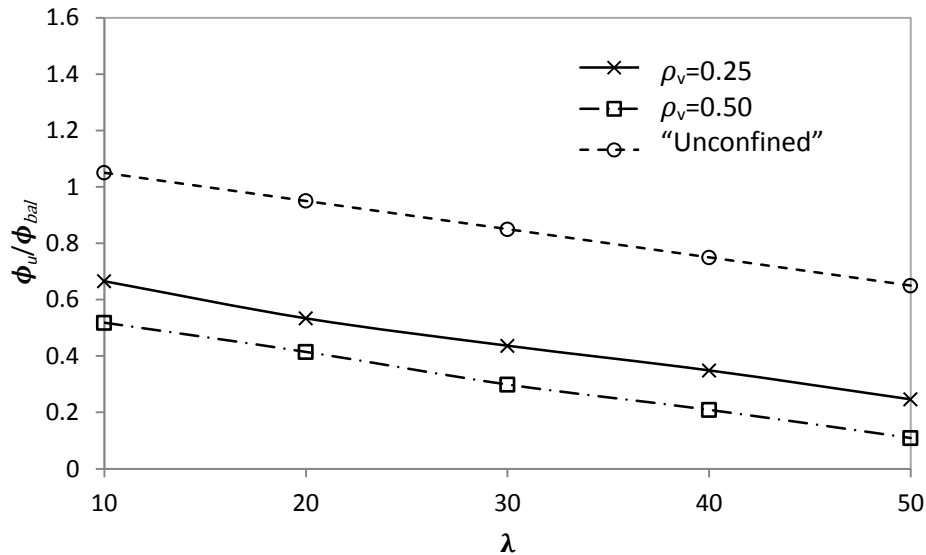


Fig. 5. Variation of factor  $\xi_2$  as a function of slenderness for a SSTT-confined HSC section

## 4 Proposed design procedure

### 4.1 A new equivalent stress block for SSTT-confined sections

Existing design codes for RC members use an ‘equivalent stress block’ with uniform compressive stresses to represent the compressive stress profile of concrete at ultimate condition. Such equivalent stress block is usually defined by the magnitude of stresses and by the depth of the stress block. To maintain force balance, the resulting equivalent stress block and the original stress profile have to resist the same axial force and bending moment. Due to the steel strap confinement, the equivalent stress block proposed by codes is inappropriate to assess the ultimate capacity of SSTT-confined HSC columns. Therefore, a parametric study is carried out to develop an equivalent stress block of SSTT-confined HSC sections. The equivalent stress block is defined by:

1) A mean stress factor ( $\alpha_1$ ), defined as the ratio of the uniform stress over the stress block to the compressive strength of SSTT-confined HSC, and

2) A block depth factor ( $\beta_1$ ), defined as the ratio of the depth of the stress block to that of the neutral axis.

To derive  $\alpha_1$  and  $\beta_1$ , section analyses are performed using the circular column model shown in Fig. 1b-c.

The stress distribution over the compression zone of a circular column is obtained using different neutral axis depths and a SSTT-confinement ratio  $\rho_v=0.50$ . The stress block parameters are determined



simultaneously from the axial load and moment equilibrium conditions so as to match the equations proposed by Warner et al. (2007):

$$N = \alpha_1 \beta_1 f'_{cc} A + \sigma_{sc} A_{sc} - \sigma_{st} A_{st} \quad (21)$$

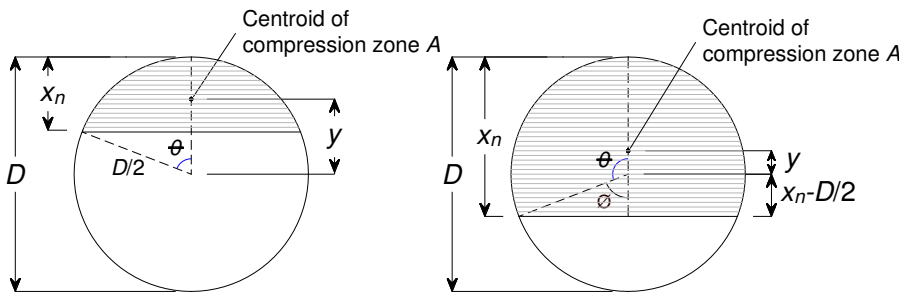
$$M = \alpha_1 \beta_1 f'_{cc} A \left( \frac{D}{2} - \frac{\beta_1 x_n}{2} \right) + (\sigma_{sc} A_{sc} - \sigma_{st} A_{st}) \left( \frac{D}{2} - d \right) \quad (22)$$

