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Automated modelling of spatially-distributed glacier ice thickness and volume

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Abstract

Ice thickness distribution and volume are both key parameters for glaciological and hydrological applications. This study presents VOLTA (Volume and Topography Automation), which is a Python script tool for ArcGIS™ that requires just a digital elevation model (DEM) and glacier outline(s) to model distributed ice thickness, volume and bed topography. Ice thickness is initially estimated at points along an automatically generated centreline network based on the perfect-plasticity rheology assumption, taking into account a valley side drag component of the force balance equation. Distributed ice thickness is subsequently interpolated using a glaciologically correct algorithm. For five glaciers with independent field-measured bed topography, VOLTA modelled volumes were between 26.5 % (underestimate) and 16.6 % (overestimate) of that derived from field observations. Greatest differences were where an asymmetric valley cross section shape was present or where significant valley infill had occurred. Compared with other methods of modelling ice thickness and volume, key advantages of VOLTA are: a fully automated approach and a user friendly graphical user interface (GUI), GIS consistent geometry, fully automated centreline generation, inclusion of a side drag component in the force balance equation, estimation of glacier basal shear stress for each individual glacier, fully distributed ice thickness output and the ability to process multiple glaciers rapidly. VOLTA is capable of regional scale ice volume assessment, which is a key parameter for exploring glacier response to climate change. VOLTA also permits subtraction of modelled ice thickness from the input surface elevation to produce an ice-free DEM, which is a key input for reconstruction of former glaciers. VOLTA could assist with prediction of future glacier geometry changes and hence in projection of future meltwater fluxes.

Keywords: ice thickness distribution, glacier volume, subglacial topography, glacier centrelines, perfect-plasticity

1 Introduction

Knowledge of contemporary regional ice thickness distribution is poor (Farinotti et al. 2009), with field measurements impractical and data requirements making larger scale modelling studies difficult. In the context of ongoing climate change, large scale assessments are important because a climate signal extracted from an individual glacier may not be representative of the entire region (Hoelzle et al. 2007). Furthermore, it is total regional ice volume that is essential for exploring the response of glaciers to climate change (Chinn et al. 2012) and for the projection of meltwater availability (Kaser et al. 2010). Ice thickness distribution is required for glacier dynamics models (e.g. Oerlemans et al. 1998) and for assessing the impact of climate change on the hydrology of glaciated catchments (e.g. Huss et al. 2008). Glacier bed topography derived via distributed ice thickness estimations can be used to reconstruct palaeoglaciers (Benn and Hulton 2010), and the resultant equilibrium-line altitudes are a widely-used source of palaeoclimatic information (e.g. Benn and Ballantyne 2005). Glacier bed topography is also of great assistance in understanding glaciological hazards, such as jökulhlaups routing from or sourced subglacially (e.g. Carrivick 2007; Staines and Carrivick 2015), and for understanding subglacial lake formation (Frey et al. 2010). Therefore, the development of methods for assessing regional ice thickness distribution is essential for improving our understanding of many glaciological, hydrological and climatological issues.

Distributed ice thickness and ice volume can either be calculated by interpolating field measurements or by modelling. Ice thickness can be measured via boreholes (e.g. Hochstein et al. 1998) or by reflection techniques such as seismics (e.g. Shean et al. 2007) or radar (e.g Singh et al. 2012). Although impractical for regional scale studies, field measurements are crucial to parameterise (e.g. Bahr et al. 1997) and validate (e.g. Li et al. 2012) models. Due to the logistical and technical difficulties of field measurement, scaling laws are often employed to estimate glacier volumes at regional scales (Bahr et al. 1997). However, scaling approaches do not account for individual glacier characteristics and do not yield information on bed topography. Furthermore, errors may be in excess of 50 % for individual glaciers, reducing to 25 % for regional volume (Meier et al. 2007).

A variety models to estimate ice thickness based on viscous flow mechanics and mass turnover are available (e.g. Farinotti et al. 2009; McNabb et al. 2012; Michel et al. 2013), and they have ability to estimate ice thickness with an accuracy of ~25 % (Farinotti et al. 2009). However, these existing models invariably require glacier specific datasets such as mass balance (e.g. Michel et al. 2013), surface velocity fields (e.g. McNabb et al. 2012) or the manual digitization of flowlines and ice flow catchments

(Farinotti et al. 2009), limiting their use for regional scale application. A neural network approach has also been developed (Clarke et al. 2009), although it has only been tested against artificial “horizontal lake-like” glaciers and is acknowledged to be computationally intensive, limiting its effectiveness for regional studies.

An alternative approach utilises the perfect plasticity assumption (Nye 1951), that glacier ice thickness (h) can be found from a glacier surface slope α by the relation:

$$h = \frac{\tau_b}{fpg \tan \alpha} \quad (1)$$

Where p is ice density (typically 900 kg m^{-3}), g is gravitational acceleration (9.81 m s^{-2}), τ_b is basal shear stress and f a ‘shape factor’ to incorporate the effect of side drag. With high resolution digital elevation models (DEMs) permitting accurate calculation of surface slope, perfect plasticity based models are proving popular, requiring just glacier outline(s), centreline(s) and a DEM. The relative simplicity of the calculations, combined with wide availability of input datasets are key advantages over viscous flow mechanics models when considering large regions. Whilst basic perfect plasticity models have been used to estimate 3D bed topography (e.g. Linsbauer et al. 2009), more complex models which account for variations in side drag have only been applied in 2D (e.g. Li et al. 2012). Furthermore, all the models to date require manual digitisation of centrelines, limiting their applicability to large regional scale studies.

The aim of this paper is to present VOLTA (Volume and Topography Automation), which is a tool for rapid estimation of distributed ice thickness. Key advantages of VOLTA are: a fully automated approach and a user friendly graphical user interface (GUI), GIS consistent geometry, automatic centreline generation, inclusion of a side drag component in the force balance equation, individual glacier basal shear stress estimation, fully distributed ice thickness and bed topography outputs, and the ability to process multiple glaciers rapidly

1.1 Volume and Topography Automation (VOLTA)

VOLTA estimates ice thickness along automatically derived centrelines and interpolates fully distributed ice thickness. It can be applied on multiple glaciers of complex geometry, requiring just glacier outline(s) and a DEM as inputs. VOLTA is written in the Python scripting language and executed via ArcGIS™ as a geoprocessing tool. VOLTA, installation instructions and manual are available for download at: https://github.com/williamjames/volta. A schematic flowchart of the VOLTA workflow is presented in Figure 1.

2 Theory

2.1 Automating centreline production

VOLTA initially estimates ice thickness along automatically derived centrelines. Whilst it would be desirable to derive centrelines which represent actual flowlines (i.e. ice trajectories), this would require fully distributed velocity fields, making regional scale applications unachievable (Kienholz et al. 2014). Whilst centrelines will usually coincide with the location of maximum ice thickness along a traverse profile, they may be offset in some circumstances, such as in the vicinity of sharp bends.

Traditionally, centreline production required manual digitization, a time consuming and subjective process. Recently, automated techniques using GIS hydrology tools (Schiefer et al. 2008; Machguth and Huss 2014), cost-distance analysis (Kienholz et al. 2014) and geometric analysis (Le Bris and Paul 2013) have been developed, although only appearing in the literature with regards to method development. To overcome the constraints of manually digitizing centrelines, VOLTA generates centreline(s) using the glacier axis concept (Le Bris and Paul 2013). VOLTA develops the algorithm by:

- Improving the axis creation technique
- Automatically adjusting the smoothing parameter.
- Automatically creating separate centrelines for multiple tributaries.
- Incorporating the algorithm with VOLTA

Firstly, a glacier axis is created to define the main direction of the glacier. Whereas Le Bris and Paul (2013) determined the axis by joining the highest and lowest points of the glacier, VOLTA uses a minimum bounding geometry (MBG) technique, followed by connecting the two farthest points (Fig. 2a). This creates an axis not influenced by individual high and low DEM cells, which may otherwise create an unrealistic axis. Perpendicular traverses are created along the axis at the resolution of the DEM, which are subsequently clipped to the glacier outline and midpoints placed on each traverse segment (Fig. 2b).

An initial centreline is constructed by iteratively joining traverse midpoints (starting at the highest midpoint) in a similar manner to that described by (Le Bris and Paul 2013). The line is then smoothed using the Polynomial Approximation with Exponential Kernel (PAEK) algorithm to remove irregularities caused by small scale variations in glacier shape. Larger glaciers require greater smoothing as wide tributaries result in a smooth course of the centreline (Kienholz et al. 2014). The amount of smoothing is controlled by glacier area in an approach similar to that of Kienholz et al. (2014):

$$l = \begin{cases} 2 \cdot 10^{-6} \cdot A + 200 & : l \le l\,max \\ l\,max & : l > l\,max \end{cases} \quad (2)$$

Where l is smoothing length, A is glacier area (m^2) and $l\,max$ is 1,000 m

2.2. Glaciers of complex geometry

A single centreline may not be suitable if the glacier is of complex geometry due to multiple tributaries or cirques (Fig. 2e). Whilst the lack of a secondary tributary centreline is the main issue, the initial centreline is also laterally deflected where the perpendicular traverses are elongated as they continue into the secondary tributary (Fig. 2d). To overcome these issues, VOLTA uses a novel 'upstream area' approach to delineate separate tributaries, allowing multiple centrelines to be generated (Fig. 2e). Iteratively working down the initial centreline, upstream area is calculated. Total area will steadily increase down-centreline, but a marked increase occurs when a new tributary enters (Fig. 3). VOLTA calculates area at an interval equal to 1 % of centreline length with a new tributary identified if area increases by > 30 % between successive points. Furthermore, any new tributary must also have an area of at least 20 % of the total. Whilst these thresholds may be altered via the GUI, testing against manually identified tributaries for 25 glaciers found these values were able to correctly delineate tributaries where appropriate, whilst not adding 'extra' tributaries due to small areas of ice adjoining. If secondary tributaries are identified, new tributary outlines are created from a subset of the original (Fig. 2e) and secondary centrelines are created in the standard manner, ignoring any midpoints of which the corresponding perpendicular crosses into another tributary. Branch order is managed in the same manner as Kienholz et al. (2014), with the initial centreline classed as the primary centreline.

