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**Monograph:**

Biswas, T. and McHardy, J. (2004) *Measuring the Balance of Intra-Regional Trade*.  
Working Paper. Department of Economics, University of Sheffield ISSN 1749-8368

Sheffield Economic Research Paper Series 2004005

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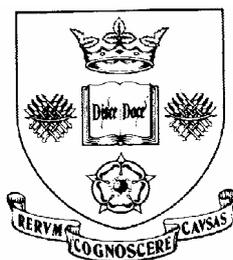
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# Sheffield Economic Research Paper Series

**SERP Number: 2004005**



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**MEASURING THE BALANCE OF INTRA-REGIONAL MIGRATION**

**June 2004**

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## **Abstract**

We introduce an original method of measuring the extent of the overall balance of migration among countries within a region which allows comparisons of the balance over time as well as between regions and various possible sub-regions.

**JEL Numbers:** F22, J61, O52

**Keywords:** Migration Balance, Perron-Frobenius Root

## 1. Introduction

In recent years, activities for regional integration have gained pace alongside the push for globalisation. In Europe in particular, the progressive liberalisation of internal labor markets (e.g. the Schengen agreement, 1995) and the recent expansion of the EU have fuelled fears in some countries of large influxes of migrants, especially from poorer countries, pressurising job markets and social services. Whilst Krugman (1987) argues that such regional integration should, in the long-run, result in more balanced migration through technology transfers and an improved distribution of labor skills, Zimmerman (1995) notes the importance of monitoring regional migration and intervening to redress balances if necessary. This paper proposes a new measure which may be used to monitor the balance of migration within a region. Section 2 introduces a measure of overall (*strong*) balance in migration. Section 3 uses this measure to examine and compare the balance of migration within two regions of the EU. Section 4 is a conclusion.

## 2. A Measure of Overall (Strong) Balance in Intra-Regional Migration

Let  $m_{ij}$  be the number of migrants from country  $i$  to country  $j$ . We define  $I_{ij}$  as the index of bilateral balance in migration,  $I_{ij} = 1 - \left[ \frac{|m_{ij} - m_{ji}|}{m_{ij} + m_{ji}} \right]$ .  $I_{ij}$  varies from 0 (perfect imbalance) to 1 (perfect balance).<sup>2</sup> Defining  $r$  as the group of countries in the region excluding country  $i$ ,  $I_{ir}$  may be used as a partial (*weak*) index of balance for each  $i$ .<sup>3</sup> However, a perfect *weak* balance ( $I_{ir} = 1, \forall i$ ) in a region is not inconsistent with perfectly

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<sup>2</sup> This measure is analogous to the Grubel-Lloyd index (1975), widely used in the intra-industry trade literature.

<sup>3</sup> Biswas and McHardy (forthcoming), use this partial measure of balance in the context of European migration.

imbalanced country-wise *bilateral* migration ( $I_{ij} = 0 \quad \forall i, j; i \neq j$ ) e.g., countries  $A$ ,  $B$  and  $C$  with equal migrant flows from  $A$  to  $B$ ,  $B$  to  $C$  and  $C$  to  $A$ . In this section, we develop an alternative (*strong*) index for intra-regional migration balance. Suppose, there are  $n$  countries in a region and the  $n \times n$  matrix  $A = [I_{ij}]$  is the matrix of bilateral migration.

**Proposition 1:** *The matrix  $A$  has the following properties: (i)  $A$  is a symmetric, non-negative matrix ( $1 \geq I_{ij} \geq 0$ ) with diagonal elements  $I_{ii} = 1$ ; (ii) If bilateral migration is perfectly imbalanced,  $A$  is an identity matrix. (iii) If bilateral migration is perfectly balanced, all the elements of  $A$  are 1 (unity).* □

The properties of  $A$  follow from the definition of  $I_{ij}$ . Let  $\rho$  be the Perron-Frobenius root (henceforth, *PFR*) of the non-negative matrix  $A$ .

**Proposition 2:** *The PFR,  $\rho$ , has the following properties: (i)  $A^* \geq A$  implies  $\rho^* \geq \rho$ , i.e.,  $\rho$  is non-decreasing for increases in any element of  $A$ . (ii)  $n \geq \rho \geq 1$ .* □

**Proof:** *Part (i) is a general property of the PFR for any non-negative matrix  $A$ . If  $A$  is indecomposable,  $\rho$  is strictly increasing in  $I_{ij}$ . The proof of part (ii) is as follows. Since,  $1 \geq I_{ij} \geq 0$  for  $i \neq j$  and  $I_{ii} = 1$ , the elements of  $A$  attain their minimum when  $A$  is an identity matrix (bilateral migration is perfectly imbalanced) and  $\rho = 1$ . The elements of  $A$  attain their maximum (each equals 1) when bilateral migration is perfectly balanced and  $\rho = n$ . To see this, note  $Ax = \rho x$  and normalise the characteristic vector  $x$  to  $\sum x_i = 1$ . Denoting the  $i^{\text{th}}$  row of  $A$  as  $A_i$  we get  $n = \sum A_i x = \rho \sum x_i = \rho$ . By part (i),  $\rho$  must lie between 1 and  $n$ .* □

The root  $\rho$  is the basis of our proposed index. The corresponding characteristic vector  $x$  may be regarded as the vector of weights given to the countries in the region.