where  $d$  is the effective depth (distance from the outmost compression fibre to the centre of the tensile reinforcement);  $\sigma_{sc}$  and  $\sigma_{st}$  are the stress of compressive and tensile reinforcements, respectively, and the rest of the variables are as defined before.

For a circular column, the shape of the compression regions can be defined using a segment as shown in Fig. 6a-b. The area  $A$  of the compression region can be computed as:

$$A = D^2 \left( \frac{\theta_{rad} - \sin\theta \cos\theta}{4} \right) \quad (24)$$

where  $\theta_{rad}$  is the angle of the compression zone as defined in Fig. 6 (in radians).



**Fig. 6.** Compression regions of circular columns under eccentric loading

The second moment of area of region  $A$  is:

$$Ay = A \left( \frac{D}{2} - \frac{\beta_1 x_n}{2} \right) \quad (23)$$

where  $y$  is the distance between the centroid of the compression region and the centroid of the column;  $\beta_1$  is the block depth ratio; and  $x_n$  is the neutral axis depth (see Fig. 6).

The angle  $\theta$  can be calculated using:

$$\theta = \arccos \left( \frac{R - \beta_1 x_n}{R} \right) \text{ if } \beta_1 x_n \leq D/2 \quad (24)$$

$$\theta = \Pi - \arccos\left(\frac{\beta_1 x_n - R}{R}\right) \text{ if } \beta_1 x_n > D/2 \quad (25)$$

Thus, the compressive concrete load  $C_c$  on a circular column is:

$$C_c = \alpha_1 \beta_1 f'_{cc} A \quad (26)$$

Likewise, the flexural moment  $M_c$  produced by the concrete in compression is:

$$M_c = \alpha_1 f'_{cc} A y \quad (27)$$

For the compressive force of the longitudinal reinforcement in compression, the strain of the steel can be calculated using strain compatibility:

$$\varepsilon_{sc} = \varepsilon_c \left(1 - \frac{d_0}{x_n}\right) \quad (28)$$

Hence, the stress in the longitudinal reinforcement in compression can be calculated as:

$$\sigma_{sc} = E_s \varepsilon_{sc} \quad \text{for } \varepsilon_{sc} \leq \varepsilon_{sy} \quad (29)$$

$$\sigma_{sc} = f_{sy} \quad \text{for } \varepsilon_{sc} > \varepsilon_{sy} \quad (30)$$

where  $\varepsilon_{sy}$ ,  $f_{sy}$  and  $E_s$  are the yield strain, yield stress and elastic modulus of the longitudinal reinforcement. The compression force of such reinforcement is:

$$C_s = \sigma_{sc} A_{sc} \quad (31)$$

where  $A_{sc}$  is the area of compressive reinforcement.

Similarly, the corresponding strain  $\varepsilon_{st}$  of the longitudinal reinforcement in tension is:

$$\varepsilon_{st} = \varepsilon_u \left(\frac{d}{x_n} - 1\right) \quad (32)$$

The stress in the tension longitudinal reinforcement can be calculated as:

$$\sigma_{st} = E_s \varepsilon_{st} \quad \text{for } \varepsilon_{st} \leq \varepsilon_{sy} \quad (33)$$

$$\sigma_{st} = f_{sy} \quad \text{for } \varepsilon_{st} > \varepsilon_{sy} \quad (34)$$