2.3 Calculating ice thickness at points along the centreline

VOLTA estimates thickness along the centreline(s) using a perfect plasticity approach (Eq. 1). For mountain valley glaciers, a shape factor (f) is required because valley sides support part of the weight of the glacier, resulting in τ_b on the centreline being lower than for an infinitely wide basin. f can be incorporated as a constant (usually 0.8: Nye 1965), a form which has been used extensively in the literature (e.g. Linsbauer et al. 2012).

Li et al. (2012) developed a more physically realistic method which dynamically adjusts f depending on the local width of the glacier (see Li et al. (2012) for a full-derivation):

$$h = \frac{0.9\,w(\frac{\tau_b}{pg\,sin\,\alpha})}{0.9\,w-(\frac{\tau_b}{pg\,sin\,\alpha})} \tag{3}$$

Where w is half the glacier width at the specified point.

Whilst Eq. (3) estimates ice thickness perpendicular to the ice surface (Fig. 4a), VOLTA estimates 'vertical' ice thickness, perpendicular to a horizontal x-axis (Fig. 4b), a form appropriate for GIS geometry:

$$h = \frac{0.9\,w(\frac{\tau_b}{pg\,tan\,\alpha})}{0.9\,w-(\frac{\tau_b}{pg\,tan\,\alpha})} \tag{4}$$

For some glacier geometries (e.g. where nunataks are present or where tributaries converge), width calculation may be inaccurate, with Li et al. (2012) cautioning against its use without cross-checking. VOLTA automatically checks for erroneous values by: (i) checking if the perpendicular line intersects another centreline (Fig. 5a) and (ii) cross checking if the resulting *f* value (Eq. 1) is realistic (> 0.445, equal to a half width to centreline thickness ratio of 1: Nye 1965, Fig. 5b). At points where either of these conditions are met, VOLTA calculates thickness using Eq. (1), with *f* set to that of the average for the tributary (calculated from points on the same tributary with accurate width calculations). The "f_type" field in the output point file indicates which method is used, with "W" denoting the independent width has been used and "A" denoting the tributary average value has been used.

2.4 Interpolating distributed ice thickness and bed topography

VOLTA interpolates distributed ice thickness using the ANUDEM 5.3 interpolation routine, which is an iterative finite difference technique designed for the creation of hydrologically correct DEMs (Hutchinson 1989). ANUDEM is implemented via the 'TopoToRaster' tool in ArcGIS, using ice thickness points as 'spot height' inputs and the glacier outline(s) as a contour input (assumed to represent zero ice thickness), with any ice divides ignored. ANUDEM generates preferably concave shaped landforms, mimicking the typical parabolic shape of (idealised) glacier beds (Linsbauer et al. 2009) and is the method of choice for interpolating both mountain valley glaciers (Fischer and Kuhn 2013) and ice sheets, such as the Bedmap2 Antarctica dataset (Fretwell et al. 2013). Interpolating of this manner is an accepted method for bed topography estimation (Farinotti et al. 2009; Li et al. 2012; Linsbauer et al. 2012), although sediment infill or the compound incision effects of multiple glaciation phases may result in modified cross sections under some circumstances (Schrott et al. 2003). VOLTA outputs the raw centreline ice thickness points, allowing bespoke interpolation by the user if required.

Once thickness h for each cell has been interpolated, total volume, V can be calculated:

$$V = \sum(c^2 h) \tag{5}$$

Where c is the cellsize.

2.5 VOLTA parameters

By default, VOLTA does not require any user specified parameters, using the DEM and glacier outline(s) to derive glacier specific values. These may be altered if required (e.g. if independent ice-thickness measurements exist). VOLTA parameters are: basal shear stress (τ_b), slope averaging distance (α_d), "effective width" slope threshold (α_{lim}) and minimum slope threshold(α_0).

2.5.1 Basal shear stress (τ_b)

τ_b is variable between individual glaciers due to many factors (e.g. basal water pressure, ice viscosity, subglacial sediment deformation), meaning no universal value should be used between glaciers. For modelling, $\tau_b$does not have to be varied longitudinally for an individual glacier as a constant value can reproduce accurate thickness estimates along the length of a centreline (Li et al. 2012).

Whilst τ_b can be "constrained reasonably from just a few ice-thickness measurements" (Li et al. 2012 p.7), in the majority of cases there are no independent measurements, requiring τ_b to be estimated. An empirical relationship between altitudinal extent and τ_b developed by Haeberli and Hoelzle (1995) is often used, although the spread of data points is large (r^2 = 0.44), with Linsbauer et al. (2012) estimating an uncertainty of up to ± 45 %. A more robust relationship was developed by Driedger and Kennard (1986a), using area and slope in an elevation band approach:

$$\tau_b = 2.7 \cdot 10^4 \sum_{i=1}^{n} \left(\frac{A_i}{\cos \alpha_i}\right)^{0.106} \tag{6}$$

Where the elevation band area (A_i) is in m^2 and shear stress (τ_b) is in Pa. This method was tested by Driedger and Kennard (1986b) as part of a volume estimation study, finding a standard deviation of error of 5 % when comparing with measured volumes. This is the default method used by VOLTA, with A_i and $\cos\alpha_i$ calculated over 200 m elevation bands. The result is a glacier specific average τ_b value which is consequently applied to each centreline point. Optionally, τ_b may be user defined via the GUI.

2.5.2 Slope averaging distance (α_d)

Analysing the centreline gradient over an appropriate distance (α_d) is required for producing reliable thickness estimates. If α_d is too low, small scale variations in the surface topography will be reproduced in the bed profile. Conversely, if α_d is too large, variations in the surface topography may be smoothed or omitted. α_d should be "several times" the local ice thickness (Paterson 1994). The value is initially set to 10 times the average glacier thickness ($\bar{h}$), with $\bar{h}$ derived from a volume area scaling approach (Radić and Hock 2010):

$$\bar{h} = \frac{0.2055A^{1.375}}{A} \tag{7}$$

Where A is the glacier area and $\bar{h}$ is average ice thickness.

2.5.3 Minimum slope threshold (α_0)

Using Eq. 4, ice thickness will tend to infinity as surface slope tends to zero, meaning thickness may be overestimated in flat regions (Li et al. 2012; Farinotti et al. 2009), To overcome this, a 'minimum slope threshold' (α_0) is used in the same manner as Farinotti et al. (2009) and Li et al. (2012), setting any lower slope values to it. The threshold was determined empirically at 5° by Farinotti et al. (2009) and 4° by (Li et al. 2012). In VOLTA, α_0 is initially set to 4°, although this may be altered via the GUI if required.

2.5.4 "Effective width" slope threshold (α_{lim})

VOLTA accounts for valley side drag by incorporating glacier width (Eq. 4). However, thin ice on higher parts of the valley wall will contribute negligible support and thus should not be included in the width calculation. An "effective width" slope threshold (α_{lim}) is used to help exclude those areas as described by Li et al. (2012). In VOLTA, α_{lim} is set by default to 30°, which is the optimal value found during analysis by Li et al. (2012), although this value may be altered via the GUI if required. Using a slope angle of 30° and shear stress values parameterised for the European Alps (130 kPa) and the New Zealand Alps (180 kPa) (Hoelzle et al. 2007), the original perfect plasticity assumption (Eq. 1), estimates ice thickness values of 27 m and 37 m respectively. These thickness values are consistent with Driedger and Kennard (1986a) who found a threshold of average glacier thickness at ~36 m where glaciers obtain a critical shear stress and contributed to deformation.

3 Results

3.1 Application to glaciers with different geometry

For comparison, VOLTA was applied to 5 separate glaciers. Glaciers were selected for: (i) ice thickness distribution that has been well constrained from field measurements; (ii) comprising a range of different spatial scales and geometries (single, multi-tributary); and (iii) occupying different regional settings and thus different thermal and dynamic characteristics. Datasets used to produce the 'field based' ice distributions are summarized in Table 1.

The Ödenwinkelkees glacier, Austrian Alps (Fig. 6a) is the smallest glacier (1.9 km^2) and is of a relatively simple geometry. South Cascade glacier (Fig. 6b) was chosen due to its exceptionally well constrained bed topography, providing an example of a simple glacier geometry in North America. Storglaciären (Sweden) is more complex, characterised by a branched accumulation area (Fig. 6c), whilst Unteraargletscher (Swiss Alps) provides an example of a larger multi-tributary glacier (Fig. 6e). The Tasman glacier, (New Zealand) and its tributaries form the largest and most complex system, covering 90 km^2 (Fig. 6d).

3.2 Generating a field based ice thickness distribution dataset

Glacier outlines and field observations of ice thickness were digitized, georeferenced and projected in a Cartesian co-ordinate system. To account for ice surface topography changes between thickness measurement and DEM capture (surface lowering, retreat etc.), ice thickness was standardised to the surface DEM by subtracting the bed elevation from the surface DEM. The following analysis is therefore correct for the time of DEM capture. Ice thickness was interpolated using the ANUDEM algorithm (Hutchinson 1989). Due to interpolation and also original error in the ice thickness measurements, the interpolated bed cannot be regarded as a truly accurate representation of the subglacial topography. Whilst the datasets used have no error analysis available, it is assumed to be ± 10 m to account for errors in the raw data collection (Pellikka and Rees 2009) and in digitization. For the Tasman, error is estimated to be ± 20 m due to the small scale of diagrams from which measurements were digitized (e.g. Hart 2014).