**Proposition 3:** *Elements of the characteristic vector  $x$  corresponding to  $\rho$  generate a scheme of weighted bilateral indices of balance for each country which sum to  $\rho$ .*  $\square$

**Proof:** Using  $x_i$  as the weight given to country  $i$ ,  $A_i x$  is the weighted bilateral index of migration. Since  $A_i x = \rho x_i$  and  $\rho \sum x_i = \rho$ , the weighted bilateral index equals the contribution of the  $i^{\text{th}}$  country to the value of  $\rho$ . If all countries have an identical balance,  $I_{ij} = k$  ( $\forall i, j; i \neq j$ ), and  $C = A_i x = \rho x_i$ , hence,  $x_i = C / \rho$ : all countries have equal weights. In the special case of an identity matrix (perfect imbalance),  $C = x_i$  and  $\rho = 1$ , therefore satisfying the above equality. In the case of a perfect balance  $C = 1$ ,  $\rho = n$  and  $x_i = 1/n$ , which also satisfies the above equality.  $\square$

Whilst any scheme of weights can be subject to criticism, the  $x_i$  weights have the desirable property that when countries should be treated equally, the weights given to the countries are the same. Moreover, since  $A_i x / A_j x = x_i / x_j$ , the country with the higher weighted bilateral index for balanced migration contributes more to the overall balance than the other country.

**Proposition 4:** *In view of propositions 1 and 2 we propose to use the index,  $\beta = (\rho - 1) / (n - 1)$ , as the measure of strong balance for intra-regional migration.*  $\square$

The value of  $\beta$  is non-decreasing with increases in any  $I_{ij}$  and lies between 0 (perfect imbalance) and 1 (perfect balance). One benefit of the normalised index  $\beta$ , is that it allows

the comparison of balance between two or more regions (or sub-regions) with varying numbers of member countries.

### 3. An Application

In this section we construct the  $I_{ij}$  indices and associated *strong* indices of regional balance  $\beta$  using Eurostat (2003) data for four southern EU countries, henceforth *SE4*: Greece, Italy, Spain and Portugal, and five northern EU countries, henceforth *NE5*: Denmark, Germany, Netherlands, Sweden and the UK.<sup>4</sup> For the period 1992-98 (the limit of available *SE4* data), the average values of  $\beta$  for these regions are  $\beta_{NE5}^{92-98} = 0.845$  and  $\beta_{SE4}^{92-98} = 0.614$ . A two-sided t-test rejects the null hypothesis of equal means at the 0.5% level: bilateral migration is significantly more balanced within the *NE5* than the *SE4*. As for changes in balance over time, Table 1 reports the time-series of  $\beta_{NE5}$  for the period 1989-2000. The slope of the OLS regression for this time-series is positive and significant at the 0.5% level: the migration balance within the *NE5* has improved significantly over time.<sup>5</sup>

**Table 1** – Time-series of  $\beta_{NE5}$  for the period 1989-2000

1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
0.822	0.838	0.828	0.806	0.842	0.847	0.869	0.849	0.844	0.861	0.859	0.890

Finally, from Proposition 3, the elements of the normalised characteristic vector associated with the *PFR* can be interpreted as weights of the contribution of each country to the overall balance  $\rho$ . The relevant figures for the nine countries in our study over the period 1992-98 are reported in Table 2. It appears the weights attached to the *NE5* countries are almost equal. Regarding the *SE4* countries, the weights are almost equal except for Portugal which seems to

<sup>4</sup> The configuration of these sub-groupings is determined by data availability.

<sup>5</sup> For the *SE4*, whose time-series is much shorter, the slope of the regression line is positive but insignificant.

be an outlier. The reason for this lies in the fact that migration for Portugal with respect to the other *SE4* countries is relatively unbalanced (the  $I_{ij}$ s are closer to zero).

**Table 2 – Average weights in the overall balance for the period 1992-98 by country**

NE5					SE4			
Denmark	Germany	Netherlands	Sweden	UK	Greece	Italy	Portugal	Spain
0.20	0.20	0.20	0.19	0.21	0.26	0.26	0.21	0.27

**4. Conclusions**

This paper uses the properties of the *PFR* to construct a new index  $\beta$  of the balance of bilateral intra-regional migration, which allows comparisons of migratory balance between regions and over time. Increases in  $\beta$  over time indicate improvements in the overall balance of bilateral migration within a region, and a region with a high  $\beta$  has a greater bilateral migration balance than a region or sub-region with a low  $\beta$ . Furthermore, normalisation of the characteristic vector for the *PFR* allows the interpretation of its elements as a measure of each country’s contribution to the overall migration balance. Finally, it is important to note that the methods introduced here are directly applicable to examinations of the overall (*strong*) balance of intra-industry trade within a region.

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