And the tensile force  $T$  in the reinforcement is:

$$T = \varepsilon_{st} A_{st} \quad (35)$$

Fig. 7a-b show the stress block factors  $\alpha_1$  and  $\beta_1$  obtained from the analysis of SSTT-confined HSC circular columns with different neutral axis depths  $x_n$ . Previous research (Ma et al. 2014a; 2015b) has shown that the SSTT is effective at confining concrete only if  $\rho_v > 0.09$ , and that values  $0 \geq \rho_v \leq 0.09$  can be used to represent “unconfined” concrete. As a result, the corresponding data results for  $\rho_v = 0.09$  are plotted at  $\rho_v = 0$  in Fig. 7a-b. It is shown that  $\alpha_1$  tends to increase with the volumetric ratio of confining steel straps  $\rho_v$ . Conversely, Fig. 7b shows that  $\beta_1$  varies only slightly for the examined  $\rho_v$  ratios, and therefore a constant value  $\beta_1 = 0.90$  can be assumed for practical design.

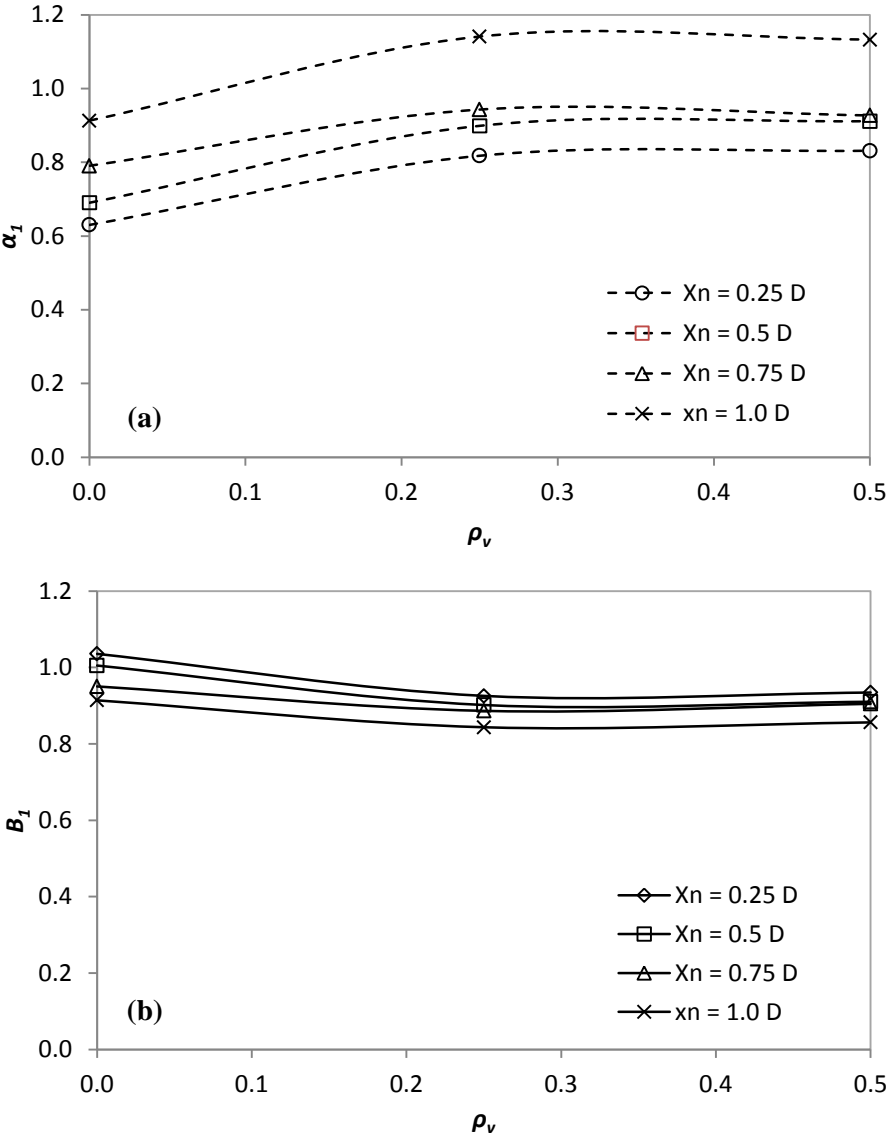
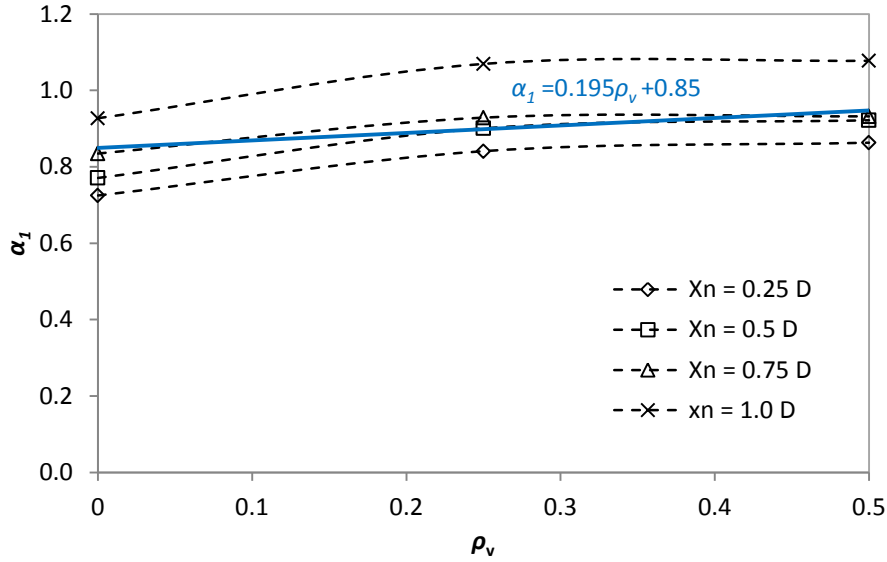