3.3 Centreline generation

VOLTA generated a single centerline for the Ödenwinkelkees and South cascade glaciers and multiple centerlines for Storglaciären, Unteraargletscher and the Tasman (Fig. 7). Due to the complex nature of the Tasman system and the format of the outline, additional ice divides were digitized between nunataks and the main outline to ensure that individual branches were defined correctly. A combination of DEM inspection and a GIS generated ridgeline network (Fig. 8) was used to inform the location of potential ice divides.

3.4 Distributed ice thickness and volume

Distributed ice thickness was estimated as described in Fig. 1, with results shown in Fig. 9 and Table 2. Overall volume was estimated to between 26.5 % (underestimate) and 16.6 % (overestimate) of that derived from field measurements (Table 2). Histograms in Fig. 9 graphically display the distribution of deviation between VOLTA and field measurements, with a positive skew (Fig. 9a, 9l) representing an overestimate by VOLTA (Ödenwinkelkees and Unteraargletscher) and a negative skew (Fig. 9f, 9i, 9o) representing an underestimate (South Cascade, Storglaciären, Tasman).

For Unteraargletscher, both the field measured (Fig. 9a) and modelled (Fig. 9b) ice thickness show a parabolic cross section with the maximum ice thickness situated close to the centreline (Fig. 10a). South Cascade glacier field data reveals an asymmetrical bed cross section (Fig. 9d and 10c), which is not particularly well defined by VOLTA (Fig. 9e and 10c), resulting in a relatively large difference between volume estimated from field data and by VOLTA (Table 2). The parabolic bed profile of Storglaciären shown by field measurements (Fig. 9g and 10e) are well reproduced by VOLTA, with longitudinal variations also defined (Fig. 9h). The good agreement between field data and VOLTA is shown by similar final volume calculations (Table 2). This is also the case of the Ödenwinkelkees, where an overdeepening evident in the field measurements (Fig. 9j) is also predicted by VOLTA (Fig. 9k). For the Tasman system, field measurements on the main branch suggest a bed cross section with a relatively flat base (Fig. 10b), whilst VOLTA predicts a parabolic shape. Despite this, VOLTA is still able to accurately predict maximum ice thickness on the centerline (Fig. 9n) and total volume (Table 2).

3.5 Regional scale modelling: New Zealand Southern Alps

The minimal input data requirements and automated approach makes VOLTA ideal for regional scale modelling. VOLTA was applied to the New Zealand Southern Alps, with Fig. 11 showing an extract for the Mt. Cook region. VOLTA was applied to 158 glaciers, from which 196 centrelines were produced. The volume of the Mt. Cook region was calculated to be 19.82 km^3 (17.82 km^3 water equivalent). Analysis was performed on a PC with a 3.0 GHz Intel Core i5 processor and 8GB of RAM, with centreline generation taking an average of 18.4 seconds per glacier and ice thickness distribution taking an average of 22.1 seconds per glacier.

4 Discussion

4.1 Sensitivity testing

To assertain to what extent uncertainties in VOLTA parameters influence volume estimates, sensitivity tests were performed (Fig. 12). Parameters τ_b, α_d, α_0 and α_{lim} were tested independantly by altering their value whilst keeping others constant. Parameters were varied within bounds that could be reasonably expected, with the baseline set to that used for initial volume estimation. For each parameter, a range of 12 values was used, so Fig. 12 reports a total of 240 model runs.

τ_b is the most sensitive parameter, showing a similar positive linear relationship for all glaciers (Fig. 12a). For example, a 10 % increase in τ_b results in a 12.2 % increase in volume for the Ödenwinkelkees. This is expected as the critical thickness at which deformation occurs will be greater as τ_b is increased (Paterson 1994). Volume is also sensitive to α_d, with smaller glaciers (Ödenwinkelkees, South Cascade, Storglaciären) showing a negative relationship (Fig. 12b). An increase in α_d results in fewer thickness estimation points as no values can be determined at each end of the centreline (to a distance equal to half of α_d). Interpolations therefore trend to zero sooner, a possible cause of the negative relationship observed for smaller glaciers. Sensitivity to α_0 is variable (Fig. 12c), with glaciers dominated by large regions of low gradient ice (e.g. Unteraargletscher) showing a negative relationship whilst steep glaciers (e.g. Ödenwinkelkees) are not sensitive to α_0. Likewise the sensitivity of VOLTA-derived volumes to α_{lim} is variable depending on glacier surface topography; glaciers with large steep accumulation zones (e.g. Tasman) are most sensitive. Overall, it is the volume of small glaciers with areas of extreme gradient (either high or low) that are likely to be most sensitive to VOLTA input parameters. In contrast, the modelled volume of larger glaciers comprising moderate gradient surfaces will be least sensitive to VOLTA input parameters. These factors influencing the sensitivity of VOLTA should inform the choice of glaciers and regions to apply the model to, and should be taken into account when interpreting results.

4.2 Centreline generation and glacier length

Fig. 7 demonstrates that VOLTA is able to produce centrelines for glaciers of different geometries and size. The main advantages compared to manual centreline production come with the reproducible and fast computation. VOLTA successfully created multiple centrelines where appropriate, although in the most complex system (Tasman) the initial model run revealed that centrelines were not created in some of the tributary branches. To rectify this, adjustment of the outline was performed, with additional ice divides added (Fig. 8). This is a quick process and is likely to affect only a very small proportion of glaciers. Application to the Southern Alps of New Zealand resulted in 18 of the 196 (i.e. 9 %) of the centrelines (and one outline polygon) requiring manual adjustment, which is comparable to the original centreline algorithm developed by Le Bris and Paul (2013), which reported adjustment in 13 % of cases.

4.3 Total volume estimation

VOLTA returned the overall volume to between 26.5 % (underestimate) and 16.6 % (overestimate) of that derived from field measurements (Table 2). For comparison, approximate errors for scaling approaches range from 30 % for large samples to 40 % when considering smaller samples (~ 200 glaciers) (Farinotti and Huss 2013). The model presented by Farinotti et al. (2009) had an accuracy of ~25 % inferred from point-to-point comparison of measured and modelled thickness values whilst Li et al. (2012) reproduced measured thicknesses (radar transects) with a mean absolute error of 11.8 %. Considering the uncertainty in the field based glacier volume estimates used for benchmarking, VOLTA can be seen to perform at a similar level to other approaches. However, the fully automated and user friendly approach offers advantages over previous models, offering the potential to rapidly model regions which would otherwise be prohibitively laborious. VOLTA is capable of regional scale ice volume assessment (Fig. 11), which is a key parameter for exploring glacier response to climate change, and which has previously been estimated using less sophisticated techniques such as volume-area scaling (e.g. Chinn et al. 2012).

4.4 Distributed ice thickness

VOLTA results show a visually and statistically good correspondence with those from field measurements. The spatial variations in thickness largely match those shown by the field measurements, with features such as overdeepenings being defined. Modelling of the Mt Cook area (Fig. 11) shows VOLTA is suitable for regional scale modelling on a standard desktop computer. Although no performance metrics are available for alternative methods, overall processing times by VOLTA are expected to be vastly superior due to the minimal input data and complete automation of the process.

Comparison of modelled and measured ice thickness distributions (Fig. 9) shows that estimating ice thickness along a centreline followed by interpolation using the ANUDEM routine is an acceptable method for determining distributed ice thickness. Where the glacier cross section is a typical parabolic shape (Harbor and Wheeler 1992), VOLTA estimates closely match the field data along the entirety of the cross section (e.g. Fig. 10a, 10e). The Tasman glacier is an example of a contemporary glacier with an infilled bed, resulting in the near-horizontal bed profile visible in the field measurements (Fig. 10b). VOLTA is able to predict the maximum thickness well (VOLTA modelled 496 m versus measured 510 m) because the perfect plasticity assumption accounts for the corresponding low bed gradient along the centreline. However, the parabolic profile generated by ANUDEM underestimates ice thickness towards the valley sides, contributing to the 16.4 % overall volume deficit for the Tasman glacier.

Whilst the centreline will usually approximate the line of maximum thickness, in some cases this may be offset due to bends in the glacier (e.g. Ödenwinkelkees, South Cascade). Whilst VOLTA can still accurately estimate the maximum thickness under such circumstances (Fig. 10c, 10d), this may be offset from the centre, resulting in skewed bed topography (Fig. 10d). This may be overcome by using actual flowlines (i.e. ice trajectories), rather than centrelines, although there is currently no feasible method to generate such inputs on a regional scale.

In general, VOLTA is able to predict maximum ice thickness values well, even if infilling or multiple past glaciations have occurred. Whilst the ANUDEM interpolation routine generally works well for linear glaciers with a bedrock base (parabolic profile), the user should be aware of potential issues when considering in-filled valleys or glaciers with sharp bends in their centreline. VOLTA outputs the original point ice thickness estimations if the user wishes to use an alternative interpolation routine.

4.5 Bed topography and 'ice-free' DEMs

Calculation of distributed ice thickness, bed topography and ice volume are all of importance for many glaciological and hydrological applications, whilst the ability of VOLTA to be run on many glaciers in a region simultaneously will be of use in investigations of spatial variability in glacier responses to climate (Carrivick and Chase 2011). Furthermore, VOLTA has the ability to produce an 'ice-free' (bed topography) surface by subtracting the ice thickness from each DEM cell, an essential input for estimating palaeoglacier volume (e.g. Golledge et al. 2012; Carrivick et al. 2015). VOLTA provides a robust approach for estimating bed topography and therefore has the potential to reduce uncertainty in palaeo ice volume estimates. 'Ice free' surfaces are also useful as a direct input for palaeoglacier reconstruction models (e.g. Benn and Hulton 2010; Schilling and Hollin 1981) and for hazard assessment (Frey et al. 2010). Palaeoglacier reconstructions driven by the 'ice free' DEM generated by VOLTA may be further used for the estimation of former equilibrium line altitudes (ELA), providing one of the few methods for estimating palaeo-precipitation (e.g. Benn and Ballantyne 2005). VOLTA can be seen as a pre-cursor for such studies, potentially improving the accuracy of palaeoclimatic models.

5 Conclusions

VOLTA is a script tool for the rapid and data-efficient 3D modelling of glaciers: specifically of distributed ice thickness, volume and bed topography. VOLTA requires just a DEM and glacier outline(s) as inputs, both of which can be obtained free of charge with an almost global coverage for research purposes (e.g GLIMS 2014; USGS 2014). Whilst the basic perfect plasticity assumption is applicable for ice sheets (Nye 1951), VOLTA is optimised for centreline based modelling where glaciers are constrained by topography. As such, VOLTA can be applied with confidence to any alpine style glaciated region where the appropriate datasets are available. Compared with other existing methods, key advantages of VOLTA are: a fully automated workflow, improved centreline generation that can accommodate glaciers with multiple branches, inclusion of a side drag component in the force balance equation, GIS consistent geometry, individual glacier basal shear stress estimation, fully distributed ice thickness and bed topography outputs and a user friendly graphical user interface (GUI). In comparison of VOLTA-derived ice thickness and volume against independent data from five glaciers where the bed topography was well constrained, total volume estimates fell between 26.5 % (underestimate) and 16.6 % (overestimate) of the volume estimated from field measurements. The greatest field based-modelled differences were where bed elevation formed an asymmetric valley shape or valley infilling had occurred.

With present knowledge of ice thickness distribution being remarkably limited, VOLTA has the potential to improve our database of estimated ice thickness and thus also of estimated glacier volume. With the ability to produce an 'ice-free' DEM, an important prerequisite for addressing a range of glaciological and hydrological issues can be provided quickly.

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References

Bahr, D. B., M. F. Meier and S. D. Peckham. 1997. The physical basis of glacier volume-area scaling. *Journal of Geophysical Research-Solid Earth,* **102**(B9), pp.20355-20362.

Bauder, A., M. Funk and G. H. Gudmundsson. 2003. The ice-thickness distribution of Unteraargletscher, Switzerland. *Annals of Glaciology,* **37**(1), pp.331-336.

Benn, D. I. and C. K. Ballantyne. 2005. Palaeoclimatic reconstruction from Loch Lomond Readvance glaciers in the West Drumochter Hills, Scotland. *Journal of Quaternary Science,* **20**(6), pp.577-592.

Benn, D. I. and N. R. J. Hulton. 2010. An Excel (TM) spreadsheet program for reconstructing the surface profile of former mountain glaciers and ice caps. *Computers & Geosciences,* **36**(5), pp.605-610.

Björnsson, H. 1981. Radio-echo sounding maps of Storglaciären, Isfallsglaciären and Rabots Glaciär, northern Sweden. *Geografiska Annaler. Series A. Physical Geography,* **63**(3), pp.225-231.

Broadbent, M. 1974. *Seismic and gravity surveys on the Tasman Glacier: 1971-2.* Geophysics Division Department of Scientific and Industrial Research.

Carrivick, J. L. 2007. Modelling coupled hydraulics and sediment transport of a high-magnitude flood and associated landscape change. *Annals of Glaciology,* **45**(1), pp.143-154.

Carrivick, J. L. et al. 2015. Decadal Scale Changes Of The Ödenwinkelkees, Central Austria, Suggest Increasing Control of Topography and Evolution Towards Steady State. *Geografiska Annaler: Series A, Physical Geography*.

Carrivick, J. L. and S. E. Chase. 2011. Spatial and temporal variability in the net mass balance of glaciers in the Southern Alps, New Zealand. *New Zealand Journal of Geography and Geophysics,* **54**(4), pp.415-429.

Carrivick, J. L. et al. 2013. Contemporary geomorphological activity throughout the proglacial area of an alpine catchment. *Geomorphology,* **188**, pp.83-95.

Chinn, T. et al. 2012. Annual ice volume changes 1976–2008 for the New Zealand Southern Alps. *Global and Planetary Change,* **92–93**, pp.105-118.
Clarke, G. K. et al. 2009. Neural networks applied to estimating subglacial topography and glacier volume. *Journal of Climate,* **22**(8), pp.2146-2160.
Driedger, C. and P. Kennard. 1986a. Glacier volume estimation on Cascade volcanoes: an analysis and comparison with other methods. *Annals of Glaciology,* **8**, pp.59-64.
Driedger, C. L. and P. M. Kennard. 1986b. *Ice volumes on Cascade volcanoes: Mount Rainier, Mount Hood, Three Sisters, and Mount Shasta*. Washington: U.S. Geological Survey Professional Paper 1365
Farinotti, D. and M. Huss. 2013. An upper-bound estimate for the accuracy of glacier volume–area scaling. *The Cryosphere,* **7**(6), pp.1707-1720.
Farinotti, D. et al. 2009. A method to estimate the ice volume and ice-thickness distribution of alpine glaciers. *Journal of Glaciology,* **55**(191), pp.422-430.
Fischer, A. and M. Kuhn. 2013. Ground-penetrating radar measurements of 64 Austrian glaciers between 1995 and 2010. *Annals of Glaciology,* **54**(64), pp.179-188.
Fountain, A. G. and I. R. W. Jacobel. 1997. Advances in ice radar studies of a temperate alpine glacier, South Cascade Glacier, Washington, VSA. *Annals of Glaciology,* **24**, pp.303 - 308.
Fretwell, P. et al. 2013. Bedmap2: improved ice bed, surface and thickness datasets for Antarctica. *The Cryosphere,* **7**(1).
Frey, H. et al. 2010. A multi-level strategy for anticipating future glacier lake formation and associated hazard potentials. *Natural Hazards and Earth System Science,* **10**(2), pp.339-352.
GLIMS. 2014. *GLIMS, and National Snow and Ice Data Center. 2005, updated 2012. GLIMS Glacier Database*. Boulder, Colorado USA.
Golledge, N. R. et al. 2012. Last Glacial Maximum climate in New Zealand inferred from a modelled Southern Alps icefield. *Quaternary Science Reviews,* **46**, pp.30-45.
Haeberli, W. and M. Hoelzle. 1995. Application of inventory data for estimating characteristics of and regional climate-change effects on mountain glaciers: A pilot study with the European Alps. *Annals of Glaciology,* **21**, pp.206-212.
Harbor, J. M. and D. A. Wheeler. 1992. On the mathematical description of glaciated valley cross sections. *Earth Surface Processes and Landforms,* **17**(5), pp.477-485.
Hart, R. 2014. *The ice thickness distribution of a debris-covered glacier: Tasman Glacier, New Zealand.* thesis, Victoria University of Wellington.
Hochstein, M. P. et al. 1998. Rapid melting of the terminal section of the Hooker Glacier (Mt Cook National Park, New Zealand). *New Zealand Journal of Geology and Geophysics,* **41**(3), pp.203-218.
Hoelzle, M. et al. 2007. The application of glacier inventory data for estimating past climate change effects on mountain glaciers: A comparison between the European Alps and the Southern Alps of New Zealand. *Global and Planetary Change,* **56**(1), pp.69-82.
Huss, M. et al. 2008. Modelling runoff from highly glacierized alpine drainage basins in a changing climate. *Hydrological Processes,* **22**(19), pp.3888-3902.
Hutchinson, M. 1989. A new procedure for gridding elevation and stream line data with automatic removal of spurious pits. *Journal of Hydrology,* **106**(3), pp.211-232.
Kaser, G., M. Großhauser and B. Marzeion. 2010. Contribution potential of glaciers to water availability in different climate regimes. *Proceedings of the National Academy of Sciences,* **107**(47), pp.20223-20227.
Kienholz, C. et al. 2014. A new method for deriving glacier centerlines applied to glaciers in Alaska and northwest Canada. *The Cryosphere,* **8**(2), pp.503-519.
Le Bris, R. and F. Paul. 2013. An automatic method to create flow lines for determination of glacier length: A pilot study with Alaskan glaciers. *Computers & Geosciences,* **53**, pp.234 - 245.
Li, H. et al. 2012. An extended "perfect-plasticity" method for estimating ice thickness along the flow line of mountain glaciers. *Journal of Geophysical Research-Earth Surface,* **117**(F01020).

Linsbauer, A., F. Paul and W. Haeberli. 2012. Modeling glacier thickness distribution and bed topography over entire mountain ranges with GlabTop: Application of a fast and robust approach. *Journal of Geophysical Research,* **117**(F3).

Linsbauer, A. et al. 2009. The Swiss Alps without glaciers – a GIS-based modelling approach for reconstruction of glacier beds. *In: Proceedings of Geomorphometry 2009, Zurich, Switzerland*.

Machguth, H. and M. Huss. 2014. The length of the glaciers in the world: a straightforward method for the automated calculation of glacier center lines. *The Cryosphere Discussions,* **8**(3), pp.2491-2528.

McNabb, R. et al. 2012. Using surface velocities to calculate ice thickness and bed topography: a case study at Columbia Glacier, Alaska, USA. *Journal of Glaciology,* **58**(212), p1151.

Meier, M. F. et al. 2007. Glaciers dominate eustatic sea-level rise in the 21st century. *Science,* **317**(5841), pp.1064-1067.

Michel, L. et al. 2013. Estimating the ice thickness of mountain glaciers with an inverse approach using surface topography and mass-balance. *Inverse Problems,* **29**(3), p035002.

Nye, J. 1951. The flow of glaciers and ice-sheets as a problem in plasticity. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences,* **207**(1091), pp.554-572.

Nye, J. 1965. The flow of a glacier in a channel of rectangular, elliptic or parabolic cross-section. *Journal of Glaciology,* **5**, pp.661-690.

Oerlemans, J. et al. 1998. Modelling the response of glaciers to climate warming. *Climate Dynamics,* **14**(4), pp.267-274.