Fig. 7. Stress block factors for SSTT-confined HSC sections

Once  $\beta_1$  is defined, the mean stress factor  $\alpha_1$  can be calculated again using the equivalent stress block approach. Fig. 8 shows the recalculated factor  $\alpha_1$  using the proposed constant value  $\beta_1=0.90$ . Based on a regression analysis the following simplified equation is proposed for design:

$$\alpha_1 = 0.195\rho_v + 0.85 \quad (36)$$



**Fig. 8.** Mean stress factor  $\alpha_1$  for STTT-confined HSC sections assuming  $\beta_1=0.90$

## 4.2 Design equations and comparison with test results

Based on the stress block factors proposed in the previous section, conventional section analysis is used to determine the axial load  $N_u$  and flexural moment  $M_u$  of STTT-confined HSC columns according to the following equations:

$$N_u = \alpha_1 \beta_1 f'_{cc} A + \sigma_{sc} A_{sc} - \sigma_{st} A_{st} \quad (37)$$

$$M_u = N_u \left( e + \frac{l^2}{\pi^2} \xi_1 \xi_2 \phi_{bal} \right) = \alpha_1 \beta_1 f'_{cc} A \left( \frac{D}{2} - \frac{\beta_1 x_n}{2} \right) + (\sigma_{sc} A_{sc} - \sigma_{st} A_{st}) \left( \frac{D}{2} - d \right) \quad (38)$$

where all variables are as defined before.

Fig. 9 compares the predictions given by equations (37) and (38) with experimental results from 18 small-scale STTT-confined HSC columns tested by Ma et al. (2014a). All columns were circular with a diameter  $D=150$  mm and were cast using HSC with a target concrete strength of  $f_{co}=60$  MPa. Three different column heights were examined 600, 900 and 1200 mm, thus leading to slenderness ratios  $\lambda=16$ ,

24 and 40. Three columns were unconfined control specimens, whereas the rest were externally confined with one or two layers of metal straps placed at clear spacing of 20 or 40 mm, which produced confinement ratios of 0.076, 0.12 and 0.178. As shown in Fig. 9, the axial load ( $N_u$ ) and moment ( $M_u$ ) predicted by the theoretical model match well (on the conservative side) the corresponding test data  $N_{ut}$  and  $M_{ut}$ . Moreover, most of the predictions remain within a  $\pm 20\%$  range of confidence, thus indicating a relatively low scatter. As a result, the proposed design approach is sufficiently accurate to calculate the maximum axial load and flexural strength of small-scale SSTT-confined HSC columns. Due to its little computational effort and good predictions, equations (37) and (38) are proposed for the rapid design and assessment of SSTT-confined HSC columns.

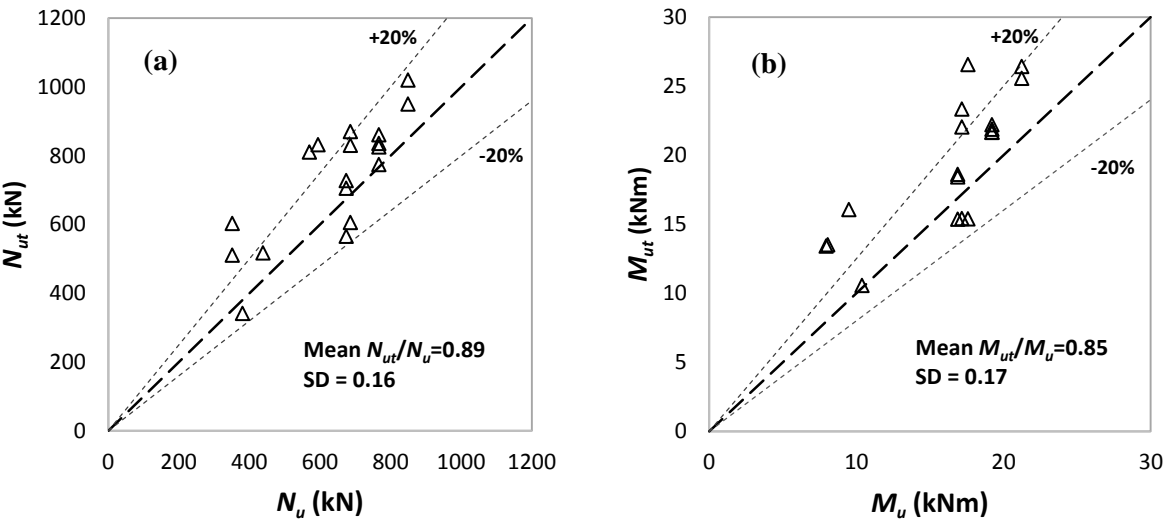


Fig. 9. Prediction given by Equations (37) and (38) vs test results by Ma et al. (2014a)

Due to the limited data used for the validation of Equations (37) and (38), future research should verify the accuracy of the proposed approach at assessing the capacity of columns with heavier strap confinement ( $\rho_v > 0.18$ ). Moreover, as for other confining techniques, the metal straps are expected to be less effective at confining rectangular sections. Consequently, further research should also verify the accuracy of the proposed approach at predicting the results of rectangular members. Future research should also verify the accuracy of Equations (37) and (38) at predicting test results from full-scale SSTT-confined HSC columns.

## 5 Conclusions

This article proposed new parameters for the practical design of circular high-strength concrete (HSC) columns confined with an innovative Steel Strapping Tensioning Technique (SSTT) using a nominal curvature approach. The new parameters were obtained from segmental analyses of slender SSTT-confined columns subjected to eccentric loads. The results obtained from the analyses were used to determine the basic parameters that influence the design of such columns. A new equivalent stress block and block depth factor equation were proposed for the design of circular SSTT-confined sections. It was found that the block depth factor can be considered as constant and equal to  $\beta_1=0.90$ , whereas the mean stress factor is variable and depends on the strap confining volumetric ratio. The use of the new proposed parameters for the design of SSTT-confined HSC columns using the nominal curvature method leads to conservative predictions of the maximum axial load and flexural strength of small-scale columns. However, future research should verify the accuracy of the approach at predicting the results of full-scale specimens. Besides, the current adopted stress-strain model was developed based on circular cylinders and hence no stress concentration at corners have been considered. The proposed design procedure can be used for rectangular or square sections columns provided that the correct stress-strain model for sections other than circular columns are used.

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