Paterson, W. S. B. 1994. *The physics of glaciers.* 3rd ed. Oxford: Pergamon.

Pellikka, P. and W. G. Rees. 2009. *Remote sensing of glaciers: techniques for topographic, spatial and thematic mapping of glaciers.* CRC Press.

Radić, V. and R. Hock. 2010. Regional and global volumes of glaciers derived from statistical upscaling of glacier inventory data. *Journal of Geophysical Research,* **115**(F1).

Schiefer, E., B. Menounos and R. Wheate. 2008. An inventory and morphometric analysis of British Columbia glaciers, Canada. *Journal of Glaciology,* **54**(186), pp.551-560.

Schilling, D. and J. Hollin. 1981. Numerical reconstructions of valley glaciers and small ice caps. *In:* P. HUGHES and D. GH, eds. *The Last Great Ice Sheets.* New York: John Wiley and Sons, pp.207-221.

Schrott, L. et al. 2003. Spatial distribution of sediment storage types and quantification of valley fill deposits in an alpine basin, Reintal, Bavarian Alps, Germany. *Geomorphology,* **55**(1), pp.45-63.

Shean, D. E., J. W. Head and D. R. Marchant. 2007. Shallow seismic surveys and ice thickness estimates of the Mullins Valley debris-covered glacier, McMurdo Dry Valleys, Antarctica. *Antarctic Science,* **19**(4), pp.485-496.

Singh, S. et al. 2012. Estimation of glacier ice thickness using Ground Penetrating Radar in the Himalayan region. *Current Science (Bangalore),* **103**(1), pp.68-73.

Span, N., A. Fischer and M. Kuhn. 2005. *Radarmessungen der Eisdicke österreichischer Gletscher [1995-1998]: Messungen 1995 bis 1998.* Zentralanstalt für Meteorologie und Geodynamik.

Staines, K. E. and J. L. Carrivick. 2015. Geomorphological impact and morphodynamic effects on flow conveyance of the 1999 jökulhlaup at Sólheimajökull, Iceland. *Earth Surface Processes and Landforms,* **40**(10), pp.1401–1416.

USGS. 2014. *USGS, and Japan ASTER Program (2003)*. Sioux Falls.

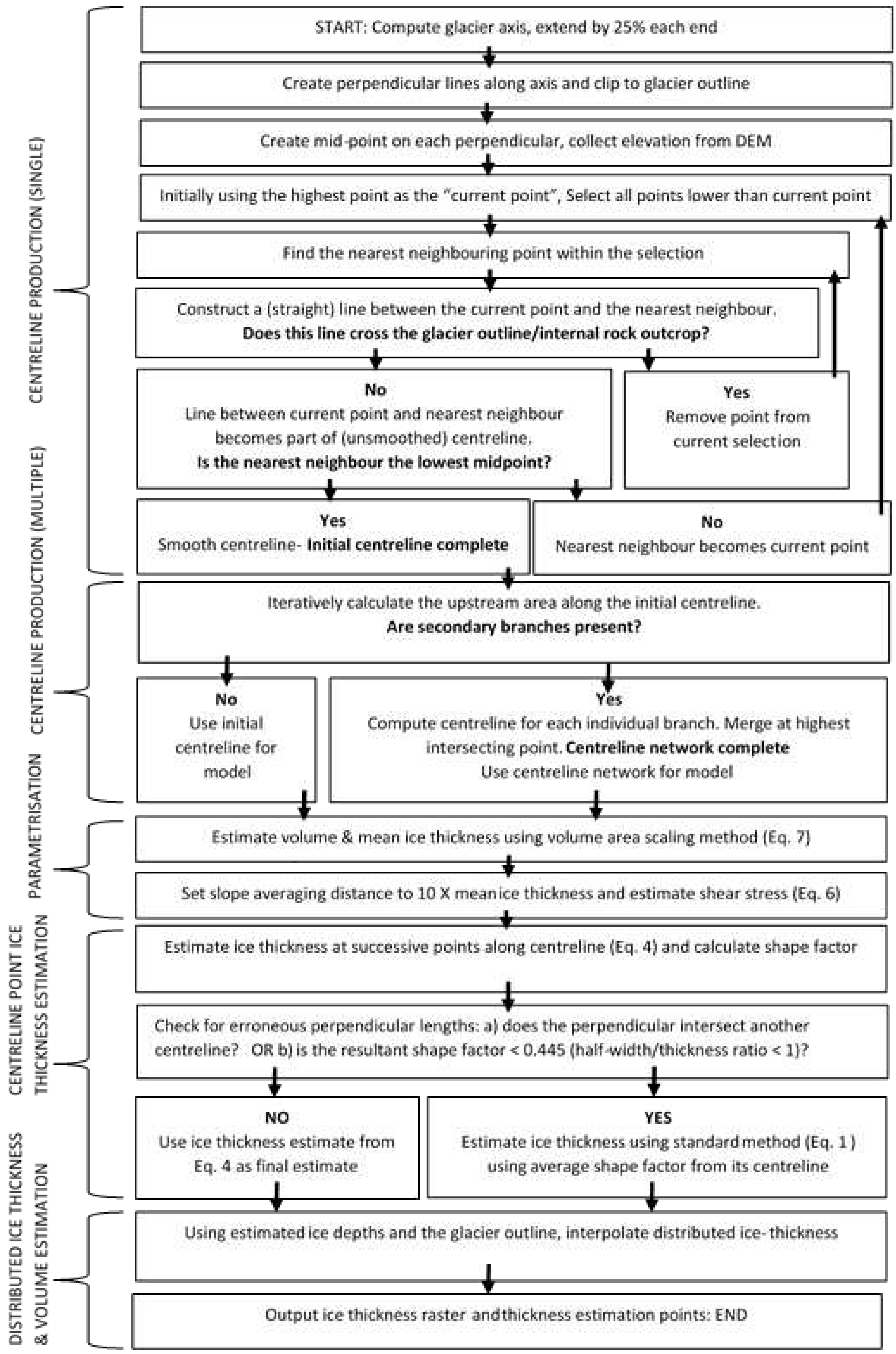

Fig. 1. Flowchart conceptually illustrating the automatic generation of glacier centrelines and estimation of distributed ice thickness and volume with VOLTA.

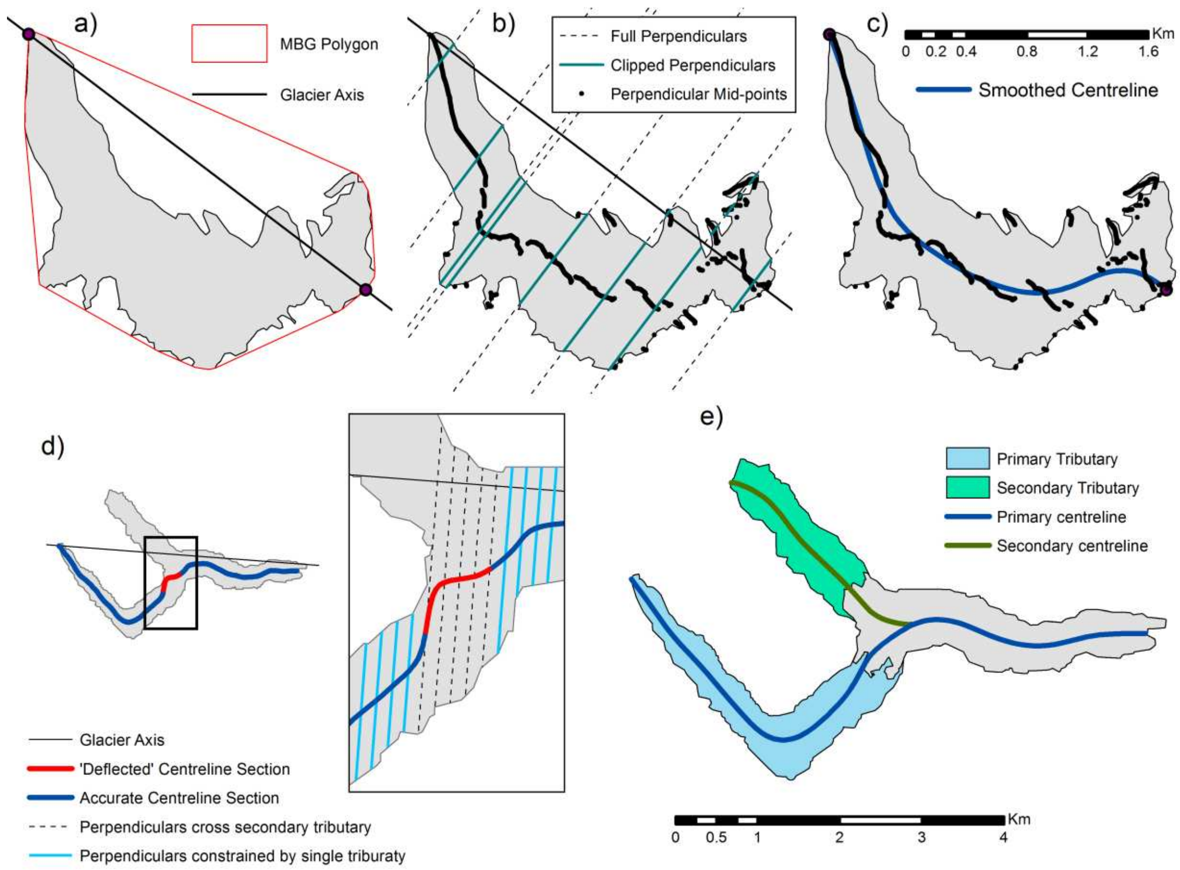

Fig. 2. VOLTA centreline production: a) definition of glacier axis via a polygon of minimum bounding geometry, b) creation of midpoints on perpendicular traverses, c) smoothed single centreline. Note that there can be unrealistic deflection of the centreline on multi-tributary glaciers (d), so VOLTA has developed a multi-centreline model (e).

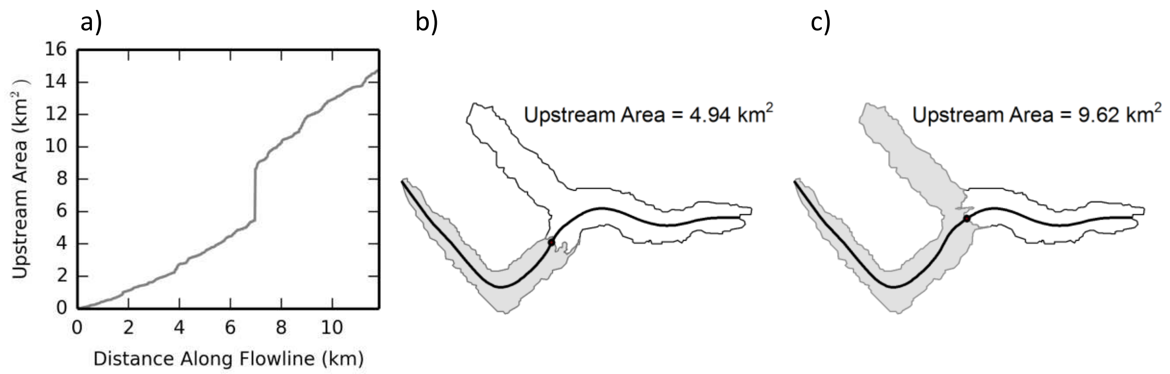

Fig. 3. a) Increase in upstream area as moving down centreline (for Unteraargletscher). Upstream area calculated before (b) and after (c) secondary branch.

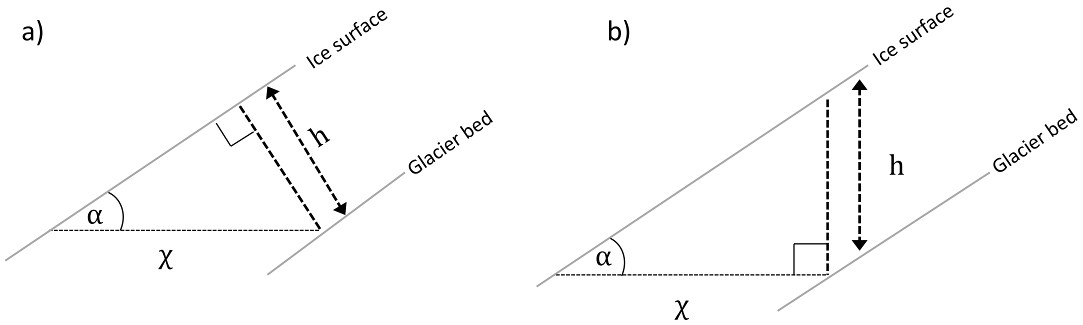

Fig. 4. a) Surface-perpendicular ice thickness calculated using the (original) sin function (Eq. 3) and b) vertical ice thickness calculated using the tan function (Eq. 4), as used by VOLTA.

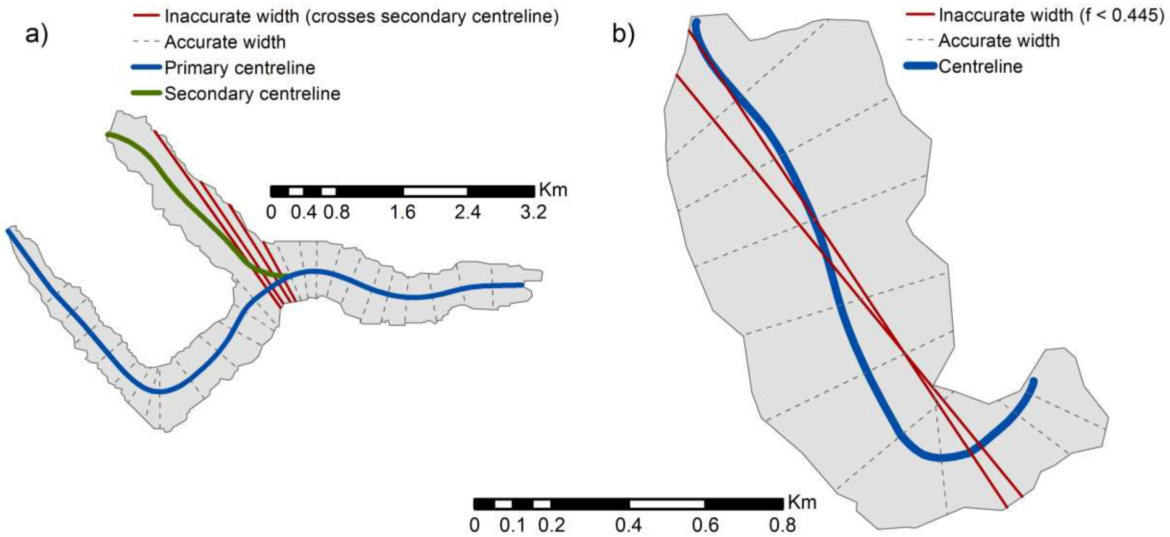

Fig. 5. Inaccurate glacier width calculations identified where: (a) the perpendicular crosses an alternative centreline or (b) the resultant f value is < 0.445

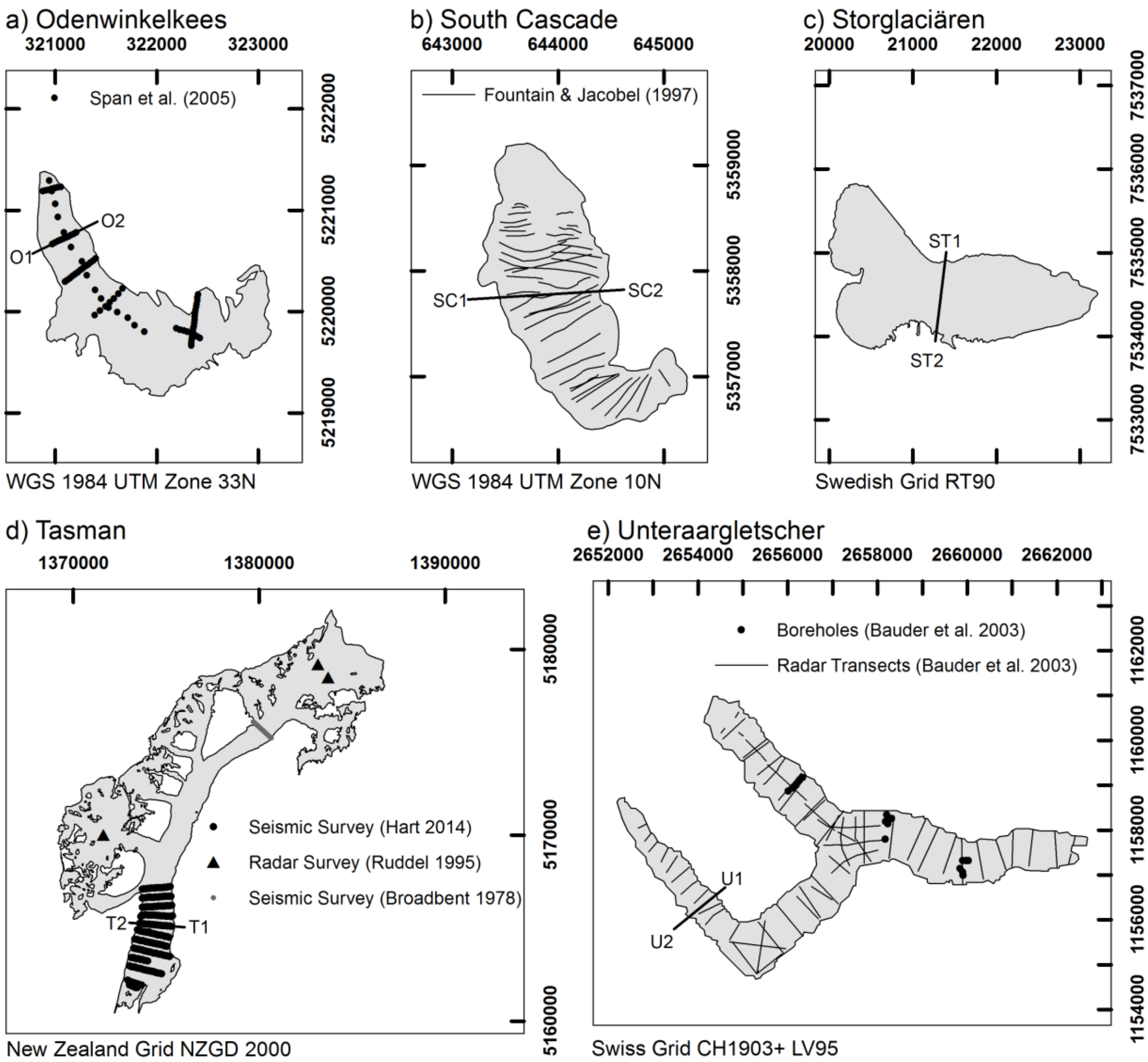

Fig. 6. Distribution of field measurements used to create bed topography datasets. Location of transects used in Fig. 10 are also shown.

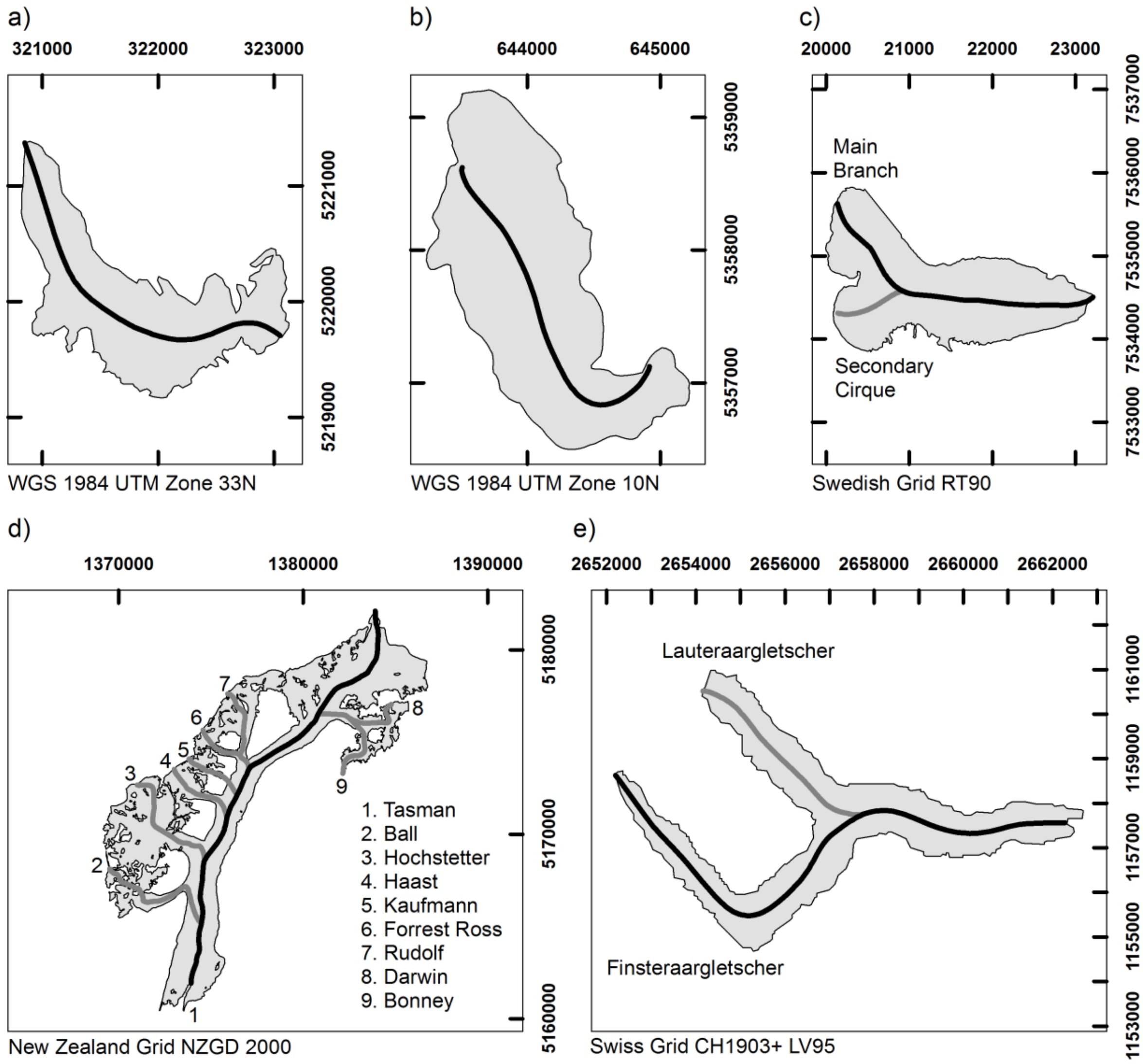

Fig. 7. Centrelines generated by VOLTA for sample glaciers. a) Ödenwinkelkees, b) South Cascade glacier, c) Storglaciären, d) Tasman glacier, e) Unteraargletscher

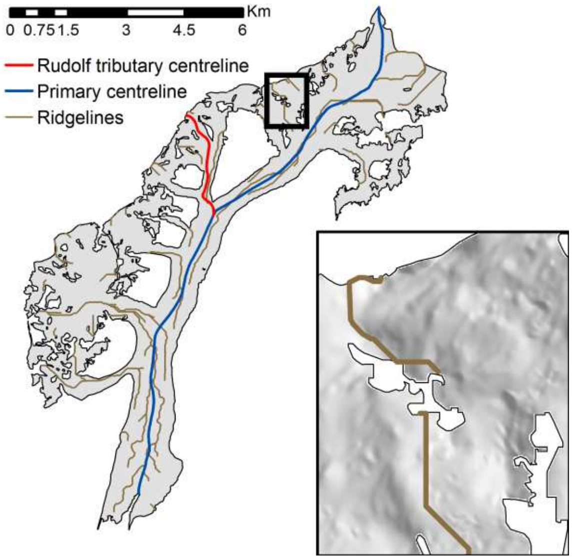

Fig. 8. GIS ridgeline network generated to indicated potential ice divide locations

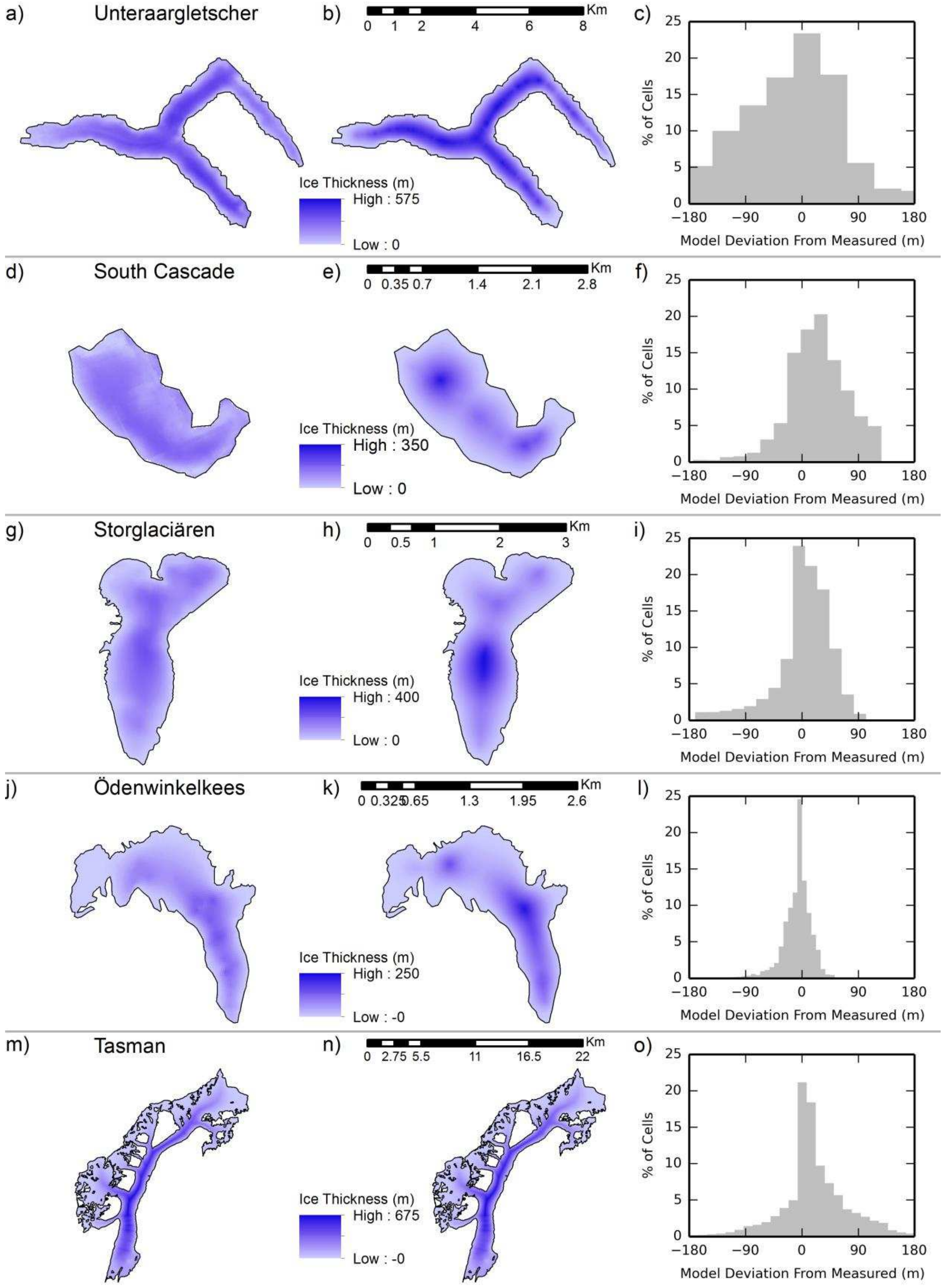

Fig. 9. Field data derived (left) and VOLTA modelled (right) ice thickness maps with accompanying histograms showing the deviation of the modelled thickness from the measured.

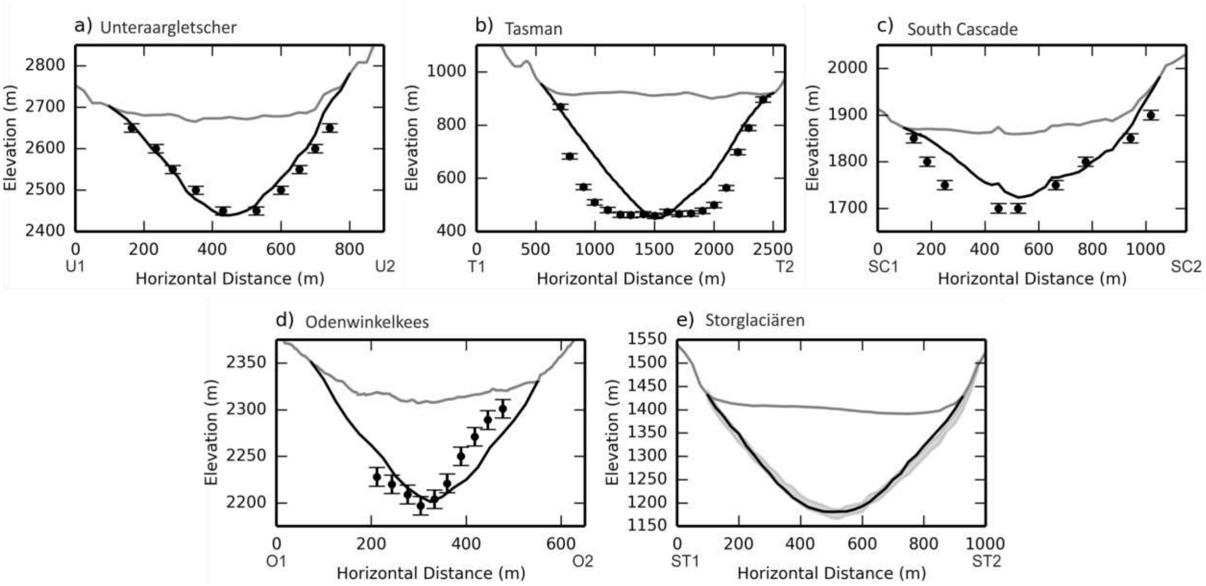

Fig. 10. Comparison of traverse glacier-bed profiles generated by VOLTA (black lines) from field measurements (black dots). Panel e) shows field measurements as a grey band as it was provided as a pre-interpolated raster surface. For transect locations see Fig. 5.

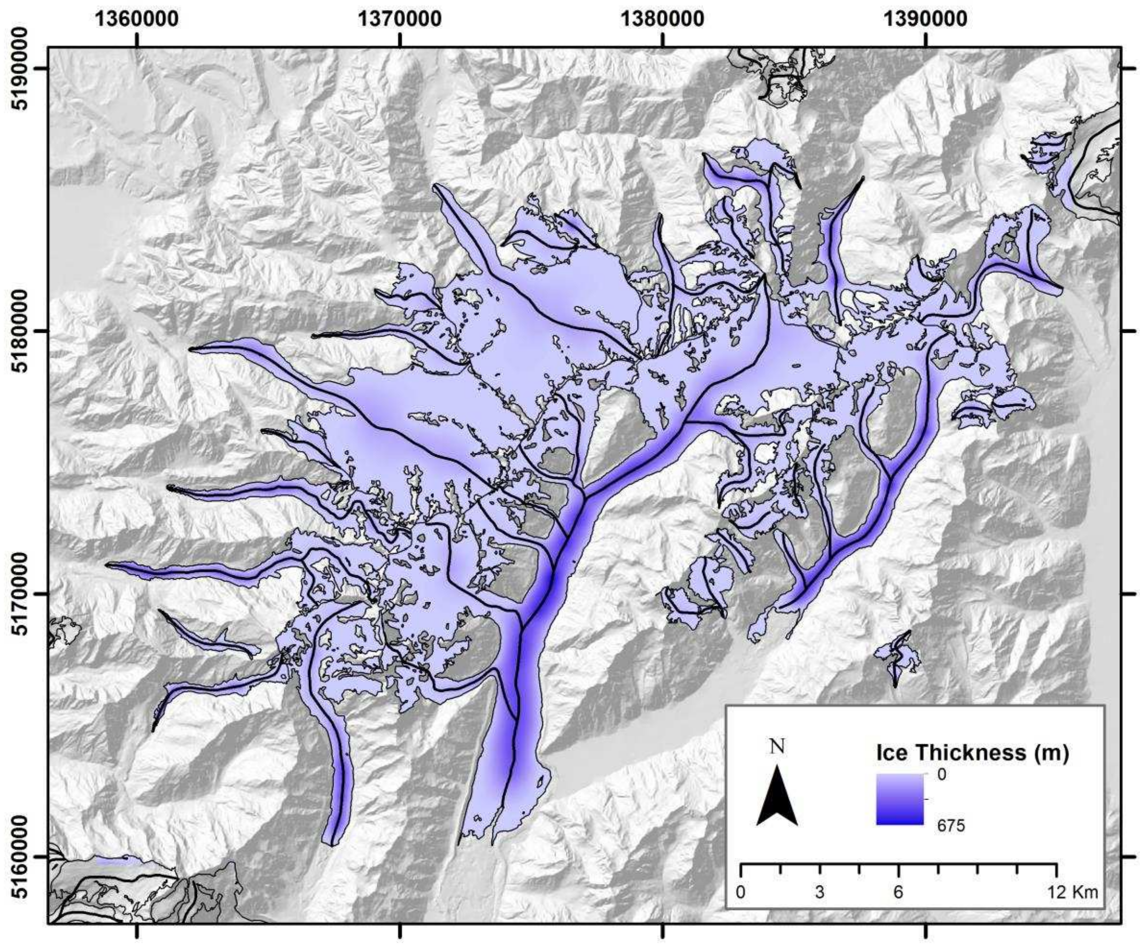

Fig. 11. Automatically generated centrelines and ice thickness distribution for the Mt. Cook region

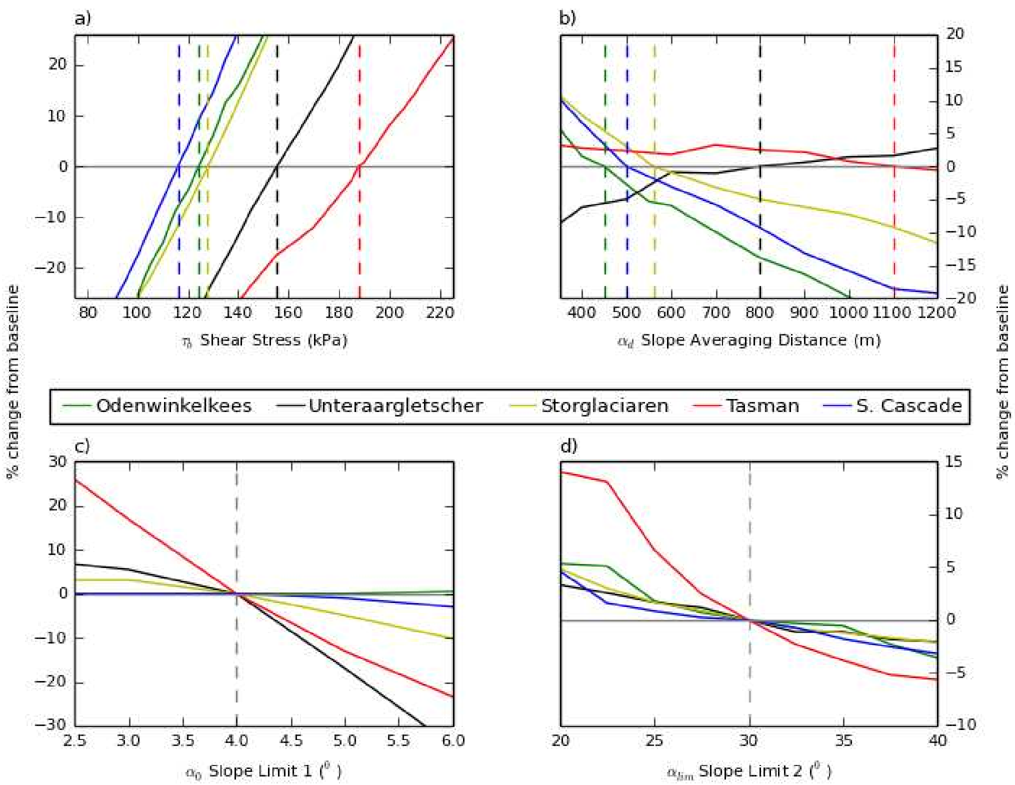

Fig. 12. Sensitivity of volume estimate to parameters: a) T_b, b) α_d, c) α_0 and d) α_{lim}. The solid grey horizontal lines represent a zero change in volume and the vertical dashed grey lines indicate the baseline value.

Glacier	Location	Area (km²)	Surface DEM	Outline	Bed topography
South Cascade	48.360° -121.058°	1.73	ASTER, 30m (USGS 2014)	Fountain and Jacobel (1997)	Interpolated from 50m bed contours (derived from 5000+ measurements from multiple GPR transects) (Fountain and Jacobel 1997)
Ödenwinkelkees	47.111° 12.643°	1.9	LIDAR, 2m (Carrivick et al. 2013)	GLIMS (GLIMS 2014)	Interpolated from 77 GPR measurements (Span et al. 2005)
Storglaciären	67.902° 18.566°	3.2	10m, Composite dataset (e.g Björnsson 1981)	Composite dataset (e.g Björnsson 1981)	Composite dataset (e.g. Björnsson 1981)
Tasman	43.6166° 170.200°	90.4	Land Information New Zealand (LINZ), 15m	GLIMS (GLIMS 2014)	Interpolated from 14 transects (seismics) (Hart 2014) & (Broadbent 1974)
Unteraargletscher	46.566° 8.216°	14.6	ASTER, 30m (USGS 2014)	Bauder et al. (2003)	Interpolated from 50m bed contours (derived from radar transects and 27 boreholes) (Bauder et al. 2003)

Table 1. Location of each glacier and details of the datasets used for determining the 'field derived' volume and VOLTA inputs.

Glacier	Volume (field measurements), km³	Volume VOLTA, km³	Volume Difference, %	Estimated shear stress, kPa	Slope averaging window, m	Median deviation, m	Deviation IQR, m
South Cascade	0.200	0.147	-26.5	115	500	25.8	59.0
Ödenwinkelkees	0.078	0.091	16.6	124	450	-2.7	22.3
Storglaciären	0.292	0.282	-3.4	127	550	7.7	42.4
Tasman	12.040	10.062	-16.4	187	1000	13.6	46.7
Unteraargletscher	2.983	3.279	9.9	155	800	-11.2	112.6

Table 2. Comparison between volumes calculated from field based observations and modelled by VOLTA. Parameters derived and summary statistics are also shown. Deviation is defined as the modelled (VOLTA) ice thickness subtracted from field measured value for each cell.